Mixed Duopoly with Motivated Teachers

Ester Manna

7. December 2013
Mixed Duopoly with Motivated Teachers

Ester Manna†

December 7, 2013

Abstract

I study the self-selection of motivated teachers between public and private schools in a mixed duopoly environment. The quality is influenced by the effort exerted by the teachers. Teachers’ motivation may have a positive impact on the levels of effort and, then, on the quality. The effect of motivation strictly depends on the degree of differentiation of the programs offered by the two schools. When both schools offer similar programs, the Nash equilibrium is the one in which both schools hire motivated teachers. This is because teachers’ motivation plays an important role in the students’ choice between the two schools. In contrast, when the two schools offer significantly different programs, the Nash equilibrium is the one in which both schools hire selfish teachers. Hiring motivated teachers would hurt schools’ profits and benefit students. However, the former effect dominates the latter.

Keywords: Intrinsic Motivation, Mixed Duopoly, School Choice, Hotelling Model.

JEL classifications: D03, D21, L13.

1 Introduction

It is often argued that preferences and work motivation of employees differ depending on the nature of the jobs. Many jobs involve helping people in need or contributing to the society at large, making these jobs attractive to people who have a strong willingness to serve the others or the public interest (see Buurman, Dur and Van den Bossche, 2009). In this paper I focus on the example of teaching. Teachers can take satisfaction from teaching and from developing strategies that are consistent with the best interest of the

---

† ester.manna1@gmail.com. I am particularly indebted to Marcello D’Amato for his careful supervision and continuous support. I also thank Georg Kirchsteiger for his support and guidance. I wish to thank Alessandro De Chiara, Salvatore Piccolo, Luigi Senatore and Damiano Silipo for many useful insights. The usual disclaimer applies.
students because they believe in the virtue of the education for the society. When this is the case, teachers are intrinsically motivated.\(^1\)

There is a positive relationship between teachers’ motivation and the quality of education. This is because the cost of eliciting a given amount of effort from a motivated teacher is lower. Therefore, when teachers are motivated the quality of education is higher. As teachers may be given different incentives to provide effort, there exists some degree of vertical differentiation between schools. Parents’ school choice depends on the differences in the *quality* of the education. A high level of quality may play a crucial role in the parents’ decision between different schools. However, it is not the only variable that parents take into account when making the schools’ choice.

Different schools may charge different tuition fees and offer heterogeneous educational services. This is particularly true if one considers the educational services provided by public and private institutions. In Italy, for example, most private schools have a strong religious connotation\(^2\) while public schools are typically secular. Furthermore, there is a number of foreign and international schools that are private, including American, French and British schools. Many international schools offer bi-lingual programs, as well as English as a Foreign Language (EFL) exams if the students’ first language is not English.

Then, private schools can be appealing to parents who want to give their children an education closer to the international standards. On top of that, most private schools can offer a more caring and protective atmosphere, as well as the opportunity of additional or intensive lessons, which some parents believe to be more conducive to learning. Parents in some of the wealthier areas of the major cities may also send their children to a private school simply for its exclusivity. Thus, there exists some degree of horizontal differentiation which impacts on the parents’ school choice.

This article analyzes the “market” for education by developing a model that focuses on the interaction between the public and private educational sectors.\(^3\) More specifically, I investigate the impact of teachers’ intrinsic motivation on the schools’ outcome in terms of quality, price and wage in a mixed duopoly environment (see De Fraja and Delbono, 1990, and Nett, 1993, for general reviews of the mixed oligopoly markets).\(^4\) To this end, I

\(^1\)Another suitable example could be the health-care system. Doctors may be interested not only in their monetary compensation but also in the impact of their work on the well-being of their patients (see for example Ma, 2004).

\(^2\)Most private schools are run by religious organizations (the majority by the Jesuits).

\(^3\)Epple and Romano (1998) also study a model of competition between public and private schools. However, their model stresses competition for students among public and private schools.

\(^4\)The issue of competition between schools has been examined by a number of authors under different assumptions pertaining the objective functions of schools and education outcomes (see for example De Fraia and Iossa, 2002, Maldonado, 2008, and De Fraja and Valbonesi, 2012). However, these papers study the impact of competition on the education outcomes when the schools’ objective functions are
develop an oligopolistic model where two schools are positioned at each end of a Hotelling line. The horizontal differentiation reflects the heterogeneity of the programs offered by the schools. While the private school maximizes its profits, the public school maximizes social welfare. The social welfare is given by the sum of the students’ utility, the teachers’ utility and the profits obtained by both schools. Each school consists of a principal and an agent, both risk neutral. The principal-agent relationship can be interpreted as the relationship between the school-principal who needs a teacher (the agent) to provide the educational service. The two schools offer imperfectly substitutable programs and they compete against each other on quality and prices. When the degree of substitutability is high, it means that the schools offer similar programs and there is more competition in the market.

The school principals non-cooperatively decide whether to hire motivated or non-motivated teachers. I show that the presence of motivated teachers can benefit or hurt public and private schools depending on the degree of differentiation between the programs offered by the two schools. If the programs offered by the schools are only slightly different and both principals hire self-interested teachers, a principal finds it profitable to deviate by hiring a motivated teacher. By doing so, the public principal increases the social welfare and the private principal obtains a comparative advantage in terms of demand and price. Then, if a principal hires a motivated teacher the best response of the other principal is to hire a motivated teacher as well. The Nash equilibrium is the one in which both schools hire motivated teachers. Teachers’ motivation plays an important role in the students’ choice between the two schools. Individuals’ intrinsic motivation increases the quality of the educational services offered by both schools. To elicit higher levels of effort to increase quality, both principals have to offer higher incentives to their teachers. Then, motivation has a positive impact on the wages offered to the teachers. This result stands in contrast to the previous literature on psychological incentives in organizations in which motivation is effective in stimulating work effort even in absence of monetary rewards (see for example Gneezy and Rustichini, 2000a,b and Bénabou and Tirole, 2003, 2006).

If the schools offer very different programs, there is less scope for quality-competition in the market and the teachers’ intrinsic motivation becomes relatively less important.

---

5 This literature shows that monetary incentives can influence negatively the individuals’ behavior in terms of their levels of contribution. The reason is that monetary incentives give the agent a selfish motive to operate. Explicit incentives from principals may change how tasks are perceived by agents (Gneezy and Rustichini, 2000a) and they may also reduce the value of generous or civic minded acts as a signal of one’s moral character (Bénabou and Tirole, 2003, 2006). If extrinsic incentives are not large enough, this change in perception can even lead to undesired effects on behavior (Gneezy and Rustichini, 2000b).
In that case, hiring motivated teachers would hurt schools’ profits and benefit students. However, the former effect dominates the latter. Then, the Nash equilibrium is the one in which both schools hire selfish teachers and students will choose the school with a program closer to their necessity.

This article is related to two strands of the literature: the literature on “mixed oligopolies” and the literature on psychological incentives in organizations. The former focuses on the competition among institutions with different objective functions (see for example Cremer et al., 1991, Grilo, 1994, and Delbono et al., 1996)\(^6\) without considering potential differences in the agents’ preferences. In this literature, some articles study the issue of competition in education markets when education providers can be public and private (see for example Cellini and Goldini, 2012, Deming et al., 2012, and Cremer and Maldonado, 2013).\(^7\) The latter focuses on the impact of monetary incentives on the level of effort exerted by motivated agents (see for instance Bénabou and Tirole, 2003, 2006; Gneezy and Rustichini, 2000a, 2000b). In this literature, some articles also study the matching of motivated employees between public and private sector (see for example Besley and Ghatak, 2005, and Prendergast, 2007). My objective is to bridge these two strands of the literature considering the interaction between intrinsic motivation and monetary incentives in a mixed duopoly environment.

The reminder of the article is as follows: in section 2 I present the set-up of the model; in section 3 I characterize the equilibrium of the model; in section 4 I study the solution of the game; and concluding remarks are given in section 5.

2 The Set-Up of the Model

I build a mixed duopoly model where two schools are positioned at each end of a Hotelling line, with locations \(x_i = 0\) and \(x_j = 1\), respectively. The school \(i\) is public and the school \(j\) is private. The public school maximizes the social welfare. In contrast, the private school maximizes his profits. Each school is constituted of a principal and an agent, both of whom are risk neutral. The principal-agent relationship can be interpreted as the relationship between the school-principal that delegates the decision about an outcome in terms of quality \(q\) to an agent (the teacher). A teacher working in the public sector

\(^6\)Cremer et al. (1991) study price competition in a market represented by a Hotelling (1929) line in which private and public firms choose first locations and then prices. Then, Grilo (1994) study a mixed competition model in which products are vertically differentiated and firms non-cooperatively choose first qualities and then prices. Finally, Delbono et al. (1996), using a model similar to Grilo (1994), introduce the possibility that the market might be uncovered.

\(^7\)Cellini and Goldini (2012) and Deming et al. (2012) empirically show that US education markets are effectively mixed. Cremer and Maldonado (2013) study a mixed duopoly model in which the quality of education depends on “peer group” effects.
cannot participate in the private sector too, and vice versa. There are a continuum of students of mass 1 distributed uniformly along the line.

The agents are wealth constrained with zero initial wealth and have a reservation wage of zero. The agents have quadratic effort costs, which are observable to the principal. The exerted effort $\epsilon$ determines the quality of the educational services offered by the two schools. For expositional convenience, I assume that quality $q$ depends linearly on the teachers’ effort: $q = \epsilon$ in both schools. There is no asymmetric information between the principal and the agent. Since quality is verifiable, the principals do not need to offer an incentive to the agents because they have all the necessary information to implement the efficient levels of quality.

In addition, the teachers’ utilities might positively depend on the benefits of the students. The teachers can be also interested in the impact of their work on the students’ utility. The measure of this utility depends on the parameter $\theta$ that represents the intrinsic motivation of the agent. It influences the optimal levels of quality and price. There are only two types of teachers: the self-interested teachers with $\theta = 0$ and the motivated teachers with $\theta > 0$. There is an infinite number of teachers of both types. The principals offer a wage that covers the cost of effort paid by the teacher minus his intrinsic motivation.

After the employment decision, the two schools offer imperfectly substitutable services, competing against each other on quality $q$ and prices $p$.

The timing of the model is as follows. At the initial stage 0, each principal decides whether to hire a motivated teacher or a non-motivated teacher. At stage 1, each principal makes an offer $(\omega, q)$ to their agent. The teachers accept any contract with an expected utility of at least their reservation utility, which I normalize to 0. If the agents accept the contract, they exert the required levels of effort; At stage 2, after agents have exerted effort determined by the contract, principals simultaneously choose prices; At stage 3, the students choose between the two schools.

### 2.1 The Objective Functions

A student enjoys a utility

$$U_i = q_i - p_i - tx$$

from the service offered by the public school $i$ and

$$U_j = q_j - p_j - t(1 - x)$$

from the service offered by the private school $j$. At location $x$, a student $i$ incurs a transport cost $tx$ for traveling to school $i$ and a cost $t(1 - x)$ to school $j$. For given $p_i, p_j, q_i, q_j$ there is a cutoff $\overline{\pi}$, such that all students with $x < \overline{\pi}$ choose the service of school $i$, and with $x > \overline{\pi}$ choose school $j$. The parameter $t$ represents the degree of differentiation of the educational services offered by the two schools. When $t$ is low the schools offer similar programs, implying fierce competition.
The key assumption of this model is that teachers can be intrinsically motivated. The teachers’ utility function consists of their own “egoistic” payoff, given by the difference between wage and effort costs, and of their intrinsic motivation. There are two types of teachers: self-interested teachers with $\theta = 0$ and motivated teachers with $\overline{\theta} > 0$.

The agents’ utility function from working in the public and private schools, respectively, can be written as:

\[
V_i = \omega_i - \frac{1}{2} q_i^2 + \theta_i \overline{U}_i \\
V_j = \omega_j - \frac{1}{2} q_j^2 + \theta_j \overline{U}_j
\]

where $\overline{U}_i$ and $\overline{U}_j$ are the utilities of the average student deciding for school $i$ and school $j$, respectively, and are equal to: $\overline{U}_i = q_i - p_i - t \frac{x}{2}$ and $\overline{U}_j = q_j - p_j - t \frac{(1-x)}{2}$.

The public school $i$ maximizes the social welfare while the private school $j$ maximizes profits. The social welfare is given by the sum of the students’ average utility, the teachers’ utility and the profits obtained by the schools. More specifically, the public school $i$ maximizes the following:

\[
\pi_i = \overline{U}_i + \overline{U}_j + V_i + p_i d_i - \omega_i + p_j d_j - \omega_j.
\]

While the private school $j$ maximizes the following profit function:

\[
\pi_j = p_j d_j - \omega_j + v,
\]

where $v$ is strictly positive and represents the funding offered to the private school.\(^8\)

The principals maximize their objective functions under the following participation constraints:

\[
\begin{align*}
\omega_i - \frac{1}{2} q_i^2 - \theta_i \overline{U}_i &\geq 0; \\
\omega_j - \frac{1}{2} q_j^2 - \theta_j \overline{U}_j &\geq 0.
\end{align*}
\]

The participation constraints guarantee that both teachers do not choose their outside option.

I make the following assumption to guarantee an interior solution.

**Assumption 1.** I restrict the attention to the case in which the parameters lie in the following intervals.

- $t \in (0, \frac{1}{4})$;
- and $\overline{\theta} \in \left(0, \frac{1-3t}{2(1+t)}\right]$.

The role of this assumption will become clear in the next sections.

\(^8\)Most private schools in Italy are either authorized or given legal recognition by the state and many receive state funding.
3 The Characterization of the Equilibrium

The equilibrium is determined by backward induction.

At stage 3, the students choose the school. A student located at \( \overline{x} \) is indifferent between the public school \( i \) and the private school \( j \) if and only if \( U_i = U_j \), or equivalently \( q_i - p_i - t\overline{x} = q_j - p_j - t(1 - \overline{x}) \). \( \overline{x} \) represents the demand for the public school \( i \) and \( (1 - \overline{x}) \) the demand for the private school \( j \):

\[
\overline{x} = d_i(q_i, q_j, p_i, p_j, t) = \frac{1}{2} + \frac{(q_i - q_j) + (p_j - p_i)}{2t};
\]
\[
(1 - \overline{x}) = d_j(q_i, q_j, p_i, p_j, t) = \frac{1}{2} + \frac{(q_j - q_i) + (p_i - p_j)}{2t}.
\]

At stage 2, the principals choose their prices to maximize their objective functions, taking qualities and wages as given. The public school maximizes the social welfare:

\[
\max_{p_i} \pi_i = (1 + \theta_i) \overline{U}_i + (1 + \theta_j) \overline{U}_j + d_i p_i - \frac{1}{2}q_i^2 + d_j p_j - \frac{1}{2}q_j^2;
\]

While the private school maximizes its profits:

\[
\max_{p_j} \pi_j = d_j p_j - \omega_j + \overline{x}.
\]

Taking the first order conditions of equations (7) and (8) with respect to \( p_i \) and \( p_j \), respectively, I obtain the following equilibrium prices:

\[
p_i = \frac{(q_j - q_i)}{2} + \frac{1}{4} t(2 - 3\theta_i - 7\theta_j);
\]
\[
p_j = (q_j - q_i) + t \left( 1 - \frac{3}{2} \theta_j \right).
\]

Substituting equilibrium prices into the equations (7) and (8), I obtain an expression for social welfare and private profits as a function of the levels of quality and wages offered by the two schools.

At stage 1, these functions are maximized with respect to \( \omega_i \), \( q_i \) and \( \omega_j \), \( q_j \), respectively. I obtain the optimal levels of quality:

\[
q_i^* = \frac{32 - 56t + 19\theta_i - 44t\theta_i + 13\theta_j - 36t\theta_j}{8(3 - 4t)};
\]
\[
q_j^* = \frac{32 - 16t + 19\theta_i + 12t\theta_i + 13\theta_j + 20t\theta_j}{8(3 - 4t)}.
\]
with wages:

\[
\omega^*_i = \frac{1}{2} \left( \frac{32 - 56t + \theta_i(19 - 44t) + \theta_j(13 - 36t)}{8(3 - 4t)} \right)^2 + \frac{-\theta_i}{2} \left( \frac{64 - 184t + 56t^2 + \theta_i(38 - 103t - 36t^2) + \theta_j(26 - 33t - 108t^2)}{16(3 - 4t)} \right);
\]

\[
\omega^*_j = \frac{1}{2} \left( \frac{32 - 16t + \theta_i(19 + 12t) + \theta_j(13 + 20t)}{8(3 - 4t)} \right)^2 + \frac{-\theta_j}{2} \left( \frac{64 - 176t + 72t^2 + \theta_i(38 - 93t - 12t^2) + \theta_j(26 - 11t - 100t^2)}{16(3 - 4t)} \right).
\]

(11)

The chosen prices at stage 2 are equal to:

\[
p^*_i = \frac{t(16 - 8t + 5\theta_i + 12\theta_i - 7\theta_j + 28\theta_j)}{4(3 - 4t)};
\]

\[
p^*_j = \frac{t(16 - 8t + 14\theta_i + 5\theta_j + 12\theta_j)}{2(3 - 4t)}.
\]

(12)

At stage 3 the demands are equal to

\[
d^*_i = \frac{8 - 24t - 5\theta_i - 12\theta_i - 11\theta_j - 4t\theta_j}{8(3 - 4t)};
\]

\[
d^*_j = \frac{16 - 8t + 5\theta_i + 12\theta_i + 11\theta_j + 4t\theta_j}{8(3 - 4t)}.
\]

(13)

and the outcomes obtained by the schools are realized:

\[
\pi_i = (1 + \theta_i) \overline{U}_i^* + (1 + \theta_j) \overline{U}_j^* + d^*_i p^*_i - \frac{1}{2} q^*_i^2 + d^*_j p^*_j - \frac{1}{2} q^*_j^2;
\]

\[
\pi_j = d^*_j p^*_j - \omega^*_j + \overline{\pi}.
\]

(14)

(15)

There are only two types of agents: the self-interested teacher with \( \overline{\theta} = 0 \) and the motivated teachers with \( \overline{\theta} > 0 \). At stage 1, I characterize the equilibrium for different degrees of intrinsic motivation and I obtain different payoffs:

- when both teachers are self-interested, i.e. \( \theta_i = \theta_j = \overline{\theta} = 0 \) (the equilibrium outcomes will be denoted by \( \pi_i^*, \pi_j^* \), respectively);

- when teachers are homogeneous and motivated, i.e. \( \theta_i = \theta_j = \overline{\theta} > 0 \) (the equilibrium outcomes will be denoted by \( \overline{\pi}_i, \overline{\pi}_j \), respectively);
• when only the teacher $i$ is motivated, i.e. $\theta_i = \overline{\theta} > 0$ and $\theta_j = \underline{\theta} = 0$ (the equilibrium outcomes will be denoted by $\hat{\pi}_i^*$, $\hat{\pi}_j^*$, respectively);

• and, finally, when only the agent $j$ is motivated, i.e. $\theta_j = \overline{\theta} > 0$ and $\theta_i = \underline{\theta} = 0$ (the equilibrium outcomes will be denoted by $\pi_i^*$, $\hat{\pi}_j^*$, respectively).

The characterization of the equilibrium for different degrees of intrinsic motivation are in the Appendix A.

### 4 The Type Choice Game

In stage zero, both firms choose simultaneously which type of agent to hire. Given prices, quantities and wages, the type choice reduces to the following game:

<table>
<thead>
<tr>
<th></th>
<th>$\theta_j$</th>
<th>$\overline{\theta}_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{\theta}$</td>
<td>$(\pi_i^<em>, \pi_j^</em>)$</td>
<td>$(\hat{\pi}_i^<em>, \hat{\pi}_j^</em>)$</td>
</tr>
<tr>
<td>$\underline{\theta}_i$</td>
<td>$(\hat{\pi}_i^<em>, \pi_j^</em>)$</td>
<td>$(\pi_i^<em>, \pi_j^</em>)$</td>
</tr>
</tbody>
</table>

Figure 1: The Type-Choice Game

I start considering the case in which there is high competition in the market ($t$ is small). I compare the benefits obtained by hiring self-interested teachers with those obtained by hiring a motivated teacher when the rival school hires a selfish agent. Furthermore, I also compare the benefits obtained by hiring motivated teachers with those obtained by hiring a non-motivated agent when the rival school hires a motivated agent.

**Lemma 1.** If $t < 0.165$, then

- $\pi_i^* < \hat{\pi}_i^*$ and $\pi_j^* < \hat{\pi}_j^*$;
- $\pi_i^* > \pi_i^*$ and $\pi_j^* > \pi_j^*$.

**Proof.** See Appendix B. 

If there is high competition between schools ($t$ is small) and both principals hire self-interested teachers, a principal finds it profitable to deviate by hiring a motivated teacher. By doing so, the public principal increases the social welfare and the private principal obtains a comparative advantage in terms of demand and price. Hence, if a
principal hires a motivated teacher the best response of the other principal is to hire a motivated agent as well.

This leads to the following proposition.

**Proposition 1.** When \( t < 0.165 \), the unique Nash equilibrium is the one in which both schools hire motivated agents.

*Proof.* See lemma 1.

When the public and the private schools offer similar programs, there is high competition in the scholastic market. In this case, the public school hires a motivated teacher because the agent’s intrinsic motivation has a positive impact on the social welfare. Then, the best response of the private school is to follow suit and to hire a motivated teacher as well. However, I find that the profits obtained by the private school when both teachers are selfish would always be higher than the profits obtained when both teachers are motivated, i.e. \( \pi^*_j > \pi^*_j \). This result is illustrated in the first graph of figure 2. The blue line represents the profits obtained by the private school when both teachers are self-interested. While the purple line represents the profits obtained by the private school when both teachers are motivated. Then, the presence of the public school, and its choice to hire a motivated teacher, “pushes” the private principal to hire a motivated agent too. This result is illustrated in the second graph of figure 2. Now, the blue line represents the profits of the private school by hiring a selfish agent when the public school hires a motivated agent. While the purple line represents always the profits of the private school when both teachers are motivated. The purple line \( \pi^*_j \) is above the blue line \( \pi^*_j \) until \( t \) is low enough. Hence, the private school finds it profitable to hire a motivated agent as well.

![Figure 2: Comparison Profits of the Private School.](image-url)
Lemma 2. If \( 0.165 < t < 0.2 \), then

- \( \pi_i^* > \pi_i^* \) and \( \pi_j^* < \pi_j^* \).
- \( \pi_i^* < \pi_i^* \) and \( \pi_j^* > \pi_j^* \).

Proof. See Appendix B.

This leads to the following proposition.

**Proposition 2.** When \( 0.165 < t < 0.2 \), the unique Nash equilibrium is the one in which the public school hires a selfish teacher, while the private school hires a motivated teacher.

Proof. See lemma 2.

As the schools tend to have differentiated educational programs, the students’ utility is reduced leading to a reduction of the social welfare. In contrast, the higher the degree of differentiation of the programs offered by the schools, the higher the profits obtained by the schools. This effect has a positive impact on the social welfare. There is a level of \( t \) for which the increase in the students’ utility due to the higher quality provided by the motivated teachers is more than offset by the reduction in the profits earned by the two schools. In this case, the public principal hires the selfish teacher. In contrast, the private principal obtains a comparative advantage by hiring a motivated agent when the public principal hires a selfish one.

Lemma 3. If \( t > 0.2 \), then

- \( \pi_i^* > \pi_i^* \) and \( \pi_j^* > \pi_j^* \).
- \( \pi_i^* < \pi_i^* \) and \( \pi_j^* < \pi_j^* \).

Proof. See Appendix B.

This leads to the following proposition.

**Proposition 3.** When \( t > 0.2 \), the unique Nash equilibrium is the one in which both schools hire selfish teachers.

Proof. See lemma 3.

If the schools offer very different educational services, there is less scope for competition in the market and the teachers’ intrinsic motivation becomes relatively less important. In that case, both schools obtain higher benefits by hiring self-interested teachers than by hiring motivated teachers. Then, the students will choose the school with a program closer to their necessity without considering the teachers’ intrinsic motivation.
5 Conclusions

In this article, I have shown that the effect of teachers’ intrinsic motivation on schools’ outcome strictly depends on the degree of differentiation of the programs. When the schools offer very similar programs, they hire motivated teachers in order to attract students. In this case, teachers’ motivation plays an important role in the students’ choice between schools. In contrast, if the schools offer different programs, the teachers’ intrinsic motivation becomes relatively less important. In that case, both schools obtain higher benefits by hiring self-interested teachers than by hiring motivated teachers. The Nash-equilibrium is the one in which both schools hire selfish teachers.

The next step of this work is to analyze the optimal position of the schools on the Hotelling line and to study different model of competition. Moreover, a possible extension of the model is the one in which the price of both schools is regulated or just the one of the public school.
A Appendix A

A.1 Characterization of the Equilibrium when Both Teachers are Self-Interested

I begin by characterizing the equilibrium when teachers are self-interested, i.e. $\theta_i = \theta_j = 0$.

At stage 1, the optimal levels of quality and wages are determined. The quality’s levels are equal to:

\[ q_i^* = \frac{(4 - 7t)}{(3 - 4t)} \quad q_j^* = \frac{2(2 - t)}{(3 - 4t)} . \]  

(16)

with wages

\[ \omega_i^* = \frac{1}{2} \left( \frac{(4 - 7t)}{(3 - 4t)} \right)^2 \quad \omega_j^* = \frac{1}{2} \left( \frac{2(2 - t)}{(3 - 4t)} \right)^2 . \]  

(17)

By hiring selfish individuals, the quality of the educational services is lower in the public school than in the private school. For this reason, the wage paid to the public teacher has to be lower than the wage paid to the teacher working in the private school.

The chosen prices at stage 2 are:

\[ p_i^* = \frac{2t(2 - t)}{(3 - 4t)} \quad p_j^* = \frac{4t(2 - t)}{(3 - 4t)} . \]  

(18)

When the schools offer similar programs (low $t$) there is more competition in the market that leads to a low price for both schools. Moreover, with a higher quality offered by the private school, the private principal sets higher price than the public school, i.e. $p_j^* > p_i^*$.

And at stage 3, the demands are realized with

\[ d_i^* = \frac{(1 - 3t)}{(3 - 4t)} \quad d_j^* = \frac{(2 - t)}{(3 - 4t)} . \]  

(19)

The improvement of the quality permits to the private school to “steal” the market to the public school, i.e. $d_j^* > d_i^* > 0$.

After some computations, the social welfare and the private school’s profits are obtained:

\[ \pi_i^* = \frac{-16 + 87t - 115t^2 + 44t^3}{2(3 - 4t)^2} \quad \pi_j^* = \frac{2(4 - 12t + 9t^2 - 2t^3)}{(3 - 4t)^2} + v . \]  

(20)
A.2 Characterization of the Equilibrium when Teachers are Homogeneous and Motivated

In this subsection, I determine the equilibrium when both teachers are homogeneous and intrinsically motivated, i.e. \( \theta_i = \theta_j = \bar{\theta} > 0 \).

The optimal levels of quality are the following, respectively:

\[
q^*_i = \frac{4 - 7t + 4\bar{\theta} - 10t\bar{\theta}}{(3 - 4t)}; \quad q^*_j = \frac{2(2 - t + 2\bar{\theta} + 2t\bar{\theta})}{(3 - 4t)}.
\]  

(21)

The quality of the services offered by the public school is always lower than the quality offered by the private school, i.e. \( q^*_i < q^*_j \). The teachers’ intrinsic motivation has a positive impact on the levels of quality offered by the two schools, i.e. \( \frac{\partial q^*_i}{\partial \theta} > 0 \) and \( \frac{\partial q^*_j}{\partial \theta} > 0 \).

In addition, the impact of teachers’ intrinsic motivation on the levels of quality is higher in the public school than the private school if the programs offered by the schools are similar, i.e. \( \frac{\partial q^*_i}{\partial \theta} > \frac{\partial q^*_j}{\partial \theta} \) if \( t < \frac{1}{6} = 0.167 \).

The wages are given by:

\[
\overline{\omega}_i = \frac{1}{2} \left( \frac{4 - 7t + 4\bar{\theta} - 10t\bar{\theta}}{3 - 4t} \right)^2 - \bar{\theta} \left( \frac{8 - 23t + 7t^2 + 8\bar{\theta} - 17t\bar{\theta} - 18t^2\bar{\theta}}{2(3 - 4t)} \right)
\]

\[
\overline{\omega}_j = \frac{1}{2} \left( \frac{2(2 - t + 2\bar{\theta} + 2t\bar{\theta})}{3 - 4t} \right)^2 - \bar{\theta} \left( \frac{8 - 22t + 9t^2 + 8\bar{\theta} - 13t\bar{\theta} - 14t^2\bar{\theta}}{2(3 - 4t)} \right)
\]

(22)

The teachers’ intrinsic motivation has a countervailing effect on the wages. On the one hand, the teachers’ intrinsic motivation has a positive impact on the wages. This is because a high \( \bar{\theta} \) leads to high levels of quality of the educational services. Then, both principals pay high wages in the way to cover the cost of effort and to improve quality performance. On the other hand, teachers’ intrinsic motivation has a negative impact on the wages. This is because motivated teachers provide a given level of quality even if they receive a low compensation for that. The overall effect is positive in both schools.\(^9\)

Furthermore, the effect of the agents’ intrinsic motivation on wages is higher in the private school than in the public school, i.e. \( \frac{\partial \overline{\omega}_i}{\partial \theta} > \frac{\partial \overline{\omega}_j}{\partial \theta} > 0 \). This is because given that the quality offered by the private school is higher, the teacher \( j \) has to receive a higher compensation for his work than the teacher \( i \), i.e. \( \omega^*_i < \omega^*_j \).

\(^9\)This result does not support the previous literature in which motivation is effective in stimulating work effort even in absence of monetary rewards (see for example Gneezy and Rustichini, 2000a,b and Bénabou and Tirole, 2003, 2006).
Prices are given by:

\[ p^*_i = \frac{t(8 - 4t - \theta + 20t\theta)}{2(3 - 4t)} \]; \quad \[ p^*_j = \frac{t(16 - 8t + 19\theta + 12t\theta)}{2(3 - 4t)} \].

(23)

The price of the public school is lower than the price of the private school, i.e. \( p^*_j > p^*_i > 0 \). Moreover, the impact of the teachers’ intrinsic motivation is different in the two school. More specifically, the impact of \( \theta \) on the price of the public school is negative, unless the programs offered by the schools are very similar, i.e. \( t < \frac{1}{20} \). In contrast, the impact of the teachers’ intrinsic motivation on the price of the private school is positive. A higher value of \( \theta \) increases the gap between the levels of quality offered by the schools increasing the price of the private school and reducing the price of the public one.\(^{10}\)

The demand in the public and private school is respectively equal to:

\[ d^*_i = \frac{1 - 3t - \theta(1 + t)}{(3 - 4t)} \]; \quad \[ d^*_j = \frac{2 - t + 2\theta(1 + t)}{(3 - 4t)} \].

(24)

A higher quality offered by the private school leads to an increase of its demand.

And the social welfare and the private school’s profits are realized:

\[ \pi^*_i = \frac{32 - 112t + 113t^2 - 20t^3 + 64\theta - 173t\theta + 88t^2\theta + 80t^3\theta + 32\theta^2 - 88t\theta^2 + 92t^2\theta^2 + 16t^3\theta^2}{2(3 - 4t)^2} + \]

\[ + \frac{(1 + \theta)(16 - 45t + 16t^2 + 16\theta - 30t\theta - 32t^2\theta)}{2(3 - 4t)}; \]

\[ \pi^*_j = \frac{16 - 48t + 36t^2 - 8t^3 + 8\theta + 44t\theta - 152t^2\theta + 64t^3\theta - 8\theta^2 + 65t\theta^2 - 56t^2\theta^2 - 80t^3\theta^2}{2(3 - 4t)^2} + \nu. \]

(25)

\(^{10}\)This result is due to the fact that the schools maximize different objective functions. In my previous article (Manna, 2013), I show that if both principals maximize their profits, the effect of \( \theta \) on the prices is negative. Motivation has a positive impact on the quality offered by the firms. It implicitly reduces the product differentiation between firms stiffening competition and reducing prices. With higher qualities, the degree of differentiation of the product becomes relatively less important, leading to fiercer competition.
A.3 Characterization of the Equilibrium when Only the Public Teacher $i$ is Motivated

Now, suppose that only the agent $i$ is intrinsically motivated, i.e. $\theta_i = \bar{\theta}$ and $\theta_j = 0$.

At stage 1, agents exert effort and the optimal levels of quality are determined:

$$\hat{q}_i^* = \frac{32 - 56t + 19\bar{\theta} - 44t\bar{\theta}}{8(3 - 4t)}; \quad \hat{q}_j^* = \frac{32 - 16t + 19\bar{\theta} + 12t\bar{\theta}}{8(3 - 4t)}.$$  \hspace{1cm} (26)

Again, the quality of the services offered by the public school is lower than the quality offered by the private school, i.e. $\hat{q}_i^* < \hat{q}_j^*$. In addition, the intrinsic motivation of the teacher hiring by the public school has a positive impact on the levels of quality offered by the two schools, i.e. $\frac{\partial \hat{q}_i^*}{\partial \bar{\theta}} > 0$ and $\frac{\partial \hat{q}_j^*}{\partial \bar{\theta}} > 0$. This is due to the fact that an increase of the level of quality offered by the school $i$ “pushes” the principal $j$ to elicit higher agent effort in order to improve quality.$^{11}$

The principals pay the following wages to their agents:

$$\hat{\omega}_i^* = \frac{1}{2} \left( \frac{32 - 56t + 19\bar{\theta} - 44t\bar{\theta}}{8(3 - 4t)} \right)^2 - \bar{\theta} \left( \frac{64 - 184t + 56t^2 + 38\bar{\theta} - 103t\bar{\theta} - 36t^2\bar{\theta}}{16(3 - 4t)} \right);$$

$$\hat{\omega}_j^* = \frac{1}{2} \left( \frac{32 - 16t + 19\bar{\theta} + 12t\bar{\theta}}{8(3 - 4t)} \right)^2.$$  \hspace{1cm} (27)

To maintain the comparative advantage in terms of quality, the principal $j$ provides more incentives. The private school produces higher levels of quality and provides stronger incentives, i.e. $\hat{\omega}_i^* < \hat{\omega}_j^*$.

The prices are given by:

$$\hat{p}_i^* = \frac{t(16 - 8t + 5\bar{\theta} + 12t\bar{\theta})}{4(3 - 4t)}; \quad \hat{p}_j^* = \frac{t(8 - 4t + 7t\bar{\theta})}{3 - 4t}.$$  \hspace{1cm} (28)

A high agent $i$’s degree of motivation has a positive effect on the price of both schools. A high $\bar{\theta}$ produces an improvement of the quality offered by both schools, which increases its price. This effect has a positive impact on the marginal profits obtained by the private school and a negative impact on the students’ utility.

The demands will be equal to:

$$\hat{d}_i^* = \frac{8 - 24t - 5\bar{\theta} - 12t\bar{\theta}}{8(3 - 4t)}; \quad \hat{d}_j^* = \frac{16 - 8t + 5\bar{\theta} + 12t\bar{\theta}}{8(3 - 4t)}.$$  \hspace{1cm} (29)

$^{11}$If the two schools maximized the same objective functions, I would have obtained that the school $i$ has a comparative advantage with respect to the private school.
The private school $j$ gains a quality comparative advantage over the public school, and obtains the “business stealing effect”: an increase of quality permits to “steal” the market share to the rival firm. This effect has a positive impact on the marginal profits obtained by the private school $j$.

At stage 3, the benefits obtained by the schools are realized:

$$
\tilde{\pi}^*_i = \frac{-512 + 2784t - 3680t^2 + 1408t^3}{64(3 - 4t)^2} + \frac{-464\bar{\theta} + 3272t\bar{\theta} - 3712t^2\bar{\theta} + 896t^3\bar{\theta} - 95t^4\bar{\theta} + 1006t^5\bar{\theta} - 608t^6\bar{\theta} - 288t^7\bar{\theta}^2}{64(3 - 4t)^2};
$$

$$
\tilde{\pi}^*_j = \frac{1024 - 3072t + 2304t^2 - 512t^3}{128(3 - 4t)^2} + \frac{1216\bar{\theta} - 2272t\bar{\theta} - 704t^2\bar{\theta} + 768t^3\bar{\theta} + 361t^4\bar{\theta} - 104t^5\bar{\theta} - 1200t^6\bar{\theta}^2}{128(3 - 4t)^2} + \bar{\eta}.
$$

(30)

Even if only the agent $i$ is motivated, the private school maintain his comparative advantage in terms of quality.

### A.4 Characterization of the Equilibrium when Only the Private Teacher $j$ is Motivated

Finally, when only the agent $j$ is intrinsically motivated, i.e. $\theta_i = \bar{\theta} = 0$ and $\theta_j = \bar{\theta}_j > 0$, agents exert effort and the following levels of quality are determined:

$$
q^*_i = \frac{32 - 56t + 13\bar{\theta} - 36t\bar{\theta}}{8(3 - 4t)}; \quad q^*_j = \frac{32 - 16t + 13\bar{\theta} + 20t\bar{\theta}}{8(3 - 4t)}.
$$

(31)

The quality of the services offered by the public school is lower than the quality offered by the private school, i.e. $q^*_i < q^*_j$. In addition, the intrinsic motivation of the teacher hiring by the public school has a positive impact on the levels of quality offered by the two schools, i.e. $\frac{\partial q^*_i}{\partial \bar{\theta}} > 0$ and $\frac{\partial q^*_j}{\partial \bar{\theta}} > 0$. In addition, this impact of $\bar{\theta}$ on the levels of quality is higher in the private school than in the public one.
The principals pay the following wages to their agents:

\[
\hat{\omega}_i^* = \frac{1}{2} \left( \frac{32 - 56t + 13\vartheta - 36t\vartheta}{8(3 - 4t)} \right)^2 ;
\]

\[
\hat{\omega}_j^* = \frac{1}{2} \left( \frac{32 - 16t + 13\vartheta + 20t\vartheta}{8(3 - 4t)} \right)^2 - \frac{1}{\vartheta} \left( \frac{64 - 176t + 72t^2 + 26\vartheta - 11t\vartheta - 100t^2\vartheta}{16(3 - 4t)} \right) .
\] (32)

In this case, the agents’ intrinsic motivation has a positive impact on the wage given to the unmotivated teacher \(i\). In contrast, the impact of \(\theta\) on the wage of the motivated teacher \(j\) depends on the value of \(t\) and \(\vartheta\). Furthermore, the wage of the public teacher can be higher than the wage of the private teacher if \(t < 0.25\). If there is competition in the scholastic market, the public principal has to pay a high wage to the unmotivated teacher \(i\) to produce a larger output. The motivated teacher \(j\) provides a given level of effort even if he receives a low compensation for that.

The prices are given by:

\[
p_i^* = \frac{t(16 - 8t + 7\vartheta - 28t\vartheta)}{4(3 - 4t)} ; \quad p_j^* = \frac{t(16 - 8t + 5\vartheta - 12t\vartheta)}{2(3 - 4t)} .
\] (33)

A high agent \(i\)’s degree of motivation has a positive effect on the price of the private school. A high \(\vartheta\) produces an improvement of the quality offered by the private school, which increases its price. This effect has a positive impact on the marginal profits obtained by the private school and a negative impact on the students’ utility. Regarding the public school, the impact of \(\vartheta\) on its price is positive if \(t < 0.25\). In this case, the public teacher receives a higher wage than the private teacher and the public principal has to increase the price to repay his agent.

The demands will be equal to:

\[
d_i^* = \frac{(8 - 24t - 11\vartheta - 4t\vartheta)}{8(3 - 4t)} ; \quad d_j^* = \frac{(16 - 8t + 11\vartheta + 4t\vartheta)}{8(3 - 4t)} .
\] (34)

The private school \(j\) gains a quality comparative advantage over the public school, and obtains the “business stealing effect”: an increase of quality permits to “steal” the market share to the rival firm. This effect has a positive impact on the marginal profits obtained by the private school \(j\).

At stage 3, the social welfare and the private school’s profits are, respectively, equal
to:
\[
\hat{\pi}_i^* = \frac{-512 + 2784t - 3680t^2 + 1408t^3}{64(3 - 4t)^2} + \\
\frac{-560\bar{\theta} + 2488\bar{\theta} - 1536\bar{\theta}^2 - 384\bar{\theta}^3 - 34\bar{\theta}^4 + 1824t^2\bar{\theta}^2 - 1568t^3\bar{\theta}^2}{64(3 - 4t)^2};
\]
\[
\hat{\pi}_j^* = \frac{1024 - 3072t + 2304t^2 - 512t^3}{128(3 - 4t)^2} + \\
\frac{-704\bar{\theta} + 5088\bar{\theta} - 9024t^2\bar{\theta} + 3328t^3\bar{\theta} - 455\bar{\theta}^2 + 1176t^2\bar{\theta} + 1232t^3\bar{\theta}^2 - 3584t^4\bar{\theta}^2}{128(3 - 4t)^2} + \bar{\pi}.
\]

(B) Appendix B

B.1 Proof lemma 1, 2 and 3.

In the first part of these lemmas, I compare the profits when both principals hire self-interested agents with the one in which only a principal hires a motivated agent.

I start by considering the explicit expression for the public school:
\[
\hat{\pi}_i^* - \pi_i^* > 0 \text{ if } \frac{-464\bar{\theta} + 3272t\bar{\theta} - 3712t^2\bar{\theta} + 896t^3\bar{\theta} - 955\bar{\theta}^2 + 1006t^2\bar{\theta}^2 - 608t^3\bar{\theta}^2 - 288t^4\bar{\theta}^2}{64(3 - 4t)^2} > 0. \tag{36}
\]

It is not immediate to see for which values of the parameters \( t \) and \( \bar{\theta} \) the inequality holds. The proof unfolds in two steps. First, I study the marginal effect of the degree of differentiation of the educational services \( t \) and of the agent’s degree of motivation \( \bar{\theta} \) on the difference between benefits. Secondly, I show that the above inequality holds also in a limit case and this completes the proof.

I begin by studying the effect of \( t \) on the difference in the benefits obtained by the public school. Differentiating equation (36) with respect to \( t \), I obtain the following:
\[
\frac{\partial (\hat{\pi}_i^* - \pi_i^*)}{\partial t} < 0. \tag{37}
\]

The degree of differentiation of the educational services \( t \) has a countervailing effect on the social welfare. On the one hand, \( t \) has a negative impact on the social welfare due to a reduction of the students’ utility. When \( t \) is high, the market is less competitive. On the other hand, \( t \) has a positive impact on the social welfare due to an increase of the schools’ profits. The overall effect of \( t \) is negative. In addition, this negative impact of \( t \) on the social welfare is higher when only the teacher \( i \) is motivated with respect to the
case both teachers are selfish.

I also analyze the effect of the agent’s intrinsic motivation $\theta$ on the differential benefits. Deriving equation (36) with respect to $\theta$, I obtain the following:

$$\frac{\partial(\tilde{\pi}^*_i - \pi^*_i)}{\partial \theta} > 0 \quad \text{if} \quad t < 0.165. \quad (38)$$

A higher motivation influences positively the social welfare when the public school is the only one to hire the motivated agent but only if $t$ is small enough. In contrast, it has no effect on the profits of the firm when both principals hire self-interested agents. Then, the sign of this derivative is positive for $t < 0.165$ and negative otherwise.

To consider a limit case, I take the maximum value for $t$ in the interval in which it affects negatively the differential benefits and a small value for $\theta$, since his impact on the differential benefits is positive. If inequality (36) holds in this limit case, it will be always satisfied for other values of these parameters in the interval $t \in (0; 0.165)$. I set $t = 0.164$ and $\theta = 0.005$ and I obtain that $\pi^*_i = 0.4214 < 0.4218 = \tilde{\pi}^*_i$.

Now, I consider the explicit expression for the private school:

$$\frac{\partial(\tilde{\pi}^*_j - \pi^*_j)}{\partial t} < 0 \quad (40)$$

The overall effect of the degree of differentiation of the product on the differential profits is negative. This is because $t$ has a positive impact on the profits of the firm when both principals hire self-interested teachers but a negative impact when only a principal hires a motivated teacher. When $t$ increases the comparative advantage by hiring a motivated agent is reduced.

I also analyze the marginal effect of the agent’s intrinsic motivation $\theta$ on the private school’s profits. Deriving equation (39) with respect to $\theta$, I obtain the following:

$$\frac{\partial(\tilde{\pi}^*_j - \pi^*_j)}{\partial \theta} > 0 \quad \text{if} \quad t < 0.2 \quad (41)$$

A higher motivation influences positively the profits when the private school is the only one to hire the motivated agent but only if $t$ is small enough. This is because the private school obtains a comparative advantage with respect to the rival school in terms of demand and price. In contrast, it has no effect on the profits of the firm when both principals hire
self-interested agents. Then, the sign of this derivative is positive for $t < 0.2$ and negative in other case.

To consider a limit case, I take the maximum value for $t$ in the interval in which it affects negatively the differential benefits and a small value of $\theta$, since it impacts positively on the differential benefits. If inequality (39) holds in this limit case, it will be always satisfied for other values of these parameters in the interval $t \in (0; 0.2)$. I set $t = 0.19$ and $\theta = 0.005$ and I obtain that $\pi^*_j = 0.19 < 0.201 = \hat{\pi}^*_j$.

If $t < 0.165$, both schools obtain higher benefits by hiring a motivated agent when the other school hires a selfish agent. If $0.165 < t < 0.2$, only the private school obtains higher profits by hiring a motivated teacher when the public school hires a selfish one. If $t > 0.2$, both schools obtain higher profits by hiring selfish individuals.

I did the same for the second part of the lemmas and I obtained the following results. If $t < 0.165$ the best response of both principals is to hire a motivated agent when the other principal hires a motivated agent. If $0.165 < t < 0.2$, only the private school obtains higher profits by hiring a motivated teacher when the public school hires a motivated one. When $t > 0.2$, both schools obtain higher profits by hiring selfish individuals even if the rival school hires a motivated agent.\(^{12}\)

\[^{12}\]All the computations and comparison are checked using Mathematica and they are available under request.
References


