On Non-Tatonnement Processes

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Abstract

The objective of this study is to examine the stability of a general equilibrium model in which trade takes place at non-equilibrium prices. Quantity constraints are perceived by the traders and effective demands and supplies are explicitly derived from utility and profit maximization. Therefore, spillover effects are taken into account whenever a quantity constraint is binding. Furthermore, we resolve the problem of bankruptcy as follows. Traders, in maximizing their objective functions, plan at the same time not to go bankrupt, by constraining their behavior explicitly. Such a constrained behavior assigns to money a role as a medium of exchange. The system is shown to be globally stable, when both prices and quantity constraints adjust in disequilibrium.

Keywords: general equilibrium; utility; profit maximization; spillover effects.

Introduction

Early studies on non-tatonnement adjustments, e.g. Hahn and Negishi (1962) and Arron and Hann (1971), examined the stability of an equilibrium where transactions take place at non equilibrium prices. However, agents are assumed to go to the market, attempting to exercise their notional demands as if they can complete their trades. The static models of the so-called disequilibrium models (Benassy (1975), Dreze (1975)) took explicitly into account the quantity constraints that agents face in disequilibrium. Therefore, subsequent studies on non-tatonnement adjustments, try to incorporate quantity constraints in a dynamic framework and prove the stability of a equilibrium. Veendorp (1975), Fisher (1978), Eckalbar (1979) and Fisher (1981) are notable attempts towards that direction. However, apart from Fisher (1981), the rest of these papers, either do not incorporate quantity constraints explicitly (Fisher (1978)) or use strong assumptions i. e. gross substitutability, constant endowments and a three-good model (Eckalbar (1979), Veendorp (1975)). Fisher's important paper takes into consideration quantity constraints and spillover effects, but the adjustment of quantity constraints that he follows is rather ad hoc as we will see later.

The model that we develop in this paper is closely related to that of Fisher's analysis in several important aspects. First, we will not assume as Fisher does that traders go to the market attempting to exercise their notional demands. This is obviously a very strong assumption in this type of model where prices are temporarily fixed, and traders are bound to be constrained in some markets. This can be true as Fisher has noticed, only if the system is close to the Walrasian equilibrium so that notional demands are assumed to be satisfied. Otherwise, quantity constraints must be perceived by traders in some way before they go to the market. If these quantity constraints are imposed by an auctioneer, or are expected by the traders themselves, we still have to specify how these quantity constraints change over the time. Furthermore, spillover effects are taken into account. The perception of quantity constraints in some goods by a trader, forces him to spill over his unsatisfied demands and supplies of these goods in all markets. In this model we assume that purchases and sales are simultaneously determined (although this follows from the construction of the model), so that during a given day, only the perceived quantity constraints affect the behavior of the traders. However, actual or realized transactions in that day, influence the behavior of the traders in the next day, through either the quantity constraints adjustment that the auctioneer will follow, or their expectations that the traders will form about the quantity constraints they might meet in the next day. Furthermore, the problem of bankruptcy is resolved.. [1]Traders in maximizing their objective functions, plan at the same time not to go bankrupt by constraining their behavior explicitly. The system is shown to be globally stable and the stability properties are due to the fact that in disequilibrium, prices and quantity constraints move in such a way so as to limit the net demands of the traders until they become zero.

1 . Description of the Economy

We consider an economy with H households labeled by h and F firms labeled by f. There are n goods labeled by I and only one medium of exchange denoted by M and called money. We will now described how trade takes place between traders using a well known idealized process. This idealization has its origins in the recent static general disequilibrium models as developed mainly by Dreze (1975) and Benassy (1975).

At the beginning of each trading day the auctioneer calls out prices in terms of money as well as quantity constraints if prices are not the equilibrium ones. Households then maximize their utility subject to their budget and quantity constraints. Firms maximize their profits subject to their production function and the imposed quantity constraints. This maximization yields effective demands for all commodities by all households and firms. Trade then takes place in that day. However, consumption and production are postponed until equilibrium is reached. At the beginning of the next day, the auctioneer will adjust both prices and quantity constraints according to some rule. If for example, there is excess demand for some commodity, the auctioneer will increase the price of that good and at

[1]This problem which is related to the Hahn process assumption is stressed by Abraham and Whittaker (1989) .

the same time he will lower the upper bound on the quantity demanded by the traders. It is clear that this procedure differs in two important aspects from that used by Fisher (1978). First, effective or active demands are explicitly derived from utility and profit maximization. Second, notional or target demands are not the starting point in the trading process. In other words, the system is not supposed to be close to the Walrasian equilibrium. Later on we will discuss a different approach in which traders have expectations about the quantity constraints they may meet in their trading process.

2 . Household and firm Behavior

Let x_{hi} be the notional demand of the h^{th} household for the i^{th} good. $\bar{x}_{hi}$ its actual holdings and p_i the money price of the i^{th} good. Each household receive a share from the profit of every firm, so that all firms are owned by households. If $b_{hf} \geq 0$ is the h^{th} household's share of the f^{th} firm, it must be that $\sum_{h}^{H} b_{hf} = 1$. Each household is assumed to maximize a strictly quasi-concave utility function $U_h\,(X_h^{'}, M_h^{'})$, which is a function of the effective demands vector $X_h^{'}$ and the effective demand for money balance $M_h^{'}$. Denoting by S_h the h^{th} household's share in the profits of the firms and by d_h the dividends that have already been paid to it, its budget constraint is:

$$\sum_i P_i\,(X_{hi}^{'} - \overline{X}_{hi}) + M_h^{'} - \overline{M}_h - S_h + d_h \equiv 0 \qquad (2.1)$$

where bars denote endowments and primes denote effective demands.

The quantity constraints sent out by the auctioneer to the households, are lower and upper bounds on their demands and are denoted by $\underset{\sim}{X}_{hi} \geq 0$ and $\tilde{X}_{hi} \geq 0$ respectively. It is assumed that households do not violate the quantity constraints they perceive. The reason for this is the following. Since purchases and sales are simultaneously determined, the households do not have any advance information about the state of the market at the beginning of each trading day. The auctioneer has the relevant information regarding quantity constraints and households must follow his rules which will be stated in section 4. Therefore, households are informed about the state of the market in a given period by the magnitude of their perceived quantity constraints. The actual state of the market, however, is known to all households after completion of transactions during that period. With regard to the behavior of households, we furthermore assume that they do not misrepresent their preferences in order to attain their goals; for example, by bidding for more, etc. On the other hand we must exclude the possibility of bankruptcy. This can be done by following Howitt (1974) and assume that all purchases must be backed up with money and that no household promises to deliver more of a good than what it actually has. At the beginning of each trading period, the h^{th} household's total money balances $\bar{r}_h$ are given by: $\bar{r}_h \equiv \bar{M}_h + S_h - d_h$. Therefore, the purchases of a household in a given period cannot exceed its total money balances. Let z_{hi}, z'_{hi} and $\hat{z}_{hi}$ be the notional net demand, effective net demand and actual or realized net demand respectively of household h for good i[2]. Then, the h^{th} household's maximization problem is the following [3].

$$\text{Max } U_h\,(X'_h, M'_h), \text{ subject to}$$

$$\sum_i P_i\,(X'_{hi} - \bar{X}_{hi}) + M'_h - \bar{M}_h - S_h + d_h \equiv 0$$

$$\underset{\sim}{X}_{hi} \leq X'_{hi} \leq \tilde{X}_{hi} \quad I = 1, 2, ..., n \tag{2.2}$$

$$\sum_{i\varepsilon B_h} P_i\, z'_{hi} \leq \tilde{r}_h \quad \text{Where } B_h = \left\{ \frac{i}{z'_{hi}} \geq -0 \right\} \tag{2.3}$$

$$z'_{hi} \geq -\tilde{x}_{hi} \quad \text{all I such that } z_{\tilde{hi}} \leq 0. \tag{2.4}$$

Constraints (2.3) and (2.4) exclude the possibility of bankruptcy and (2.3) also demonstrates the use of money as a medium of exchange. Note that the above maximization problem is

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[2] Actual transactions in a given period, are by definition equal to the change of the actual holdings within that period . The difference between notional and effective demands will be explained below.

[3] For more details on such a maximization problem as well as some comparative results , see Mackay and Weber (1977)

the second round of decision making according to Clower (1965). In the first round, households maximize their utility subject to all of the above constraints except those in (2.2). The net demands thus derived are called notional demands. If the h^th household is not constrained at all; if $\underset{\sim}{x}_{hi} < x_{hi} < \tilde{x}_{hi}$ all i, then its notional net demands are those which will be exercised in the market and in this case the notional demands are equal to the effective ones. On the other hand if the household is constrained in at least one market, say for good i; i.e., if $x_{hi} > \tilde{x}_{hi}$ or $x_{hi} > \underset{\sim}{x}_{hi}$, then the second round of decision making takes place and the h^{th} household in maximizing its utility, will take into account the constraint for good i. In this case then, the demands and supplies of all other goods will be affected. In other words, the unsatisfied demand or supply of some good will spill over to all the other markets.

It is clear that in the above problem we have followed Dreze's (1975) approach in which agents do not violate the constraints they perceive. As Grangmont (1977, p.560) has pointed out, there is no satisfactory theory explaining why traders should violate the constraints they perceive in some markets as Grossman (1971) and Benassy (1975) have assumed.

Coming up to the discussion of firms, we assume that each firm attempts to sell outputs and buy inputs in a competitive market. Since no production takes place until equilibrium is reached, firms issue provisional contracts to buy or sell certain quantities of goods at given prices. The contracts are provisional since they are binding only if the stated prices turn out to be the equilibrium ones. Otherwise new contracts will have to be established. Hence, net demands and supplies by firms are in the form of contracts. Let $Y_{fi} \geq 0$ denote the notional supply of good i by the f^{th} firm. Using the conventional notation inputs are measured negatively and outputs positively. If $\bar{Y}_{fi}$ denotes the realized or actual contracts of the f^{th} firm for the i^{th} good, then $E_{fi} \equiv Y_{fi} - \bar{Y}_{fi}$ is the net notional supply (if positive) or net demand (if negative) of the i^{th} commodity by the f^{th} firm. We assume that money is used in the production process as an input and denote by M_f and $\overline{M}_f$ the notional and actual money holdings of the f^{th} firm. The production relation of the f firm is denoted by $\phi_f\,(Y_f^{'}, M_f^{'}) = 0$, where $Y_f^{'}$ denotes the vector of its effective demands and supplies and $M_f^{'}$ denotes the effective demand for money for production purposes.

The quantity constraints perceived by the f^{th} firm are upper bounds on its net supplies and lower bounds on its net demands. We denote these bounds by $\tilde{E}_{fi} \geq 0$ and $\underset{\sim}{E}_{fi} \leq 0$ respectively. What has said about such constraints in the case of households, applies also in the case of firms. Here, we also have to exclude the case of bankruptcy. At any given period of time, the f^{th} firm has realized a certain amount of profits through past trading denoted by $\overline{\pi}_f$. An explicit account on these profits will be given below. Out of these profits, the f^{th} firm has to pay its shareholders. Denoting by q_f the total payments that the f^{th} firm makes to its shareholders, the total money held by the f^{th} firm is given by $\overline{r}_f \equiv \overline{\pi}_f - q_f - \overline{M}_f$. We now assume that all purchases of the f^{th} firm must not exceed its transactions balances and that it never promises to deliver more of a good than it can produce. Formally then, the f^{th} firm's maximization problem is the following:

$$\max\ \pi_f = \sum_i P_i \left(Y_{fi}^{'} - \overline{Y}_{fi}\right) + M_f^{'} - \overline{M}_f + \overline{\pi}_f \ \text{ subject to,}$$

$$\phi_f\left(Y_f^{'}, M_f^{'}\right) = 0$$

$$\underset{\sim}{Y}_{fi} \leq Y_{fi}^{'} \leq \tilde{Y}_{fi} \quad \text{all I} \tag{2.5}$$

$$-\sum_{i \in B_f} P_i\, E_{fi}^{'} \leq \overline{\pi}_f - q_f \ \text{ where } B_f = \left\{\frac{i}{E_{fi}^{'}} \leq 0\right\} \tag{2.6}$$

$$E_{fi}^{'} \leq Y_{fi}^{'}$$

where $\tilde{Y}_{fi} \geq 0$ and $\underset{\sim}{Y}_{fi} \leq 0$ are upper and lower bounds on transactions and primes denote effective demands. Constraints (2.6) and (2.7) constitute the no bankruptcy condition and the profits $\overline{\pi}_f$

which have realized in the past, up until the present time t are given by

$$\bar{\pi}_f\ (t) = \int_0^t \left[\sum_i P_i\ (\tau)\ \dot{\bar{Y}}_{fi}\ (\tau) + \dot{\bar{M}}_f\ (\tau) \right] d\tau + \bar{\pi}_f(0)$$

where 0 is some initial date.

Since trade takes place at disequilibrium prices, the f[th] firm will not be able at any moment of time to realize its profits π_f. Instead the f[th] firm will receive an amount $\bar{\pi}_f$ which must be greater than the payment q_f that it will make to its shareholders. Later on we will see that in equilibrium all profits are realized and distributed to the households, so that $\bar{\pi}_f = q_f$. However, in disequilibrium $q_f \leq \bar{\pi}_f$ and the difference $\bar{\pi}_f - q_f$ represents the amount of money held for transactions purposes; i.e., for the purchase of inputs. It is now clear that the h[th] household's share of firms profits S_h and the dividends d_h are defined as follows:

$$S_h \equiv \sum_f b_{hf}\ \pi_f \qquad\qquad d_h \equiv \sum_f b_{hf}\ q_f\ .$$

Note that there is no rule about the payments q_f that the firm makes to its shareholders. It is only required that $\bar{\pi}_f \geq q_f$ and that there must exist some money balances to be used for the purchase of inputs. Hence, the no bankruptcy condition (2.6) determines the amount q_f that the f[th] firm will distribute. This is absent in Fisher's (1974) model. On the other hand, the firm in maximizing its profits takes into account the fact that its contracts will be binding when equilibrium is reached and production begins. Furthermore, spillover effects affect the demands and supplies of every firm which perceives quantity constraints that limit its notional demands or supplies. Hence, the above maximization is the second round of decision making by firms. The first round is similar to that described in the case of households. We also assume that the above maximization problem yields unique demands and supplies by the firm so that constant returns to scale are excluded. The reason for this requirement will become clear in section 6.

As will be seen below, the auctioneer follows certain rules in adjusting quantity constraints in disequilibrium situations. Since the auctioneer can have a knowledge of the endowments and the actual contracts of the agents without knowing their preferences and production functions, the quantity constraints that he sends must satisfy the following consistency conditions:

$$\tilde{X}_{hi} \leq \sum_f \overline{Y}_{fi} + \sum_h \overline{X}_{hi} \qquad \text{all I, h} \tag{2.8}$$

$$-\underset{\sim}{Y}_{fj} \leq \sum_h \overline{X}_{hi} + \sum_f \overline{Y}_{fi} \qquad \text{all j, f} \tag{2.9}$$

$$\underset{\sim}{X}_{hi} \geq 0, \quad \tilde{Y}_{fi} \geq 0 \qquad \text{all h, f, I} \tag{2.10}$$

Condition (2.8) states that a household cannot demand more of a good than what can actually be supplied by all traders and (2.9) states that a firm cannot demand more of a input than can actually be supplied by all traders. Note that the auctioneer has no way of restricting the supplies of households or firms, since he does not know either their preferences or their production functions and therefore he does not know their effective demands. Hence, (2.10) requires only that $\underset{\sim}{X}_{hi}$ and $\tilde{Y}_{fi}$ be non-negative in order to be meaningful.

3. <u>Assumptions and Trading Rules</u>

Since production and consumption are postponed until equilibrium is reached, we postulate the following:

<u>Assumption 3.1</u>

$$\sum_h \dot{\overline{X}}_{hi} - \sum_f \dot{\overline{Y}}_{fi} \equiv 0 \quad i = 1, 2, ..., n .$$

Assumption 3.1 states that any change in the endowments of households is due to a change in actual contracts of firms. A similar condition can be applied to money whose quantity is assumed to be fixed:

Assumption 3.2

$$\sum_h \dot{\overline{M}}_h - \sum_f \dot{\overline{M}}_f - \sum_f \dot{q}_f \equiv 0.$$

Having each household and firm complete their transactions during a trading period, their money balances at the end of the period or at the beginning of the next period will depend on the actual transactions of the previous period. We have seen that $\overline{M}_h + S_h - d_h$ is the amount of money that the h[th] household spends for the purchase of goods. Hence, the rate of decrease of these balances during a trading period, depends on the actual purchases of that period; i.e.,

$$\dot{\overline{M}}_h + \dot{S}_h - \dot{d}_h \equiv -\sum_{i \in B_h} P_i \dot{\overline{X}}_{hi} \text{ all h.} \tag{3.1}$$

However, the difference between initial and end-of-period money balances depends on the actual transactions performed.
Hence, we have:

Assumption 3.3

$$\dot{\overline{M}}_h - \dot{d}_h \equiv -\sum_i P_i \dot{\overline{X}}_{hi} \text{ all h.}$$

A similar reasoning applies in the case of firms. The money balances held by the f[th] firm for the purchase of inputs is given by $\overline{\pi}_f - q_f$. The rate of decrease of these balances during a trading period, depends on the actual purchases made:

$$\dot{\overline{\pi}}_f - \dot{q}_f \equiv -\sum_{i \in B_f} P_i \dot{\overline{Y}}_{fi} \text{ all f} \tag{3.2}$$

However, the total change of money balances, positive or negative, within a trading period, depends on the actual transactions of that period. [4]

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[4] Assumption 3 . 4 can be derived by differentiating the definition of realized profits on p . 9 .

Assumption 3.4

$$\dot{\bar{\pi}}_f \equiv \sum_i P_i \dot{\bar{Y}}_{fi} + \dot{\bar{M}}_f \quad \text{all f}$$

By combining (3.2) with Assumption 3.4 it is interesting to see that $\dot{q}_f \equiv -\sum_{i \notin B_f} P_i \dot{\bar{Y}}_{fi} + \dot{\bar{M}}_f$ which states that the larger the sales of the f[th] firm, the larger its revenue will be and therefore the larger the amount that it will distribute to its shareholders.

We are now in a position to derive Walras' Law for this economy. Summing up over all households the budget constraint (2.1) and taking into account the definitions of profits, shares and dividends we get Walras' Law:

$$\sum_i \sum_h P_i Z'_{hi} - \sum_i \sum_f P_i E'_{fi} + \sum_h \left(M'_h - \bar{M}_h\right) - \sum_f \left(M'_f - \bar{M}_f\right) + \sum_f \left(q_f - \bar{\pi}_f\right) \equiv 0 \qquad (3.3)$$

4. Price-Quantity Adjustments and the Hahn Process Assumption

As we stated in the beginning, after the end of each trading period, the auctioneer has the task of adjusting both prices and quantity constraints if the economy is not in equilibrium. The rules that he follows are stated in the following assumption. First we define $Z'_i \equiv \sum_h Z'_{hi} - \sum_f E'_{fi}$.

Assumption 4.1

(i) $$\dot{P}_i = \begin{cases} 0 & \text{if } P_i = 0 \quad \text{and} \quad Z'_i < 0 \\ H_i\left(Z'_i\right) & \text{otherwise.} \end{cases}$$

(ii) $$\dot{\tilde{X}}_{hi} = \begin{cases} -F_{hi}\left(Z'_{hi}\right) & \text{if} \quad Z'_{hi} > 0 \quad \text{and} \quad Z'_{hi} = \tilde{Z}_{hi} \\ 0 & \text{if} \quad Z'_{hi} \le 0 \end{cases}$$

$$\dot{\underset{\sim}{X}}_{hj} = \begin{cases} -G_{hj}\left(Z'_{hj}\right) & \text{if} \quad Z'_{hj} < 0 \quad \text{and} \quad Z'_{hj} = Z_{hj} \\ 0 & \text{if} \quad Z'_{hj} \ge 0 \end{cases}$$

(iii) $$\dot{\tilde{Y}}_{fi} = \begin{cases} -F_{fi}\left(E'_{fi}\right) & \text{if} \quad E'_{fi} > 0 \quad \text{and} \quad E'_{fi} = \tilde{E}_{fi} \\ 0 & \text{if} \quad E'_{fi} \leq 0 \end{cases}$$

$$\dot{\underset{\sim}{Y}}_{fj} = \begin{cases} -G_{fj}\left(E'_{fj}\right) & \text{if} \quad E'_{fj} < 0 \quad \text{and} \quad E'_{fj} = E_{fj} \\ 0 & \text{if} \quad E'_{fj} \geq 0 \end{cases}$$

where F (....) and G (....) are sign preserving functions in their arguments and furthermore $H_i(0)= 0$. Moreover, we assume that quantity constraints are stationary for those goods which are in excess supply and their prices are zero.

The price adjustments rule in (I) is well known and needs no explanation[5]. The quantity adjustment rule in (ii) and (iii) states that, in general, upper and lower quantity constraints which are binding can become stricter but not looser. Quantity constraints which are not binding in the maximization process of the agents, remain stationary since they are not restrictive. Hence, for example, in situations of a positive net effective demand for some good by a household, the upper bound on the household's demand will be decreased. The Hahn process Assumption that will be stated in a moment will help us to elaborate more on this rule. First, note an asymmetry between the price adjustment rule and the quantity adjustment one. In the former, prices respond to the aggregate net effective demands whereas in the latter quantity constraints respond only to individual ones. This is, however, done as the following example shows, in order to exclude some unnecessary adjustments from taking place. Suppose that all but one agent's effective net demands for all goods are zero. For some household h we assume that there is a good i such $Z'_{hi} > 0$ and $\tilde{Z}_{hi} = Z'_{hi}$. We furthermore assume that all quantity constraints for good i are binding for all traders. Then, if quantity constraints

[5] Champsaur , Dreze and Henry (1977) have shown that under fairly weak assumptions , this rule does not create any discontinuities in the trajectories of the system . Their method can also ensure that the quantity adjustment rule in (ii) and (iii) does not violate the Lipschitz conditions .

respond according to individual net effective demands as in Assumption 4.1 (ii), (iii), the auctioneer will only limit the hth household's transaction for the ith good. On the other hand, if we suppose that quantity constraints respond to aggregate net effective demands, the auctioneer would have to adjust in this case all agent's quantity constraints, which demand this good, even if their respective net individual demands for this good are zero. This is the reason for not allowing a quantity adjustment rule of the latter type, although in every other respect both rules are equivalent (as we shall see) if the Hahn process Assumption holds. The functions F and the G which are different for each household and firm, may include any form of rationing imposed by the auctioneer.

The next Assumption that will be stated is the usual Hahn process Assumption as stated in Hahn and Negishi (1960), which is, however, applied to effective demands.

Assumption 4.2

$Z'_{hi} \neq 0$ implies $Z'_{hi} Z'_i > 0$

and $E'_{fi} \neq 0$ implies $E'_{hi} Z'_i < 0$ all h, f, I and all $t \geq 0$.

Assumption 4.2 states that if *ex post* a household or firm has excess demand for some good, then the aggregate net effective demand for this good is positive. This Assumption must hold at all times and it is a natural consequence of trade provided that every supplier of a good can find a demander of this good and *vice versa*. It is different than the modified Hahn process Assumption as stated in Fisher (1978, p.21) which postulates that $Z_{hi} Z'_i \geq 0$. This Assumption turns out to be pretty strong as Fisher has indicated when the long side of a market switches to a short one and *vice versa*. In essence, Assumption 4.2 precludes the coexistence of net effective demand and net effective supply in the market of any good. What this implies then, is that an agent is constrained only if he is in the long side of the market and the actual constraint that he perceives in this case is more severe than the one sent out by the auctioneer. On the other hand, traders in the short side of the market do not perceive any actual quantity constraint even if some of the respective constraints imposed by the auctioneer are binding. Therefore those agents always realize their plans, in the sense that their effective net demands will be met.

It is now clear in the quantity adjustment rule (ii) and (iii) in Assumption 4.1, that although quantity constraints respond to individual net effective demands, implicitly they respond to the aggregate ones, since by Assumption 4.2 individual net effective demands have the same sign as the aggregate ones. Therefore, we have allowed individual net effective demands rather than aggregate to activate the quantity adjustment mechanism since this kind of adjustment is weaker as it was seen on p.43.

Coming back now to our discussion concerning the quantity adjustment rule in Assumption 4.1, we see that in situations of a positive net effective demand for some good, the upper bounds on households' transactions and lower bounds on firms' transactions will get stricter, provided their quantity constraints for this good are binding. However, the lower and the upper bounds on households and firms' transactions respectively remain unchanged. This implies that those agents who *ex ante* had excess supply of this good will not have the opportunity to increase it. The reason for the establishment of such a rule is due to Assumption 4.2 which implies that only one side of the market is binding. That is why the auctioneer in situations of an aggregate net effective demand for some good, will only restrict the transactions of net demanders for this good since the quantity constraints sent by the auctioneer to the net suppliers of this good are irrelevant to the long side of the market, even if some of them may be binding. Therefore, quantity constraints are adjusted only if it is necessary; i.e., only if they belong to the long side of the market. Quantity constraints which belong to the short side of the market are not adjusted since agents in this side realize their plans. Hence, each time quantity constraints are adjusted so as to make agents always worse off in the sense that more severe constraints will have to be imposed on traders in the long side of the market.

5 . Equilibrium and Disequilibrium

Definition 5.1

The economy is in a competitive equilibrium, if and only if $Z_i^{'} \leq 0,\ \ M_h^{'} = \overline{M}_h$ and $M_f^{'} = \overline{M}_f$ for all h, f, i. Furthermore, for every h, f and i, $Z_{hi}^{'} = \hat{Z}_{hi}$ and $E_{fi}^{'} = \hat{E}_{fi}$.

It is seen from the definition of equilibrium, that since effective and not notional demands are zero, persistent disequilibrium in Varian's (1975) terminology will always arise. Using the definition of equilibrium and Walra's Law (3.5), we see that since prices are no-negative and $\overline{\pi}_f \geq q_f$ we must have P_i=0 for $Z_i^{'} < 0$ and $q_f = \overline{\pi}_f$ all f, for Walras' Law to hold. Also, using the definition of profits we see that $\pi_f = \overline{\pi}_f$ so that all profits are distributed to the households. Furthermore, the only money balances held by the firms in equilibrium, are those needed in the production process since $\overline{\pi}_f = q_f$. From definition 5.1 and Assumption 4.2 it is clear that individual demands will also be zero for all non-free goods. This implies that no actual transfer of goods takes place in equilibrium, since everyone's demands are equal to his actual holdings; i.e., $X_{hi}^{'} = \overline{X}_{hi}$ and $Y_{fi}^{'} = \overline{Y}_{fi}$ all h, f, i. Hence, we may say that there is no trade at equilibrium, even though trade has been taking place in the past up until the establishment of equilibrium. However, there is a case in which trade cannot take place at all and the system is in equilibrium. This case which we call "artificial'' equilibrium is described as follows. At time t=0, the auctioneer knows the actual holdings of all agents without knowing their preferences or production functions. In such a case he can eliminate every possibility for trading by sending to all agents quantity signals equal to their actual holdings; i.e., $\overline{X}_{hi}\,(0) = \tilde{X}_{hi}\,(0) = X_{hi}\,(0)$ and $\left|\overline{Y}_{fi}\,(0)\right| = \left|\tilde{Y}_{fi}\,(0)\right| = \left|\underset{\sim}{Y}_{f}\,(0)\right|$ all h, f, i. It is evident that this is sufficient to establish since all agents' demands are restricted to their initial holdings and the maximization procedures eg households and firms described in section 2 will produce demands and supplies such that $X'_{hi} = \overline{X}_{hi}$ and $Y'_{fi} = \overline{Y}_{fi}$. For example, if a households has excess demand for good i, then its best choice is $\tilde{X}_{hi}$ which is equal to is endowment of good i. Therefore, we must exclude such a peculiar case which may also rise at some t>o. In other words, the auctioneer may wish at any time to establish an artificial equilibrium as defined below.

Definition 5.2

An artificial equilibrium at time t≥0 , is a state in which all upper and lower quantity constraints by all agents coincide with their actual holdings.

A sufficient condition to exclude an artificial equilibrium is given below.

Assumption 5.1

For every t≥0 out of equilibrium, there is at least a household h, a firm and a good i, such that for household h, $\overline{X}_{hi}(t) < \tilde{X}_{hi}(t) > X'_{hi}(t)$ and $Z'_{hi}(t) > 0$ and for firm f, $\overline{Y}_{fi}(t) < \tilde{Y}_{fi}(t) > Y'_{fi}(t)$ and $E'_{fi}(t) > 0$.

Assumption 5.1 states that out of equilibrium, not all quantity constraints are binding and that not all effective excess demands are zero. The latter is sufficient to allow trading to take place at t=0 and the former is sufficient to exclude an artificial equilibrium at t>0.

Lemma 5.1

Under Assumption 5.1 an artificial equilibrium is impossible.

Proof

At time t=0 trade will obviously take place since there is a supplier of good i and a demander for good i whose net demands are not zero. At some t>0, suppose that an artificial equilibrium is established. By Assumption 4.1 upper quantity constraints of households which are binding , always decline. Hence, there is a t' close to t and t'<t, such that for all agents whose excess demand were not zero at t', $Z^{'}_{hi}(t') = \tilde{Z}_{hi}(t')$ and $E^{'}_{fi}(t') = \tilde{E}_{fi}(t')$ which contradicts Assumption 5.1..

We now proceed to show that the profits of firms decline out of equilibrium. This is so because prices and quantity constraints change in such a way so as to take firms worse off. For example, some goods that firms are unable to sell become cheaper, whereas some others that they cannot acquire become expensive. On the other hand those quantity constraints which are binding become stricter. Assuming that the sets $\tilde{Q}^{f} = \left\{ \frac{j}{E_{fj} > \tilde{E}_{fj}} \right\}$ and $Q^{f} = \left\{ \frac{k}{E_{fk} > \underset{\sim}{E}_{fk}} \right\}$ are not null, so that firms cannot transact all of their notional demands, we have the next Lemma.

Lemma 5.2

$\dot{\pi}_f < 0$ out of equilibrium and $\dot{\pi}_f = 0$, if and only if the economy is in equilibrium.

Proof

Since each firm is a profit maximize and in disequilibrium it is adversely affected by changes in prices and quantity constraints, to show that profits decline out of equilibrium i.e. essentially the same as showing that the maximum profit of each firm declines. The Lagrangean of the fth firm is given by:

$$L_f = \sum_i P_i\left(Y^{'}_{fi} - \overline{Y}_{fi}\right) + M^{'}_f - \overline{M}_f + \overline{\pi}_f - \lambda_f\left(\phi_f\left(Y^{'}_f, M^{'}_f\right)\right) - \sum_{k \in \underset{\sim}{Q}f} \mu_{gk}\left(\underset{\sim}{Y}_{fk} - Y^{'}_{fk}\right) +$$

$$\sum_{j \in Qf} \rho_{fj}\left(\tilde{Y}_{fj} - Y^{'}_{fj}\right) + \theta_f\left(\overline{\pi}_f + q_f + \sum_{i \in B_f} E^{'}_{fi}\right) + \sum_i \sigma_{fi}\left(Y^{'}_{fi} - E^{'}_{fi}\right).$$

We know that at the optimum, L_f is a function of prices, actual holdings and quantity constraints and it is known as the profit function. Differentiating then this function with respect to time yields:

$$\dot{L}_f = \sum_i \dot{P}_i \left(Y'_{fi} - \bar{Y}_{fi}\right) - \left(\sum_i P_i \dot{\bar{Y}}_{fi} + \dot{\bar{M}}'_f - \dot{\bar{\pi}}_f\right) - \left(\sum_{k \in \underset{\sim}{Q} f} \mu_{fk} \dot{\underset{\sim}{Y}}_{fk}\right) + \left(\sum_{j \in \tilde{Q} f} \rho_{fj} \dot{\tilde{Y}}_{fj}\right) +$$

$$+\theta_f \left(\dot{\bar{\pi}}_f - \dot{q}_f - \sum_{i \in B_f} P_i \dot{\bar{Y}}_{fi}\right) + \theta_f \sum_{i \in B_f} \dot{P} E'_{fi} - \sum_i \sigma_{fi} \dot{\bar{Y}}_{fi}$$

The first term is negative out of equilibrium by Assumptions 4.1 and 4.2 and zero in equilibrium. For the same reason the sixth term is also non-positive. The second and fifth terms are zero by Assumption 3.4 and (3.2) respectively. The third and fourth terms are negative out of equilibrium and zero in equilibrium, since $\dot{\underset{\sim}{Y}}_{fk} \geq 0$ and $\dot{\tilde{Y}}_{fj} \leq 0$ by Assumption 4.1 and $\mu_{fk} = -\dfrac{\partial L_f}{\partial \underset{\sim}{V}_{fk}} < 0$ and $\rho_{fi} = \dfrac{\partial L_f}{\partial \tilde{Y}_{fj}} > 0$.

Finally the last term is negative out of equilibrium and zero in equilibrium since $\sigma_{fi} > 0$ and $\dot{\bar{Y}}_{fi} \equiv \hat{E}_{fi} > 0$ all i. The multiplier $\mu_{fk}\left(\rho_{fj}\right)$ represents the marginal decrease (increase) in profits by tightening (loosening) the kth (jth) quantity constraint. Therefore, bearing in mind the previous remarks, the profits of firms always decline out of equilibrium and are stationary if and only if the economy is in equilibrium.

The above results will be helpful in proving the next Lemma. First we define the following two sets $\tilde{Q}^h = \left\{\dfrac{j}{Z_{hj} > \tilde{Z}_{hj}}\right\}$ and $\underset{\sim}{Q}^h = \left\{\dfrac{k}{Z_{hk} < \underset{\sim}{Z}_{hk}}\right\}$ which we assume to be non-null, so that notional demands and supplies of some goods cannot be transacted.

<u>Lemma 5.3</u>

Let V_h denote the indirect utility function of the h^{th} household. Then, $\dot{V}_h < 0$ out of equilibrium and $\dot{V}_h = 0$ if and only if the economy is in equilibrium.

<u>Proof</u>

The Lagrangean of the h^{th} household is:

$$V_h \equiv U_h\left(X_h^{'}, M_h^{'}\right) - \lambda_h\left(\sum_i P_h\left(X_{hi}^{'} - \overline{X}_{hi}\right) + M_h^{'} - \overline{M}_h - S_h + d_h\right) - \sum_{j\in \tilde{Q}^h} \mu_{hj}\left(X_{hi}^{'} - \tilde{X}_{hj}\right) -$$

$$- \sum_{k\in \underset{\sim}{Q}^h} \rho_{hk}\left(X_{hk}^{'} - \underset{\sim}{X}_{hk}\right) + \left(\overline{M}_h + S_h - d_h - \sum_{i\in B_h} P_i Z_{hi}^{'}\right) - \sum_i \sigma_{hi}\left(X_{hi} + Z_{hi}^{'}\right).$$

At the optimum this Lagrangean is a function of prices, endowments, quantity constraints, shares and dividends and is known as the indirect utility function. Differentiating it with respect to time yields:

$$\dot{V}_h \equiv -\lambda_h \sum_i \dot{P}_i\left(X_{hi}^{'} - \overline{X}_{hi}\right) + \lambda_h\left(\sum_i P_h \dot{\overline{X}}_{hi} + \dot{\overline{M}}_h - \dot{d}_h\right) + \lambda_h \dot{S}_h + \sum_{j\in \tilde{Q}^h} \mu_{hj} \dot{\tilde{X}}_{hj} +$$

$$+ \sum_{k\in \underset{\sim}{Q}^h} \rho_{hk} \dot{\underset{\sim}{X}}_{hk} + \theta_h\left(\dot{\overline{M}}_h + \dot{S}_h - \dot{d}_h + \sum_{i\in B_h} P_i \dot{\overline{X}}_{hi}\right) - \theta_h \sum_{i\in B_h} \dot{P} Z_{hi}^{'} - \sum_i \sigma_{hi}\left(\dot{\overline{X}}_{hi} - \dot{\overline{X}}_{hi}\right).$$

The first and seventh terms are negative by Assumption 4.1 and 4.2 unless there is equilibrium in which case they are both zero. The second and sixth terms are zero by Assumption 3.3 and (3.1) respectively. The third term can be written as $\lambda_h \sum_f b_{hf} \dot{\pi}_f$ and since $\lambda_h > 0$ and $b_{hf} > 0$, by Lemma 5.2 it is negative out of equilibrium and zero in equilibrium. the fourth and fifth terms are non-positive out of equilibrium since $\dot{\tilde{X}}_{hj} \leq 0$ and $\dot{\underset{\sim}{X}}_{hk} \geq 0$ by assumption 4.1 and $\mu_{hj} = \dfrac{\partial V_h}{\partial \tilde{X}_{hj}} > 0$ and $\rho_{hk} = \dfrac{\partial V_h}{\partial \underset{\sim}{X}_{hk}} < 0$. At equilibrium, however, $\dot{\tilde{X}}_{hj} = \dot{\underset{\sim}{X}}_{hk} = 0$ by the same Assumption. The last term is also zero. The multiplier $\mu_{hj}\left(\rho_{hk}\right)$ represents the marginal increase (decrease) in utility by loosening (tightening) the $j^{th}\left(k^{th}\right)$ quantity constraint.

6. Stability

Note that the previous Lemma holds for every regime which will be defined below. Since different agents will be constrained in different markets, the rate of change of the state variables of the system (i.e., prices, endowments and quantity constraints) will be governed by a different differential equation depending on the prevailing market conditions. A particular regime then defines a particular market condition. Since market condition change continuously during the adjustment process, stability of the system must be examined in all possible market conditions or regimes. The next theorem is a generalization of theorem 2 in Eckalbar (1980) and is the main tool in showing the stability of our system. Consider the following system of autonomous equations defined on the different regimes S^i as follows:

$$\dot{X} = \begin{cases} f^i(X) & \text{if} \quad X \in S^i \subset R^n \quad i = 1, 2, \ldots, m \\ f^i(X) = f^j(X) = \ldots & \text{if} \quad X \in S^i \cap S^j \cap \ldots \end{cases} \qquad \text{(A)}$$

where X belongs to an open set $W \subset R^n$ and $U_i S^i = W$. To each regime S^i we associate a Lyapunov function which is defined as follows:

<u>Definition 6 . 2</u>

Let V : W $\rightarrow$ R such that,

$$V(X) = \begin{cases} V^i(X) & \text{if} \quad X \in S^i, \quad i = 1, 2, \ldots, m \\ V^i(X) = V^j(X) = \ldots & \text{if} \quad X \in S^i \cap S^j \cap \ldots \end{cases}$$

satisfying:

a) $D^+V^i(X) < 0$ in W\E, $\quad$ i = 1, 2, ..., m.

$D^+V^i(X) = 0$ only if X is an equilibrium of (A) i = 1, 2, ..., m

where $D^+V^i(X) = \lim\limits_{h \to 0} + \sup \dfrac{V^i\left(X + hf^i(X)\right) - V^i(X)}{h}$ denotes the right-hand time derivative of V^i and $E = \left\{ \dfrac{X}{D^+V^i(X)} = 0, \quad X \in S^i \right\}$ denotes the equilibrium set of (A).

Theorem 6 . 1

Consider system (A) and assume that it satisfies the Lipschitz conditions. Suppose there exists a locally Lipschitzian Lyapunov function as defined above and let X(t) be any bounded solution of (A). Then $\lim(X)\rightarrow E$ as $t\rightarrow\infty$, i.e., solution path approaches the equilibrium set of (A)[6].

Proof

The trajectory X (t) may pass from one or more regimes depending on the initial conditions. Since it is bounded, there is a convergent subsequent such that $X(t_n)\rightarrow X^*$ as $t_n\rightarrow\infty$. Since V(X(t)) is decreasing and bounded from below, there is some $C\in R$ that $V(X(t_n))\rightarrow C$ as $t_n\rightarrow\infty$. By the continuity of V in all regimes we must have $V(X^*)=C$. But V is decreasing and therefore $V(X(t_n))\rightarrow C$ as $t_n\rightarrow\infty$ for every limit point X*. Hence; $D^+V(X^*)=0$ and using b) $X^*\in E$. Since X(t) is bounded for $t\geq 0$, $X(t)\rightarrow E$ as $t\rightarrow\infty$. This shows that every solution path of (A) converges to the set of equilibria.

We can now proceed by showing that our economy has a quasi-globally stable adjustment process. First we need the following result.

Lemma 6 . 1

The time path of prices, actual holdings and quantity constraints is bounded.

Proof

Fisher (1974) has shown that prices and actual holdings are bounded and his proof is applicable in our model. The upper bounds

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[6]Using the properties of limit set , a more elegant form of this Theorem is known as the Invariance Principle . See LaSalle and Lefschetz (1961 , Section 13) and Lasalle (1968) . Uzawa (1961 , Theorem 1) has shown that the Lyapunov function need not be decreasing as long as it is convergent . His proof can also be used to establish the theorem . See also Arrow and Hahn (1971 , ch . 11 , thoerem 3)

$\tilde{X}_{hi}$ and $\tilde{Y}_{fi}$ are also bounded since $\tilde{X}_{hi} \geq 0$ and $\tilde{Y}_{fi} \geq 0$ and $\dot{\tilde{X}}_{hi} \leq 0$, $\dot{\tilde{Y}}_{fi} \leq 0$ in disequilibrium by Assumption 4.1. The lower bound $\underset{\sim}{X}_{hj}$ is bounded from below since it is positive. The firms' lower bounds $\underset{\sim}{Y}_{fj}$ are also bounded from below by (2.9) Both $\underset{\sim}{X}_{hj}$ and $-\underset{\sim}{Y}_{fj}$ are increasing in disequilibrium by Assumption 4.1, up to the point where $Z_j^{'} = 0$. At that point, however, individual net demands are zero, so that $\underset{\sim}{X}_{hi} - \overline{X}_{hj} = 0$ and $\underset{\sim}{Y}_{fj} - \overline{Y}_{fj} = 0$. Clearly then the lower bounds are also from above since endowments are bounded.

To show quasi-global stability we assume that there are J different regimes or market conditions denoted by S^r, r = 1, 2, ..., J. In what follows a superscript r on a function denotes the corresponding regime.

Theorem 6 . 2

The economy has a quasi-globally stable adjustment process, i.e., every limit point is a equilibrium.

Proof

Let $V^r(t) \equiv \sum_h V_h^r$ if V_h^r belongs to regime S^r, r = 1, 2, ..., J. Since Lemma 5.3 holds for every regime, it is easy to show that $V(t)=V^r(t)$ r = 1, 2, ..., J is a Lyapunov function. Since endowments are bounded, $V^r(t)$ is bounded from below by the initial endowments of households. By Lemma 5.3, $\dot{V}^r(t) < 0$ if and only if $Z_i^{'} \neq 0$ all i, r. At equilibrium, however, $\dot{V}^r(t) = 0$ by the same Lemma. Hence, $V^r(t)$ satisfies the assumptions of Theorem 6.1 and therefore every path of prices, endowments and quantity constraints converges to some equilibrium.

Although we have shown that the economy will approach some equilibrium from any initial point, it is desirable to know whether the adjustment process converges to a unique equilibrium starting from a given initial point. A nice way to do this is that originated by Arrow and Hahn (1971) and generalized later on by Fisher (1974). The method of proof uses the expenditure minimization and profit maximization to show that all limit points of the process are the same. The same method of proof can be employed here. However, in order to prove that prices converge to the same limit point we need the following Assumption.

Assumption 6 . 1

At any equilibrium, for any pair of goods (including money) with strictly positive prices, at least one of the following is true: a) There is some h and two goods i and j, such that $X'_{hi} > 0$ and $X'_{hj} > 0$. b) There is some f and two goods i and j such that $Y'_{fi} \neq 0$ and $Y'_{fj} \neq 0$.

Assumption 6.1 which has been used by Fisher (1974), effectively rules out corner solutions, so that some agent's marginal rate of substitution between any two goods with positive prices determines their respective price ratio.

Theorem 6 . 3

The economy has a globally stable adjustment process, i . e., starting from a given initial point it converges to a unique equilibrium.

Proof

Since the adjustment process is quasi-globally stable and the path of prices, endowments and quantity constraints is bounded, we need only show that all limit points of the process are the same. By Lemma 5.3 every household's utility approaches a limit, say $\overline{U}_h$. Let $P^*, \overline{X}^*, \overline{M}^*, \tilde{X}_h^*, \underset{\sim}{X}_h^*$ be the limit points of the path of prices, endowments and quantity constraints and let $X_h^{'*}$ and $M_h^{'*}$ be the respective limit points of effective demands. Suppose now that there is another set of limit points denoted by $X_h^{'**}$ and $M_h^{'**}$. Since the limiting value $\overline{U}_h$ is the same for every limit point:

$$U_h\left(X_h^{'*}, M_h^{'*}\right) = U_h\left(X_h^{'**}, M_h^{'**}\right) = \overline{U}_h \text{ all h .}$$

Since U is strictly quasi-concave,

$$\sum_i P_i^* X_{hi}^{'*} + M_h^{'*} < \sum_i P_i^* X_{hi}^{'**} + M_h^{'**} .$$

In equilibrium all profits are distributed to the households, so that using the budget constraint (2.1) we get:

$$\sum_i P_i^* \overline{X}_{hi}^* + \overline{M}_h^* < \sum_i P_i^* \overline{X}_{hi}^{**} + \overline{M}_h^{**} \tag{6.1}$$

On the other hand, since $Y_f^{'*}$ and $M_f^{'*}$ is a unique profit maximizing choice (note that we have excluded constant returns to scale)

$$\sum_i P_i^* Y_{fi}^{'*} + M_f^{'*} > \sum_i P_i^* Y_{fi}^{'**} + M_f^{'**}$$

and by the fact that $Y_{fi}^{'*}, Y_{fi}^{'**}, M_f^{'*}$ and $M_f^{'**}$ are equilibrium points,

$$\sum_i P_i^* \overline{Y}_{fi}^* + \overline{M}_f^* > \sum_i P_i^* \overline{Y}_{fi}^{**} + \overline{M}_f^{**} \tag{6.2}$$

Summing up (6 . 1) over all households and (6.2) over firms and then summing both inequalities up and rearranging yields:

$$\sum_h\sum_i P_i^*\left(\overline{X}_{hi}^{**} - \overline{X}_{hi}^*\right) + \sum_h\left(\overline{M}_h^{**} - \overline{M}_h^*\right) + \sum_f\sum_i P_i^*\left(\overline{Y}_{fi}^* - \overline{Y}_{fi}^{**}\right) + \sum_f\left(\overline{M}_f^* - \overline{M}_f^{**}\right) > 0$$

(6.3)

By Assumptions 3.1 and 3.2 we get

$$\sum_h \dot{\overline{X}}_{hi} - \sum_f \dot{\overline{Y}}_{fi} + \sum_h \dot{\overline{M}}_h - \sum_f \dot{\overline{M}}_f - \sum_f \dot{q}_f \equiv 0 \tag{6.4}$$

Since in equilibrium $q_f = \pi_f$, then $\dot{q}_f = \dot{\pi}_f = 0$ by Lemma 5.2. Summing (6.2) over all goods yields:

$$\sum_h\sum_i \dot{\overline{X}}_{hi} - \sum_f\sum_i \dot{\overline{Y}}_{fi} + \sum_h \dot{\overline{M}}_h - \sum_f \dot{\overline{M}}_f \equiv 0$$

which contradicts (6.3) . Hence, there is one set of limit points of effective demands and therefore one set of limit points of actual holdings. Obviously then, quantity constraints which are binding will be the same in both equilibria.

Now consider the convergence of prices. The first order conditions for utility maximization imply that for household h and some good $j \in \tilde{Q}^h$

$$\left(\frac{\partial U_h}{\partial X'_{hj}} - \lambda_h \, P_j - \mu_{hj} - \theta_h \, P_j \right) X'_{hj} = 0 .$$

By Assumption 6 .1 the marginal rate of substitution between any such good and money (which is the same at both equilibria) determines a unique money price since the Lagranfe multipliers μ^{*}_{hj} and μ^{**}_{hj} must be the same at both equilibria; i.e., $\mu^{*}_{hj} = \dfrac{\partial V^{*}_{h}}{\partial \tilde{X}^{*}_{hj}} = \dfrac{\partial V^{*}_{h}}{\partial \tilde{X}^{**}_{hj}} = \mu^{**}_{hj}$; for the quantity constraints $\tilde{X}^{*}_{hj}$ and $\tilde{X}^{**}_{hj}$ must be binding at both equilibria (and therefore are equal) otherwise μ^{*}_{hj} and μ^{**}_{hj} are both zero by the Kuhn-Tucker conditions. Similarly θ^{*}_{h} and θ^{**}_{h} must be the same if they are not zero. The same is true for every good $k \in \underset{\sim}{Q}^h$ and similar reasoning applies in the case of firms. Hence, prices converge to a unique limit point and thus the proof is completed.

7 . An Alternative Approach

In the model we have already examined, the auctioneer had the dual role of adjusting both prices and quantity constraints after each trading day. In the present section we will assume that the role of the auctioneer is limited to adjusting only the prices. However, when false trading takes place and prices are fixed during the trading day, it is unrealistic to suppose, as Fisher (1978) admits, that notional demands of agents are a starting point in the trading process, especially when quantity constraints have been perceived by the agents in the past. Hence, we assume that at the beginning of each trading day, the auctioneer calls out a given price vector which will remain fixed in that day. Agents are assumed to have expectations about the quantity constraints that they will meet in their transactions during the day. These expectations are assumed to be held with certainty and that they follow a simple rule which will be stated in a moment. Each agent maximizes his objective function subject to the relevant constraints. The maximization procedure are the same as in section 2 and need not be repeated. Essentially all of our previous Assumption hold, except for Assumption 4.1 which now reads as follows.

(i) $$\dot{P}_i = \begin{cases} 0 & \text{if } P_i = 0 \quad \text{and} \quad Z_i^{'} < 0 \\ H_i\left(Z_i^{'}\right) & \text{otherwise.} \end{cases}$$

(ii) $$\dot{\underset{\sim}{X}}_{hi} = \begin{cases} -\alpha\left(Z_{hi}^{'} - \hat{Z}_{hi}\right) & \text{if} \quad 0 < Z_{hi}^{'} > \hat{Z}_{hi} \quad \text{and} \quad Z_{hi}^{'} = \tilde{Z}_{hi} \\ 0 & \text{if} \quad Z_{hi}^{'} = \hat{Z}_{hi} \end{cases}$$

$$\dot{\underset{\sim}{X}}_{hj} = \begin{cases} -\beta\left(Z_{hj}^{'} - \hat{Z}_{hj}\right) & \text{if} \quad 0 > Z_{hj}^{'} < \hat{Z}_{hj} \quad \text{and} \quad Z_{hj}^{'} = \underset{\sim}{Z}_{hj} \\ 0 & \text{if} \quad Z_{hj}^{'} = \hat{Z}_{hj} \end{cases}$$

(iii) $$\dot{\underset{\sim}{Y}}_{fi} = \begin{cases} -\gamma\left(E_{fi}^{'} - \hat{E}_{fi}\right) & \text{if} \quad 0 < E_{fi}^{'} > \hat{E}_{fi} \quad \text{and} \quad E_{fi}^{'} = \tilde{E}_{fi} \\ 0 & \text{if} \quad E_{fi}^{'} = \hat{E}_{fi} \end{cases}$$

$$\dot{\underset{\sim}{Y}}_{fj} = \begin{cases} -\delta\left(E_{fj}^{'} - \hat{E}_{fj}\right) & \text{if} \quad 0 > E_{fj}^{'} < \hat{E}_{fj} \quad \text{and} \quad E_{fj}^{'} = \underset{\sim}{E}_{fj} \\ 0 & \text{if} \quad E_{fj}^{'} = \hat{E}_{fj} \end{cases}$$

where α, β, γ, δ are positive constants.

The quantity adjustment rule in (ii) and (iii) shows that change in those expected quantity constraints which are binding tend to be unfavorable, in the sense that they always become stricter. The reason for this is the following. Suppose that a household attempts to exercise its excess demand for some good without violating its expected upper bound. If its effective demand is greater than its actual or realised transaction, then the household realizes that its actual constraint was more severe than the one that was expected. Therefore, the household will have to adjust the upper bound downwards provided that it was binding; other-wise the upper bound remains unchanged. On the other hand if it succeeds in completing its transaction, so that its effective and actual transaction for this good coincide, then there is no reason to adjust its upper bound, no matter whether it was binding or not. It might actually decide to buy more than its effective demand if it wishes to do so. However, such a behavior would not be optimal, since the optimal net demands are given as solutions to the maximization problem of each agent subject to the relevant constraints. On the other hand, since purchases and sales are simultaneously determined, such a behavior is ruled out.

At this point, we should note that the quantity adjustment rule in (ii) and (iii) is different than the one presented by Fisher (1981). [7]Fisher's adjustment rule only requires that quantity constraints which are close to binding become stricter. However, this implies that an agent will keep adjusting his upper and lower bounds of some goods even if his effective net demands coincide with his actual ones; i.e., even if his net demands for these goods are satisfied. The reason for this peculiar result is that agents in Fisher's model do not adjust their quantity constraints according to the market conditions but according to whether or not these constraints are binding.

It is now easily seen, that using Assumption 7.1 instead of 4.1, all of our previous results hold. Hence, the system is again globally stable when prices are adjusted by the auctioneer and quantity constraints are adjusted by the traders themselves. The reason for such a stability result is the same as in the previous sections. Prices and quantity constraints move unfavorably in disequilibrium so that each time the net effective demands of agents decline until they become zero.

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[7]Actually, Fisher's quantity adjustment rule is stated explicitly only in his M.I.T. discussion paper No.231 (1979), which is an extended version of his published work (1981). Our discussion is concerned with the former.

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