# Fight or Flight?

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# Fight or Flight? Defending Against Sequential Attacks in the Game of Siege*

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## Abstract

This paper examines theory and behavior in a two-player game of siege, sequential attack and defense. The attacker's objective is to successfully win at least one battle while the defender's objective is to win every battle. Theoretically, the defender either folds immediately or, if his valuation is sufficiently high and the number of battles is sufficiently small, then he has a constant incentive to fight in each battle. Attackers respond to defense with diminishing assaults over time. Consistent with theoretical predictions, our experimental results indicate that the probability of successful defense increases in the defenders valuation and it decreases in the overall number of battles in the contest. However, the defender engages in the contest significantly more often than predicted and the aggregate expenditures by both parties exceed predicted levels. Moreover, both defenders and attackers actually increase the intensity of the fight as they approach the end of the contest.

*Keywords*: Colonel Blotto, conflict resolution, weakest-link, game of siege, multi-period resource allocation, experiments.

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# 1. Introduction

Environments, such as cyber-security (Moore et al., 2009), pipeline systems (Hirshleifer, 1983), complex production processes (Kremer, 1993), and anti-terrorism defense (Sandler and Enders, 2004) can be characterized as weakest-link systems. In each of these cases an attacker only needs to disrupt one component of the system to create a total failure. Defenders are forced to constantly protect the entire system while attackers are encouraged to seek the weakest point.

Recently, a number of theoretical papers emerged trying to model the optimal strategies of those who wish to protect weakest-link systems and those who wish to destroy them. Most of the theoretical work has been focused on the case where the attacker and the defender simultaneously decide how much to invest in each potential target, a variation of the Colonel Blotto game.[1] For example, Clark and Konrad (2007) and Kovenock and Roberson (2010) both provide a theoretical analysis of a multi-battle two-player game where the attacker and the defender simultaneously commit resources to concurrent multiple battles in order to win a prize.[2] To receive the prize, the attacker needs to win at least one battle, while the defender must win all battles. Another class of attack and defense games, distinct from the simultaneous multi-battle game, assumes that battles proceed sequentially. Most of such models originated with the seminal R&D paper of Fudenberg et al. (1983).[3] The theoretical model studied in our paper, however, is most closely related to Levitin and Hausken (2010). Both papers consider a contest in which a defender seeks to protect a network and an attacker seeks to destroy it through multiple sequential attacks.[4] Levitin and Hausken (2010) model the probability of winning a given battle with a lottery contest success function. Due to complexity of their model, most of the paper's theoretical results are based on numerical simulations. In contrast, our paper uses an all-pay auction format, allowing us to explore sequential attacks in a weakest-link network theoretically. We also conduct a series of controlled laboratory experiments and compare observed behavior to the theoretical benchmarks.

Sequential attacks in a weakest-link network can be viewed as a "game of siege" where the defender attempts to hold an asset such as a fort or a landing strip against repeated assault. Arguably, the most famous siege, whether it is true or not, was the battle of Troy in which the Greeks finally ended a prolonged siege by hiding in a wooden horse according to Greek mythology. In this type of game, the attacker and defender decide how much to invest in each battle after learning the outcome of any previous battle. The side making the larger investment wins that battle, creating a series of all-pay auctions. The attacker only needs to be successful once, while the defender must repel each successive assault to win, and hence the game has a weakest-link structure. Our theoretical model predicts that if the defender's valuation is sufficiently high and the number of battles is sufficiently small, then the defender has a constant incentive to fight in each battle and otherwise he folds immediately. Thus, defenders exhibit a response pattern of "fight or flight." Attackers respond to defense with diminishing assaults over time. Consistent with theoretical predictions, our experimental results indicate that the probability of successful defense increases in the defender's valuation and it decreases in the overall number of battles in the contest. However, the defender engages in the contest significantly more often than predicted and the aggregate expenditures by both parties exceed predicted levels. Also, contrary to theoretical predictions, both the defender and attacker actually increase the intensity of the fight as they approach the known end of the game.

Identifying the predictive success of contest models, such as the one described in the current study, is of social value. However, the usual concerns about unobservable information are present with studies of naturally occurring data and conducting field tests could be extremely costly in this context, making laboratory experiments an ideal tool for empirical validation. Our study adds to the experimental literature on multi-battle contests. To date there are only a few experimental studies that investigate games of multiple contests. Avrahami and Kareev (2009) and Chowdhury et al. (2011) test several basic predictions of the original Colonel Blotto game and find support for the major theoretical predictions. Kovenock et al. (2010) study a multi-battle contest with asymmetric objectives and find support for the theoretical model of Kovenock and Roberson (2010) but not Clark and Konrad (2007). Our study contributes to this literature by investigating both theoretically and experimentally the dynamic multi-battle contest which we call the "game of siege."

## 2. The Game of Siege

Before introducing the general model of sequential attack and defense (or game of siege), it is useful to review the simple one shot contest, or all-pay auction, between two asymmetric players as in Baye et al. (1996). Assume that two risk-neutral players compete for a prize in a contest. The prize valuation for player 1 is $v_1$ and for player 2 it is $v_2$, where $v_1 > v_2 > 0$. Both players expend resources $x_1$ and $x_2$, and the player with the highest expenditures wins. In case of a tie, the winner is selected randomly. Irrespective of who wins the contest, both players forfeit their expenditures. It is well known that there is no pure strategy equilibrium in such a game (Hillman and Riley, 1989; Baye et al., 1996). The mixed strategy Nash equilibrium is characterized by the following proposition due to Baye et al (1996).

**Proposition 1.** In the mixed strategy equilibrium of a contest between two asymmetric players, with valuations $v_1 > v_2 > 0$:

(i) Players randomize over the interval $x \in [0, v_2]$, according to cumulative distribution functions $F_1^*(x) = \frac{x}{v_2}$ and $F_2^*(x) = 1 - \frac{v_2}{v_1} + \frac{x}{v_1}$.

(ii) Player 1's expected expenditure is $E(x_1) = \frac{v_2}{2}$ and player 2's is $E(x_2) = \frac{v_2^2}{2v_1}$.

(iii) Player 1's expected payoff is $E(\pi_1) = v_1 - v_2$ and player 2's is $E(\pi_2) = 0$.

(iv) Player 1's probability of winning is $p_1 = 1 - \frac{v_2}{2v_1}$ and player 2's is $p_2 = \frac{v_2}{2v_1}$.

We now turn to the case of two players, attacker and defender, competing in multiple sequential contests. The objective of the attacker $A$ is to win a single battle, in which case he receives a valuation of $v_A$. The objective of the defender $D$ is to win all $n$ battles, in which case he receives a valuation of $v_D$, where $v_D > v_A > 0$. As the battles occur sequentially, both players first simultaneously allocate their respective resources $x_A^1$ and $x_D^1$ in battle 1. If $x_A^1 > x_D^1$, then the contest stops and the attacker receives $v_A$. However, if the defender is successful in battle 1, the contest proceeds to battle 2. Again, if $x_A^2 > x_D^2$, then the contest stops and the attackers receives $v_A$. This process repeats until either the attacker wins one battle or the defender wins all $n$ battles. The net payoff of player $A$ if he wins is equal to the value of the prize minus the expenditures spent during the competition in each battle up to that point, e.i. $\pi_A = v_A - \sum_{k=1}^{l} x_A^k$, where $l$ is the battle won by the attacker. If player $A$ is never successful, this payoff (loss) is the negative sum of his expenditures, i.e. $\pi_A = -\sum_{k=1}^{n} x_A^k$. The payoff to player $D$ is similar, i.e. $\pi_D = v_D - \sum_{k=1}^{n} x_D^k$ if player $D$ wins all the battles and $\pi_D = -\sum_{k=1}^{l} x_D^k$ if he loses battle $l$.

To analyze this game we apply backward induction and identify the subgame perfect equilibrium. As will be shown, the expected contest winner depends on the size of $n$ for a given $v_A$ and $v_D$. Specifically, the expected outcome depends on whether $v_D \geq nv_A$, $nv_A > v_D > (n-1)v_A$ or $(n-1)v_A \geq v_D$. First, consider the contest in battle $n$. In the last battle, the value of winning the contest for player $D$ is $v_D$ and the value for player $A$ is $v_A$, with $v_D > v_A$. Therefore, this is a simple one-stage contest between two asymmetric players as characterized by Proposition 1. In such a contest, the expected expenditure of player $D$ in battle $n$ is $E(x_D^n) = \frac{v_A}{2}$ and the expenditures of player $A$ is $E(x_A^n) = \frac{v_A^2}{2v_D}$. According to Proposition 1, the expected payoff of player $D$ in battle $n$ is $E(\pi_D^n) = v_D - v_A$ and the expected payoff of player $A$ is $E(\pi_A^n) = 0$.

Next, we consider the contest in the penultimate battle. The defender's continuation value of winning battle $n-1$ is $v_D - v_A$ (his expected payoff from competing in battle $n$) and his value of losing is 0 (since the contest stops if the attacker wins even a single battle). On the other hand, the value to the attacker of winning battle $n-1$ is $v_A$ (since the attacker only needs a single victory) and the value of losing is 0 (the expected payoff from competing in battle $n$). Given, these expected payoffs, the contest in battle $n-1$ is again a simple single stage contest between two asymmetric players as characterized by Proposition 1. However, this time the continuation value of player $D$ is $v_D - v_A$ and the value of player $A$ is $v_A$. If the defender's continuation value is sufficiently higher than the attackers value, i.e. $v_D - v_A > v_A$, then the defender has the advantage and his expected payoff in battle $n-1$ is $v_D - 2v_A$, while the attacker's expected payoff is 0.

A similar exercise can be performed for battle $n-k$, the results of which are reported in Panel A of Table 1. Note that in generating Panel A of Table 1, we assume that $v_D \geq n v_A$ (or alternatively that $n \leq \frac{v_D}{v_A}$), i.e. the defender's valuation is sufficiently high relative to the number of battles $n$ and attacker's valuation $v_A$. In such a case, the defender always randomizes between 0 and $v_A$ and the expected expenditure of the defender in each battle $n-k$ is $E\left(x_D^{n-k}\right) = \frac{v_A}{2}$. On the other hand, the expenditure of the attacker $E(x_A^{n-k}) = \frac{(v_A)^2}{2(v_D - kv_A)}$ is decreasing in $n-k$, which means that the attacker's aggression decreases in number of battles won by the defender.[5] We summarize these findings in the following proposition:

**Proposition 2.** In the subgame perfect equilibrium, if $v_D \geq nv_A$, then in battle $n-k$:

(i) Player $D$ randomizes over the interval $x_D \in [0, v_A]$, according to the cumulative distribution function $F(x_D) = \frac{x_D}{v_A}$, and player $A$ randomizes according to the cumulative distribution function $F(x_A) = 1 - \frac{v_A}{v_D - kv_A} + \frac{x_A}{v_D - kv_A}$.

(ii) Player $D$'s expected expenditure is $E\left(x_D^{n-k}\right) = \frac{v_A}{2}$ and player $A$'s expected expenditure is $E\left(x_A^{n-k}\right) = \frac{(v_A)^2}{2(v_D - kv_A)}$.

Proposition 2 is based on the assumption that the defender has a relatively high valuation.[6] Now consider the case that $nv_A > v_D > (n-1)v_A$ (or alternatively that $\frac{v_D}{v_A} < n < \frac{v_D}{v_A} + 1$). As shown in Panel B of Table 1, in this special case, the disadvantaged defender in battle 1 receives expected payoff of zero. The attacker, on the other hand, receives positive expected payoff of $nv_A - v_D$ and should attack. Although the defender does not entirely give up in this case, his expected expenditures in battle 1 are lower than the expenditures of the attacker. Should the defender win this initial battle, he would have the advantageous position in battle 2 and all subsequent battles and the game would progress as in Proposition 2.

If the player $D$'s valuation $v_D$ is sufficiently small or the number of battles is sufficiently high, then the defender will give up, by expending 0 resources in the first battle.[7] To demonstrate this, assume that in battle 2 the continuation value of the defender $v_D - (n-2)v_A$ is not enough to cover the current valuation of the attacker $v_A$, i.e. $v_D - (n-2)v_A \leq v_A$ (or alternatively that

$(n-1)v_A \geq v_D$ or $n \geq \frac{v_D}{v_A} + 1$). In such a case, the attacker has an advantage in battle 2 over the defender. According to Proposition 1, the expected payoff to the attacker is $(n-1)v_A - v_D$, which for now assume is strictly positive, and the expected value to the defender is 0. Therefore, when making a decision in battle 1, the defender is expecting to receive 0 payoff in battle 2. Obviously, in such a case, the defender has a strictly dominant strategy to make no expenditure in battle 1 assuring a payoff of 0 rather than incurring a costly bid that will result either in a defeat yielding a reward of 0 or a continuation of the game yielding an expected subsequent payoff of 0.[8] On the other hand the attacker's valuation of winning is still $v_A$. If ties are broken randomly, then the attacker should make any minimal possible expenditure $\varepsilon$ (0.1 francs in our experiment) to guarantee the victory.[9] We summarize these results in Panel C of Table 1 and in the following proposition:

**Proposition 3.** In the subgame equilibrium, if $(n-1)v_A \geq v_D$, then in battle 1:

(i) Player $D$ makes an expenditure of 0 and player $A$ makes a minimal expenditure of $\varepsilon$.

(ii) Player $D$'s expected payoff is 0 and player $A$'s expected payoff is $v_A - \varepsilon$.

A straight forward implication of Proposition 3 is that for any values the two players have, there is some critical number of battles, $n^*$, above which the defender should immediately fold.[10] This is formalized in the following corollary:

**Corollary 1:** For any contest with at least $n^* = \frac{v_D}{v_A} + 1$ battles, player $A$ wins in battle 1 in equilibrium.

Another result that springs from Propositions 2 and 3 along with the intermediate case discussed above is that attackers never give up. This is formalized in the following corollary:

**Corollary 2:** For any finite contest, in equilibrium, player $A$ attacks with positive probability in every battle until the ultimate outcome is resolved.

To summarize, the main prediction of our model is that the attacker always engages in each battle. The defender engages in the battle only if $v_D > (n-1)v_A$. However, if the number of battles $n$ is sufficiently high or the defender's valuation $v_D$ is sufficiently small, i.e. $(n-1)v_A \geq v_D$, then the defender gives up with probability one, by expending zero resources in the first battle. Stated another way, for a given set of values $v_D > v_A$, if the horizon is sufficiently short then the defender will fight while the attacks grow weaker, but if the horizon is long the defender will simply give up. The number of battles the defender is willing to endure is determined by the relative size of $v_D$ and $v_A$, with the defender's endurance increasing in $v_D$.

Our model is for a game with a finite horizon, but we now briefly consider the case of an infinite horizon game. As it turns out, in an infinite horizon game the defender will surrender immediately. While this result matches the finite story well, the reasoning is somewhat different. The intuition is that winning a battle results in the defender being in the same value position as he was prior to the battle and losing results in a negative payoff, since fighting is costly, while surrender yields a certain profit of 0. The defender's position is thus characterized by a Bellman equation where the optimal choice is to surrender. The attacker's behavior is characterized by a similar Bellman equation and the attacker will find it optimal to attack in a battle if $v_A > 0$.[11]

Given that our model suggests that extended contests favor attackers, why don't we observe more attacks in naturally occurring settings? One possible explanation is that our players face no resource constraint, nor are there any costs that are non-productive in the sense of not increasing the likelihood of winning, nor are there any alterative targets for attack. It is important to keep in mind that the parameters in our model can be interpreted as economic profits and costs, which account for opportunity costs. In practice, the relative value of $v_A$ to

$v_D$ may be quite small. Based on the expected bids shown in Panel A of Table 1, as $v_A \to 0$ the expected bids of both players approach 0 so long as the defender finds it optimal to defend.

## 3. Experimental Design and Procedures

Our experimental design employs three treatments, by manipulating the number of battles and the valuation of the defender. In all treatments, the valuation of the attacker is kept constant at $v_A = 50$ experimental francs, the experimental currency. In the baseline treatment N3-V150, the number of battles is $n = 3$ and the defender's valuation is $v_D = 150$ francs. The subgame perfect equilibrium prediction for this treatment is that the defender engages in the competition with the attacker, and the defender wins the contest with probability 0.31, the joint probability of winning all three battles.

The other two treatments are designed to increase the attacker's advantage. In treatment N4-V150, the number of battles is increased to $n = 4$. The defender should not be willing to fight four battles and thus should invest 0 in battle 1 and concede the contest. Should this not occur, and the defender actually wins the first battle 1, then behavior in battle $1 + k$ in N4-V150 should be identical to behavior in battle $k$ in N3-V150 for $k \in \{1,2,3\}$. Obviously, in the subgame perfect equilibrium the defender's joint probability of winning all three battles is 0, but should behavior not follow the subgame perfect equilibrium path during the first battle, it is expected to follow the equilibrium path from that point forward creating the identical predictions for the last three battles, should they occur. The third treatment is N3-V100, which is similar to the baseline N3-V150 except that the defender's value is reduced from $v_D = 150$ to $v_D = 100$ francs. This has the effect of reducing the continuation value of the defender in every battle just as if extra battles had been inserted into the contest. With these values, defenders should be unwilling to engage in three battles and give up in battle 1, but would have the upper hand and fight should the contest reach battle 2. Our choice of a 50 franc reduction was so that the strategic situation was the same in battle $k$ in N3-V100 as in battle $k-1$ in N3-V150, when it exists, and battle $k$ in N4-150. The predicted average investment, expected payoff, and probability of winning the contest are reported in Table 2 for all three treatments.

The experiment was conducted at the Economic Science Institute at Chapman University. The computerized experimental sessions were run using z-Tree (Fischbacher, 2007). Six sessions each involving 16 undergraduates were run, for a total of 96 unique participants. Some students had participated in other economics experiments that were unrelated to this research.

Each experimental session involved subjects playing 20 contests each in one of the three treatments, thus we have a between subjects design. This was done to give the subjects maximum experience with a set of parameters during the sixty minute session given the sophisticated backwards induction required to solve this game. Before the first contest in each session subjects were randomly and anonymously assigned as attacker or defender, which we called participant 1 and participant 2.[12] All subjects remained in the same role assignment for the first 10 contests and then changed their assignment for the last 10 contests. Subjects of opposite assignments were randomly and anonymously re-paired each contest to form a new two-player group. In each contest, subjects were asked to choose how many francs to allocate in a given battle, which we called a round. Subjects were not allowed to allocate more than the value of the reward in any battle and were informed that regardless of who won the contest, both participants would have to pay their allocations.[13] At the end of each battle, the computer displayed one's own allocation, one's opponent's allocation, and the winner of that battle. The contest ended when the attacker won one battle or the defender won all the battles.

At the end of the experiment, 2 out of the first 10 contests and 2 out of the last 10 contests were randomly selected for payment. The sum of the earnings for these 4 contests was exchanged at rate of 25 francs = $1. Due to institutional constraints, actual losses cannot be extracted from subjects. This creates the potential for loss of experimental control as a subject is indifferent between small and large losses. We follow the standard procedure of endowing subjects with money from which losses can be deducted, in this case $20.[14] Subjects were paid privately in cash and the earnings varied from $13.25 to $27.5.

# 4. Results

## 4.1. Treatment Effects

Table 2 provides the aggregate results of the experiment. We start our analysis with the general description of treatment effects. The model predicts the probability of the defender winning the contest decreases with the defender's value. Under the parameters used in our experiment, the equilibrium probability the defender wins the contest is 0.31 in the N3-V150 treatment and it is 0 in the N3-V100 treatment. The observed probabilities in the experiment are 0.41 and 0.29, respectively. Although the observed probabilities are inconsistent with the theoretical point predictions, they comply with comparative statics predictions. Specifically, consistent with theoretical predictions, the probability of successful defense is higher in the N3-V150 treatment than in the N3-V100 treatment. This difference is significant based on the estimation of a random effect probit model where the dependent variable is the defender winning the contest and the independent variables are a period trend and a treatment dummy-variable ($p$-value $< 0.05$).[15]

**Result 1:** Consistent with theoretical predictions, the probability of successful defense increases in the defender's valuation.

The theory also predicts the probability of the defender winning the contest decreases in the number of battles. The equilibrium probability of the defender winning the contest is 0.31 in the N3-V150 treatment and it is 0 in the N4-V150 treatment. The observed probabilities in the experiment were 0.41 and 0.27, respectively. Again, despite off theoretical point predictions, qualitatively, this difference is in the predicted direction and significant based on the estimation of a random effect probit model similar to the one described above ($p$-value < 0.05).

**Result 2:** Consistent with theoretical predictions, the probability of successful defense decreases in the number of battles.

## 4.2. Within Treatment Behavior

Although the qualitative predictions of the theory are supported by the data, the quantitative predictions are clearly rejected. One notable feature of the data is the considerable over-expenditure in all treatments. This can be seen from the fact that both the attacker and the defender earn significantly lower payoffs than predicted.[16] Such significant over-expenditure is not uncommon in experimental literature on contests and all-pay auctions (Davis and Reilly, 1998; Potters et al., 1998; Gneezy and Smorodinsky, 2006; Lugovskyy, et al., 2010; Sheremeta, 2010a, 2010b, 2011). Still, we rarely observe defenders spending more in the contest (over all three rounds) than the value of winning. In fact, such over-dissipation by defenders only occurs in 1% of the contests in N3-V150 and N3-V100 and 4% of the contests in N4-V150. For attackers the rate is higher, although still not large at 4% in N3-V100 and 15% in N3-V150 and

N4-V150. This difference in attackers and defenders is unsurprising given that the value of winning is much higher for defenders.

**Result 3:** Contrary to theoretical predictions, there is considerable aggregate over-expenditure in all treatments by both attackers and defenders.

One explanation for the over-expenditure is that subjects fall prey to a sunk cost fallacy. For the payoff maximization problem, expenditures in previous battles are sunk costs and should be ignored, but evidence from various behavioral studies suggests people incorporate sunk costs in their decision-making (Friedman et al. 2007).[17] Several other possible explanations, proposed in the literature, include subjects having a non-monetary value of winning (Sheremeta, 2010a, 2010b; Price and Sheremeta, 2011), having spiteful preferences (Herrmann and Orzen, 2008) or making mistakes (Potters et al., 1998; Sheremeta, 2011) and judgmental biases (Sheremeta and Zhang, 2010).

Camerer (2003) argues that subjects can learn to play equilibrium strategies with experience. Figure 1 shows the total expenditure (sum of expenditures in all battles) over time. There is no clear trend in any of the three treatments, suggesting that on aggregate subjects consistently employ similar strategies across all periods of the experiment. A regression of the total expenditure on a time trend, estimated separately for each treatment, shows that there is no significant relationship between the two variables (p-values > 0.10). Separating the data by player type and battle, we again find no consistent patterns (see Figures A1, A2, and A3 in the online appendix).[18]

Another readily apparent feature of the data is that defenders do not surrender in the first battle in N3-V100 or N4-V150, see Panel B of Table 1. While the average investment is lower in these two periods than in the subsequent periods, it is not 0. In fact, defenders spend 0 in less than 5% of the battles in which they are predicted to do so.[19] Defenders' behavior is counteracted by attackers who invest more than the minimal amount predicted in equilibrium. A simple random effect model, estimated separately for each treatment, finds that the average bid in the first battle is significantly higher than 0 for both the attacker and the defender (p-values $<$ 0.05).

**Result 4:** Contrary to theoretical predictions, the defenders do not give up and the attackers expend substantial resources in the first battle.

In all other battles the expected expenditure by defenders should be the same. However, defenders are actually increasing their defenses as the end of the contest approaches. Attackers also increase the intensity of their assault as the end of the contest approaches, the exact opposite of the pattern predicted by the theory. These trends are statistically significant based on the estimation of the panel regression models.[20] Moreover, such patterns persist throughout all 20 periods of the experiment, as indicated by Figures A1, A2 and A3 in the online appendix.

**Result 5:** Contrary to theoretical predictions, both defenders and attackers increase the intensity of the fight as they approach the end of the contest.

Results 4 and 5 are clearly inconsistent with the theoretical predictions, which are largely based on a well-known phenomena in the all-pay auction literature – a "discouragement effect."[21] In particular, the defender should be discouraged in the first battle in treatments N3-V100 and N4-V150 because his relative valuation is so much lower than the valuation of the attacker. This discouragement effect also causes the attacker's aggression to decrease in the number of battles won by the defender.[22] Although our results are clearly inconsistent with these predictions, we do find some support for a discouragement effect. In particular, consistent with the theoretical predictions, we find the probability of the attacker winning each consecutive battle decreases, while the probability of the defender wining increases (p-values < 0.05).[23]

**Result 6:** Consistent with theoretical predictions, with each successful defense, the probability of the defender winning the next battle increases, while for the attacker it decreases.

## 4.3. Guerillas In Our Midst

While it is clear that subjects are not behaving in strict accordance with the theoretical predictions, is there some consistency to how they behave? For attackers, there is anecdotal evidence to suggest that many people are behaving like guerillas, focusing their investments on one intense attack. Figure 2 plots the largest and second largest attacks for every contest lasting at least three battles for each treatment.[24] Nearly half of these contests are such that the largest attack is at least 10 times greater than the next largest attack.[25] For comparison, the ratio of the largest defense to the second largest defense is less than 2 for more than 90% of these same contests. Kovenock et al. (2010) also report behavior consistent with guerilla attacks in the simultaneous weakest-link contest. Chowdhury et al. (2011) report similar behavior in constant-sum Colonel Blotto games between asymmetric players. Together these results suggest such behavior is a robust strategy when attacking weakest-link systems.

# 5. Conclusions

Numerous systems in society can be described as weakest-link networks, where a single breach can destroy the entire system. For example, in preventing airplane hijackings, passenger screening inside the terminal at Los Angeles International Airport (LAX) is only valuable if a terrorist cannot freely walk up to planes on the tarmac at Northwest Arkansas Regional Airport

(XNA). Recently, attention has been given to modeling the optimal strategies of those who wish to protect weakest-link systems and those who wish to destroy them. However, that work has been focused on the case where the attacker is deciding among targets and the defender has to protect all potential targets concurrently. In this paper we consider the case where battles occur sequentially, a game of siege. For example, an employer has to retain its skilled employees every period and it is not enough for the army to prevent the overthrow of the government once.

In our model, a battle is won by the party investing more, but the defender has to win the entire series of battles to win the contest while the attacker needs to win only once. Within this structure, the continuation value of the defender is increasing within each battle won as the number of future battles that must be won is decreasing. If the horizon is too long, the defender should optimally choose to concede in the first battle. If the horizon is sufficiently short, then the defender will put up a fight in every battle, but the intensity of the defense should not change as the end approaches. Thus, the decision of defenders when they first come under assault is one of "fight or flight." Somewhat counter intuitively, when facing a fight the intensity of the assault should decrease over time. These predictions are dramatically different from the existing literature on simultaneous battle contest where attackers concentrate on a single target and defenders are forced to randomize their protection of each target.

This study also reports the results of a series of laboratory experiments designed to test the theoretical predictions of our model. In our baseline treatment, defenders should fight. Our two alternative treatments have either more battles or a lower payoff to the defender for winning, both of which should cause defenders to prefer flight. We find that contest outcomes are largely consistent with the theory. Results 1 and 2, respectively, show that defenders are more likely to be successful when the defender's value is larger and as the number of battles in the contest is smaller. Result 6 shows that defenders are more likely to win battles as the contest progresses. However, Results 3, 4, and 5, which deal with individual behavior, are not consistent with the theoretical predictions. What we observe is that subjects in both roles tend to over invest, driving profits down (Result 3). Further, defenders are reluctant to flight when they should (Result 4) and tend to actually increase their effort as the contest progresses (Result 5). Attackers also increase their investments as the contest progresses (Result 5), contrary to the theoretical predictions. It also appears that attackers engage in concentrated assaults suggesting that the guerilla behavior reported by Kovenock et al. (2010) and Chowdhury et al. (2011) is a robust phenomenon when people are attacking weakest-link systems.

We believe that the connection between behavior and theory is an important area for future research. A partial explanation of observed behavior for which there is some support from previous laboratory experiments (Sheremeta, 2010a, 2010b; Price and Sheremeta, 2011) is that subjects place a positive value on "winning" battles distinct from the prize money even when no one receives public recognition for victories. Such motivation could explain why defenders are reluctant to flight, but cannot explain guerilla attacks since a utility for winning simply shifts payoffs. Another way in which subjects may differ from the bidders in the model is in their risk attitudes. While the theory assumes risk neutral agents, many laboratory studies have found that people behave as if they are risk averse (Holt and Laury, 2002; Sheremeta and Zhang, 2010; Sheremeta, 2011). However, risk aversion should encourage defenders to flight and not to fight. Similarly, previous experiments have found that people have difficulty backwards inducting in new situations, but typically learn to do so with modest experience (Camerer, 2003). However, we do not observe improved performance with experience. Observed behavior may be due to some form of a gambler's fallacy or spitefulness, both of which are commonly observed in the field and in the laboratory experiments (Croson and Sundali, 2005; Sheremeta and Zhang, 2010). Another potential explanation may be that bidders act as if they or their opponent have a resource constraint across all battles in a contest.[26] For example, due to psychological reasons bidders may be unwilling to spend more in total than the value of the contest. These conjectures are intriguing and they suggest a number of avenues for future theoretical work.

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# Appendix: Tables and Figures

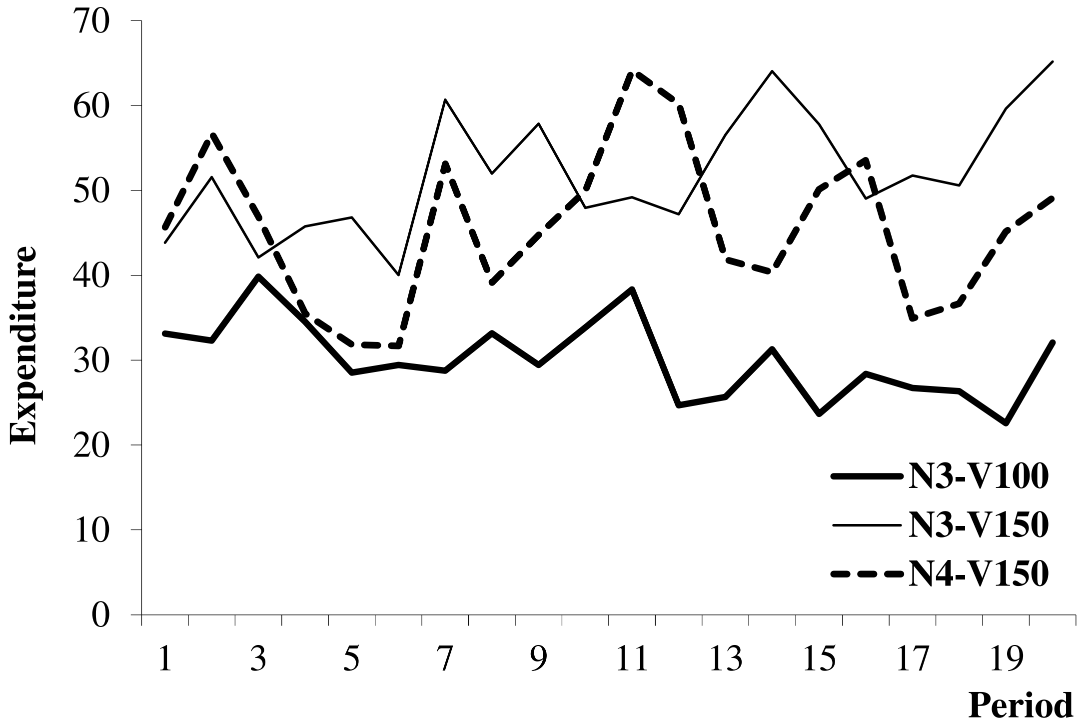
**Table 1: Equilibrium Payoffs and Expenditures**

| Battle | Expected Payoff | | Expected Expenditure | | Probability of Winning |
|---|---|---|---|---|---|
| | Player $D$ | Player $A$ | Player $D$ | Player $A$ | Player $D$ |
| Panel A: Battle $n-k$ for $v_D \geq nv_A$ | | | | | |
| $n-k$ | $v_D-(k+1)v_A$ | 0 | $\frac{v_A}{2}$ | $\frac{(v_A)^2}{2(v_D-kv_A)}$ | $1-\frac{v_A}{2(v_D-kv_A)}$ |
| Panel B: Battle 1 for $nv_A > v_D > (n-1)v_A$ | | | | | |
| 1 | 0 | $nv_A-v_D$ | $\frac{(v_D-(n-1)v_A)^2}{2v_A}$ | $\frac{v_D-(n-1)v_A}{2}$ | $\frac{v_A}{2(v_D-(n-1)v_A)}$ |
| Panel C: Battle 1 for $(n-1)v_A \geq v_D$ | | | | | |
| 1 | 0 | $v_A-\varepsilon$ | 0 | $\varepsilon$ | 0 |

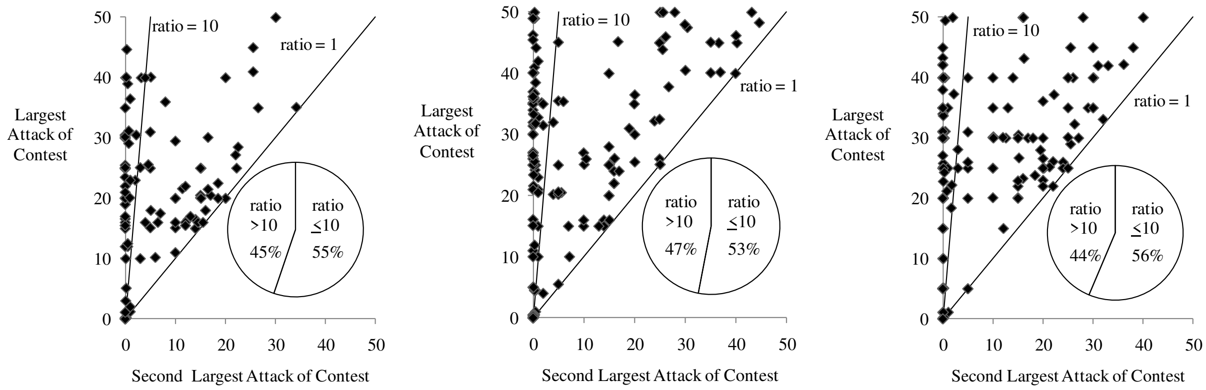
**Table 2: Equilibrium Predictions and Aggregate Statistics**

| Treatment $(n, v_D, v_A)$ | Battle Number | Average Allocation | | | | Payoff | | | | Probability of Winning a Battle | | Probability of Winning the Game | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Equilibrium | | Actual | | Equilibrium | | Actual | | Equilibrium | Actual | Equilibrium | Actual |
| | | *D* | *A* | *D* | *A* | *D* | *A* | *D* | *A* | *D* | *D* | *D* | *D* |
| N3-V100 (3, 100, 50) | 1 | 0.0 | 0.1 | 15.0 | 12.4 | 0 | 50 | -7.5 | 11.6 | 0.00 | 0.55 | 0.00 | 0.29 |
| | 2 | 25.0 | 25.0 | 20.1 | 11.1 | | | | | 0.50 | 0.70 | | |
| | 3 | 25.0 | 12.5 | 26.7 | 14.6 | | | | | 0.75 | 0.75 | | |
| N3-V150 (3, 150, 50) | 1 | 25.0 | 25.0 | 26.8 | 18.0 | 0 | 0 | -6.7 | -6.0 | 0.50 | 0.65 | 0.31 | 0.41 |
| | 2 | 25.0 | 12.5 | 32.6 | 13.0 | | | | | 0.75 | 0.79 | | |
| | 3 | 25.0 | 8.3 | 39.2 | 17.0 | | | | | 0.83 | 0.80 | | |
| N4-V150 (4, 150, 50) | 1 | 0.0 | 0.1 | 18.4 | 13.1 | 0 | 50 | -17.4 | 3.2 | 0.00 | 0.63 | 0.00 | 0.27 |
| | 2 | 25.0 | 25.0 | 22.5 | 12.0 | | | | | 0.50 | 0.74 | | |
| | 3 | 25.0 | 12.5 | 28.6 | 15.2 | | | | | 0.75 | 0.73 | | |
| | 4 | 25.0 | 8.3 | 34.8 | 16.8 | | | | | 0.83 | 0.79 | | |

**Figure 1: Total Expenditure across All Periods (All Treatments)**



## Figure 2: Attacks Lasting for at least Two Battles by Treatment

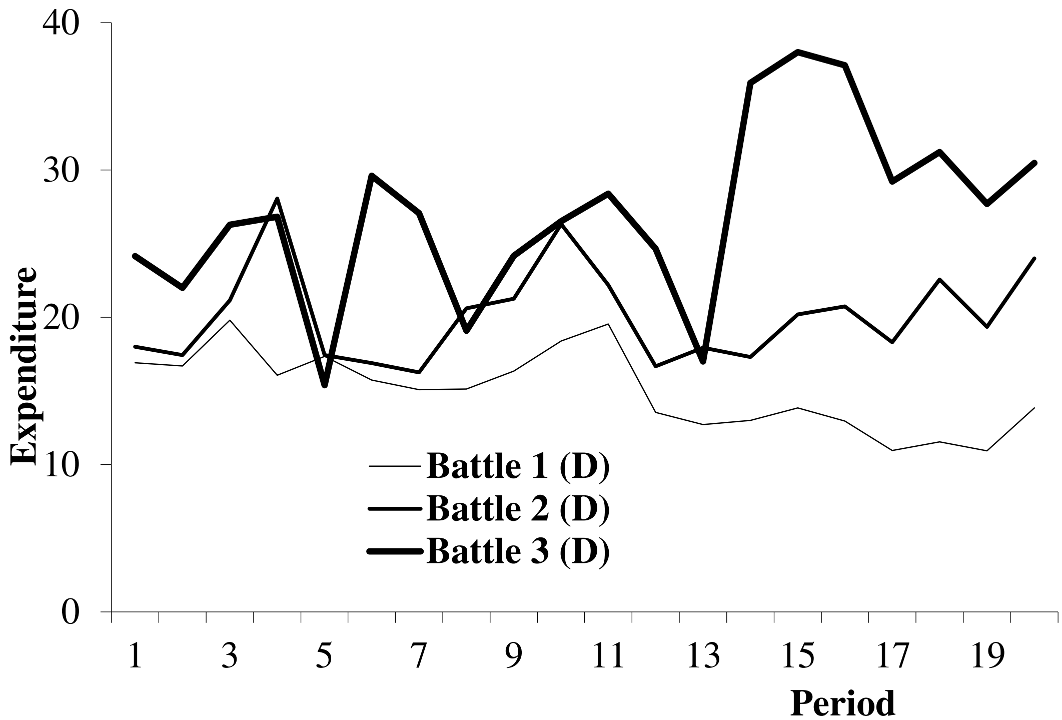



**N3-V100** **N3-V150** **N4 -V100**

# Online Appendix A: Additional Figures

**Figure A1: Expenditures in Each Battle across All Periods (N3-V100 Treatment)**

Defender Expenditures




Attacker Expenditures

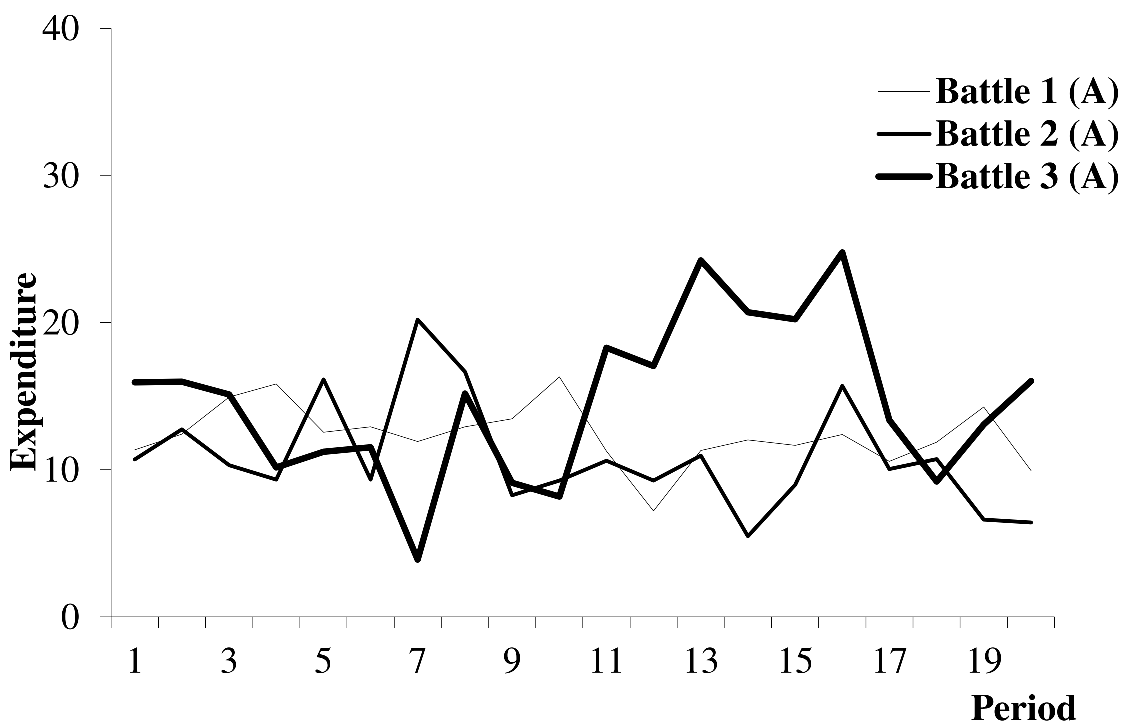
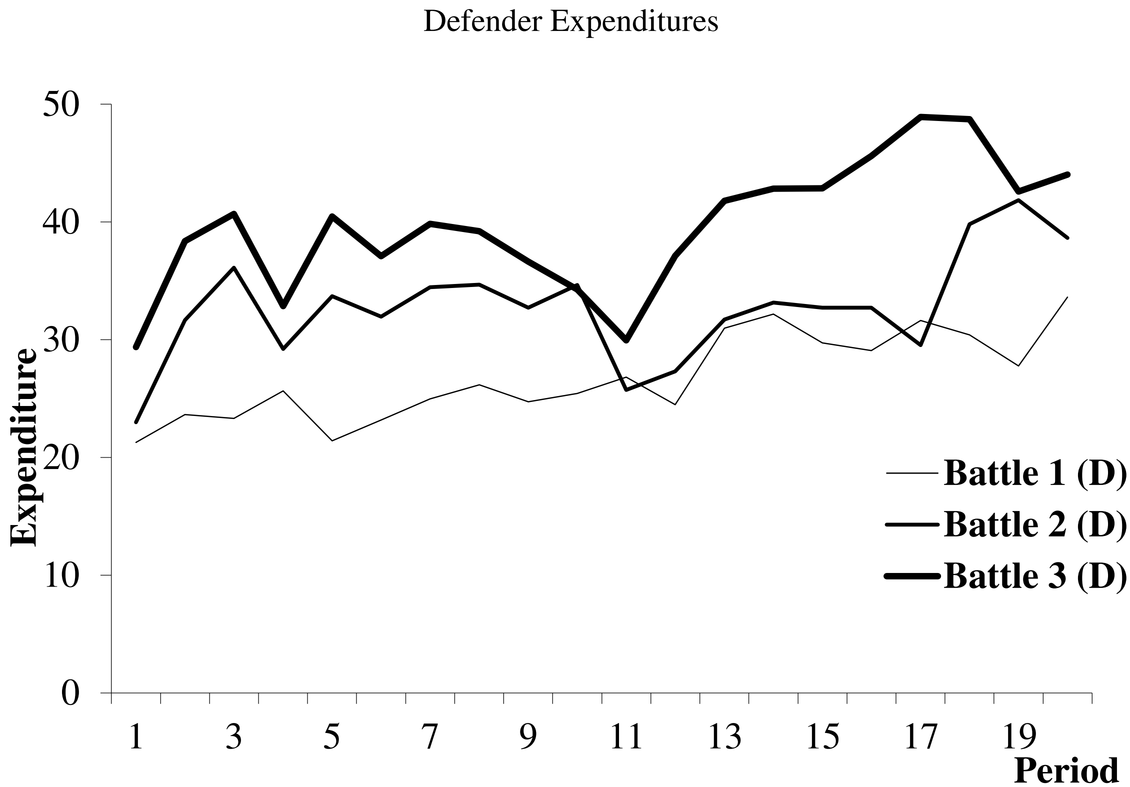


**Figure A2: Expenditures in Each Battle across All Periods (N3-V150 Treatment)**



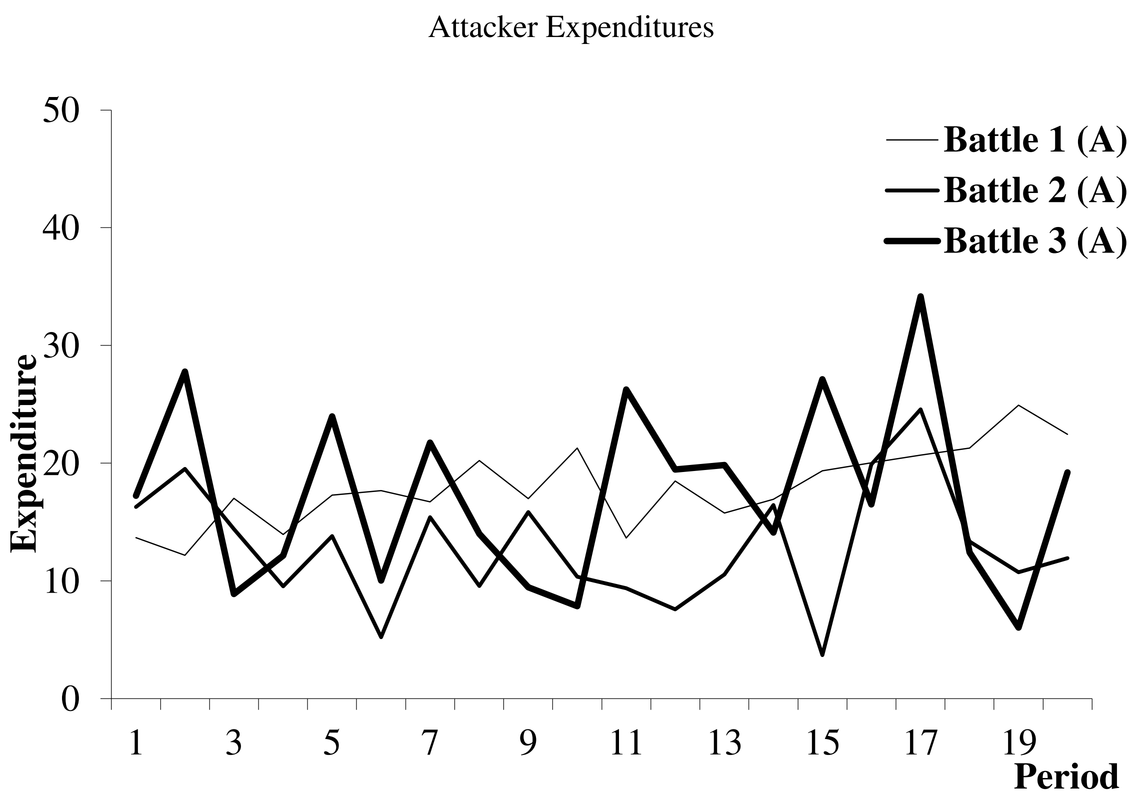
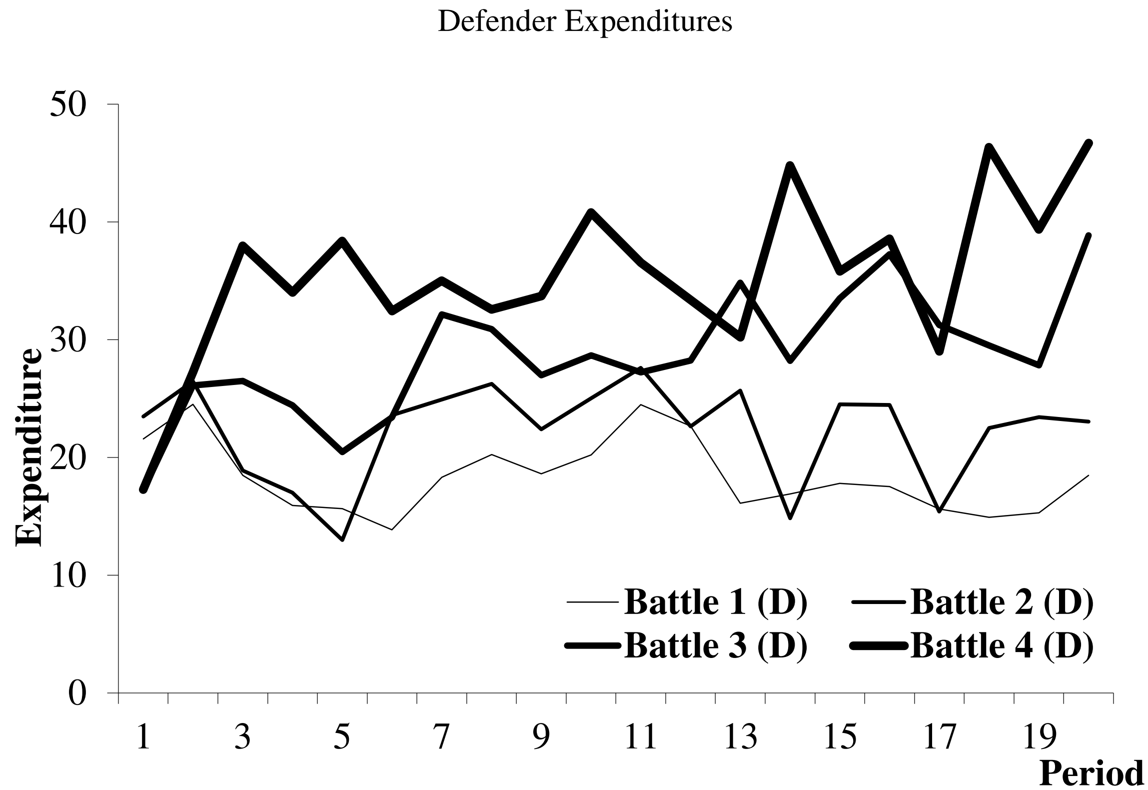
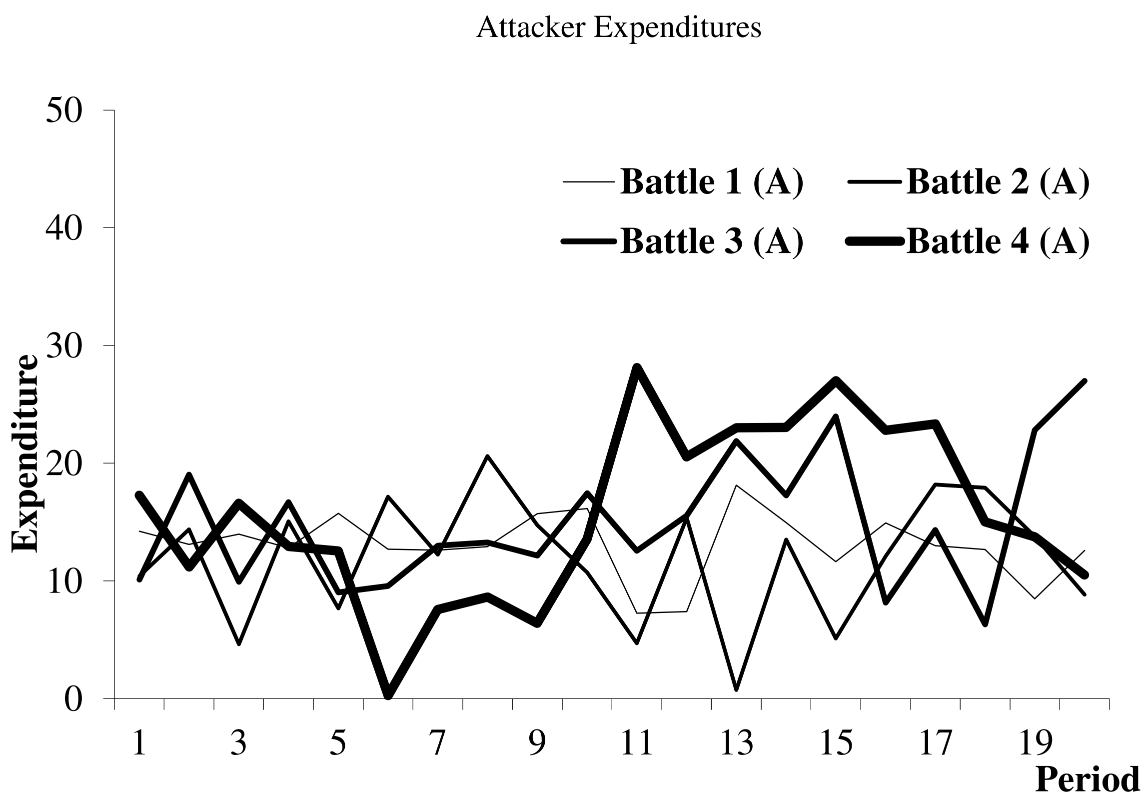



**Figure A3: Expenditures in Each Battle across All Periods (N4-V150 Treatment)**



# Online Appendix B: Instructions for N3-V100

**General Instructions**

This is an experiment in the economics of decision making. Various research agencies have provided the funds for this research. The instructions are simple and if you follow them closely and make careful decisions, you can make an appreciable amount of money.

The currency used in the experiments is called Francs. At the end of the experiment your Francs will be converted to US Dollars at the rate **25 Franks = US $1**. You are being given a **$20** participation payment. Any gains you make will be added to this amount, while any losses will be deducted from it. You will be paid privately in cash at the end of the experiment.

It is very important that you do not communicate with others or look at their computer screens. If you have questions, or need assistance of any kind, please raise your hand and an experimenter will approach you. If you talk or make other noises during the experiment you will be asked to leave and you will not be paid.

**Instructions for the Experiment**

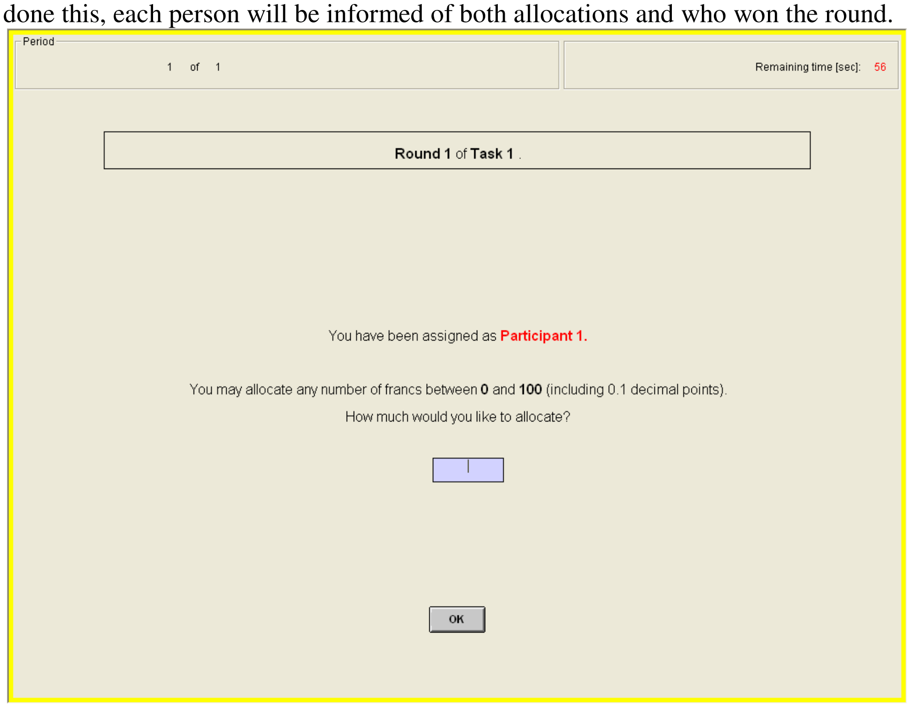
The experiment consists of 20 decision tasks. At the beginning of the first task you will be randomly assigned the role of **participant 1** or **participant 2**. You will remain in this role for the first 10 decision tasks and then change your role assignment for the last 10 tasks of the experiment. For each task you will be randomly paired with another participant in the experiment who is in the opposite role. There are **16 participants** in the experiment so 8 are in each role. For each task you are equally likely to be paired with any of the 8 participants in the other role, but no participant will be able to identify if or when he or she has been paired with a specific person.

The Decision Task

For each decision task there is a reward in Francs for participant 1 and a reward in Francs for participant 2. These rewards are not the same for the two participants. Only one of the participants will receive the reward for a given task. The reward to **participant 1 is 100 Francs** and the reward to **participant 2 is 50 Francs**.

Each task involves up to **3 rounds**. In each round, both participants allocate Francs, and whoever allocates more Francs wins that round with ties being broken randomly. A participant's allocation cannot exceed his or her reward so allocations can be anything from [0, 0.1, 0.2, …, the reward]. So for example, if participant 1 allocates 11.4 Francs and participant 2 allocates 11.3 francs, then participant 1 will win the round.

To enter your allocation, you simply type it in the box on your screen and press OK. After both participants have done this, each person will be informed of both allocations and who won the round.

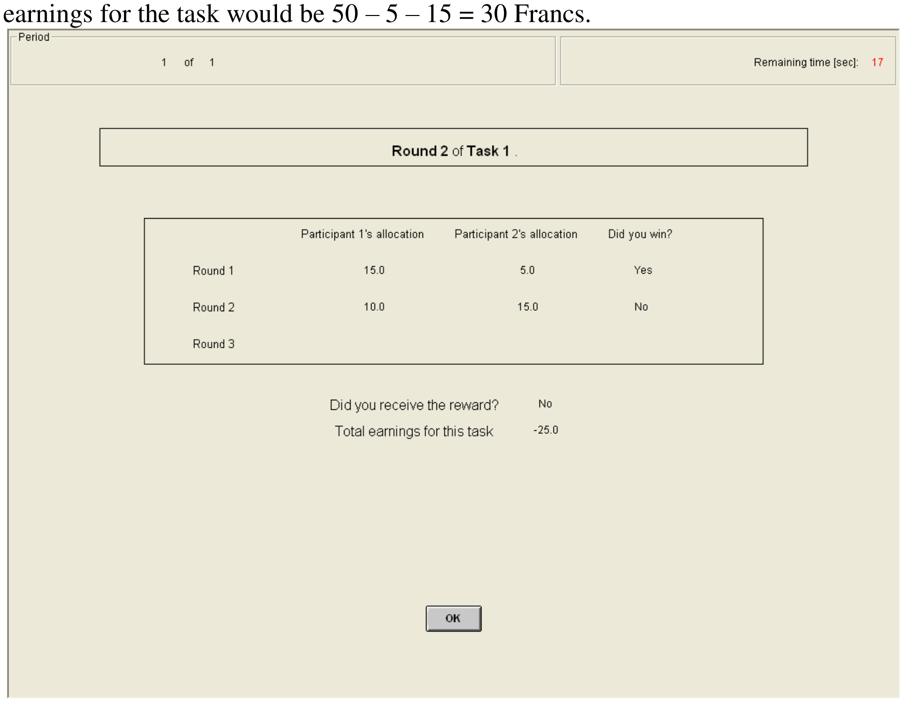


Your Earnings

If participant 1 wins **all 3 rounds** then he or she receives the reward. However, if participant 2 wins **any** round, then participant 2 receives the reward. Since participant 2 only needs to win a single round, the task will end if this occurs. Notice that a single reward is received for the whole task; there is not a reward for each round.

Any Francs allocated in a round are deducted from your payment regardless of whether or not you won the round or the reward. This means that if both participants allocate Francs in a task, then one will lose Francs. This is why each participant is being given a participation payment of **$20**, which corresponds to 500 Francs.

Consider the following example where the reward to participant 1 is 100 and the reward to participant 2 is 50 and the task involves 3 potential rounds. If participant 1 allocates 15 Francs in round 1 and participant 2 allocates 5 Francs in round 1 then participant 1 wins the round and both participants lose their allocations. If in the second round participant 1 allocates 10 Francs and participant 2 allocates 15 Francs then participant 2 wins the round and hence receives the reward. Participant 1's earnings for the task would be –15 – 10 = –25 Francs and Participant 2's earnings for the task would be 50 – 5 – 15 = 30 Francs.




After each task, you will be shown your payoff (positive or negative) in francs for that task. You should record this information on your Personal Record Sheet. At the conclusion of the experiment, 2 out of the first 10 tasks and 2 out of the last 10 tasks will be randomly selected. Your experimental earnings will be the sum of your earnings on those four tasks. This amount will be added to your participation payment.

# Notes

[1] For a review of the literature see Roberson (2006). Some examples of simultaneous games of attack and defense are Gross (1950), Cooper and Restrepo (1967), Shubik and Weber (1981), Snyder (1989), Coughlin (1992), Szentes and Rosenthal (2003), Bier et al. (2007), Kvasov (2007), and Hart (2008).

[2] The main distinction between the two papers is that Clark and Konrad (2007) assume the probability of winning a given battle is proportional to investment, while Kovenock and Roberson (2010) assume that victory is deterministic.

[3] The subsequent papers of Harris and Vickers (1985, 1987), Leininger (1991), Budd et al. (1993), and Konrad and Kovenock (2009) investigate different factors that affect behavior in the sequential multi-battle contests.

[4] Similar problems have been studied in the "shoot-look-shoot problem" literature (for a review see Glazebrook and Washburn, 2004). However, those models assume that the probability of winning a battle does not depend on the defender's and the attacker's efforts.

[5] This is mainly because the defender's valuation of the overall contest in early battles is relatively low, since the defender has to be successful in each battle and there are still many battles to go. However, as the defender wins early battles, his valuation for continuing the contest increases and thus the attacker becomes discouraged. As a result, the probability of winning future battles by the attacker decreases, while the probability of winning future battles by the defender increases.

[6] It is interesting to compare our results to the simultaneous battle game of attack and defense by Kovenock and Roberson (2010). In particular, when $v_D \geq nv_A$, the expected payoffs of the attacker and the defender are exactly the same under sequential and simultaneous structures.

Nevertheless, the strategic behavior in two games is quite different. In particular, Kovenock and Roberson (2010) find that the attacker utilizes a stochastic guerilla warfare strategy in which, with probability one, the attacker engages in only one single battle. On the contrary, in our model, the attacker always has an incentive to fight in each battle.

[7] Konrad and Kovenock (2009) call such a break point a 'separating state.' Their theoretical model of a contest with intermediate prizes is more general than the model studied in the current paper. In fact, some of the results provided in our paper can be derived from Konrad and Kovenock (2009).

[8] Note that this is never the case in the simultaneous game of attack and defense by Kovenock and Roberson (2010). In particular, under all parameters, the optimal strategy for the defender is to stochastically fight with positive probability in all battles, allocating random, but positive, resource levels in each battle. On the contrary, in our model, when $(n-1)v_A \geq v_D$, the defender gives up with probability one, by allocating zero resources in the first battle.

[9] To demonstrate that $(0, \varepsilon)$ is a Nash equilibrium strategy profile for players $D$ and $A$, see that the expected payoff in battle 2 for player $D$ is 0 and the expected payoff for player $A$ is positive, i.e. $v = (n-1)v_A - v_D > 0$. Therefore, in battle 1, players $D$ and $A$ compete in a simultaneous move all-pay auction, where player $D$'s value is 0 and player $A$'s value is $v$. It is a strictly dominant strategy for player $D$ to make an expenditure of 0, which assures that played $D$ receives a payoff of 0. Making any expenditure $\varepsilon > 0$ guarantees a sure loss of $\varepsilon$, because losing the battle yields no benefit and costs $\varepsilon$ and winning the battle has an expected value of 0 and the cost is still $\varepsilon$. On the other hand, player $A$'s dominant strategy is to assure the victory of the prize value $v$ at the lowest possible cost $\varepsilon$. The expenditure of 0 (instead of $\varepsilon$) would be a strictly

dominant strategy for player $A$ if the rule was to allocate a victory to player $A$ with certainty in case of a tie (Roberson, 2006; Konrad and Kovenock, 2009). However, given that ties are broken randomly, player $A$'s dominant strategy is to make the lowest possible expenditure of $\varepsilon > 0$ (0.1 francs in our experiment) to guarantee the payoff of $v$ at the minimal cost of $\varepsilon$. This choice is better than making an expenditure of 0 and receiving an expected payoff of $v/2$, i.e. $v - \varepsilon > v/2$. Therefore, neither of the players has an incentive to deviate from the equilibrium profile $(0, \varepsilon)$.

[10] This suggests that attackers want to stretch out the duration of the contest. Indeed, this tactic has often been applied in historic games of siege. However, various constraints such as a food supply or weather often prevent he attacker from forcing the contest to continue indefinitely.

[11] For the defender, the value of being at period $t$ is $v_D^t = max\{0, pv_D^{t+1} + (1-p)0 - d\}$, where $p$ denotes the chance of the defender winning the battle given both players' investments and $d$ denotes the expected investment by the defender. Since $v_D^t = v_D^{t+1}$ this equation implies that $v_D^t = v_D^{t+1} = 0$ and the defender should surrender so that $p = 0$. For the attacker, the value of being at period $t$ is $v_A^t = max\{0, pv_A^{t+1} + (1-p)v_A - a\}$, where $a$ is the attacker's expected investment. The attacker will choose to attack if $v_A^t$ is positive, which is the same condition as $v_A$ being positive since $p = 0$ and $a \geq 0$ can be arbitrarily small.

[12] The experimental instructions used context neutral language. The instructions are available in an online appendix.

[13] Placing a theoretically nonbinding upper limit on bids may have some psychological impact on behavior (Sheremeta, 2011; Price and Sheremeta, 2011); however concerns regarding the

potential loss of control due to bankruptcy (as described at the end of this section) were considered to be more important.

[14] By randomly selecting periods for payment, the size of the endowment is smaller than it would be if subjects were paid for each contest. Given the restriction that bids could not exceed value in any round and the other features of the experimental design it was not possible for subjects to go bankrupt.

[15] We used two different variables for a period trend, one for the first 10 periods and one for the last 10 periods of the experiment. The two variables were used since subjects changed their role assignments after the first 10 periods of the experiment.

[16] A standard Wald test, conducted on estimates of panel regression models, rejects the hypothesis that the average payoffs in N3-V100, N3-V150, and N4-V150 treatments are equal to the predicted theoretical values in Table 4 (p-values < 0.05). The panel regression models included a random effects error structure, with a random effect for each individual subject, to account for the repeated measures nature of the data. The standard errors were clustered at the session level to account for session effects. The two separate period trends were used to control for learning for the first 10 periods and the last 10 periods of the experiment.

[17] In our experiment, subjects who get to the last battle have already made some expenditures in the previous battles. If the sunk cost hypothesis is true, it will entail that subjects who expend more in previous battles are also more likely to expend more in the last battle – to recoup some of their expenditure. A simple random effect model finds that for the defender there is a positive relationship between expenditure in battle 3 and total expenditure in the previous battles 1 and 2 (p-value < 0.05). However, for the attacker such correlation is negative (p-value < 0.05).

Therefore, we conclude that the sunk cost fallacy is not likely to be the main consistent force driving the over-expenditures in our experiment.

[18] Of course, it may be that changes to behavior due to learning require more experience than provided in our experiment.

[19] A reluctance to bid 0 could be due to the active participation hypothesis (see Lei et al., 2001), which argues that subjects who come to laboratory experiments want to do something.

[20] We estimate a panel regression model separately for each treatment and player type. Each model included random effects for each individual subject and standard errors were clustered at the session level. The two separate period trends were used to control for learning for the first 10 periods and the last 10 periods of the experiment. The independent variable is bid and the main dependent variable is the battle number. For the defender, the battle number variable is positive and significant in all treatments (p-values $< 0.05$). For the attacker, the battle number variable is positive and significant in treatments N3-V100 and N4-V150 (p-values $< 0.05$), but not in treatment N3-V150.

[21] Theoretically, this discouragement effect is the driving force behind the predictions of our model (Baye et al., 1996). The idea behind the discouragement effect is straightforward: the player with the higher valuation imposes a strong discouragement effect on the player with the lower valuation. As the result, the player with the lower valuation reduces his expenditures.

[22] The defender's valuation for continuing the contest increases in the number of battles won and thus the attacker becomes discouraged.

[23] We estimate a probit panel regression model separately for each treatment, using subject random effects and two period trends. The independent variable is an indicator whether the

defender won the battle and the main dependent variable is the battle number. The battle number variable is positive and significant in all treatments (p-values < 0.05). Obviously, the probability of the attacker winning each battle is simply one minus the probability of winning that battle by the defender. So, the same statistical conclusions carry on to the attackers.

[24] Any contest that ends after the first battle is trivially consistent with a guerilla attack. Also, any attacker following the equilibrium strategy in N3-V100 or N4-V150 and winning the second battle would appear to be a guerrilla based upon the metric used in Figure 2. Therefore the three panels in Figure 2 only include contests that lasted at least 3 battles, although they are qualitatively unchanged if contests lasting for only two battles are included.

[25] For defenders, individual behavior is similar to the aggregate pattern discussed above where defense tends to increase with each successive battle.

[26] We thank a reviewer for suggesting this explanation.