Limited Information Aggregation and Externalities - A Simple Model of Metastable Market

Gong, Zheng and Tian, Feng and Xu, Boyan

Peking University, Nankai University, Peking University

4 December 2013

Limited Information Aggregation and Externalities - A Simple Model of Metastable Market*

Zheng Gong†
Feng Tian‡
Boyan Xu§

Abstract

We analyze a model in which agents' decisions to enter or exit investments are influenced from their individual and external parties' transaction histories. Actual investment outcomes are unknown to all participants until the end of decision periods, but outcomes do change depending on the number of participating players in the market and the market's current state of condition. In this particular model, agents have access to external parties' information from those who are within their specific social network. Our study of limited information aggregation mainly focuses on market responses to investors' decisions of exiting the investment. With social structures complicating investment outcomes, we present a model that describes how markets can enter relatively stable statuses long enough for exiting participants to return, which brings the investment back to normal conditions. Our model also supports previous studies that limited information aggregation can cause the exogenous shock effect of global collapse.

Keywords: Information aggregation, Social structure, Internet Externality, Simulation

* **Acknowledgement:** We would like to thank Yifan Zhou, Rixin Li, Fan Zhang and Scott Pogoda for their helpful advices and comments.

† China Center for Economics Research and School of Physics, Peking University. *E-mail address:* gzidiablo@163.com

‡ Department of Economics, Nankai University. *E-mail address:* 813969011@qq.com

§ School of Physics, Peking University. *E-mail address:* xby@pku.edu.cn

1 Introduction

Network externalities affect the outcomes in markets by influencing participants' decisions. Take investments in solar battery technology for example. As more people invest, the stronger the scale effect becomes which reduces the unit cost and increases the profit of every investor.

Now consider a risky situation that has a network externality and agents repeatedly investing into the market upon different entry periods. Jeitschko and Taylor (2001) analyzed such a situation, but agents could only base their decisions to invest or exit around individual investment outcomes. What was proven in their AER paper was that a global economic collapse could occur even if the investment was in a state of Pareto-efficient equilibrium and could bring positive returns. Yet, in the situation where an agent received a negative outcome, they will assume other investors also experienced similar results and will most likely exit the market without the intention of returning. In this situation, if multiple investors exited, this would cause a domino effect of a simultaneous pullout because the lesser the number of agents in the market, the more negative the network externality becomes, causing more agents to exit as a result. Hence, even an investor experiencing positive outcomes will be discouraged after some time, and the simultaneous market collapse - in other words, avalanche - is inevitable in this kind of model.

Though the conclusion of Jeitschko and Taylor's paper accurately display how an investment that does not allow information sharing between participants can lead to investors simultaneously pulling out from a positive valued asset due to the network externality effect, their model doesn't acknowledge how investments can also rebound from drops in economic activity by remaining in a relatively weak status long enough for exiting investors to return. By examining an investment situation that has agents exiting and reentering the market at different entry periods, we show a market that has multiple agents simultaneously exit the investment enter into a vulnerable status for a period of time and then re-stabilize to a prosperous condition due to the exiting investors' return. Furthermore, we support the view of Jeitschko and Taylor (2001) by showing investments that cause agents' decisions to exit triggering a global collapse. With that being said, there are countless of possibilities an investment can result in once network externalities affect investors' decisions. We specifically observe situations that allow the transaction outcomes between specific parties to be shared - this could represent investments involving friends and family members or business groups. In this situation, investors that exit markets based on their initial negative outcome could then draw two conclusions which would cause different effects to the market activity: one, they are reconfirmed of their decision to exit from other players' outcomes and will not reenter, leaving the investment in a vulnerable condition that could cause future investors to exit and trigger a possible global collapse. Two, they return to the market after observing their relatives' gains, which could restore the investment back to normal operating conditions. In this case of aggregate information, the network externality is proven to cause either a full out market collapse or the re-stabilization of the investment.

As reference, Gunay (2008) has previously linked information aggregation into the coordination avalanche model of Jeitschko and Taylor (2001) and presented how agents could update their information according to the shock and every other player's investment behavior of the previous period. Gunay's model focused on complete information aggregation which provides interesting results to agents' decisions being made on the investment outcomes of the entire investment group. However, our goal is to focus on a limited information aggregation model based off of Jeitschko and Taylor's (2001) avalanche model.

In this paper, we extend the model of Jeitschko and Taylor (2001) into a limited information aggregation model to explain two basic features of a risky market with repeated investment. First, investments can stay in a relatively vulnerable, yet stable prosperous status for a lengthy period of time. Second, to analyze that what caused market instability within a limited information aggregation investment model, can also be leading factor of a full market collapse under the presence of a small exogenous shock.

This is a Stag-Hunt game of multiple players: at the beginning of each decision period τ, each player must decide whether to take a risky action of (investing) or not (exiting), and these decisions will remain unchanged throughout the transaction periods until the following decision period. An investor who chooses to exit the market will receive a payoff of zero. If the investor selects to invest in this period, he will experience a probability p to win and a probability $1-p$ to lose in every transaction periods until the following decision period. The outcome is independent across players and over time.

There are three key characteristics in our model. Firstly, there exists limited information aggregation. In particular, every investor can observe their own investment outcome and transaction history as well as several other investors' within a social network. Investors can take use of the available information of the last decision period to strategize at the beginning of decision periods.

Secondly, players are uncertain about the actual value of p. Investors can only predict what the value of p will be based on their outcome and transaction histories. Specifically, in our paper, we assume that people only consider the outcome history of the last decision period and ignore the outcome history *ex ante*, since they believe that the probability of p changes with time, and previous outcome records cannot serve as an accurate indicator of the next winning probability.

Lastly, there exists a network externality or complementarity of participants' actions. In particular, as the number of active investors decreases, the profit of successful investors drops as well. The expected return of investment depends on both the number of active investors and the state of economic nature.

There are three fields of research that are mainly related to our work, namely network externality, herding behavior and social network. The main difference between our paper and the paper of Gunay (2008) is that we do not consider the condition of complete information aggregation, but instead consider information aggregation is shared through three different scales of social structure. This enables our model to be applied into a variety of situations, adding to its complexity. The second main difference is that we assume people will not rapidly change their decisions in short periods of time. The third difference is that we recognize how a small exogenous shock affects the market.

Jeitschko and Taylor (2001) conclude that market collapses can happen simply out of fear. According to our case study on different social structures, markets can also remain stable for relatively long periods of time due to information aggregation. The degree of stability is largely determined on the structure of the social network.

As for herding theory, for example, various papers focus on the information cascade theory, one being Bikhchandani, Hirshleifer and Welch (1992). This paper in particular shows that information cascades occur when individuals have made decisions following the behavior of their predecessors. The main difference between their paper and ours is that they suggest individuals have a sequence in making investment decisions. In many ways, our paper is similar to Bikhchandani, Hirshleifer and Welch (1998) since we also agree that information cascades are based on social learning experiences. However, the individual players in our model not only focus on the entry and exiting behaviors of others, but also make decisions based on information from their neighbors' transaction outcomes.

Lastly, in our model, the information exchange between different investors depends on social structures in the market, which is related to models in literature on the social network and social structure field. The work of Zollman (2008) supports our assumption that limited information aggregation is a common reality in investments and he used a similar approach of social structure to study the effects of conformity.

A paper that highly complements ours is Acemoglu *et al.* (2010). They develop a model that presents how information is exchanged in social networks. They test when asymptotic learning comes into effect and how it influences the aggregate welfare. The main difference between our paper and theirs is that they focus on the existence of asymptotic learning and its effects on welfare whereas we focus on the effect of network externalities on market coniditions.

Section 2 will be a brief introduction to the model we use in this paper. In Section 3, we will analytical propose a mechanism to explain how the three characteristics of our model can lead to a relatively stable status of an investment that loses a number of participating players, but not the rapid collapse as predicted by Jeitschko and Taylor (2001). In Section 4, we will propose a mechanism to explain how a small exogenous shock that influences a limited number of investors in the market can create a severe market collapse. In Section 5, we will turn to examine the effect of different social topology structures amongst investors. Programming methods are applied to simulate the mechanism of Section 3 and Section 4, and we will discuss under which topology structure the market will reach its most robust status. Brief concluding remarks will appear in Section 6.

2 Model

We assume that there exist M individual investors within a specific market named $\{P_1, P_2, ..., P_M\}$. Individual investors are risk neutral and aim to maximize their expectation return, which is on an infinite timespan and the discount factor δ is zero.

There are two types of periods in our model: a transaction period, t, and a decision period, τ, which satisfies $\tau = kt, k = 1, 2, ..., n$. At the beginning of every decision period, each investor decides from an Action Set $\{IN, OUT\}$, which represents whether the investor enters the market or not. If OUT is chosen, the agent receives a payoff of 0 in each transaction period, $t = 1, 2, ..., k$ within the decision period τ. If an agent selects IN, he will face a WIN probability of p

and *LOSE* probability of $1-p$. The return $U(m)$ is a function of m, which is the number of investors that choose IN in the period τ, as shown below:

$$U(m_\tau) = \begin{cases} f(\frac{m_\tau}{M}) - c & \text{if WIN} \\ -c & \text{if LOSE} \end{cases}$$

$f(\cdot)$ is a strictly increasing function that is defined on $[0,1]$, where $f(0)=0$, $f(1)=1$. c remains a constant variable that represents the fixed cost of transaction, and $c \in (0,1)$

Another important element in our model is that investors cannot observe the true value of the winning probability p. Instead, players' decisions are based on information from transaction histories that reveal previous win and lose rates to estimate the winning probability p in the market. At the end of each decision period investors update their belief of p based on the information they gained from the last decision period, and all other aspects of the market condition is freely accessible knowledge in the model.

Different from what is assumed by Jeitschko et al (2001), a key element of our model is that each investor can observe the transaction history of a few selected interrelated investors. For example, A and B are two investors in our model. If B can observe the transaction history of A, we name that A is a sender of B, and B is a receiver of A, and denote such a relation group as $A \to B$. Specifically, if transaction history is shared between A and B, *i.e.*, $A \to B$ and $B \to A$, we denote it as $A \leftrightarrow B$ and we name A as a friend of B, and likewise, B is a friend of A. For an investor P_i that belongs to the set $\{P_1, P_2, ..., P_M\}$, we denote $P_i = P_i(s_i, r_i)$. s_i represents the number of investors submit their outcome to P_i, and r_i denotes the number of investors receiving information from P_i.

To ensure that there exists a Nash Equilibrium, all investors play IN during the first decision period and we should consider the initial belief p. The expected initial belief of p is denoted as $\overline{p}$ and $\overline{p} = E(p)$. As proven by Jeitschko et al (2001), there should be $\overline{p} - c > 0$ to ensure that all players enter the market in the first round.

3 Market Structure - Relatively Stable Status

If an investor selects IN to enter the market, the agent's expected rate of return to the market depends on two factors: his believed value of p and the number of investors who also play IN in the same period. As shown in Model section, every investor enters the market during the first decision period.

Let's begin with a simple outline of our model. Consider an investor, A, whose social network is shown below:

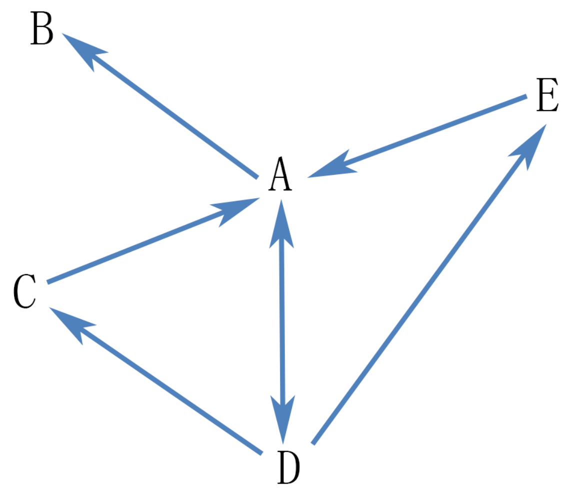

Figure3.1 An example of the social structure

After the first decision period, A will update his/her belief on probability p according to their individual experience and from the related investors' transaction information, *i.e.*, C, D and E. In a particular case, if A experiences more failure than success in the first period, it is possible that their level of pessimism on the value of p will lessen because C, D and E received positive returns from the market (*i.e.*, A is influenced to select IN). Of course, it is also possible that even after taking others' information into consideration, A will still believe that the probability of p's outcome will be too low and exits the market after the first period. To extend this example into a more concrete situation, we formalize the following stopping lemma:

Lemma 1. *NECESSARY AND SUFFICIENT CONDITION OF STOP.*

Differentiating between investors, $P_i \in \{P_1, P_2, ..., P_M\}$, who has the number of s_i senders.After the first decision period, the necessary and sufficient condition that he will select OUT is:

$$\sum_{i=0}^{s_i} \sum_{t=1}^{t=k} U_t(m_\tau) < 0$$

Then selecting OUT will be the dominate strategy to the investor P_i.

As shown in the previous section, the initial winning probability p is higher than the fixed cost of transaction, c. Let $B(n|k,p)$ be the binomial distribution with parameter k and p. $B(n|k,p)$ acts as the probability of experiencing n or less $WINs$ within k periods. Suppose that N is the largest integral that satisfies:

$$N(1-c)-[(s_i+1)k-N]c<0$$

Then, $B(N|(s_i+1)k,p)$ represents the exiting probability of P_i with the mass of s_i senders after the first decision period. The expected gross return to an investor who selects IN in next decision period will drop to:

$$pf(1-B(N|(s_i+1)k,p))$$

In particular, if s_i equals to zero, the exiting possibility of P_i is then equals to $B(n|k,p)$. We can now formalize the exiting LEMMA:

Lemma 2. *EXITING LEMMA*
Suppose $F(k)=B(N(k)|k,p)$*, and we have* $1>p>c$ *and* $N(k)=\lfloor kc \rfloor$*, then:*
(1) If $N(k+1)=N(k)$*, we have* $F(k+1)<F(k)$*.*
(2) If $N(k+1)=N(k)+1$*, we have* $F(k+1)>F(k)$
(3) $\lim_{k\to\infty} B(N(k)|k,p)\to 0$
A brief proof is shown in Appendix.

LEMMA 2-(1), (3) illustrates that to a specific investor P_i, generally speaking, the more information senders to P_i, the less possibility P_i will exit the market after the first decision period (*i.e.*, the market structure is more stable under the circumstances of a social network). And LEMMA 2-(2) implies that there will be fluctuations within this declining procedure because of the discontinuity of binominal distribution, which is also clearly shown in Section 5.

Let's trace this to the second decision period. As is shown in previous paragraphs, after the first decision period, the expected number of investors that choose OUT is:

$$O_1=\sum_{i=1}^{M} B(N|(s_i+1)k,p)$$

We are interested to examine whether the quitting number will increase or not after this stage. Consider an investor in this market, P_i, who has the number of s_i' information senders and decides to quit the market in the second decision period. Keep in mind, this investor cannot observe his own transaction outcome in this period from selecting OUT. However, unlike Jeitschko and Taylor (2001), P_i can still observe the outcome of the information senders and shape a belief of the winning probability p. Suppose that N' is the biggest integral that satisfies:

$$N'[f(1-B(N|(s_i'+1)k,p))-c]-(s_ik-N)c<0$$

Then, $B(N'|s_ik,p)$ represents the probability that the investor P_i will remain in an OUT status after the second decision period. In this way, the expectation number of investors that return to the market after the second decision period is:

$$R_2=\sum_{i=1}^{M}\{B(N|(s_i+1)k,p)\times[1-B(N'|s'k,p)]\}$$

Also presented is a mechanism that shows investors who quit the market in the decision period of τ will return to the market with a possibility of $1-B(N'|s'k,p)$

In respect, there also may be investors who selected IN on the second decision period that then decide to exit the market after the second stage. The expectation number of new investors that exit the market after the second decision period is:

$$O_2=\sum_{i=1}^{M}\{[1-B(N|(s_i+1)k,p)]\times B(N'|(s'+1)k,p)\}$$

Denote ΔO_2 as the expected difference between the number of investors that select OUT after the second decision period and the number of investors that select OUT after the first decision period. We now have:

$$\Delta O_2=\sum_{i=1}^{M}\{[1-B(N|(s_i+1)k,p)]\times B(N'|(s'+1)k,p)-B(N|(s_i+1)k,p)\times[1-B(N'|s'k,p)]\}$$

$$\Delta O_2>0$$

We can also define $\Delta O_i, i=1,2,...,n$ in this way. It appears difficult to analytically solve the above function of ΔO_i. However, the following LEMMA shows essential features of ΔO_i.

Lemma 3. *CHARACTERS OF $DeltaO_i$*

(1) To any given integral s_i, if we have $p<c$, $\Delta O_i<0$

(2) $\lim_{s_i\to\infty}\Delta O_i=0$

Though LEMMA 3 shows that the predicted number of investors who select OUT will increase as time goes on, we still say that the market can reach a relative stable status under the assumption of limited information aggregation. Compare with O_1, ΔO_2 is a high order quantity and it is possible that the sum of the series

$\{\Delta O_1, \Delta O_2, ..., \Delta O_i\}$ converges. Remarkably, in Section 5, we will show that when f(·) is not entirely convex, the market will not collapse in only a few periods, instead, the number of active investors will fluctuate around a certain equilibrium value.

4 Market Collapse Mechanism

In this section, we will propose a market collapse mechanism that occurs under a small exogenous shock. There are many situations where a small shock impacts an entire market. For example, a sudden hurricane might affect the supply of crude oil and many investors will suffer a great loss, and a drop in market demand will devastate suppliers heavily during the period. In our model, we define a shock as an exogenous negative factor that makes the winning probability p of all the investors that it affects drop to zero, which lasts for one decision period.

We use a simple example to explain the collapse mechanism in our model. This market is consisted of 5 investors, and their social networks are shown below:

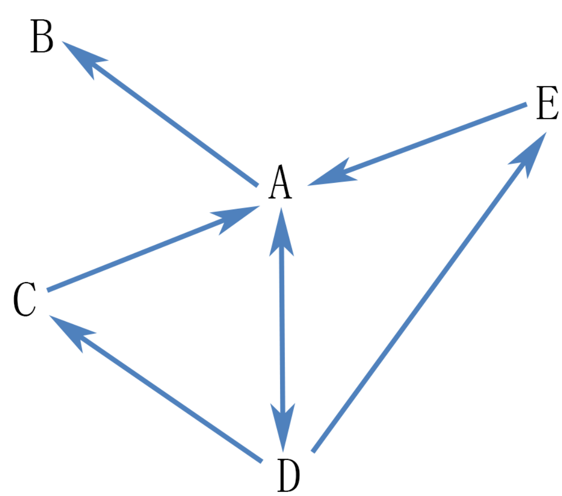

Figure4.1 An example of the social structure

Before the first decision period, all participants in the market select IN. We simply assume that A suffers from a shock during the first period, which, as defined in previous paragraph, makes the winning probability of A drop to zero. Let $B(n|k,p)$ represents the probability of experiencing n or less $WINs$ in k periods and $B(n|k,p)$ is a binominal distribution with parameter k and probability p. Suppose N_A is the biggest integral that satisfies:

$$N_A(1-c)-(4k-N_A)c<0$$

Then, $B(N_A|3k, p)$ represents the possibility of A exiting after the first decision period. That makes $B(n|k, p)$ a strictly decreasing function of k. Therefore, with the shock, the exiting probability of A, $B(N_A|3k, p)$, will be significantly higher than the exiting probability of A without an exogenous shock, known as $B(N_A|4k, p)$. Specifically, if $N_A < 3k$, we have $B(N_A|3k, p) = 1$ and A is surely to exit the market after the first decision period.

Consider the other investors who also receive the same investment outcome as A in this market, namely, B and D. Both B and D have only one sender and denote N_{BD} as the largest integral that satisfies:

$$N_{BD}(1-c) - (2k - N_{BD})c < 0$$

Similar to A, $B(N_{BD}|k, p)$ represents the exiting probability of B and D after the first decision period, and we have: $B(N_{BD}|k, p) < B(N_{BD}|2k, p)$, which means that the probability of B and D exiting rises even though they did not experience the exogenous shock of A. We denote this phenomenon as "Pass Effect" (*i.e.*, The receivers of A experience the shock through A's outcome). This is the first step of our Collapse Model. From this example, we can formalize a LEMMA of Pass Effect without any further proof:

Lemma 4. *PASS EFFECT*

Consider an investor $P_i(s_i, r_i)$ who receives a shock.

(1) The quitting probability of P_i rises from $B(N|(s_i+1)k, p)$ to $B(N|s_i k, p)$, if k is large enough.

(2) The set of all receivers of P_i is $\{P_{r1}, P_{r2}, ..., P_{ri}\}$. To a certain receiver $P_{rj}(s_{rj}, r_{rj})$, $j \in 1, 2, ..., i$, the quitting probability of P_{rj} rises from $B(N|(s_i+1)k, p)$ to $B(N|s_i k, p)$.

The second step of our Collapse Model is that the increased numbers of exiting members causes a further negative network externality to active investors. As is shown in LEMMA 3, a shock of small proportion can be passed to a larger scale of investors, which will further influence their probability of quitting the market at the next decision period. If too many investors fear playing IN in the second decision period, an avalanche can occur because the negative network externality brought about from the absence of cautious investors. Typically, a collapse effect will occur 3 to 5 decision periods after receiving the initial exogenous shock. The following example helps to illustrate how a market collapse happens, and examines our assumption of how limited information aggregation helps in accelerating the collapse procedure.

Example 4.1. *A Dig Gold Game*

Consider a dig gold game with 100 participants. The more participants taking part in this game, the higher the profit every participant will gain from a successful digging (because of the scale effect). Of course in a real world there will also be a crowding externality as is discussed in Clark and Polborn (2006), yet we ignore such effect to simplify this example. The fixed cost of every digging is $c = 0.2$ *and a successful digging brings a profit of* $0.5p$*, where* p *denotes the proportion of people who join into the digging game. After ten digging periods, all participants will be allowed to decide whether to stay in the game or not. To simplify the example, we simply assume that every participant was successful 5 times and lost 5 times in the first round. Unfortunately, the digging equipment of 10 participants suffers from a temporary breakdown so that those players cannot receive any return in the first stage. We consider three different information sharing situations among the participants in this game.*

(1) No one shares their own outcome with others. In this situation, the 10 unfortunate participants receive a return of -2*, who will likely quit the game after the first stage. It is easy to know that the other 90 participants receive an outcome of 0.5. In the second round of the dig gold game, since the proportion of active participants falls to* 90%*, the return also falls to 0.25 for each person, but they still have enough motivation to stay in this game since the expectation return is positive.*

(2) Every player's outcome information is completely shared among all the participants. In this situation, the 10 participants with the equipment breakdown receive a return of -2 *after the first stage, and the other 90 players receive a return of 0.5. However, since all of them can observe the information of every other player, all of the participants will know that the average return of the first period is 0.25 in the first stage. With this knowledge, it is worthwhile for them to stay in this game since the expectation return is positive. Thus, no one will quit the game at the second round.*

(3) Each player shares their outcome information with three other participants in their social network. In this situation, each player can make decisions based on five players' outcome history. To simplify the discussion, we assume that there is no relationship between any two players whose equipment broke, and no common member is shared among any two players with broken equipment. Then, 40 participants will observe 1 player with 10 failures, and 3 players with 5 successes and 5 failures each. To these 40 players, they will witness an expectation return of -0.125 *after the first round. Hence, even players who had positive outcomes in the first*

period will quit the game in the second round because of the negative information gained from the related player. Since there is a network externality, the expectation return of the second round game for the other 60 players is -0.5 *each. This negative experience will cause a further avalanche in the third stage,* i.e.*, every player will refuse entering in the third round of the Dig Gold Game.*

Is an avalanche inevitable under the limited information aggregation model if an exogenous shock is experienced? The answer is no. Actually, several important factors shall be taken into consideration: the shock intensity; the social structure; and the shape of the network externality function f(·). Firstly, if the shock intensity is not strong enough to cause a wide-spread panic to exit the game, then the network externality in the following decision period will not be strong enough to trigger a market collapse. Let's look back to the previous dig gold game example. If only one player suffers from an equipment breakdown in the first round, only 4 players will quit the game in the second round, but this will not cause an avalanche since the expectation return of the other 96 players remains positive.

Secondly, we found that the social structure is of paramount importance to the stability of a certain market. To illustrate, consider once again the previous example. Compared with the third situation where every participant is linked to a limited amount of players, the non-information aggregation situation (first situation) is relatively stable. However, the second situation that allows access to each player's information proves to be even more stable than the first situation, which is corresponding to the conclusion of Gunay (2008).

Finally, whether a collapse will be triggered or not also depends on the shape of f(·). For example, if f(·) is concave, the network externality generated by the absence of those shocked receivers may be so small that $pf(\frac{m}{M}) < c$ still exists. Therefore, a sudden market collapse is unlikely, and it is even more possible that the market will generally recover to an equilibrium state. On the other hand, if f(·) is convex, the shock receivers that exit can thereby trigger an avalanche since the expectation return of selecting IN in this situation is negative.

Though we do not provide a complete analytical solution to the collapse procedure due to the many factors that could be considered, we find that a collapse may occur even when the underlying fundamentals actually favor investment (*i.e.*, the expectation return of investment remains positive). We find that the avalanche occurs because the limited access of information between players in our model boosts the collapse procedure in two ways: firstly, it will cause the "Pass Effect" and secondly, it will cause investors who receive negative information to overestimate the severity of the shock and exit an otherwise optimal market. In other words, imperfect information increased the likelihood of a market collapse.

5 Simulation and Analysis

In this section, we will use programming methods to analytically simulate our model and examine the collapse procedure of the market. This Section will consist of three subsections. First, we will illustrate how vulnerable markets enter and maintain a relatively stable status when there exists no exogenous shock, and we will also show market responses that suffer from sudden shocks that affect a number of participants. Secondly, we will examine how the different factors in our model affect the stability of the market, *e.g.*, the fixed cost, the probability of winning or the number of people who suffer from a sudden shock. Finally, we will research the influence of different social network structures, and we will compare our model to those of Jeitschko and Taylor (2001) and Gunay (2008).

5.1 The Market Collapse Procedure

Consider a market of 400 participants who form a square array social network (20×20). Each player has a standing relationship with the bordering four players. If a player is on the edge of the square, the relationship extends to players on the other side of the row or column. For example, the player whose coordinates are $(0,0)$ has four neighbor players, being $(0,1),(1,0),(0,19),(19,0)$.

In this simulation, we set the return function as below:

$$U(m_\tau) = \begin{cases} \frac{m_\tau}{400} - 0.52 & \text{if WIN} \\ -0.52 & \text{if LOSE} \end{cases}$$

In this function, m_τ represents the number of participants in this decision period. We set the winning probability of p to 0.7. When there is an exogenous shock, 10% of investors in this market will be influenced, *i.e.*, 40 participants in this case. We assume that there are 10 transactions during each decision period. We use the Hinton diagram to display our result. A sample of the Hinton diagram is shown below:

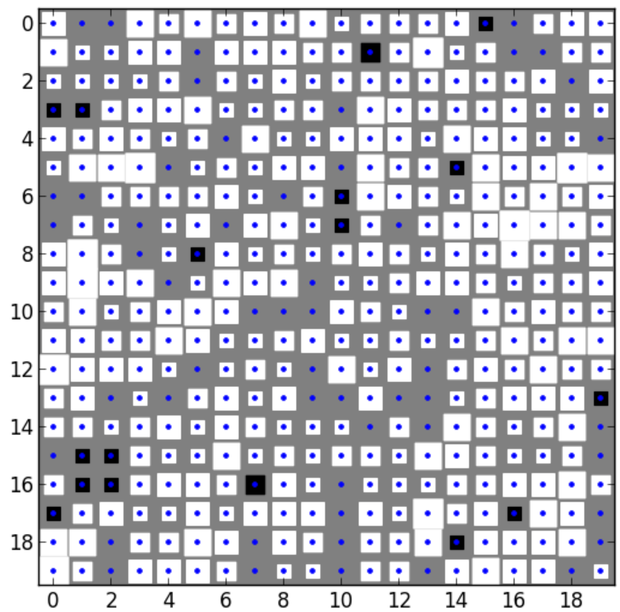

Figure5.1 A sample of Hinton diagram when the market suffers from an exogenous shock.

In Figure 5.1, each small square represents an investor in this market. The white square represents a player that has profited in this decision period, while the black square implies a loss. The area of the small square represents the net profit (net loss) during this period. If a square is nonexistent for a certain investor, it indicates that this player did not select IN for this decision period (like the player $(13,15)$). If there is a blue dot in the coordinate, it implies that this player will select IN for the next decision period. Otherwise, he will choose OUT. An orange dot implies that this player suffered from a shock during this period, which brings his winning probability down to zero.

Figure 5.2 below displays a typical market collapse procedure. The x-axis represents the decision periods, while the y-axis represents the number of investors that select IN in any period. We introduce a shock at the 201^{st} period and 601^{st} period. There is a small trough beginning in the 200^{th} period and a market collapse at the 600^{th} period. This graph clearly displays that the market is fluctuating around an equilibrium value of about 395 during the periods between 0 to 200 and 200 to 600, which supports our previous proposition in Section 3 that the market will converge to a relatively prosperous status for a long period of time when there is no further outside shocks.

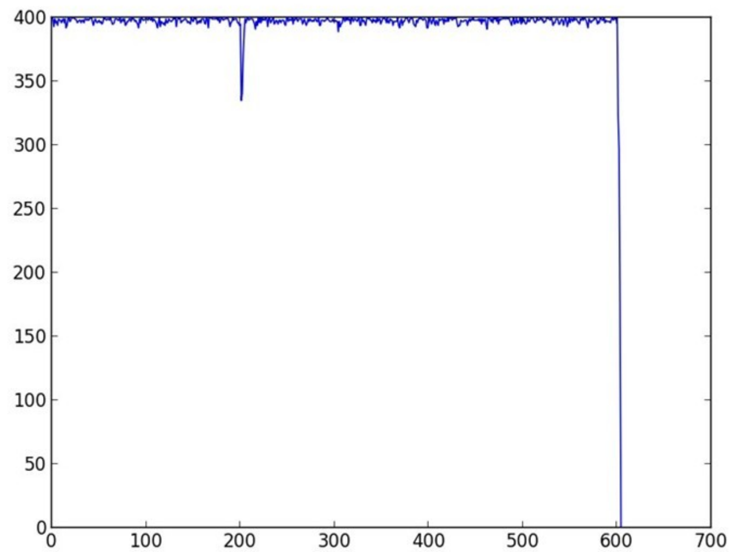

Figure5.2 Time - players number graph.

To better illustrate how the market survived from the exogenous shock at the 200^{th} period until the market collapse procedure at the 600^{th} period, we display the panel data of all investors around these two shocks in the following Figure 5.3 and Figure 5.4.

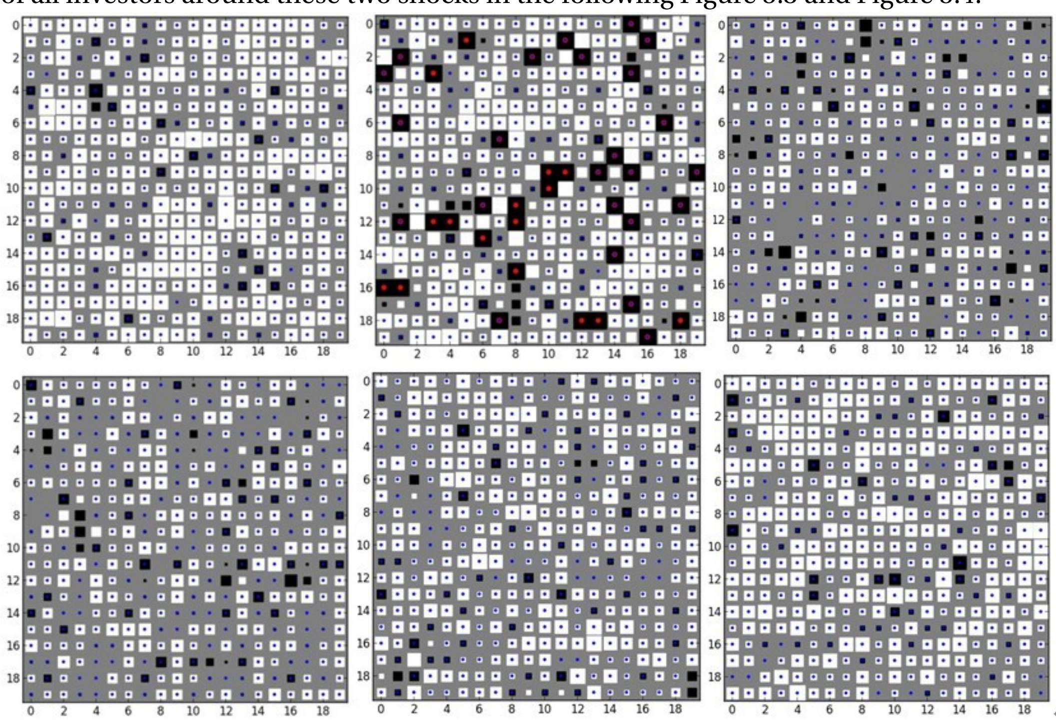

Figure5.3 Top left: period 200; top middle: period 201; top right: period 202. Bottom left: period 203; bottom middle: period 204; bottom right: period 205.

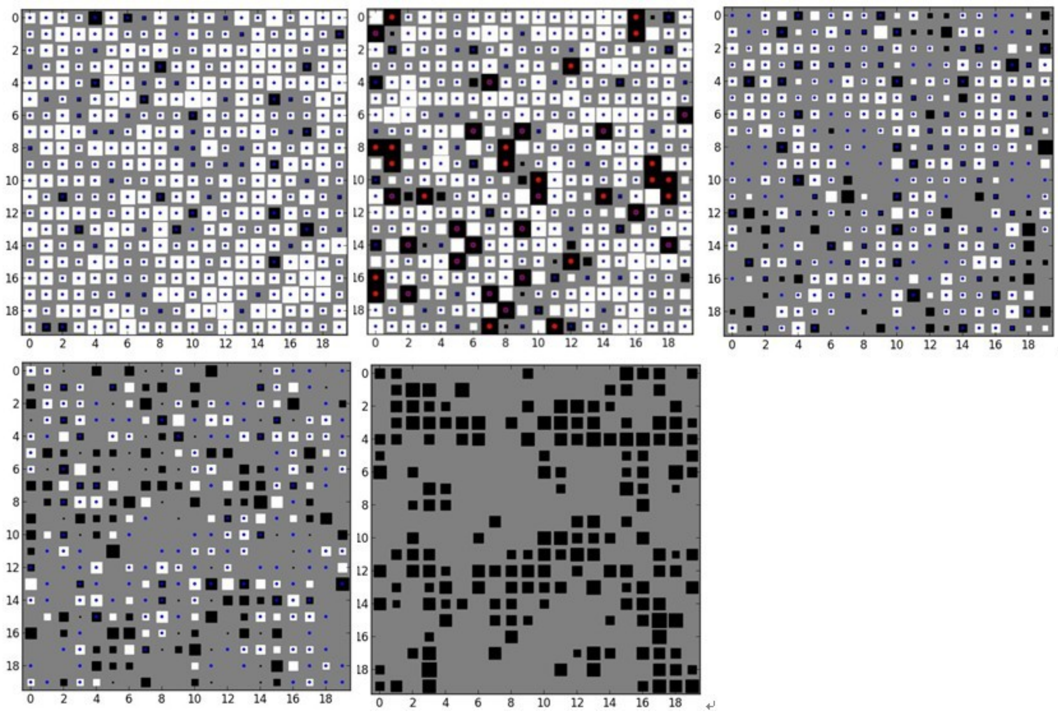

Figure5.4 Top left: period 600; top middle: period 601; top right: period 602. Bottom left: period 603; bottom middle: period 604.

Figure 4.3 illustrates how a market resisted the shock at period 201. Observe how the 40 random investors who suffered from an exogenous shock exited the market at period 202 (top right graph of figure 4.3) since their winning probability was brought down to zero. However, this downtrend did not further occur in the following several periods: in fact, exiting investors returned to the market at periods 203 and 204 and the market returned to its original level of stableness at period 205.

Figure 5.4, however, shows a different result. It clearly illustrates how a market avalanche happens under an exogenous shock. Again, 40 random investors suffered from a shock at period 601. Differing from Figure 5.3, this shock successfully triggered the domino effect in the following periods: the shock caused a huge loss for some investors in the market (period 601); their loss frightened the players bordering them and caused many of their neighbors to then exit the market as well (period 602); the initial exit of a majority of investors decreased the payoff function of the remaining IN players, which resulted in a wide-range loss (period 603); since no one could earn a profit in the market, the market collapsed (period 604).

From this example, we show that we cannot prove whether or not "there will be a collapse" or "there will not be a collapse" in our model. Actually the market behavior follows a complex stochastic process and we only know under which circumstance the market is more likely to collapse. In the following part we will use the Monte-Carlo method to estimate the expected probability of collapse under a different combination of factors, which then leads to different expectations and outcomes. In this way our model discovers how different causation factors affect the stability of a specific market.

5.2 Factor Analysis

In this section, using the market simulating program, we examine the gradients of the collapse probability Pc with fixed costs c, winning probability p, the number of transaction periods within a decision period k and the number of investors suffers from a shock Ns.

Figure 5.5 displays the relationship between the probability of a collapse and fixed costs when other factors are fixed. In this figure, the winning probability p is set at 70% and every investor trades 10 times in each decision period, with 40 out of 400 investors suffering from a shock. We set the interval of c as $0.49 < c < 0.54$, and we pick up 26 points of c within this interval with a step length of 0.002. To better estimate the collapse probability of a given c, we repeat the collapse procedure 500 times for each c point.

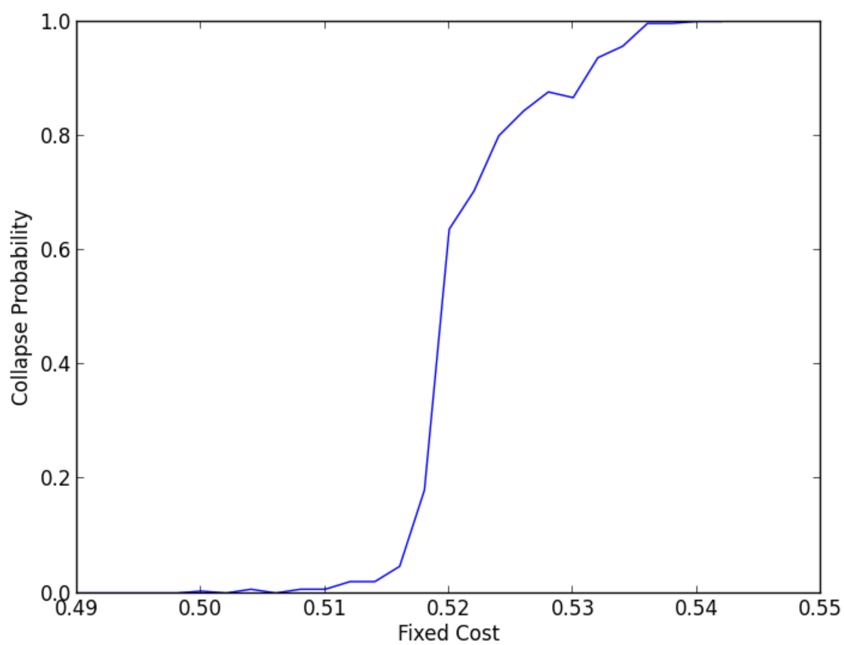

Figure5.5 Correlations between the collapse probability and fixed transaction cost

A detailed examination on this figure reveals an extreme sensitivity of a collapse probability to the fixed transaction cost. The collapse probability tends to be zero when fixed cost is 0.51, yet the probability tends to be 1 when a fixed transaction cost equals 0.54. There is a varying collapse probability within a narrow interval of the fixed cost c. Such observation implies that even a tiny rise in the fixed transaction cost will lead to an unstable market status and increases the likelihood of a collapse under a shock.

Figure 5.6 shows the overall trends of a collapse probability to the number of transaction times within a decision period when other factors are fixed. The winning probability is set at 70%, 40 out of 400 investors suffer from a shock, and the fixed cost of every transaction is set to 0.52. In this figure, the x-axis represents the transaction times within a decision period which varies from 4 to 18, and the y-axis denotes the collapse probability of a certain market. We set a 500 times repeating game to estimate the collapse probability of each given k.

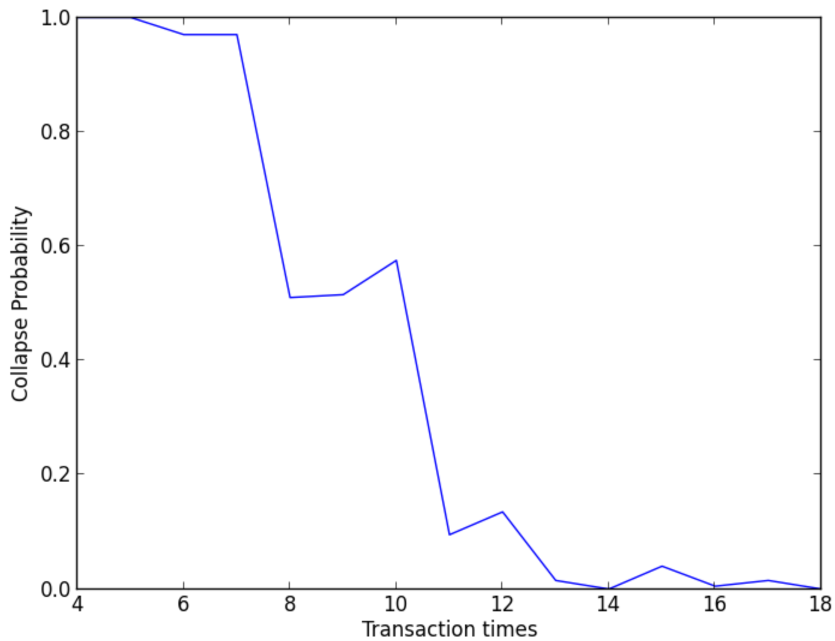

Figure5.6 Correlations between the probability of collapse and the number of transaction times

The figure above shows that there exists a generally negative relationship between the number of transaction times within a decision period and the collapse probability. Yet, we also observe several fluctuations at $k = 10, 12, 15 and 17$, which is resulted from the discontinuity of binominal distribution and is previously proved in LEMMA 2. This downward trend relation coincides with an agent's intuition: since the expectation return of each transaction is positive when less people quit, a higher number of transaction times means that each investor in the market is more likely to receive a positive return after the decision period, which leads to a more stable market status and the collapse probability tends to be valued at zero when each investor trades multiple times within a decision period.

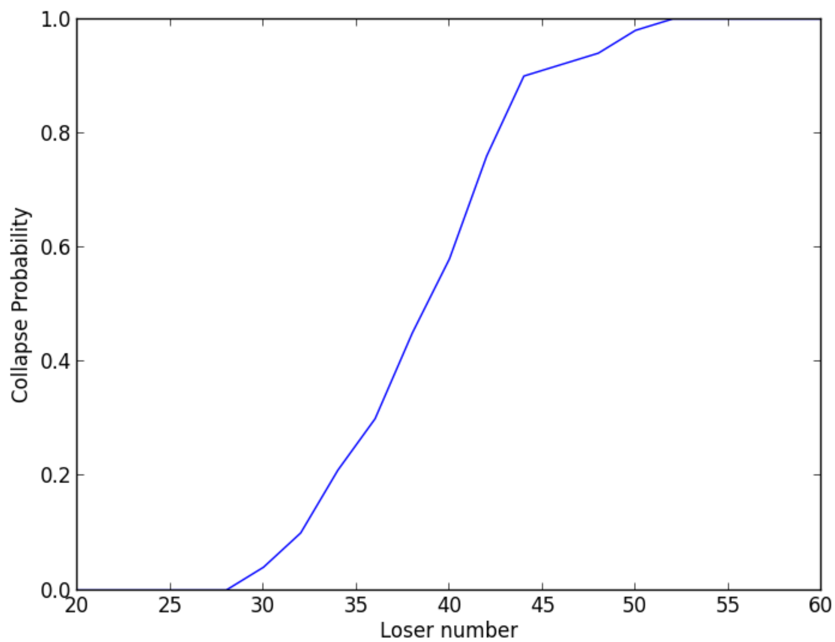

Figure5.7 The relationship between the number of investors who suffer from a shock and the collapse probability

Figure 4.7 displays the correlation between the number of investors that suffer from a shock with the probability of a market collapse. The fixed cost is set at 0.52, while the winning probability is 0.7 and in this case, every investor trades 10 times within a decision period. The number of suffering investors various from 20 to 60 in this simulation and the step length is 2. To better estimate the accuracy of the market collapse probability, we play a 500 times repeated game for each point in this graph.Obviously, the figure shows a clear uptrend relationship between the number of investors experiencing the shock and the collapse probability. The more people that are influenced by the market shock, the more investors - not only shocked investors, but also neighboring players - will become frightened and exit the market. As a result, the market completely collapses.

Figure 5.8 below displays the negative trend between the winning probability and market collapse probability. The step length is 0.01 and we hold other factors fixed as in our previous figures. The sudden rapid downturn shown in Figure 4.8 implies that the winning probability is vitally important to the market's stability, similar to the affects fixed costs had. This conclusion also fits with our intuition that when people have a higher chance of winning in a certain market, their return expectation will grow as a reasonable result and fewer participants will exit the market even though the market is suffering from a shock.

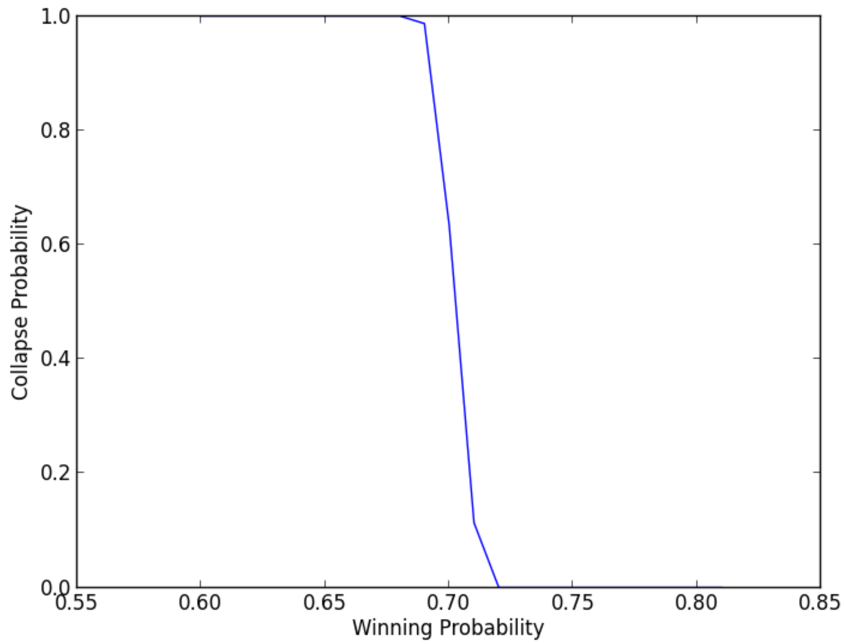

Figure5.8 Correlation between the winning probability and collapse probability

5.3 Social Structure and Model Comparison

In this section, we will change the structure of the social network in our model to simply analyze how different social networks can affect the stability of a certain market. Then we will compare our simulation results to the predictions in a number of theoretical models from Jeitschko and Taylor (2001) and Gunay (2008).

In previous sections, we used an uncomplicated social structure that everyone in the market has four neighboring friends and with whom they share transaction information to. We will extend this model to a more complex situation: each investor is randomly given a group of people to share their trading information with, and another group of random people whom they receive outcome information from. Consider a market with 400 participants and each player can receive the information of k other players. The fixed cost of each transaction is 0.5, the winning probability p is held at 0.7 and 40 out of 400 players suffer from an exogenous shock at period 200, 600 and 800. The Figure 5.9 below will show a typical time - players number graph when $k = 0, 1, 2, 3, 4$ and 399.

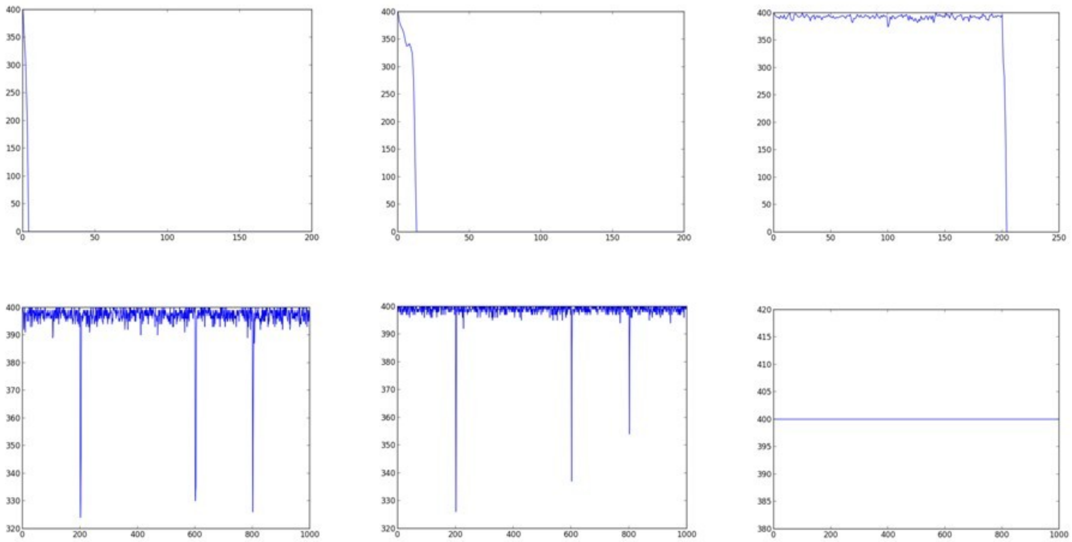

Figure5.9 Top left: k=0; top middle: k=1; top right: k=2. Bottom left: k=3; Bottom middle: k=4; Bottom right: k=399.

It is clearly shown from Figure 5.9 that as each market member receives more information from other players, the more stable the market is. Consider the top left graph of the above panel $k = 0$ situation. The market collapse happens unexpectedly fast, only within a few periods even though selecting IN creates Nash equilibrium for every market participant. In this case, every participant can only observe their own information and no one will re-enter the market as a result. Such a market avalanche procedure is thoroughly analyzed and predicted by Jeitschko and Taylor (2001). When $k = 1$, the collapse procedure slows down because of the information aggregation, however the market also collapses in a short period of time. The $k = 2$ situation is relatively robust since it experienced a longer period of prosperousness from period 0 to 200, yet, a shock at period 200 destroyed the market in this case.

Consider the graph of $k = 3$ and $k = 4$. They both show stability when facing a shock. Nevertheless, the case of $k = 4$ has less volatility than the case of $k = 3$, which implies that the more players one can observe information from, the more stable the market is. An extreme example is shown in the case of the bottom right graph in which each player can observe every trading outcome within the market, which is discussed by Gunay (2008). In this case, the market is extremely stable and even the influence of shocks is dismissed by the information aggregation and in result, our research highly supports the idea of Gunay (2008) that the information aggregation procedure will lead to a more stable market.

6 Conclusion

The limited information aggregation market collapse model in this paper is innovative in some respect. To begin with, we have proved that under the circumstance of limited information aggregation and internet externality, it is possible for a market to remain in a metastable status for a long period of time. Moreover, a market collapse may happen when there exists an exogenous shock that only influences a small number of investors in the market. Specifically, this collapse can happen even if selecting IN creates Nash equilibrium for each participant at a given decision period.

We present analytical derivations to analyze the mechanism of a market's metastable status and market's collapse. To better illustrate our model and analyze the influence of different factors, we use programming methods to simulate the relationship between different factors and collapse probability. Additionally, we compared our work to previous works in this area, such as Jeitschko and Taylor (2001) and Gunay (2008), which helps prove the applicability of our model.

There are several ways to extend our analysis. The first is to apply an everyday social network occurrence into the model. Secondly, it is worthwhile to consider the situation that investors are risk averters or low level-k rationality types. Finally, it would be interesting to consider how the collapse of a market negatively influences outside markets and how a world-wide crisis could happen under such a mechanism. We hope that our work provides future reference for further research in this field.

Appendix

PROOF OF LEMMA 2

(1) Assume $A(N|k,p)$ denotes the probability of $NWINs$ within k rounds, and $A(N|k,p)$ corresponds to a binominal distribution, then:

$$B(N|k,p) = \sum_{i=0}^{N} A(i|k,p)$$

and we have:

$$A(i|k+1,p) = A(i|k,p) \times (1-p) + A(i-1|k,p) \times p$$

, if $i > 0$

Specfically, if $i = 0$, then:

$$A(0|k+1,p) = A(0|k,p) \times (1-p)$$

Then,

$$B(N|k+1,p) = \sum_{i=0}^{N} A(i|k+1,p) = \sum_{i=0}^{N} A(i|k,p) \times (1-p) + \sum_{i=0}^{N-1} A(i|k,p) \times p$$

And we have:

$$B(N|k+1,p) = B(N-1|k,p) + A(N|k,p) \times (1-p) = B(N|k,p) - A(N|k,p) \times p$$

Since $A(N|k-p) > 0$, $B(N|k+1,p) < B(N|k,p)$ if and only if $N(k+1) = N(k)$. $\square$

(2) If $N(k+1) = N(k)+1$, then we need to compare $A(N+1|k+1,p)$ with $A(N|k,p) \times p$
Suppose $A(N+1|k+1,p) > A(N|k,p) \times p$, then:

$$\frac{(k+1)!}{(N+1)!(k-N)!} p^{N+1}(1-p)^{k-N} > \frac{k!}{N!(k-N)!} p^{N}(1-p)^{k-N} \times p$$

Which means:

$$\frac{(k+1)!}{(N+1)!(k-N)!} > \frac{k!}{N!(k-N)!}$$

Then we have $k+1 > N+1$, which is contradict to our basic assumption. Hence, we have proved that if $N(k+1) = N(k)+1$, $F(k+1) > F(k)$. $\square$

(3) Since $p > c$, according to laws of large numbers, we have:

$$\lim_{k\to\infty} E(B(k,p)) = \lim_{k\to\infty} k \times (p-c) \to \infty$$

Since binominal distribution converges to normal distribution when $\lim_{k\to\infty}$, and we have $B(N|k,p)$ represents the probability that $B(k,p) < 0$, we have:

$$\lim_{k\to\infty} B(N|k,p) = 0. \quad \square$$

Reference

Acemoglu D, Bimpikis K, Ozdaglar A. Dynamics of information exchange in endogenous social networks[R]. National Bureau of Economic Research, 2010.

Bikhchandani S, Hirshleifer D, Welch I. A theory of fads, fashion, custom, and cultural change as informational cascades[J]. Journal of political Economy, 1992: 992-1026.

Bikhchandani S, Hirshleifer D, Welch I. Learning from the behavior of others: Conformity, fads, and informational cascades[J]. The Journal of Economic Perspectives, 1998, 12(3): 151-170.

Gunay H. The role of externalities and information aggregation in market collapse[J]. Economic Theory, 2008, 35(2): 367-379.

Jeitschko T D, Taylor C R. Local discouragement and global collapse: a theory of coordination avalanches[J]. American Economic Review, 2001: 208-224.

Zollman K J S. Social structure and the effects of conformity[J]. Synthese, 2010, 172(3): 317-340.

Clark C R, Polborn M K. Information and crowding externalities[J]. Economic Theory, 2006, Vol. 27, No. 3: 565-581

Guarino A, Huck S, Jeitschko T D. Averting economic collapse and the solipsism bias[J]. Games and Economic Behavior 2006, 57: 264-285