Dynamic conditions for smooth convergence in the Ricardo–Mill model under commitment of trade and continuum of goods

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2 December 2013

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December 12, 2013

Abstract

Under general conditions, it has been proved that free trade improves the welfare of open economies. However, the conditions to attain the free trade equilibrium are non trivial: when the productive process is planned, industries do not know the price that will prevail, while the production is not available in the world markets, generating a "general price uncertainty" due to the time–consuming nature of productive process. Consequently, additional assumptions is required to construct the time path driving economies from the autarky to the free trade. Thus, we assume commitment of trade and continuum of goods with the aim of handle with such a problems. This paper finds the general conditions for a smooth time path stable, monotonic and that guarantees a successful process of liberalization.

Keywords: Ricardo–Mill model, general equilibria, dynamic models, dynamic welfare
JEL Codes: C62, F11, F22, J61

1 Introduction

The theory of comparative advantage is a very powerful tool to describe the international distribution of production, and an elegant argument supporting the free trade. Also,

*This research was supported by the Universidad del Pacífico, Ecuador
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the Ricardo–Mill model guarantee that countries have enough incentives to open their economies in order to improve the welfare of their inhabitants. Thus, this model is seeming the simplest and most basic general equilibrium model of international trade, since the most important properties of the model depends only on the relative price of a single and immobile production factor. Then, comparing this relative price before and after to trade we can predict the geographical distribution of production and its consequences over the welfare. So that each prior to trade price generates a particular geographical distribution of production.

This paper shows that the path from autarchy to free trade equilibria is not defined in the classical theory. And, given the timing consuming nature of the model, this path is not trivial, excepting in the perfect mobile factor case. Consequently, a necessary question to be answered is if there is a time path driving economies from the autarky to the free trade equilibrium. And, if such a time path exists, what conditions must be satisfied to guarantee that exists a stable and monotonic time path leading to a equilibrium that does not overshoots initial values, backing economies to the autarky.

2 Theoretical framework

This paper assumes the general features of the Ricardo–Mill model in continuum of goods (*RMC*). Let denote by $k \in K = (0,1) \subseteq \mathbb{R}_+$ a commodity, defined in a continuous of goods (Dornbusch et al., 1977; Wilson, 1980). Then, any differentiable function in $\mathbb{R}$ is also a differentiable function in K. Let assume constant return to scale technology to produce each k, depending on a unique production facto, labor L, which is independent of time. Then, we can assume that there are a differentiable function $a^c(k)$ that represent the amount of labor need to produce one unit of commodity k in country $c = 1,2$. Under previous assumption, a small change in labor generates a different commodity k; and, the amount of labor required to produce k is constant and, in particular, does not change over time. Finally, let assume that each k defines an industry.

The function $a^c(k) \subseteq \mathbb{R}_+$ is defined to be strictly positive (non free lunch), differentiable, strictly decreasing and well defined function $a(k) = \frac{a^1(k)}{a^2(k)}$ of k, consistent with the previous assumptions. So that, this function ranks countries by the relative amount of labor required to produce a commodity k: if $a(k) \to 0$ then, country 1 is relatively more efficient in the production of commodity k.

Since $a(k)$ is independent from the pattern of trading and time, we can assume the technical hypothesis that the inverse function a^{-1} is a differentiable an monotone function. Then, commodities are ordered by the across counties relative amount of labor required to produce a commodity k. Under this assumption we can define a border commodity $\bar{k}(t)$ defining the geographical distribution of production, i.e., given the commodity $\bar{k}(t)$, any $k > \bar{k}(t)$ is produced in country 1, and the opposite occurs for any commodity $k > \bar{k}(t)$.

Given the price of each unity of labor, $\omega^c(\cdot)$ —a function of time which properties will be defined along this paper—, the unit cost of produce a commodity k is $d^c(k,\cdot) = a^c(k)\omega^c(\cdot)$. Also, we can define the relative price $\omega(\cdot) = \frac{\omega^1(\cdot)}{\omega^2(\cdot)}$, and the function $d(k,t) = a(k)\omega(\cdot)$ is the relative cost of production of commodity k. Notice that factor prices $\omega(\cdot)$ is defined as function of the argument "$\cdot$" that represents the price dynamics, and will be defined and explained in the next sections.

The relative cost $d(k,\cdot)$ ranks the k industries by relative marginal cost of production. Consequently, at any time t, this relative cost defines the geographical distribution of production, and the border commodities $\bar{k}(t)$ satisfies the condition $d(\bar{k},\cdot) = 1$. So that, for a fixed t, there is a $\bar{k}(t)$ for a given $\omega(\cdot)$ (maybe not unique), that generates a unique partition of the set K in two sets $K^c(t)$ for $c = 1,2$. For instance, for any commodity $k \in K^1(t)$ it satisfies that $d(\bar{k},\cdot) \leq 1$, and is relatively less costly produce such a commodities in country 1, or equivalently, country 1 has comparative advantage in the production of such a commodities.

Also, given a relative price $\omega(\cdot)$, the properties of $A(k)$ guarantee that exists an unique border line industry, $\bar{k}(t) = A^{-1}[\omega(\cdot)]$, defining a partition of the set K on two sets, $K^1(t) = [0,\bar{k}(t)]$ for set of commodities produced in country 1, and $K^2(t) = (\bar{k}(t),1]$ for the set of commodities produced in country 2, describing completely the geographical distribution of production.

3 The equilibria and the commitment trade

Returning to the above discussion, in the theory of comparative advantage arises two different ideas of marginal cost: the marginal cost of production and the marginal cost used to determine the comparative advantage (Dixit, 1976). the question to be answered is what marginal cost? In the previous section we define by $\cdot$ the argument of the factor price function.

In absence of trade, the price of commodities k in the country c is determined exclusively by the domestic production price $\omega^c(t)$ and and the amount of labor used to produce the commodity $a^c(k)$. Since individuals supply all the available labor inelastically, the domestic factor price is the numerary of the economy. Consequently, under autarchy, economies produces all commodities at $p^c(k) = d^c(k,t)$, the marginal cost of produce k and the commodity price is a function of t.

But, if countries are trading, commodities price are determined by the international relative factor price $\omega(\cdot)$ where the time path structure is not defined in the Ricardo–Mill model. Under partial equilibria, prices jumps from autarchy to the free trade prices. This jump is the core of the Ricardo–Mill theory to explain the comparative advantage and the welfare improvement. Nevertheless, the simplest Ricardo–Mill theory does not explain how economies are driven out from the initial equilibrium (prior–trade) to the final free trade equilibrium (post–trade), settled somewhere between prior–trade prices (Dixit and Norman, 1980). These price changes are often dealt with through comparative statics, but without checking the essentials of the dynamic path from autarky to free trade.

Given a relative factor supply $S(t)$, the equilibrium in the factor market implies that exists a relative factor price that equals the implicit relative factor demand and the factor supply, even if the labor supply is fixed. Thus, the marginal costs used to determine the comparative advantage is calculated upon an observed relative price (prior–trade price), but the marginal cost of production is a unobserved price (post–trade price) during the process of liberalization (Dixit and Norman, 1980).

Then, during this process of liberalization (driving economies from the autarky to the free trade), for a given observed prior–trade price at the beginning of t, industries decide if must to close or if must to supply an unbounded amount of commodities. After trade is realized, at the end of t, industries know the post–trade price, and decide if continue producing or close, given the observed prior–trade price at $t+1$. Notice that, at the post–trade prices all markets are cleaned. So that, there is an equilibrium conditional to a particular geographical distribution of production that could not exhaust the comparative advantage between countries, even if markets are cleaned. And, if and only if both prices coincide (prior and post–trade prices), economies reach the free trade equilibrium, exhausting the comparative advantage, and the transitional dynamics leads to an equilibrium.

Thus, in the theory of comparative advantage we should distinguish between the free trade equilibrium and the conditional trade equilibrium. Explicitly, the free trade equilibrium (FT) is defined by a vector price ω^* that equilibrates all markets given a geographical distribution of production that eliminates the comparative advantage. And, the conditional trade equilibrium (CT) is defined by a vector price $\omega(t|\bar{k})$ that equilibrates all markets, conditional to a particular geographical distribution of production defined for $\omega(t)$, that does not eliminate the comparative advantage. Then, economies are in a free trade equilibrium if and only if $\omega(t|\bar{k}) = \omega(t) = \omega^*$

The question is how agents deal with prices defined prior to consumer decisions. A natural assumption arising from the advantage theory is that production decisions concerning to supply or not supply a commodity are taken using prior–trade prices. The prior–trade price $\omega(t)$ determine the relative marginal cost $d^c(k,t)$, and the geographic distribution of production, $\bar{k}(t)$. Thus, for a given $\bar{k}(t)$, bids of commodities are thrown on the world market, leading economies to the CT equilibrium. Notice that at the beginning of t, producers do not know exactly how many units of output will be traded in the world market, since the prior–trade price equilibrates all markets only if $\bar{k}$ exhausts the comparative advantage.

At this point, we need to assume that industries decide to produce at t and do not change this decision until $t+1$. Notice that if an industry located in country c decides to closes at t, this industry is opened in the country $|c-1|$. However, an industry k can be opened in both countries if prior–trade marginal cost equals for this industry.

During the period t industries adjust the commodity supply to the commodity demand and labor supply and, at the end of period t, the k commodity price is $p(k,t|\bar{k})$, the post–trade vector prices that equilibrate all markets for a given $\bar{k}$. This kind of prior and post–trade equilibrium is a general equilibrium relative of what can be called "commitment trade" or commitment of tradeRuffin (1974). In this models, a country is seen as a simple economic agent that decides to exchange physical quantities of commodities before knowing the terms of trade (Ruffin, 1974; Bhagwati et al., 1998; Pomery, 1994). And, under constant return to scale technology, industries commit to produce zero or an unbounded quantity of commodities at post–trade prices.

3.1 The dynamics of the model under the general price uncertainty

The commitment trade hypothesis explains the industries decision–making mechanism. Nevertheless, when the productive process is planned, industries do not know the price that will prevail, while the production is not available in the world markets, generating a "general price uncertainty" due to the time–consuming nature of productive process

(Ruffin, 1974). In fact, at period t producers only know $\omega(t-j)$ and $\omega(t-j|\bar{k})$ for $j>0$. The prior–trade relative price is foreseen, given the available information and post–trade relative price is assess for the given foresee prior–trade price t. If the available information is perfect, in the sense that future information can be perfectly foreseen then, the set $\omega(t)=\omega(t|\bar{k})$ is a consistent hypothesis, in other case, this assumption is not correct. The utmost case of perfect information is done by perfect mobile factor, since factor price are equal in both countries, and the relative factor price is constant, $\omega(t)=\omega(t|\bar{k})=1$.

But, if the information is not perfect, in the sense that agents only know the past post–trade factor price, economies do not know how to jump from the autarky to the free trade equilibrium. In such a case, the foreseen prior–trade relative price determines a time path with dynamic properties that can strongly affect the properties of the Ricardo–Mill model during the transitional dynamics.

Let assume agents foresee the prior–trade relative price $\omega(t)$, using a differentiable prediction function F of known information. Under differentiability of all the functional forms (see section 2), the non–linear difference equations can be linearly approximate, simplifying the analysis for a point enough near from the FT equilibrium. Let consider the prediction function being a differentiable function F on the most recent known relative prices:

$$\omega(t)=F[\omega(t-1),\omega(t-1|\bar{k})] \tag{1}$$

The price $\omega(t-1)$ embodies information relatively to the geographical distribution of production, $\bar{k}(t-1)$, and $\omega(t-1|\bar{k})$ that relatively to the world market demands and labor market equilibrium.

Let define by $\delta_\omega(t)=\omega(t)-\omega^*$ the difference between the prediction $\omega(t)$ and the FT equilibrium factor price, by $\delta_{\omega|\bar{k}}(t)=\omega(t|\bar{k})-\omega^*$ the difference between the the CT and FT relative factor price equilibrium, and by F_ω and $F_{\omega|\bar{k}}$ the partial derivatives of F with respect to the prior and post–trade relative prices assess at ω^*. Then, the unbalance function $\delta_\omega(t)$ can be linearly approximate by means of the differential for any point enough near from the FT equilibrium ω^*:

$$\delta_\omega(t)=\delta_\omega(t-1)F_\omega+\delta_{\omega|\bar{k}}(t-1)F_{\omega|\bar{k}}+e_F(t) \tag{2}$$

Notice that $e_F(t)$ is the mathematical error due to the approximation. The error $e_F(t)$ is not stochastic, since it depends on the functional form F and, specially, on the closeness of $\delta_\omega(t)$ to the FT equilibrium. The differentiability guarantees that $e_F(t)\to 0$ if $\delta_\omega(t)\to 0$.

Consequently, for a prediction enough near from the CT equilibrium, we can consider $e_F(t) = 0$.

3.2 The market equilibria dynamics

In the previous sections we analyze the main conditions and hypothesis of the *RMC* model, and we construct the dynamic model under commitment of trade hypothesis, in order to solve the unexplained jump from the autarky to the free trade equilibrium. This section is devoted to find the equilibrium in the commodities and labor markets, for a given prior–trade factor price forecast.

Let define by $x^c(k|t|\bar{k})$ the total demand of a commodity k at t in c, and by $y^c(k,t|\bar{k})$ the k industry supply located in the country c at t. In section 2 we assume $a(k)$ the amount of labor need for the production of one unit of commodity k. Therefore, $y^c(k,t|\bar{k}) = {}^{L^c(k,t|\bar{k})}\!/_{a(k)}$ is the total production of commodity k produced in c at t. In the demand side, $x^c(k,t|\bar{k}) = {}^{N^c m^c(t)\alpha(k)}\!/_{p(k,t|\bar{k})}$.

At the price $p(k,t|\bar{k}) = d^c(t,\bar{k})$, the total labor demand calculated from the commodity market CT equilibrium is:

$$L^c(t|\bar{k}) = \int_{k=l_1}^{k=l_2} L^c(k,t|\bar{k})dk = \frac{N^1 m^1(t) + N^2 m^2(t)}{\omega^c(t|\bar{k})} \int_{k=l_1}^{k=l_2} \alpha(k) \tag{3}$$

with l_1 and l_2 the limits of the partitions K^c. Denoting by $v(k,t|\bar{k}) = \int_{k=0}^{k=\bar{k}} \alpha(k)$ the fraction of national income spent in acquiring commodities produced in country 1 the relative labor demand of labor is:

$$L(t|\bar{k}) = \frac{L^1(t|\bar{k})}{L^2(t|\bar{k})} = \frac{\omega^2(t|\bar{k})}{\omega^1(t|\bar{k})} \frac{v(t|\bar{k})}{1 - v(t|\bar{k})} = \omega(t|\bar{k}) \frac{v(t|\bar{k})}{1 - v(t|\bar{k})} \tag{4}$$

Notice that, the relative factor CT equilibrium is $S(t) = L(t|\bar{k})$; and, $\bar{k}(t) = d^{-1}(\omega)$, i.e., the border commodity is a function of the unknown prior–trade prices and the prior–trade price is foreseen by industries. Thus, the CT equilibrium post–trade price depends on the foreseen price $\omega^c(t)$ and, on the relative labor supply:

$$\omega(t|\bar{k}) = \frac{1}{S(t)} \frac{v(t|\bar{k})}{1 - v(t|\bar{k})} = G(\omega(t), S(t)) \tag{5}$$

Bhagwati et al. (1998) remark that there are at least to ways to interpret the G schedule.

Seen one way, it is a representation of demand conditions. Accordingly, it says that an increase in the range of commodities produced in the country 1 increases the demand for production factor in that country and, hence requires an increase in its relative factor price to clear the labor market at the fixed factor supply. The second interpretation of G is in terms of trade balance conditions. Since the total remuneration at t is $L^c(t|\bar{k})\omega^c(t|\bar{k})$, the schedule G is the balanced pattern of trading, i.e., G is the relative fraction of income spent in commodities that assures the balance in trade for a given $\omega(t)$ and geographic distribution of production.

In section 3.1 we analyze the prediction function F of the prior–trade $\omega(t)$ and the linear approximation from any point enough near from the FT equilibrium ω^*, measuring the convergence of predictions to the FT equilibrium. In this section, we analyze the function G. The function G is the function of the CT factor price equilibrium, then the linear approximation to the ω^* measures que convergence of the CT to the FT equilibrium.

Let denote $\delta_S(t)$ the difference between the CT equilibrium and the FT equilibrium in the labor market; and, by G_ω and G_S the partial derivative of G with respect to $\omega(t)$ and S respectively. Then, the linear approximation of $\delta_{\omega|\bar{k}}(t)$ under differentiability of G is:

$$\begin{aligned}\delta_{\omega|\bar{k}}(t) = \omega(t|k) - \omega^* = [\omega(t) - \omega^*]G_\omega + [S(t) - S^*]G_S + F_G \\ = \delta_\omega(t)G_\omega + \delta_S(t)G_S + e_G(t)\end{aligned} \tag{6}$$

The $e_G(t)$ accounts for mathematical error due to linear approximation and, given the differentiability of G, $e_G(t) \to 0$ if $\omega(t|\bar{k})(t) - \omega^* \to 0$. The linear approximation of the the forecast function F can be assess as a function of the equilibrium in the labor markets. Plugging equation (6) in equation (2) we obtain the function :

$$\begin{aligned}\delta_\omega(t) =& \delta_\omega(t-1)F_\omega + [\delta_\omega(t-1)G_\omega + \delta_S(t-1)G_S + e_F(t)]F_{\omega|\bar{k}} + e_G(t) \\ =& [F_\omega + G_\omega F_{\omega|\bar{k}}]\delta_\omega(t-1) + G_S F_{\omega|\bar{k}}\delta_S(t-1) + F_{\omega|\bar{k}}e_F(t) + e_G(t)\end{aligned} \tag{7}$$

Returning to the discussion relative to the concept of general price uncertainty, we cannot consider the unknown $\omega(t)$ non stochastic. The uncertainty implies that $\omega(t)$ is a random variable. In others words, even if F is a linear function and the error is $e_F(t) = 0$, it does not means perfect predictions, due to the general price uncertainty. So that, the aleatory variable $\varepsilon_\omega(t)$ accounts for this uncertainty and the following equation for the prior–trade price holds:

$$\begin{aligned}\delta_\omega(t) = [F_\omega + G_\omega F_{\omega|\bar{k}}]\delta_\omega(t-1) + G_S F_{\omega|\bar{k}}\delta_S(t-1) \\ + F_{\omega|\bar{k}} e_F(t) + e_G(t) + \varepsilon_\omega(t)\end{aligned} \tag{8}$$

Additionally, we assume perfect past predictions leading to a perfect current predictions: if agents correctly predict at t —i.e., $\omega(t) = \omega(t|k)$—, the predictions for any $t+j$ with $j > 0$ are perfect, or $\omega(t+j) = \omega(t+j|\bar{k}^*)$. Consequently, the best prediction for ω^* is the FT equilibrium, or $\omega^* = F(\omega^*)$. Assuming $E_{t-1}\varepsilon_\omega(t) = 0$, expectations are rational (Attffeld et al., 1991), and the expectational error $\omega(t) - E_{t-1}\omega(t) = \varepsilon_\omega(t) - E_{t-1}\varepsilon_\omega(t) = \varepsilon_\omega(t)$ is independent from the period at which the forecast is done.

4 Conditions for the convergence of the conditional trade equilibrium to the free trade equilibrium

The equation 7 is a function of the best prediction function F on the best information available at t. This information accounts for the geographical distribution of production at $t-1$ and the labor market equilibrium (or market commodities equilibrium) at $t-1$. Since agents are rational, industries use a convergent F function to the CT equilibrium.

For sake of simplicity, let assume non population growth, and a point enough near from the the CT equilibrium: the error e_F and e_G are insignificant, and the labor unbalance δ_S equals to zero. Solving the first order difference equation 7 for an initial unbalance $\omega(0) = \omega_0$ and assuming convergence to the CT equilibrium, we find:

$$\omega(t) = (F^*_\omega + F^*_{\omega|\bar{k}} G^*_\omega)^t \omega_0 \tag{9}$$

where, "*" denotes the derivatives evaluated at the CT equilibrium ω^*.

The derivative $G_\omega(t)$ a negative function of $\omega(t)$, since the integral $v(t|\bar{k})$ is a increasing function of $\bar{k}$, and $\bar{k}$ is, by assumption, a decreasing function of $\omega(t)$. The figure 4 illustrates the main properties of the time path for a given G_ω. Industries predictions of $\omega(t)$ are convergent if and only if $|F_\omega + F_{\omega|\bar{k}} G_\omega| < 1$; if $F_\omega + F_{\omega|\bar{k}} G_\omega < 0$ the time path is an alternating function of time t (i.e., if $\omega(t) - \omega(t+1) > 0$ then $\omega(t+1) - \omega(t+2) < 0$); and, the time path does not overshoots the initial values $\omega(0)$ if and only if $|F_{\omega|\bar{k}} G_\omega| < 1$ (Ogata, 1990; Nise, 1991).

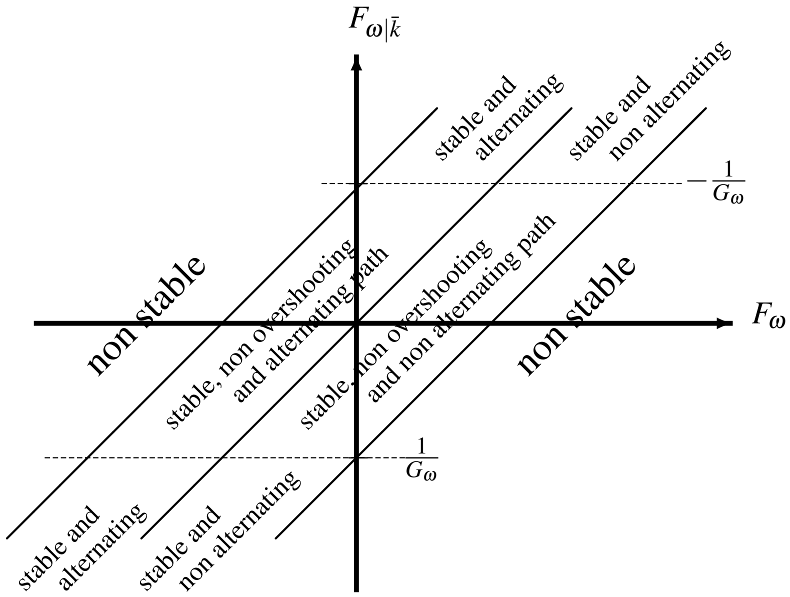

Figure 1: Time path properties

Under rationality, we should assume convergent prediction, since industries need a convergent prediction function F to achieve the CT equilibrium. Nevertheless, we can not assume that rationality implies non alternating path. Neither assume non overshooting condition.

First and foremost, let understand the Figure 4 obtained from the solution found in equation 9. This solution (and the figure) assumes convergence and known derivatives at t^*, the period at which prices reach the FT equilibrium. Nevertheless, excepting in the case of linear functions, derivatives are not constant and, for this reason, unknown. In particular, we cannot assume the derivative G^*_ω a constant function of $\omega(t)$: this derivative depends on the functional form of $\alpha(k)$ and $a^{-1}(\omega)$, not necessarily constants on its arguments. Obviously, if $\omega(t)$ is very near from the FT equilibrium, derivatives converge to those assess at ω^*.

Additionally, if the time path does not alternate, industries that have been closed at t does not opens at $t+j$ for $j>0$ and one of the two countries. For instance, let asume that $\bar{k}(t)<\bar{k}^*$, then industries that are closed at t in country 2 will not be opened at $t+j$, but some industries closed in country 1 will open at $t+j$. Thus, $\bar{k}(t)$ converges monotonically to $\bar{k}^*$. This time path behavior is consistent with the commitment of trading hypothesis and with rationality in the sense that countries knows the direction of the convergence.

However, this monotonic time path behavior is inconsistent with the hypothesis that the CT equilibrium is very close from the FT equilibrium: industries k very close from the $\bar{k}^*$ have no reasons for close (or open) if they are not sure about the true ω^*, in particular for industries that are producing the border line commodity $\bar{k}^*$. It is more true taking into account the random nature of $\omega(t)$. Then, we can expect alternating path (even non alternating path), for geographical distribution of production very near from the FT equilibrium or, equivalently, industries does not perfectly forecast the direction of trade when they are very near from the free trade equilibrium.

The argument to support overshooting or non overshooting time path is not trivial. The non overshooting condition implies that prices $\omega(t)$ never overshoots $\omega(0)$ for any $t > 0$ or, mathematically, $|F_{\omega|\bar{k}} G_\omega| < 1$. Technically, as bigger $|G_\omega|$ is, smaller the $|F_{\omega|\bar{k}}|$ should be, in order to avoid overshooting. Returning to the discussion relative to G_ω properties, this derivative depends on the particular structure of consumer tastes and relation between industries technology. If tastes growth very fast as $k \to 1$ (or k is strongly preferred to k' if $k > k'$), then the $F_{\omega|\bar{k}}$ must be small enough to compensate tastes. Similar result is obtained if the relation between technologies is strongly decreasing in k, i.e. if country 2 increasing in competitiveness growths very fast as $k \to 1$. One other interpretation for non overshooting condition is neither optimistic nor pessimistic predictions.

The above conditions (stability, non alternating path and non overshooting) can be seen in figure 4 and, it implies $F_\omega > 0$, $F_{\omega|\bar{k}} \in (-\frac{F_\omega}{G_\omega}, \frac{1-F_\omega}{G_\omega})$ and $F_{\omega|\bar{k}} \in (-\frac{1}{G_\omega}, \frac{1}{G_\omega})$ for a given G_ω value.

5 Conclusion

The theory of comparative advantage is an excellent tool to explain the incentives for trading. But, usually, arguments supporting the advantages of free trade compares autarky with free trade equilibrium. This paper shows that conditions to attain the free trade equilibrium are non trivial.

Consequently, we should not compare these equilibrium if, given the particular conditions of an economy, we are not sure how to driven an economy from autarky to free trade. In particular, the time path strongly depends on the available information and on the mobility of the factor. If the information is not perfect in the sense that agents cannot perfectly forecast the ex–post or international prices, the system does not jumps from the autarky to free trade equilibrium. Excepting in the case of perfectly mobile factor. If it is not the case, the time path strongly depends on the prediction function.

In section 3, under the general conditions analyzed in the theoretical framework, and given a particular prediction function, we obtained a set of conditions that completely characterize the dynamical behavior of the system for a small shock around the equilibrium. In general, this paper shows that the conditions to achieve the free trade with a non overshooting and non alternating path can be very hard, in particular if tastes and technology are not smooth functions of the k.

Finally, the factor supply plays an important role in the process of liberalization: if the factor supply is perfectly mobile, economies jump from autarky to free trade. Consequently, under regular conditions, the mobility of the factor (or migration) is a good prescription for smooth the process of liberalization.

Find the dynamic conditions for the smooth convergence is an interesting question, since prices defines the time path for individuals and countries welfare gains from trading. In particular, overshooting can impose strong conditions for liberalization.

The labor market structure strongly changes results as well. We show that perfect mobility of labor leads economies to the free trade, since factor prices will be equal in both countries. It is partially true, since it means that differences in technology is due only to technical reason. But, it is absolutely unreal assume that difference between countries is not due to the technological advances embodied in the labor factor. Also, we do not consider population growth. In this case, for a population growth constant and equal to $\rho \in [0,1)$ the system converges to the FT equilibrium and, is a non alternating function of time if the system is not alternating for $\rho = 0$. Nevertheless, the conditions for the non overshooting time path are not simple or intuitive.

Finally, the analyze is done for strong assumptions. Assume the mathematical errors due to the linear approximation equal to zero is a very unreal assumption. In particular, for the G function, since this function is clearly non linear. Also, non linear equations can not often be solved explicitly, and a rule for solving must be constructed. Consequently, in such a cases, the linear approximation is not a innocuous assumption.

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