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Lach, Łukasz

AGH University of Science and Technology, Department of Applications of Mathematics in Economics

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ŁUKASZ LACH*

FIXED CAPITAL AND LONG RUN ECONOMIC GROWTH: EVIDENCE FROM POLAND

In this paper, the results of testing the causal interdependences between the gross fixed capital formation and GDP in Poland are presented. Some recent theoretical deliberations (like alternative method of testing for direction and sign of long run causality) and econometric tests (bootstrap procedure, nonlinear causality test) were applied. The dataset included quarterly data for the period Q1 2000–Q4 2009, however the research was also performed for non-crisis subsample (Q3 2002–Q2 2008). Due to theoretical (formulation of growth models) and practical (omission of important variables) reasons employment was chosen as an additional variable.

The significant evidence of short run feedback between fixed assets and GDP as well as between fixed assets and employment were found in both periods analyzed. However, the long run positive impact of fixed capital on economic growth was found only for reduced sample. Moreover, the robustness of these results was also approved.

The results of this paper provided evidence to claim that fixed capital in Poland is still under its growth-maximising level. Although participation in UE stimulated rise in fixed assets, economic crisis have distorted this process. The main policy recommendation resulting from this paper is that the Polish government and private sector should definitely increase the level of fixed investment.

Keywords: *fixed assets, economic growth, employment, Granger causality, impulse response*

1. Introduction

There is no doubt that the fixed capital is one of the fundamentals of each economy. The economic activity of society requires machines, devices, equipment, roads, track, power grids, airports, water systems, telephones and other forms of fixed assets. Thus, it seems obvious that economic growth and development are strongly dependent on available fixed capital. However, investment in this specific type of capital reduces investment in other types of capital, e.g., the human capital. Moreover, existing fixed assets must be paid for (purchase, investment, maintenance, repairing, modernization, utilization, etc.), which increases current expenditures and additionally reduces other investment and savings. Therefore, it is fully justified to ask whether gross fixed capital formation in specific economy is below, under or about its growth-maximising level. This question is of a great importance especially for economies in transition from Central Europe which aim to improve

the level and quality of post-Soviet fixed assets, on the one hand, but on the other hand tend to invest reasonably (especially in public infrastructure) to avoid the risk of slowing down the pace of economic growth.

In this paper, the case of Poland is analyzed, because this country is the largest in Central Europe. Furthermore, Poland was the first of former Eastern Bloc countries which started its transition process in 1988 and was the only European country whose GDP growth rate in 2001 and 2008 remained positive despite the global economic crises and only little fiscal stimulus (as in most CESEE countries). To the best of the author's knowledge there are no contributions concerning gross fixed capital formation and economic growth for transitional country from Central Europe which would use reliable up-to-date quarterly data together with recent theoretical solutions and econometric techniques¹. Such an analysis may be of interest for policy makers both in Poland (in terms of maintaining its economic development and increasing fixed investment, especially in the public infra-

* Department of Applications of Mathematics in Economics, Faculty of Management, AGH University of Science and Technology, Al. A. Mickiewicza 30, 30-059 Cracow, Poland, tel.: +48 12 6174218, fax: +48 12 6367005, e-mail: lukilach1983@o2.pl

¹ Due to the lack of reliable datasets of sufficient size, most of the previous papers used panel datasets instead analysing time series for individual country (see Section 2).

structure) as well as in other transition economies, not necessarily in the Central Europe.

The remainder of the paper is organized as follows. In the next section a literature overview is given. In Section 3, the main research conjectures concerning the interrelations between gross fixed capital formation and economic growth in Poland are presented. Section 4 contains details of the dataset applied. The methodology is briefly discussed in Section 5. The empirical results are presented and discussed in Section 6. Section 7 concludes the paper.

2. Literature overview

The early contributions concerned with the dynamic interactions between fixed capital and economic growth focused mainly on the largest economies of the world. In general, previous empirical papers can be clustered into two major groups: contributions related to public expenditure (in particular, investment in public infrastructure) and papers dealing with the role of fixed capital (machines, equipment, etc.) in the development of manufacturing process.

Aschauer [3] examined the dynamic interrelations between aggregate productivity and government-spending variables in the case of the USA. The findings of this paper confirmed significant positive impact of non-military public capital stock and infrastructure (e.g., roads, airports, water systems, etc.) on productivity. In his later study, Aschauer [4] examined the dynamic links between productivity and different forms of government spending in the case of G-7 countries. He found that shift in government spending from investment in fixed assets (mainly infrastructure) to consumption led to significant decrease in the level of output.

The positive impact of fixed assets on economic growth has been reported by many authors. De Long and Summers [12] found strong causal link running from equipment investment to economic growth. In their cross section of nations each percent of GDP invested in equipment raised GDP growth rate by 1/3 of a percentage point per year, which was interpreted by the authors as a fact revealing that the marginal product of equipment is about 30 percent per year. In their later contribution, De Long and Summers [13] applied more recent data and found that equipment positively affects productivity. Easterly and Rebelo [17] applied cross country data and found that fixed investment in transport and communication has a positive effect on

economic growth. Similar links between telephones (one of the physical measures of infrastructure, which, in general, is a dominating component of gross fixed capital formation) and economic growth were established by Canning et al. [9]. The cross country contribution by Sanchez-Robles [36] in turn proved that electricity generating capacity and road length were found to have a positive impact on GDP.

On the other hand, there are also empirical studies which provide basis to claim that causality runs in opposite direction, i.e., from GDP growth to fixed capital. Blomstrom et al. [7] performed an analysis of 101 countries using five-year averages of fixed investment shares and GDP growth for the years 1965–1985. They found evidence of unidirectional causality from GDP growth rates to investment rates, but not vice versa. Carrol and Weil [11] performed analogous analysis for 64 countries obtaining a similar general conclusion.

Finally, there is a group of papers which report causality running from gross fixed capital formation (or its major components) to GDP, however, the sign of this impact is found to be negative. The latter means that in some economies fixed capital is above its growth-maximising level, which causes slowdown of the rate of economic growth. Devarajan et al. [14] examined 43 low- and middle-income countries using data covering the period from 1970 to 1990. They found that fixed capital expenditures – usually thought to be the foundation of economic development – may have been excessive in the economies examined. Moreover, their results confirmed that governments in developing countries being analyzed have been misallocating their resources. Ghali [19] examined the case of Tunisia – a developing country implementing the IMF debt-stabilization programmes – and showed that in the economy examined public investment was having a negative short-run impact on private investment and a negative long-run impact on both private investment and economic growth.

The papers presented above are only representatives of the main research streams. The body of literature concerned with importance of fixed assets in economic growth is expressly larger. Nevertheless, one can see that the previous empirical literature provided solid evidence to claim that dynamic interrelations between investment in fixed capital and economic growth are not unique and strongly depend on characteristics of specific economies. Previous papers have shown that simple assumption that the more the fixed assets the higher the economic growth may be clearly false, even for developing economies and long run perspective. Therefore, the examination of an

issue discussed in the case of Poland seems to be fully justified.

3. Main research conjectures

In this paper, abbreviations are used for all the variables. Table 1 contains some initial information².

To summarize, the empirical investigations of this paper were conducted in two research variants, i.e., a three-dimensional approach involving GDP , $EMPL$ and $GFCF$ in Poland in the period Q1 2000–Q4 2009 (full sample) and a two-dimensional approach involving GDP_{pc} and $GFCF_{pc}$ in Poland in the period Q3 2002–Q2 2008 (non-crisis subsample). Some additional details justifying the choice of variables for both periods are presented in the Appendix.

Table 1. Units, abbreviations and short description of variables examined.

Description of variable	Unit	Period	Abbreviation for variable
Real quarterly gross domestic product in Poland	mIn PLN	Full sample (Q1 2000–Q4 2009)	GDP
Employment in Poland based on quarterly Labour Force Survey	thousands	Full sample (Q1 2000–Q4 2009)	$EMPL$
Real quarterly gross fixed capital formation	mIn PLN	Full sample (Q1 2000–Q4 2009)	$GFCF$
Real quarterly gross domestic product per capita in Poland	mIn PLN	Non-crisis subsample (Q3 2002–Q2 2008)	GDP_{pc}
Real quarterly gross fixed capital formation per capita in Poland	mIn PLN	Non-crisis subsample (Q3 2002–Q2 2008)	$GFCF_{pc}$

The empirical research was performed for two samples, i.e., the full sample (Q1 2000–Q4 2009) and non-crisis subsample covering the period Q3 2002–Q2 2008. The motivation to analyze this specific subsample is twofold. First, the subsample examined covers the time of significant rise in gross fixed capital formation in Poland³. Secondly, in the case of full sample the application of recent method of testing for direction and sign of long run impact of fixed capital on GDP growth adopted from Canning and Pedroni [10] and modified in this paper was impossible⁴.

The main goal of this paper is to examine the structure of causal dependences between economic growth and fixed capital in Poland in the last decade. The first step in causality analysis is test for the stationarity of all the variables under study. This is the crucial precondition of traditional causality testing. Since GDP, the situation in the labour market and the size of fixed capital were, in general, dynamically changing in the last decade, one may formulate the following:

Conjecture 1: *All time series under study (for respective periods) are nonstationary.*

² Details on dataset applied are presented in Section 4.

³ Starting from the third quarter of 2002 Polish economy has gone out from crisis which started in 2001. Moreover, at the Copenhagen European Council held on 12 and 13 December 2002, the negotiations between Polish government and EU authorities were formally ended. These facts clearly opened a gate for increase in investment in fixed assets (modernization of equipment, investment in public infrastructure). However, the upward tendency was stopped by the crisis of September 2008.

⁴ This approach (see the Appendix) is based on two theorems which originally (see [10]) were related to infrastructure capital. This paper contains not only adaptation of these theorems to overall fixed capital but also slight extension of the idea discussed (see the Appendix, point T7). In general, the use of both theorems requires fulfilment of three assumptions, which are formally verified for both periods analyzed.

Economic theory (production functions, growth models) predicts a strong dependence between labour input and production output as well as between various forms of capital (especially fixed capital) and output. Therefore, by analogy, one can presume the existence of short run causality between these two sets of variables in the Granger sense. Since dependences based on production functions are usually expressed by monotone increasing functions (with respect to employment or capital) feedback, i.e., a mutual Granger causality between employment and GDP as well as between fixed capital formation and economic growth, can be expected. The existence of these links

implies short run feedback between employment and fixed capital. However, taking into account the law of diminishing marginal returns it is hard to expect that these dependences are of linear nature. Therefore, it seems reasonable to formulate:

Conjecture 2: *In the period 2000–2009 there was a short run feedback between GDP and GFCF as well as between EMPL and GFCF. The nature of these dependences was rather nonlinear.*

As already mentioned the period 2000–2009 was the time of dynamic changes in Polish economy. The Stock Market Crash of 2000–2002 (IT bubble) caused the loss of 5 trillion USD in the market value of companies from March 2000 to October 2002. On 11 September 2001 terrorist destruction of the World Trade Center Twin Towers speeded up the stock market crash. The NYSE suspended trading for four sessions. Moreover, in the year 2001 Polish budget deficit exploded and reached by the beginning of August 2001 the level of 100 billion USD. Release of this unexpected information by Polish finance minister Jarosław Bauc caused immediately deep drop of exchange rate of Polish zloty, capital flight from Poland and crash on Warsaw Stock Exchange. In consequence, the investment outlays have been cut and GDP growth rate dropped. The aforementioned consequences of EU accession and crisis of September 2008 caused further fluctuations in gross fixed capital formation in Poland. However, contrary to the fixed capital formation, the GDP in Poland exhibited a stable upward tendency in the last decade.

These facts lead to some doubts about the possibility of existence of long run impact of GFCF on economic growth in Poland. Thus, it is likely that the following conjecture might be true:

Conjecture 3: *In the period 2000–2009 there was no significant long run impact of GFCF on GDP.*

On the other hand, in the non-crisis subsample one could observe a stable rise in GFCF, EMPL and GDP time series. Moreover, in this case all modelling assumptions required for application of recent theoretical method of testing for direction and sign of long run causal dependences developed by Canning and Pedroni [10] were fulfilled⁵. At this place it is also important to underline that examining the long run impact of fixed capital on GDP is especially important, since short run causal link may be related to business cycle or multiplier effects and die out without having a lasting effect on economic growth.

Therefore, it seems to be especially important to examine the following:

Conjecture 4: *In the non-crisis subsample (Q3 2002–Q2 2008) there was a significant and positive long run feedback between $GFCF_{pc}$ and GDP_{pc} in Poland.*

Beside the use of modification of Canning and Pedroni's [10] procedure the standard impulse response (IR) methodology was additionally applied in order to test Conjecture 4, despite the fact that vector autoregression based estimates of IR analysis are often criticised due to high sensitiveness to misspecification of the underlying unit root and cointegration properties of the data, which lead to serious inaccuracy of results, especially for long horizons (see, e.g., [18] and [35]). On the other hand, contrary to Canning and Pedroni's [10] procedure the IR analysis allows the magnitude of shocks to be measured. Therefore, the application of these two different methods of measuring the long run impact of fixed capital formation on economic growth seems to be especially important in terms of robustness and validation of empirical results.

In addition, testing the above conjectures is believed to provide some details about impact of economic crises and EU accession on the structure of causal dependences between fixed assets and GDP in Poland in the last decade. In general, the suppositions reflected in hypotheses 2–4 lead to formulation of the last conjecture:

Conjecture 5: *The economic crises of August 2001 and September 2008 as well as process of integration with economic and political structures of European Union had a significant impact on the nature of causal dependences between gross fixed capital formation and GDP in Poland in the last decade, especially in terms of long run equilibriums.*

The hypotheses listed above will be tested by some recent causality tests and procedures. The details of the testing methods will be shown later (Section 5). The test outcomes depend to some extent on the testing methods applied, thus testing the robustness of all empirical findings is one of the main goals of this paper. Before describing the methodology, in the next section brief characteristics of all the time series included in this study are presented.

4. The dataset and its properties

The first part of this section contains a description of the dataset applied. In subsection 4.2, the stationarity prop-

⁵ See the Appendix.

erties of all the time series are examined. The identification of the orders of integration of the time series under study is a crucial stage of causality analysis.

4.1. Description of the dataset

The chosen dataset includes quarterly data on GDP, employment and gross fixed capital formation in Poland in the period Q1 2000–Q4 2009⁶. Moreover, the specification of the aforementioned theoretical procedure (see the Appendix) encourages application of GDP per capita and gross fixed capital formation per capita for reduced sample (Q3 2002–Q2 2008). Thus, the dataset contains five variables with 40 (full sample) or 24 observations (non-crisis subsample). In order to remove the impact of inflation, GDP and gross fixed capital formation were calculated at constant prices of the year 2000.

Since the time series of GDP, employment and gross fixed capital formation were all characterized by significant quarterly seasonality, and this feature often leads to spurious results of causality analysis, the X-12 ARIMA procedure (which is currently used by the U.S. Census Bureau for seasonal adjustment) of Gretl software was applied to adjust each variable⁷. Next, the *GDP* per capita and *GFCF* per capita time series were constructed using seasonally adjusted series of GDP, gross fixed capital formation and employment⁸. Finally, each variable was transformed into logarithmic form, as this operation allows application of linear form of classical growth models. Moreover, this Box–Cox transformation may stabilize variance and therefore improve the statistical properties of the data, which is especially important for parametric tests.

The important point that distinguishes this paper from previous contributions on fixed capital and economic growth is the application of (less aggregated) quarterly data. This is mainly because the data necessarily covered only the recent few years and therefore

a causality analysis based on annual data could not have been carried out due to the lack of the degrees of freedom. However, as shown in some papers ([23]) the application of low frequency data (e.g., annual) may seriously distort the results of Granger causality analysis because some important interactions may stay hidden.

The originality of this paper is also related to another fact. As far as the author knows this is the first study which analyses dynamic interactions between quarterly gross fixed capital formation and economic growth in Poland, which is a leading country in the CEE region. As already mentioned, majority of the previous papers were based on application of panel datasets and annual data. The lack of reliable time series of sufficient size is a common characteristic of most post-Soviet economies and causes serious problems to the researcher. However, the application of reliable quarterly data and modern econometric techniques (described in detail in Section 5) provided a basis for conducting this leading research for one of the transitional European economies.

The initial part of data analysis contains some descriptive statistics of all the variables. Table 2 contains suitable results obtained for seasonally adjusted and logarithmically transformed time series.

In order to conduct a comprehensive preliminary analysis the charts for all the variables under study should also be analyzed. Figure 1 contains suitable plots.

In the years 2000–2009 there was a relatively stable development of the Polish economy since *GDP* exhibited an upward tendency. One cannot forget that the Polish economy was one of the few that managed to avoid an undesirable impact of the 2001 and 2008 crises. However, until the third quarter of 2002 and after September 2008 one could observe a relatively low rate of growth of the Polish economy in comparison to other quarters. Similarly, for *EMPL* in the period analyzed there was a stable rise between 2003 and 2008, while slight drops were observed until the end crisis of 2001 and after beginning the crisis of September 2008. These tendencies are even more evident for *GFCF* time series. In this case, both crises caused significant drops in gross fixed capital formation. However, as Fig. 1 shows the finalisation of negotiations with EU and first four years of participation in European systems and structures (political and economic) was a time of significant rise in both employment and *GFCF* in Poland. Finally, one should note that these conclusions are also reflected in charts of *GDP_{pc}* and *GFCF_{pc}*, where significant upward tendency is clearly present.

⁶ The Central Statistical Office in Poland (GDP, employment) and The International Monetary Fund (gross fixed capital formation) provided the data applied in this paper.

⁷ The seasonal adjustment is also important for assumptions reflected in (2) and (3) in the Appendix. Namely, the quarterly seasonality (e.g., parts of the year, annual settlements, etc.) may be an important factor determining the amount of funds going to fixed investment during a year. However, when variables under study are seasonally adjusted, equations (2) and (3) are indeed reasonable, as they describe the dynamics of general trends. All these suppositions were verified using regression-based approach (see the Appendix).

⁸ As suggested in [10] the workforce was identified with population.

Table 2. Descriptive statistics of variables examined.

Variable Quantity	Full sample			Non-crisis subsample	
	<i>GDP</i>	<i>EMPL</i>	<i>GFCF</i>	<i>GDP_{pc}</i>	<i>EMPL_{pc}</i>
Minimum	12.11	9.51	10.41	9.51	7.79
1 st quartile	12.15	9.53	10.45	9.60	7.83
Median	12.26	9.57	10.58	9.63	7.86
3 rd quartile	12.41	9.63	10.83	9.67	8.10
Maximum	12.49	9.68	10.93	9.70	8.18
Mean	12.28	9.58	10.62	9.63	7.92
Std. deviation	0.12	0.09	0.18	0.04	0.12
Skewness	0.27	0.48	0.42	-0.42	0.74
Excess kurtosis	-1.40	-1.12	-1.36	-0.71	-0.97

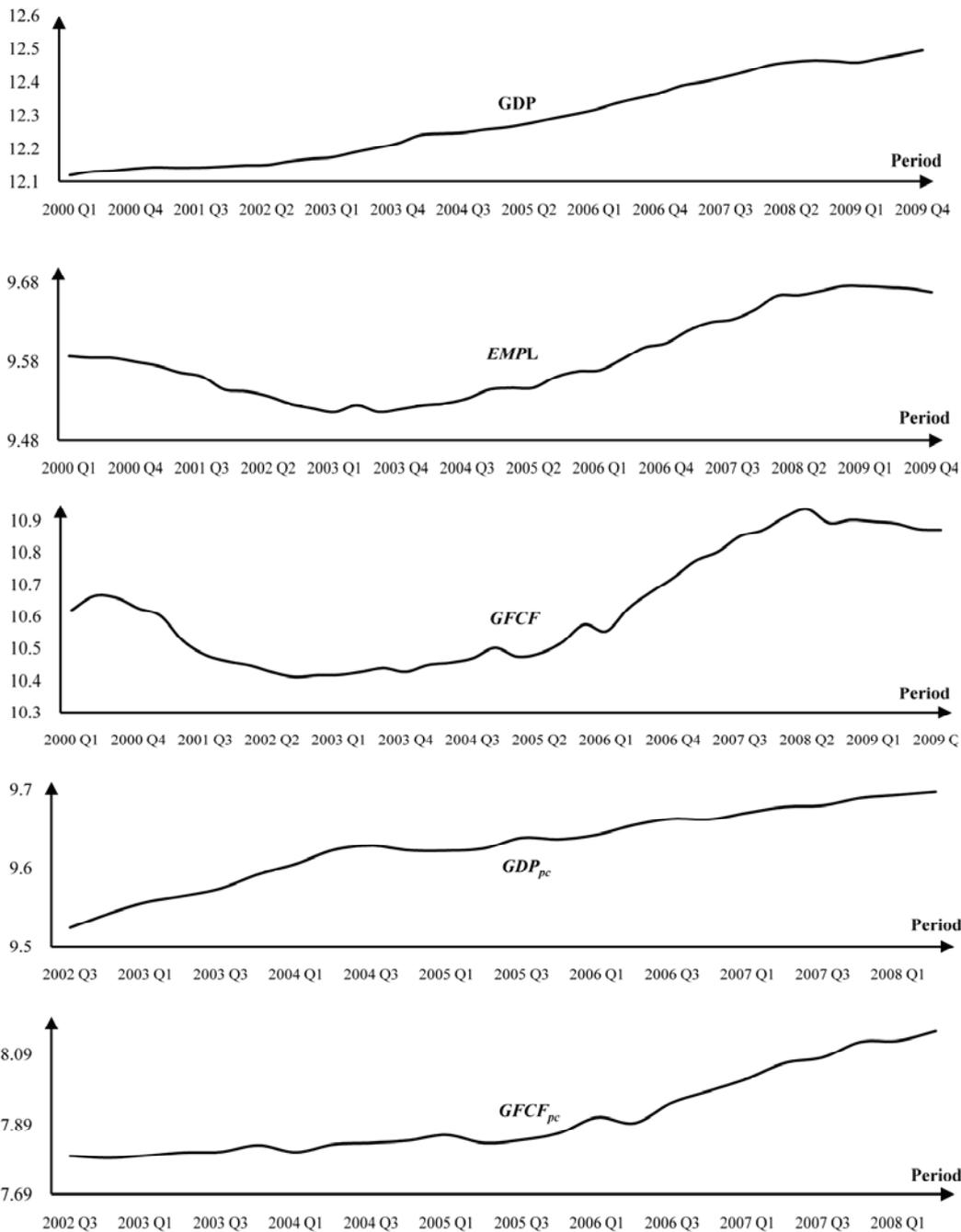


Fig. 1. Plots of time series examined.

In the next subsection, the descriptive analysis of the time series included in the dataset will be extended by stationarity testing. This is a crucial stage of empirical research of this paper (it is a precondition for traditional causality testing as well as for theoretical procedure described in the Appendix), thus it should be carried out with great precision.

4.2. Stationarity properties of the dataset

In the first step of this part of research, an Augmented Dickey–Fuller (ADF) unit root test was conducted. Before conducting the test, the maximal lag length was set at a level of 6 and then the information criteria (namely, the AIC, BIC and HQ) were applied to choose the optimal lag. However, the application of the ADF test is related to two serious problems. First, this test tends to under-reject the null hypothesis pointing at nonstationarity too often⁹. Second, the outcomes of ADF test are relatively sensitive to an incorrect establishment of lag parameter. This is why the Kwiatkowski–Phillips–Schmidt–Shin (KPSS)¹⁰ test was conducted to confirm or undermine the results of the ADF one. If two unit root tests lead to contradictory conclusions, a third test must be applied to make a final decision about the stationarity of time series. In this paper, the Phillips–Perron (PP) test was additionally applied. A non-parametric method of controlling for serial correlation is used when testing for a unit root by means of PP test. In this case the null hypothesis once again refers to nonstationarity.

Table 3 contains summary of the results of the stationarity analysis. Bold face indicates finding nonstationarity at a 5% level.

Table 3 shows that all time series were found to be nonstationary around constant at a 5% level. All three tests pointed at nonstationarity for every analyzed time series except for *EMPL*. In this case, nonstationarity was confirmed by two of the three tests carried out. Anyhow, Conjecture 1 should clearly be accepted. Some further calculations (conducted for first differences) confirmed that all variables under study are $I(1)$ ¹¹.

5. Methodology

In this paper, several econometric tools were applied to test for both linear and nonlinear Granger causality between GDP and gross fixed capital formation in Polish economy. The main part of the research was conducted for full sample in a three-dimensional variant involving *GDP*, *EMPL* and *GFCF* as well as for non-crisis subsample in a two-dimensional variant involving GDP_{pc} and $GFCF_{pc}$.

5.1. Linear short and long run Granger causality tests

The idea of Granger [20] causality is well known and has been commonly applied in previous empirical studies, thus there is no need to explain it in detail. By and large, this concept is used to investigate whether a knowl-

Table 3. Results of stationarity analysis.

Test type Variable	ADF				KPSS		PP	
	with constant		with constant and linear trend		with constant ^a	with constant and linear trend ^b	with constant	with constant and linear trend
	<i>p</i> -value	Optimal lag	<i>p</i> -value	Optimal lag	Test statistic		<i>p</i> -value	
<i>GDP</i>	0.99	1	0.19	1	1.08	0.23	0.98	0.52
<i>EMPL</i>	0.00	4	0.00	4	0.78	0.25	0.92	0.60
<i>GFCF</i>	0.53	3	0.63	0	0.52	0.18	0.90	0.66
GDP_{pc}	0.17	1	0.19	0	0.69	0.16	0.09	0.21
$GFCF_{pc}$	0.99	0	0.89	0	0.64	0.19	0.92	0.91

^a critical values: 0.347 (10%), 0.463 (5%), 0.739 (1%).

^b critical values: 0.119 (10%), 0.146 (5%), 0.216 (1%).

⁹ Low power against stationary alternatives has been frequently reported by many authors, see, e.g., [1].

¹⁰ In contrast to the ADF test, the null hypothesis of a KPSS test refers to the stationarity of the time series.

¹¹ It should be underlined that detailed results of all computations which are not presented in the text (usually to save space) in detailed form are available from the author upon request.

edge of the past and current values of one stationary variable may improve prediction of future values of another one or not. Stationarity is a crucial precondition in this context, as unfulfillment of this assumption may lead to misleading conclusions by a traditional linear causality test, which has been pointed out in previous empirical ([21]) and theoretical ([34]) deliberations.

Since all the variables were found to be integrated of order one three econometric methods suitable for testing for linear short and long run Granger causality in this context, namely, a traditional analysis of the vector error correction model (VECM), the sequential elimination of insignificant variables in VECM and the Toda–Yamamoto method were applied. Application of a variety of methods is believed to be especially important in terms of robustness and validation of empirical findings. All these methods are well known, thus they will only be briefly described.

In this paper, the Trace and Maximal Eigenvalue variants of Johansen cointegration test were applied¹². As shown by Granger [22] the existence of cointegration implies long run Granger causality in at least one direction. To establish the direction of this causal link one should estimate a suitable VEC model and check (using a *t*-test) the statistical significance of the error correction terms. Testing the joint significance (using an *F*-test) of lagged differences provides a basis for short run causality investigations.

It is important to note that causality testing based on the application of an unrestricted VEC model has got a serious drawback. In practical applications it is often necessary to use a relatively large number of lags in order to model the dynamic multidimensional process in a proper way and avoid the consequences of the autocorrelation of residuals. However, the more lags the less degrees of freedom, which in turn may have an undesirable impact on test performance, especially for small samples. Furthermore, testing for linear causality using a traditional Granger test often suffers because of possible multicollinearity, especially for dimensions higher than two. This is why the sequential elimination of insignificant variables is often additionally performed for each VECM equation separately in order to test for short and long run linear Granger causality. Each step of this procedure leads to omission of the variable with the highest *p*-value (*t*-test). The procedure ends when all remaining variables have a *p*-value no greater than a fixed value (in this paper, it was 0.10). The reader may find more technical details of this approach in [25].

¹² These tests have been commonly applied in recent empirical studies. The reader may find a detailed description of these methods in [29] and [30].

An alternative method for testing for linear Granger causality was formulated by Toda and Yamamoto [37]. This approach has been commonly applied in recent empirical studies (see, e.g., [38]) as it is free of complicated pretesting procedures and relatively simple to perform, which may be of great advantage when dealing with nonstationary variables. An important feature of the Toda–Yamamoto (TY) approach is the fact that this procedure is applicable even if the variables under study are characterized by different orders of integration or when the cointegration properties of the data are uncertain¹³. On the other hand, TY approach does not enable us to examine long run causality as it is in the case of VEC-based analysis.

Toda–Yamamoto approach requires the establishment of a parameter p_1 (order of Vector AutoRegression (VAR) model), parameter p_2 (highest order of integration of all variables examined) and then a calculation of the standard Wald test applied for the first p_1 lags of the augmented VAR($p_1 + p_2$) model. After ensuring that some typical modelling assumptions¹⁴ hold true for the augmented model the test statistic has the usual asymptotic $\chi(p_1)$ distribution ([37]). However, since the sample examined is relatively small the asymptotically *F*-distributed variant of TY test statistic was applied as it performs better in such cases ([33]).

The aforementioned parametric methods have got two serious drawbacks. First, the application of asymptotic theory may lead to spurious results if suitable modelling assumptions do not hold. Secondly, the distribution of the test statistic may be significantly different from an asymptotic pattern when dealing with extremely small samples, regardless of the modelling assumptions.

One of the possible ways of overcoming these difficulties is the application of the bootstrap technique. This method is used for estimating the distribution of a test statistic by resampling data. Since the estimated distribution depends only on the available dataset it seems reasonable to expect that bootstrapping does not require such strong assumptions as parametric methods. However, this approach is likely to fail in some specific cases and therefore cannot be treated as a perfect tool for solving all possible model specification problems ([28]).

¹³ In such cases, a standard linear causality analysis cannot be performed by the direct application of a basic VAR or VEC model. On the other hand, differencing or calculating the growth rates of some variables allows the traditional approach to be used, but it may also cause a loss of long run information and lead to problems with the interpretation of test results.

¹⁴ A list of these assumptions may be found in [33].

Since heteroscedasticity may have an undesirable influence on the bootstrap test ([28]) the resampling procedure was based on leveraged residuals¹⁵. In recent years, the academic discussion on the establishment of the number of bootstrap replications has attracted considerable attention (see, e.g., [28] and [32]). In this paper, the procedure of establishing the number of bootstrap replications recently presented by Andrews and Buchinsky [2] was applied. In each case, the aim was to choose such a value of the number of replications which would ensure that the relative error of establishing the bootstrap critical value (at a 10% significance level) would not exceed 5% with a probability equal to 0.95¹⁶.

5.2. Impulse response analysis

As a complement to standard linear Granger causality tests, an Impulse Response (IR) analysis was also performed. Linear Granger causality tests provide an opportunity for the establishment of the direction of any causal link between variables, but they do not say anything about the signs (and magnitudes) of this relationship. Therefore, the linear Granger causality testing is often supplemented with the impulse response analysis as it allows predicting the reaction of the dynamic system to the shock in one or more variables¹⁷. In order to examine the nature of this reaction (which is transmitted through the dynamic structure of the VAR model) the residual impulse response function was based on one standard deviation shocks¹⁸.

5.3. Nonlinear Granger causality test

Generally, the motivation to use nonlinear methods in testing for Granger causality is based on two facts. First, the traditional linear Granger causality was found to have extremely low power in detecting certain kinds of nonlinear causal interrelations ([8], [24]). Second, since linear methods are mainly based on checking the statistical significance of suitable parameters only in a mean equation, testing for causality in higher-order structures (like variance) is impossible ([15]).

In this paper, a nonlinear causality test proposed by Diks and Panchenko [16] was applied with some typical values of the technical parameters, which have been commonly used in previous papers (e.g., [16], [25]). The bandwidth (denoted as b_{DP}) was set at a level of 0.5, 1 and 1.5 while the common lag parameter (denoted as l_{DP}) was set at the order of 1 and 2¹⁹.

Since previous studies provided evidence that the presence of heteroscedasticity leads to over-rejection of the nonlinear test discussed ([16]), all the time series examined were tested for the presence of various heteroscedastic structures (using, inter alia, White's test and a Breusch–Pagan test).

6. Empirical results

In this section, the results of short and long run linear Granger causality analysis as well as the outcomes of nonlinear causality tests are presented. The main goal of these empirical investigations was to examine the structure of the dynamic relationships between fixed capital and GDP in Poland with special attention paid to examination of five research hypotheses presented in subsection 3. The empirical research was performed for the full sample (Q1 2000–Q4 2009) and non-crisis subsample (Q3 2002–Q2 2008).

6.1. Results obtained for full sample

As already mentioned in the case of the period Q1 2000–Q4 2009 the adaptation of Canning and Pedroni's [10] procedure was impossible due to unfulfillment of required assumptions (see the Appendix for details). Thus, the examination of short and long run causalities between GDP and fixed assets was performed with application of traditional econometric methods described in Section 5. As the two-dimensional approach (only *GDP* and *GFCE*) may lead to spurious results due to omission of important variables, the research was performed in a three dimensional framework²⁰. Since *GDP*, *EMPL* and *GFCE* were all found to be I(1) a cointegration analysis was first performed for these variables. All five

¹⁵ A detailed description of resampling procedure applied in this paper may be found in [26].

¹⁶ The Gretl script including implementation of all the linear methods mentioned with asymptotic- and bootstrap-based variants is available from the author upon request.

¹⁷ See, e.g., [23].

¹⁸ Since in each case analyzed no Wold instantaneous causality was found, no Cholesky decomposition was used. The reader may find the theoretical background of these concepts in [33] and [27].

¹⁹ The reader may find a detailed description of the role of these technical parameters and the form of test statistic in [16]. Moreover, practical suggestions presented in [25] and concerned with the nonlinear procedure discussed were also used in this paper.

²⁰ The three-dimensional approach (involving employment) not only improves the statistical properties of the model analyzed but is also strongly justified by economic theory (basic growth models).

Table 4. Results of cointegration analysis for *GDP*, *EMPL* and *GFCF* variables.

Hypothesized number of cointegrating vectors	Eigenvalue	Johansen Trace test		Johansen Maximal Eigenvalue test	
		Trace statistic	<i>p</i> -value	Maximal Eigenvalue statistic	<i>p</i> -value
Zero	0.46	35.51	0.00	22.02	0.03
At most one	0.31	13.49	0.09	13.49	0.06
At most two	0.00	0.00	0.96	0.00	0.96

Table 5. Analysis of causal links between *PAT*, *GDP* and *EMPL* variables (VEC models).

Short run								
Null hypothesis	<i>p</i> -value							
	Unrestricted				Sequential			
	Asymptotic		Bootstrap ^a		Asymptotic		Bootstrap ^a	
<i>GFCF</i> → <i>GDP</i>	0.33		0.37		0.00		0.00	
<i>GDP</i> → <i>GFCF</i>	0.23		0.31		0.07		0.02	
<i>GFCF</i> → <i>EMPL</i>	0.21		0.18		0.00		0.00	
<i>EMPL</i> → <i>GFCF</i>	0.36		0.26		0.05		0.00	
<i>GDP</i> → <i>EMPL</i>	0.49		0.31		0.00		0.00	
<i>EMPL</i> → <i>GDP</i>	0.98		0.81		NCL		NCL	
Long run								
Equation	<i>p</i> -value of <i>EC</i> ₁ component				<i>p</i> -value of <i>EC</i> ₂ component			
	Unrestricted		Sequential		Unrestricted		Sequential	
	Asymptotic	Bootstrap ^a	Asymptotic	Bootstrap ^a	Asymptotic	Bootstrap ^a	Asymptotic	Bootstrap ^a
<i>GDP</i>	0.22	0.31	NCL	NCL	0.94	0.69	NCL	NCL
<i>GFCF</i>	0.00	0.00	0.08	0.01	0.46	0.29	NCL	NCL
<i>EMPL</i>	0.00	0.00	0.02	0.00	0.01	0.00	0.05	0.08

^a Number of bootstrap replications established using Andrews and Buchinsky (2000) method varied between 1549 and 2759.

possibilities listed in [31] were analyzed to specify the type of deterministic trend. In view of the results presented in subsection 4.2 (no trend-stationarity) Johansen's third case was assumed, i.e., the presence of a constant in both the cointegrating equation and the test VAR. In the next step, the information criteria (i.e., AIC, BIC, HQ) were applied to establish the appropriate number of lags. The final lag length was set at a level of 5²¹. Table 4 contains the results of Johansen cointegration tests.

One can see that both variants of Johansen test provided solid evidence (at a 10% significance level) for claiming that the dimension of cointegration space is equal to two. Moreover, the hypothesis that the smallest eigenvalue is equal to zero was clearly accepted (last row in Table 4), which additionally validates the results of the previously performed unit root

tests²². Next, a suitable VEC model was estimated model assuming 4 lags (for first differences) and two cointegrating vectors²³. Table 5 contains *p*-values obtained while testing for linear short and long run Granger causality using an unrestricted VEC model and the sequential elimination of insignificant variables²⁴.

The results obtained for the unrestricted VEC model did not provide a basis for claiming that short run Granger causality run in any direction in the period under study. On the other hand, the sequential elimination of insignificant variables led to the conclusion that in the short run there was a feedback between *GDP* and *GFCF* as well as between *EMPL* and

²¹ The maximal lag length (for levels) was set at a level of 6. BIC criterion pointed at one lag, however the results of Ljung-Box Q-test confirmed that in the case of one lag residuals were significantly autocorrelated, which in turn may lead to serious distortion of the results of the causality analysis.

²² It is a well known fact that the case of full rank refers to stationarity of all time series considered ([33]).

²³ The first vector (denoted as *EC*₁) involved *GDP* and *GFCF* while the second one (*EC*₂) involved *EMPL* and *GFCF*.

²⁴ Throughout this paper, the notation "*x* → *y*" is equivalent to "*x* does not Granger cause *y*". Moreover, the symbol "NCL" is the abbreviation of "No coefficients left". Finally, bold face always indicates finding a causal link in a particular direction at a 10% significance level.

Table 6. Analysis of causal links between the *GFCF*, *GDP* and *EMPL* (TY approach).

Null hypothesis	p-value	
	Asymptotic	Bootstrap ^a
<i>GFCF</i> \rightarrow <i>GDP</i>	0.72	0.63 (N = 1679)
<i>GDP</i> \rightarrow <i>GFCF</i>	0.51	0.35 (N = 2179)
<i>GFCF</i> \rightarrow <i>EMPL</i>	0.36	0.47 (N = 1839)
<i>EMPL</i> \rightarrow <i>GFCF</i>	0.08	0.15 (N = 1659)
<i>GDP</i> \rightarrow <i>EMPL</i>	0.81	0.61 (N = 1659)
<i>EMPL</i> \rightarrow <i>GDP</i>	0.91	0.83 (N = 2059)

^a Parameter *N* denotes the number of bootstrap replications established according to the Andrews and Buchinsky (2000) procedure.

Table 7. Analysis of nonlinear causal links between *GFCF*, *GDP* and *EMPL* variables.

Null hypothesis	p-value											
	$b_{DP} = 0.5, l_{DP} = 1$		$b_{DP} = 1, l_{DP} = 1$		$b_{DP} = 1.5, l_{DP} = 1$		$b_{DP} = 0.5, l_{DP} = 2$		$b_{DP} = 1, l_{DP} = 2$		$b_{DP} = 1.5, l_{DP} = 2$	
<i>GFCF</i> \rightarrow <i>GDP</i>	0.07	0.35	0.18	0.42	0.24	0.27	0.24	0.29	0.30	0.49	0.26	0.71
	0.59		0.39		0.46		0.17		0.07		0.06	
<i>GDP</i> \rightarrow <i>GFCF</i>	0.65	0.57	0.56	0.41	0.70	0.29	0.71	0.55	0.57	0.82	0.92	0.58
	0.09		0.89		0.34		0.51		0.32		0.42	
<i>GFCF</i> \rightarrow <i>EMPL</i>	0.10	0.35	0.65	0.52	0.82	0.71	0.65	0.39	0.29	0.48	0.34	0.19
	0.55		0.48		0.43		0.35		0.31		0.08	
<i>EMPL</i> \rightarrow <i>GFCF</i>	0.86	0.48	0.51	0.59	0.57	0.38	0.24	0.48	0.17	0.72	0.08	0.25
	0.45		0.37		0.58		0.19		0.27		0.27	
<i>GDP</i> \rightarrow <i>EMPL</i>	0.91	0.81	0.75	0.39	0.75	0.33	0.71	0.46	0.63	0.37	0.45	0.52
	0.21		0.45		0.78		0.71		0.39		0.18	
<i>EMPL</i> \rightarrow <i>GDP</i>	0.89	0.45	0.82	0.32	0.81	0.24	0.32	0.63	0.25	0.35	0.82	0.55
	0.78		0.32		0.53		0.29		0.62		0.48	

GFCF, which also proves that for small samples and large lags results of estimation of unrestricted VECM may indeed be significantly different from outcomes of sequential elimination. Moreover, *GDP* was found to Granger cause *EMPL*. It is worth mentioning that all the results for sequential elimination were found in asymptotic- and bootstrap-based research variants.

In all the research variants, the *EC*₁ and *EC*₂ components were found to be insignificant in the *GDP* equation, which provides a basis for claiming that *GFCF* and *EMPL* did not have a long run impact on economic growth in Poland in the period analyzed. This way important hypothesis 3 should also be accepted. On the other hand, *GDP* was found to have a long run impact on fixed capital. Furthermore, both the *GDP* and *GFCF* were found to cause employment in the long run.

For the sake of comprehensiveness the Toda–Yamamoto approach for testing for causal effects between *GFCF*, *GDP* and *EMPL* was additionally applied. The outcomes of the TY procedure are presented in Table 6.

In general, the results presented in Table 6 are in line with outcomes presented in the previous table. In augmented VAR model applied in TY approach six

lags had to be used which caused (together with small sample) that no dynamic links were found to be statistically significant at a 10% level (causality from *EMPL* to *GFCF* was found to be significant only in asymptotic variant).

The final part of the causality analysis was based on nonlinear test and was performed for the residuals resulting from all linear models, i.e., the residuals of unrestricted VECM, the residuals resulting from individually (sequentially) restricted equations and the residuals resulting from the augmented VAR model applied in the Toda–Yamamoto method²⁵. Table 7 presents results obtained while testing for nonlinear Granger causality between *GFCF*, *GDP* and *EMPL*. For each combination of *b*_{DP} and *l*_{DP} three *p*-values are presented²⁶. Since in all the cases examined no

²⁵ Since the structure of linear connections had been filtered out after an analysis of linear models, the residuals are believed to reflect strict nonlinear dependencies (see, e.g., [5]).

²⁶ For each pair of *b*_{DP} and *l*_{DP} *p*-values are presented according to the following rule: Upper row: *p*-value for residuals of unrestricted VEC model (left), *p*-value for residuals of sequentially restricted equations (right); Lower row: *p*-value for residuals of TY procedure.

significant evidence of heteroscedasticity was found, no filtering was used.

As one can see test results (performed for residuals from augmented VAR model of TY procedure) provided evidence to claim that there was a nonlinear feedback between *GFCF* and *GDP*. Moreover, the analysis of residuals from unrestricted VECM confirmed nonlinear feedback between fixed capital formation and employment.

In general, the results of all the methods provided relatively strong support for claiming that in the short run there was a feedback between *GFCF* and *GDP* as well as between *GFCF* and *EMPL*. These dependences were indicated by analysis of sequentially restricted equations and nonlinear tests. The low power of significance tests applied for small samples was also reported since both these feedback effects were not supported by outcomes of unrestricted linear models. Therefore, Conjecture 2 should also be accepted. Moreover, it is worthwhile to underline that this conclusion, in general, was confirmed by the results of two completely different methods (a two-stage analysis of the VEC model and the TY approach with respective nonlinear tests), which validates this major conclusion and confirms its robustness when exposed to statistical tools.

On the other hand, the lack of long run impact of *GFCF* on *GDP* does not seem to be a consequence of small sample and power properties of significance tests applied, as there were no differences between results of long run causality testing in unrestricted and sequentially restricted variants of VEC-based analysis. The natural explanation for this phenomenon is that in analyzed period there were two economic crises (in August 2001 and in September 2008), finalization of negotiations with EU and first five years of participation in the structures of European Union. These events and processes caused significant shocks to outlays on fixed assets in Poland. Simultaneously, there was a stable and continuous rise in GDP during the whole period analyzed. It seems that because of these reasons the long run effect of fixed investment on economic growth in Poland was not reported in the period Q1 2000–Q4 2009.

6.2. Outcomes obtained for non-crisis subsample

As already mentioned, in the period Q3 2002–Q2 2008 general trends in *GDP* and *GFCF* time series were relatively similar. This was one of the factors which caused that all assumptions required for

the use of procedure presented in the Appendix were, in general, fulfilled. Therefore, in this part of the empirical research the modification of recent method of testing for the direction and sign of long run causal effects between economic growth and fixed capital developed by Canning and Pedroni [10] and modified in this paper was applied. In the first step, the cointegration analysis was performed for GDP_{pc} and $GFCF_{pc}$ variables²⁷. Table 8 contains results of Johansen tests performed under the assumption of Johansen's third variant and 3 lags (for variables in first differences).

As one can see, the variables examined were found to be cointegrated (at 5% significance level). In the next step, a suitable VEC model (with 3 lags and one cointegrating vector) was estimated. Finally, the theorems presented in the Appendix were used to investigate the long run dependences between both variables. Table 9 contains results of estimation of VEC model constructed for GDP_{pc} and $GFCF_{pc}$ variables as well as the main research conclusion.

Outcomes contained in Table 9 (estimation results) provided a basis for claiming that for GDP_{pc} and $GFCF_{pc}$ there was a significant feedback in the long run. Moreover, the estimation results and suitable theorems (see points T6 and T7 of Theorem 2 in the Appendix) lead to the conclusion that in the non-crisis period the signs of both these causal links were positive, which clearly supports Conjecture 4.

As already mentioned, Canning and Pedroni [10] used their procedure instead of applying standard IR response analysis as this way of measuring the sign of long run impact may often be more accurate. However, the magnitudes of dynamic impacts cannot be measured using this approach (it measures the directions and signs only). Therefore, IR analysis was additionally performed for GDP_{pc} and $GFCF_{pc}$ variables. Figure 2 presents responses for the period of 20 quarters.

As one can see the positive long run impact of per capita fixed capital formation on GDP_{pc} was confirmed by the results of standard IR analysis, i.e., a rise in fixed assets per capita causes growth of per capita GDP. Moreover, a rise in GDP_{pc} causes a permanent positive change in $GFCF_{pc}$. Both these results are in line with outcomes presented in Table 9, which were based on recent theoretical deliberations described in the Appendix.

²⁷ The preliminary part of cointegration analysis (i.e., specification of the type of deterministic trend, lag selection procedure) was performed in exactly the same way as in the case of three-dimensional model.

Table 8. Results of cointegration analysis for GDP_{pc} and $GFCF_{pc}$ variables.

Hypothesized number of cointegrating vectors	Eigenvalue	Johansen Trace test		Johansen Maximal Eigenvalue test	
		Trace statistic	<i>p</i> -value	Maximal Eigenvalue statistic	<i>p</i> -value
Zero	0.49	17.25	0.02	16.51	0.02
At most one	0.03	0.73	0.39	0.73	0.39

Table 9. Directions and signs of long run causal dependences between GDP_{pc} and $GFCF_{pc}$.

Equation	Coefficient of error term ^a	Ratio	Conclusion
GDP_{pc}	$\lambda_2 = -0.12$ [0.01; 0.00]	$-\frac{\lambda_2}{\lambda_1} = 0.38$	Positive long run feedback between per capita gross fixed capital formation and GDP per capita.
$GFCF_{pc}$	$\lambda_1 = 0.31$ [0.04; 0.02]		

^a Numbers in square brackets denote *p*-values obtained for asymptotic and bootstrap variants, respectively.

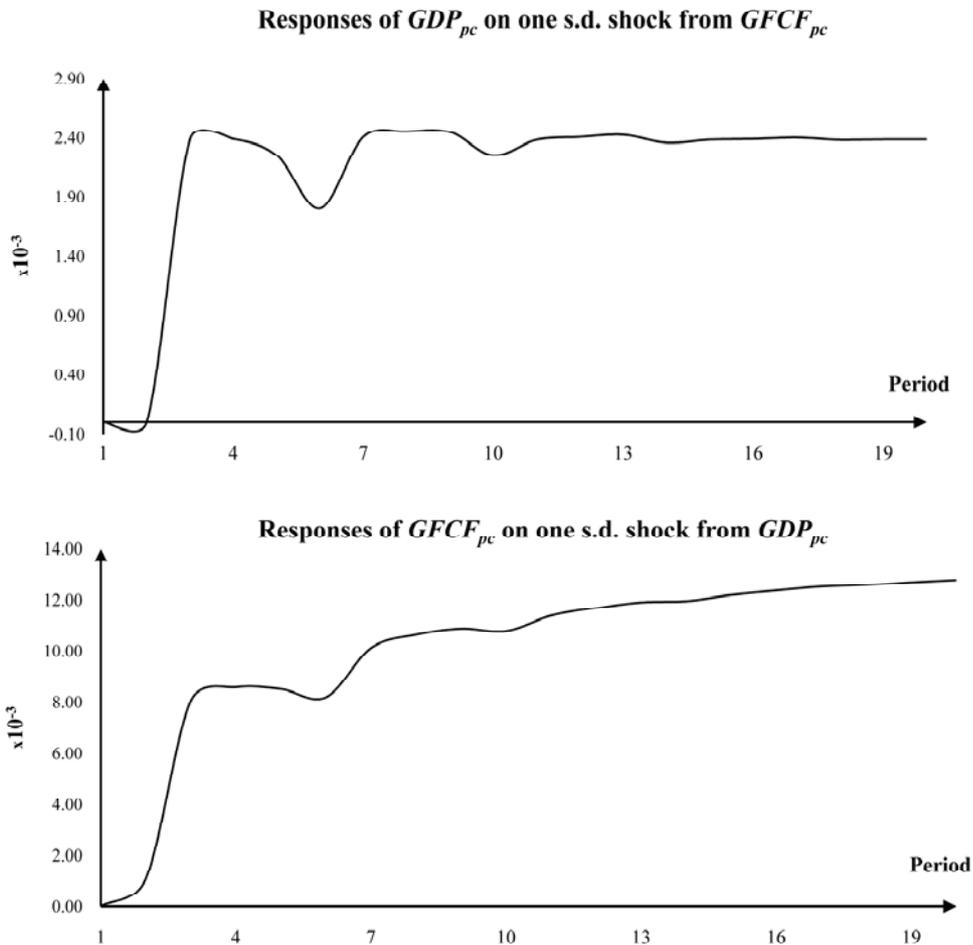


Fig. 2. Impulse response analysis for GDP_{pc} and $GFCF_{pc}$.

For the sake of comprehensiveness standard short run causality tests were also performed. Table 10 contains results of this analysis.

In general, results presented in Table 10 lead to conclusion that the short run causality run from $GFCF_{pc}$ to GDP_{pc} (evidence of causality in the opposite direc-

Table 10. Results of short run causality analysis between GDP_{pc} and $GFCF_{pc}$.

Null hypothesis	p-value					
	VEC-based approach				TY-based approach	
	Unrestricted		Sequential		Asymptotic	Bootstrap ^a
Asymptotic	Bootstrap ^a	Asymptotic	Bootstrap ^a			
$GFCF_{pc} \nrightarrow GDP_{pc}$	0.07	0.04	0.01	0.00	0.12	0.03
$GDP_{pc} \nrightarrow GFCF_{pc}$	0.19	0.24	NCL	NCL	0.09	0.19

tion was rather weak). This phenomenon was reported by both linear tests for short run causality (VEC- and TY-based approach). Finally, it should be noted that nonlinear causality was not found for residuals resulting from any analyzed model, thus results of Diks and Panchenko's tests are not presented in a separate table.

To summarize, standard econometric methods (IR analysis) confirmed outcomes obtained after application of theoretical procedure adopted from [10] and modified in this paper. This is especially important in terms of robustness of empirical results and surely validates major findings of this paper. In addition, the significant difference in structure of causal dependences between fixed capital and GDP in Poland in both periods analyzed (especially for long run links) provided strong evidence in favour of Conjecture 5.

7. Concluding remarks

The main goal of this paper was the examination of causal interdependencies between gross fixed capital formation and GDP in Poland. The research was performed on the basis of quarterly data for the period Q1 2000–Q4 2009 (full sample) as well as period Q3 2002–Q2 2008 (non-crisis subsample). In addition, the data on employment was used in empirical research in three- (variables in levels) and two-dimensional (per capita measures) approach, since application of workforce is required both from theoretical (production functions, growth models) and practical (statistical problems arising due to the omission of important variables) point of view. In order to conduct a comprehensive causality analysis both traditional methods (linear and nonlinear causality tests) as well as some recently developed econometric tools (an alternative way of measuring direction and sign of long run influence) were applied.

The analysis of full sample provided relatively strong support for claiming that for fixed capital and GDP as well as for fixed capital and employment

there was a feedback in the short run. Moreover, relatively solid evidence was found to claim that the nature of these links was rather nonlinear, which seems to be in line with the law of diminishing marginal returns. On the other hand, no long run effect of fixed assets on economic growth was found. This significant result was most likely caused by the fact that analyzed period was extremely dynamic time for Polish economy. The two economic crises (of 2001 and 2008) and first years of accession in political and economic structures of European Union caused significant shocks in the level of private and public investment purposed for fixed assets. On the other hand, one could observe stable rise in the GDP in the last decade. Both these phenomena caused that long run causality from fixed capital to economic growth was not indicated by traditional statistical tools applied (based on concept of cointegration, IR analysis) or recently developed alternative methods (theorems based on growth models).

The results obtained for non-crisis subsample confirmed the supposition that economic crises had seriously distorted the structure of causal dependences between per capita gross fixed capital formation and per capita GDP, especially in the long run. The application of recent alternative method of testing for long run dependences confirmed significant positive long term causal feedback between fixed assets and output. Establishing these crucial causal links is the most important finding of this paper. Moreover, solid evidence of robustness of these major empirical findings when exposed to statistical tools was also found.

The results of this paper have some important policy implication. First, the existence of positive long run influence of fixed assets on GDP in non-crisis period proves that this type of capital in Poland is still under its growth-maximising level. Therefore, the opinion that increasing fixed capital formation may hamper economic growth as it is extremely costly (higher current expenditures, long term investment, etc.) is clearly false. In other words, the sustainable economic growth in Poland seems to require increased investment in fixed capital. Sec-

ond, the differences in results of causality analysis (especially in the long run) between full and reduced sample confirm that in face of economic crises both the private and public sector reduce the size of investment in fixed capital in the first place. In addition, the results of this paper seem to prove that completion of negotiation with EU in 2002 was a trigger for rapid increase (as a result of EU financial support) in investment in various forms of fixed assets which in turn had a positive impact on economic growth in Poland in following years. Finally, since major part of gross fixed capital formation in Poland is purposed for investment in infrastructure, the main conclusions of this paper, although directly formulated for overall fixed capital, may be, at least to some extent, adopted for causal links between public infrastructure and economic growth. The latter seems to be a fine starting point for future research using disaggregated data on fixed investment.

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Appendix

A stylized growth model applied in this paper was adapted from [6]. Moreover, the suggestions of Canning and Pedroni [10] were also taken into consideration as they provided the basis for testing the sign of long run effects between specified type of capital and GDP without the need of using IR analysis (which often produces biased results, especially for small samples). The original paper of Canning and Pedroni [10] examined the links between infrastructure capital and GDP. In this paper, the approach mentioned was adopted to the case of overall outlays on fixed assets. Moreover, an additional observation (T7) was formulated and proved.

In order to shed some light on this issue consider the following model (all formulas presented in this subsection are for illustrative purposes only – the estimation procedure applied in this paper allows for more general structures, e.g., application of lags higher than one, etc.)

$$Y_t = A_t K_t^\alpha G_t^\beta L_t^{1-\alpha-\beta}, \quad (1)$$

where:

Y_t – aggregate output at time t ;

A_t – total factor productivity at time t ;

G_t – fixed capital at time t ;

K_t – other capital at time t ;

L_t – employment at time t .

For simplicity one should assume a constant savings rate (s), and that G_t and K_t fully depreciate each period. Moreover, one should assume that fixed capital formation and other capital in period t are a proportion of previous output, i.e.,

$$G_t = \tau_{t-1} Y_{t-1}. \quad (2)$$

$$K_t = (1 - \tau_{t-1}) s Y_{t-1}, \quad (3)$$

Combining equations (1), (2) and (3) gives

$$Y_t = A_t s^\alpha (1 - \tau_{t-1})^\alpha \tau_{t-1}^\beta Y_{t-1}^{\alpha+\beta} L_t^{1-\alpha-\beta}. \quad (4)$$

Dividing both sides of equation (4) by L_t gives

$$\left(\frac{Y}{L}\right)_t = A_t s^\alpha (1 - \tau_{t-1})^\alpha \tau_{t-1}^\beta \left(\frac{Y}{L}\right)_{t-1}^{\alpha+\beta} \left(\frac{L_{t-1}}{L_t}\right)^{\alpha+\beta}. \quad (5)$$

As one can see, the evolution of per capita output is determined by technical progress, the share of output going to fixed capital formation and the size of employment. The dynamic interrelations between output per capita and fixed capital formation per capita in the framework of the model examined may be summarized in the following theorem.

Theorem 1. *Let the following assumptions hold true:*

Assumption 1: *The log of total factor productivity is described by the following model*

$$\log(A_t) = a_0 + \sigma t + \varepsilon_t \quad (6)$$

where $\varepsilon_t = \delta \varepsilon_{t-1} + w_t$ for some $\delta \in [0, 1]$ and w_t is a zero mean stationary time series.

Assumption 2: *The amount of output going to fixed capital formation is described by the process*

$$\tau_t = \bar{\tau} + \mu_t \quad (7)$$

where $\bar{\tau}$ is a constant and μ_t is a zero mean stationary time series.

Assumption 3: *The growth rate of population is given by the following process*

$$\log\left(\frac{L_t}{L_{t-1}}\right) = \bar{\eta} + \eta_t \quad (8)$$

where $\bar{\eta}$ is a constant and η_t is a zero mean stationary time series.

Then:

T1: The log of per capita output (i.e., $\log\left(\frac{Y}{L}\right)_t$) con-

tains a unit root whenever $\delta = 1$ and $\alpha + \beta < 1$, or $\delta < 1$ and $\alpha + \beta = 1$.

T2: If $\delta = 1$ and $\alpha + \beta < 1$, or $\delta < 1$ and $\alpha + \beta = 1$, then $\log\left(\frac{Y}{L}\right)_t$, and log per capita fixed capital series,

$\log\left(\frac{G}{L}\right)_t$, will each be non-stationary and integrated

of order one. Moreover, there will exist a cointegrating vector for these variables. Shocks to productivity will have a long run positive effect on log per capita output.

T3: If $\delta = 1$ and $\alpha + \beta < 1$, then shocks to per capita fixed assets, μ_t , have no long run effect on per capita output.

T4: If $\delta < 1$ and $\alpha + \beta = 1$, then shocks to per capita fixed assets, μ_t , will have a non-zero long run effect on per capita output. For small shocks, the sign of this effect will be positive if $\bar{\tau} < \frac{\beta}{\alpha + \beta}$, and negative if

$$\bar{\tau} > \frac{\beta}{\alpha + \beta}.$$

Proof: See [10] for detailed proof. \square

An important research question is which (if any) version of the growth model (with respect to parameters α , β , δ , etc.) fits best to Polish data. In order to answer this question (i.e., examine conditions (1)–(8)) one should analyze individual properties of input data and verify results of suitable regressions. The next theorem describes method of testing for the directions and signs of long run dependences between GDP and fixed capital.

Theorem 2. Let assumptions 1–3 from Theorem 1 hold true. Moreover, assume that $\log\left(\frac{Y}{L}\right)_t$ and

$\log\left(\frac{G}{L}\right)_t$ are individually nonstationary but together

are cointegrated. Let the estimated error correction representation of these series be of the form:

where \hat{e}_{t-1} stands for estimated error correction mechanism.

Then:

T5: The coefficient λ_2 is zero if, and only if, innovations to log per capita fixed assets have no long run effect on log per capita output.

T6: The ratio of the coefficient $-\frac{\lambda_2}{\lambda_1}$ has the same sign as the long run effect of innovations to log per capita fixed assets on log per capita output.

T7: The long run effect of innovations to log per capita fixed assets on log per capita output implies long run effect in opposite direction. Both effects have the same signs.

Proof: For proofs of points T5 and T6 see [10]. For proof of point T7 assume a positive shock in

$\log\left(\frac{Y}{L}\right)_{t_0}$. From equation (2) and assumption 2 this

implies a positive shock in the next period fixed capital $\log\left(\frac{G}{L}\right)_{t_0+1}$. The latter implies a long run effect on

log per capita output with the same sign as $-\frac{\lambda_2}{\lambda_1}$,

which stems directly from point T6. Finally, the permanent change in per capita output implies permanent change in fixed capital per capita, which flows directly from equation (2) and assumption 2. Since $\bar{\tau} > 0$, the sign of this effect depends only on the sign of $-\frac{\lambda_2}{\lambda_1}$. \square

To summarize, using Theorems 1 and 2 one may easily test whether gross fixed capital formation in examined economy is below, under or about its growth-maximising level (comp. T6). Moreover, whenever shocks to fixed assets are significantly influencing output the opposite long run impact takes place with the same sign (comp. T7). However, in order to formulate any conclusion based on presented theorems all assumptions have to be fulfilled. In [10], none of these assumptions were examined. In this paper, assumptions 1–3 were formally verified.

Since GDP_{pc} exhibited an upward tendency in the whole analyzed period while $GFCE_{pc}$ only in the pe-

$$\begin{cases} \Delta \log\left(\frac{G}{L}\right)_t = c_1 + \lambda_1 \hat{e}_{t-1} + \sum_{i=1}^p \phi_{11i} \Delta \log\left(\frac{Y}{L}\right)_{t-i} + \sum_{i=1}^p \phi_{12i} \Delta \log\left(\frac{G}{L}\right)_{t-i} + \varepsilon_{1t} \\ \Delta \log\left(\frac{Y}{L}\right)_t = c_2 + \lambda_2 \hat{e}_{t-1} + \sum_{i=1}^p \phi_{21i} \Delta \log\left(\frac{Y}{L}\right)_{t-i} + \sum_{i=1}^p \phi_{22i} \Delta \log\left(\frac{G}{L}\right)_{t-i} + \varepsilon_{2t} \end{cases} \quad (9)$$

riod Q3 2002–Q2 2008, it is hard to expect that proportion τ_t is stationary around constant. The results of unit root tests performed for τ_t and full sample (available from the author upon request) formally confirmed this supposition. Since assumption 2 was unfulfilled for full sample the methodology of examining long run properties of output and fixed capital based on Theorems 1 and 2 was inapplicable in this case.

However, both theorems examined should work well for non-crisis subsample as in this case time series of per capita gross fixed capital formation was characterized with small shocks (note that too large fluctuations of G_t could conceivably move the system into a different regime). All assumptions of Theorems 1 and 2 were empirically verified in the case of reduced sample (Q3 2002–Q2 2008). The results of suitable regressions (available from the author upon request) confirmed that in this case assumptions 1–3 were all fulfilled.

The number of patents registered in The Patent Office of Poland was taken as a proxy for total factor productivity. An analysis of regression outcomes confirmed that assumption 1 holds true. Moreover, parameter δ was found to be significantly smaller than one, which indicates that there indeed may exist a long run impact of fixed capital on output (see condition T4). The formal examination of this supposition was presented in Section 6.

As already mentioned assumption 2 was also found to be fulfilled for reduced sample. Finally, results of unit root tests additionally performed for non-crisis subsample confirmed that logarithm of employment is nonstationary around constant but is stationary in first differences, which supported assumption 3.

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