Inflation Skewness and Price Indexation

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Abstract

One of the two price adjustment (indexation) schemes for the intermediate good producers, in the staggered price Dynamic Stochastic General Equilibrium (DSGE) models is the indexation to the average inflation. In this essay we show that using average of inflation as index multiplier may lead to the deviation from the optimal price for intermediate good producer. Although there is no problem with this indexation method as far as the distribution of inflation is symmetric, when we have a skewed distribution for inflation (as we have in the U.S. economy and most of the G7 countries), indexation to average inflation does not reflect the profit maximizer firm’s decision making process. After showing the deficiencies of this method we introduce the Median of the distribution of inflation ($\text{Med}(\pi)$) as an index multiplier, explain its advantage and support our claim by simulating the intermediate firm’s profit, in two alternative scenarios of using average inflation and Median of the historic distribution of inflation. Our results suggest that using $\text{Med}(\pi)$ as index multiplier in staggered price framework, helps intermediate good producer to increase its profit.

Keywords: DSGE; Inflation Skewness; Long run inflation; Non-Linearity; Calvo Pricing.

JEL classification: E5,E3.

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1 Introduction

Assume that there is a price setter firm in a monopolistically competitive market and this firm cannot re-optimize its price in each period to get the maximum profit, but rather the firm is allowed to re-optimize the price in equal distances in equal intervals of time. Since fluctuations are the inherent characteristics of the markets, the optimum price which can guarantee the maximum profit for the firm is varied during the time. And the firm which cannot re-optimize its price faces some loss from not charging the optimum price.

The idea which was introduced above is the core idea of the new Keynesian models which consider the staggered prices (wages). To analyze an economy with the staggered price some frameworks have been suggested in the literature. First is introduced by Calvo (1983). Calvo suggests the pricing procedure in which the price setter firm which is active in the monopolistically competitive market, sets its price, knowing that the probability of re-optimizing the price in each period is $1 - \alpha$.

The Calvo’s model argues the model which the firm is not allowed to change its declared price until the re-optimization signal is received, but there are some more advanced Calvo type models that allow the firm to change its price. But still this change does not mean the re-optimization. Indeed the mechanism of changing the price in each period should be declared in the price optimization (current) period. This change in price is called adjustment instead of re-optimization. Since price change can be reflected on the inflation rate, all developments of the Calvo pricing procedure focus on the inflation rate.

There are two distinguished forms of price adjustment in the literature. Adjustment to the lagged inflation which is suggested by Christiano et al. (2005) and adjustment to the average inflation ($\bar{\pi}$) that Yun (1996) employed in his model. Christiano et al. (2005) incorporates both these two adjustment methods in their model, where they assume that $\alpha$ fraction of firms that they can not re-optimize their price, are divided into two groups. The firms in the first group adjust their previous period’s prices by $\bar{\pi}$ (multiplying the price by $\bar{\pi}$), where $\bar{\pi}$ is the average inflation (mean) and the firms in the second group adjust their prices by the most recently observed inflation ($\pi_{t-1}$), so they change their prices by multiplying the previous period’s price by previous period’s inflation rate.

By implementing one of these two schemes, firm can adjust its initial price so that the price in the upcoming periods is more likely to be close to the price which is optimum at the time and hence increase the accumulative profit.

The solution which is applied by the firms in the first group to increase their accumulative
profit, is plausible, because the positive inflation is the inherent nature of the almost all economies since the world war two and firms may expect encounter some positive inflation in each period, so adjusting the initial price by the positive trend inflation rate is more likely to lead into higher profit comparing with the strategy which states, “keep the price unchanged” and since we assume that the firm is profit maximizer, increasing the initially optimum price in each period by some fixed rate, say average, trend or long-run inflation is reasonable.

Although this mechanism of changing the price is more reasonable behavior for the firm comparing with the strategy of keeping the price unchanged, when it comes to the question of ” what is the optimum rate of adjustment? “ there is no straightforward answer. The first answer is the one which was explained above: the average rate of inflation through the recent periods. Indeed it is the only answer which one may find in the literature and to some extent it works properly. But if it is optimum rate of adjustment? The answer is: it depends on some assumptions about the inflation. The most important one is about the symmetry’s property of the probability distribution of inflation. Although it is vital assumption it is surprisingly neglected in the literature. And it is more important in Analyzing of the post world war II inflation, when data show that probability distribution of inflation is positively skewed in almost all countries and particularly in U.S, no matter if it is measured quarterly or monthly. Aizenman and Hausmann (1994) studied monthly inflation data for 56 countries in the period of 1979-1993 and indicated that the monthly inflation distribution is positively skewed in almost all these countries. Ruge-Murcia. (2012) investigated the U.S. quarterly inflation data from 1960 to 2001 and stressed the positive skewness in the inflation distribution.

in this paper we try to answer questions like:
1) although using the average inflation(\(\bar{\pi}\)) as an index multiplier is plausible assumption as long as we suppose that inflation is symmetrically distributed, if this assumption still holds when the inflation distribution is positively skewed?
2) Why firms that they are not allowed to re-optimize and they multiply their previous period’s prices by a constant rate, should adjust their prices by \(\bar{\pi}\)?
3) Is there any alternative for average inflation (\(\bar{\pi}\)) to be used as an indexation multiplier?

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1Trend inflation, Average inflation and long-run inflation are used in this paper in the same meaning

2like what we assume in the simple Calvo model

3or as it is called in this paper optimum index multiplier
To answer these questions, we develop a dynamic stochastic general equilibrium (DSGE) model in section 2, then investigate the price setting behavior of intermediate good producer under the alternative scenarios for inflation distribution in part 3. Part 4 is dedicated to explaining the benefits of employing the median of inflation distribution (\(\pi_{med}\)) as index multiplier. Part 5 is calibration of our DSGE model. Part 6 includes the analyzing of the simulation and its results and finally part 7 is the conclusion part. Section 8 is the Appendix.

2 Model

In this section we employ a small new Keynesian Dynamic Stochastic General Equilibrium model which has four sections; Household, Final Good Producer, Intermediate good producer and the Government. The model is based on the model which developed by Ascari and Ropele (2009).

2.1 Household

The utility function for the Ricardian household is:

\[
U_t(C, M/P, N) = C^{1-\sigma_c} - \frac{1}{1-\sigma_m} \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1-\sigma_m} - \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \tag{1}
\]

\(U_t\) stands for the household’s utility, \(C_t\) Consumption, \(M_t\) the money stock which household holds, and \(N_t\) Labor force which is provided by household, all in period \(t\). In the above equation \(\sigma_c, \sigma_n\) and \(\sigma_m\) respectively represent the inversed Intertemporal elasticity of substitution of Consumption, Labor and Money.

Considering the constraint on household budget we have:

\[
P_tC_t + M_t + B_t \leq W_tN_t + M_{t-1} + (1 + i_{t-1})B_{t-1} + F_t + TR_t \tag{2}
\]

In this equation \(W_t\) is the wage, \(B_t\) is the holding of the Bonds and \(i_t\) is the interest rate. \(F_t\) represents the profit from the firms which household gains in each period as share holder and \(TR_t\) is the Transfers from the government to the household.

Maximizing equation 1 in the infinite horizon framework, constrained to the Budget constraint:

\[
\max_{C_t, M_t/P_t, N_t, B_t} \sum_{t=0}^{\infty} \beta^t \left( C^{1-\sigma_c}_t - \frac{1}{1-\sigma_m} \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1-\sigma_m} - \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right) \tag{3}
\]
Subject to:

\[ P_tC_t + M_t + B_t \leq W_tN_t + M_{t-1} + (1 + i_{t-1})B_{t-1} + F_t + TR_t \]  \hspace{1cm} (4)

And solving the 3 considering the 4, we get the FOCs:

\[ \chi_n \frac{N_t^{\sigma_n}}{C_t^{\sigma_C}} = \frac{X_t}{P_t} \]  \hspace{1cm} (5)

\[ \chi_m \frac{(M_t/P_t)^{-\sigma_m}}{C_t^{\sigma_C}} = \frac{i_t}{1 + i_t} \]  \hspace{1cm} (6)

and the Euler equation:

\[ 1 = \beta E_t \left\{ \frac{C_t^{\sigma_C}}{C_{t+1}^{\sigma_C}} (1 + i_t) \frac{P_t}{P_{t+1}} \right\} \]  \hspace{1cm} (7)

2.2 Final Good Producer

Suppose that Final good producers are producing in perfectly competitive market, then production function can be defined as:

\[ Y_t = \int_0^1 Y_t(i) \frac{\theta-1}{\theta} di \]  \hspace{1cm} (8)

Since the final good producer is in a competitive market, it is price taker. Also it chooses the quantity to maximize its profit. By maximizing the profit function with respect to the quantity, we get the demand for the inputs which themselves are the production of the intermediate good producers. This demand function for intermediate goods will be as follows:

\[ Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\sigma} Y_t \]  \hspace{1cm} (9)

2.3 Intermediate Good Producer

Here we may assume that we have monopolistic competitor firms, prices are sticky (Staggered price) and they are determined by intermediate good producers using some adjusted version of the Calvo (1983) pricing procedure. The technology which the firm uses is \( Y_t(i) = N_t(i) \), which means the firm transforms the Labor(\( N_t(i) \)) to the \( i^{th} \) good. From the previous part we know that the intermediate good producer faces the demand function:

\[ Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{\theta} Y_t \]  \hspace{1cm} (10)
Using the Calvo pricing procedure firm can re-optimize its price by the probability of $1 - \alpha$. Although there is no restriction which explicitly has been pointed out in Calvo’s formalism, it has not been usual to consider the trend inflation in this framework before Ascari and Ropele (2009). They introduce the trend inflation ($\bar{\pi}$) as an index multiplier, which means that firms use $\bar{\pi}$ to index their prices in each period and here, we use their formalism too ($\bar{\pi} = \tilde{\pi}$). So in each period, we have firms which they re-optimize their prices and they who don’t re-optimize it. Introducing $\alpha \in [0, 1]$ we have:

$$\text{Total share of the Firms} = \begin{cases} \text{Share of the Firms re-optimize their price} & \alpha \\ \text{Share of the Firms which index their price} & 1 - \alpha \end{cases}$$

And then for firms who don’t re-optimize we have two groups. The firms which adjust(index) their price and the the other group of firms which keep their prices unchanged. So assuming $\epsilon \in [0, 1]$ we have:

$$\text{Total share of firms they don’t re-optimize} = \begin{cases} \text{Share of the firms they index using trend inflation} & \epsilon \\ \text{Share of the firms they they don’t re-optimize price} & 1 - \epsilon \end{cases}$$

Knowing the Total real cost function as:

$$TC^r_t(Y_t(i)) = wY_t(i) \quad (11)$$

And plugging them into the profit function, maximize it with respect to the price as below:

$$\max_{p_t^*(i)} E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j}[p_t^*(i)\tilde{\pi}(i) - TC^r_{t+j}(Y_{t+j}(i))]$$

(12)

Considering 11 and noting that $\tilde{\pi} = \bar{\pi}$ in the work of Ascari and Ropele (2009).

$$Y_{t+j}(i) = \left(\frac{p_t^*(i)\bar{\pi}(i)}{p_{t+j}}\right)^{-\theta}Y_{t+j} \quad (13)$$

The FOC will be:

$$p_t^*(i) = \frac{\theta}{1 - \theta} \frac{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j}[p_t^\theta(i)Y_t(i)MC^r_{t+j}(i)\bar{\pi} - \theta\epsilon_j]}{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j}[p_t^\theta(i)Y_t(i)\bar{\pi}(1-\theta)\epsilon_j]} \quad (14)$$

Now that we have the First Order Conditions, we are ready to run the model using the standard
software packages like *Dynare⁴*. But before that, we will investigate in the next section, the different scenarios under which we can detect different profit maximization behavior based on the cost benefit analysis.

### 3 Profit Maximization and Uncertainty

In a simple new Keynesian DSGE model we can recognize four different parts including The Household, Final good producer, Intermediate good producer and Government. The final good producers are active in the competitive market and hence they are price takers. But the intermediate good producers are price setter firms. They set the price to reach their goal which is maximizing the profit in the current and all the future periods. In the Figure 1 the profit function of intermediate good producer firm is depicted as it is in the simple new Keynesian DSGE model in which it is assumed the labor as the only factor in producing goods.

![Figure 1: Profit function of intermediate good producer is highly non-linear and asymmetric with respect to real price. If we plot profit, we have \( p_t(i) \) on the horizontal axis, where \( p_t(i) \) is the firm’s price which is set in the period \( t \) and \( P_t \) is the price index of the whole economy at the current period. On the vertical axis we have the "profit".](image)

Since the profit function of the intermediate good producer in the new Keynesian framework is highly non-linear and asymmetric with respect to the real price⁵ and because of the skewed historical distribution of inflation, using the expected inflation(\( \bar{\pi} \)) as the index multiplier(\( \tilde{\pi} \)) is

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⁴we use Dynare 4.3.3

⁵One of the sources of this nonlinearity is the elasticity of substitution between intermediate goods in the CES production function of the final goods. This elasticity of substitution, usually calibrated around 11 in the U.S. economy.
no longer guarantees the maximum profit for intermediate good producers. The reason is: the probability of surprising by inflation rate less than expected inflation is more than half and hence, the real price after $n$ period ($\frac{p_t(i) \cdot \pi}{P_t \cdot \pi_t + n}$) systematically deviates from the optimal real price. Note that skewness implies that $Med(\pi) < \bar{\pi}$. And it means in each period firm’s indexed price deviates more and more from the optimal price. it is depicted in the Figure 2.

\[ \text{Figure 2: The indexed real price in the period } t+1 \text{ (when } n = 1) \text{ is } p_t(i) \cdot \frac{\pi}{P_t \cdot \pi_t + 1} \text{ and since by positive skewness, } \text{Prob}(\pi_{t+1} < \bar{\pi}) > \text{Prob}(\pi_{t+1} > \bar{\pi}) \text{ it is more likely that the real price deviates in each period to the right or in other words “increases”. So if the } \rho \text{ is the probability of changing in real price by same amount, } 1 - \rho > \frac{1}{2} \]

3.1 Deterministic Optimization

If the firm has access to all the information and there is no uncertainty about the inflation in the next period, the firm simply sets the price where the maximum level of profit is gained in the current period; And in each forthcoming periods, the firm multiplies this price by the certain level of inflation which is known, to reach the optimum prices in the next periods too. since both the $p_t(i)$ and $P_t$ are multiplied by the same rate (inflation), there is no change in the real price in forthcoming periods. In the figure 3, $A$ is the deterministic price and it will be same in the next period, so the firm will gain the maximum profit in each period.

3.2 Optimization under uncertainty

If there is uncertainty about the price in the next period and the firm can not change its price in the next period by the probability of $\alpha$, this uncertainty deviates the price from the deterministic price. We can define two effects here. one of them is the effect which can be rooted in the variance of inflation distribution and another one in the skewness of it.
3.2.1 Variance effect

If the inflation of the next period ($\pi_{t+1}$) is uncertain and it is distributed normally, then standard error ($\sigma$) of this distribution which by definition is "the variation of inflation around its mean" ($\bar{\pi}$) makes the expected profit maximizer firms to select a price in which they can reach the expected maximum of some alternative scenarios. Since of the asymmetries of the profit function, the optimum price differs from the deterministic one. the deviation depends on the shape of the profit function. analytically we can say: since the slope of the profit function is higher in the left hand side of the deterministic maximum, to avoid the loss from unexpected inflation, the firm chooses the optimum price in the right hand side of the deterministic optimum price. Note that the price in the next period, in the uncertain scenario will be $p_t^*(i) \frac{\bar{\pi}}{\pi_{t+1}}$ and since of the difference between $\bar{\pi}$ and $\pi_{t+1}$ the price will differs from $p_t^*(i)$. And it means that if firm chooses the deterministic expected maximum when there is chance the real price will be deviated to the left or right by the same amount, since of the asymmetric profit function, the deviation to the left ($\pi_{t+1} - \bar{\pi} > 0$) harms the firm much more than the deviation to the right. It is why the firm chooses the optimum price bigger than the deterministic optimum price. This price($B$) is shown in Figure 4.

3.2.2 Skewness effect

The mechanism by which the firm is allowed to index its price by expected inflation ($\bar{\pi}$) first suggested by Yun (1996) and developed by Ascarì and Ropele (2009) and had not been employed in the Calvo’s work. If we consider the problem for $n$ periods we will have $p_t^*(i), \pi^n = \frac{p_t^*(i), \pi^n}{\pi_{t+1}, \pi_{t+2}, \pi_{t+3}, ..., \pi_{t+n}}$. Since $P_t$ is known at the time when the firm makes the decision, the only source of uncertainty is
Figure 4: The price here is set assuming that the inflation is normally (symmetrically) distributed. As a result only the variance is important. This price (B) is more than the deterministic price (A).

the forthcoming inflations. ⁶ If the distribution of inflation is skewed the other problem will be aroused and needs to be considered by the firm. To address this problem we need to explain the meaning of skewness. If the distribution is positively skewed then what usually is the case is that \( \text{Med}^7 < \text{Mean} \). By \( \text{Median} \) we mean the observation in our data which separates the data into two equal block of information. The block below median contains all the observations less than median and the other block contains observations more than median. For example, if the stochastic variable is inflation, \( \text{Med}(\pi_{t+n}) \) is the inflation rate which the chance of occurring some inflation more than that is 50 percent and the chance of facing an inflation below that, is 50 percent as well. Since the Mean of the positively skewed distribution is located in the right hand side of the Med (\( \text{Med} < \text{Mean} \)) we expect that the chance of occurrence of an inflation more than the \( \bar{\pi} \) is less than 50 percent (say 30) and the chance of an inflation below the mean is more than 50 percent (say 70).

In the case of infinite horizons Profit Maximization problem, since of the skewness we expect that in the 70 percent of times the actual inflation is less than the predicted one (\( \bar{\pi} \)) and since we multiplied the current period’s optimal price by \( \bar{\pi} \) in each period, the real price which was defined as

\[
\frac{p_t^*(\bar{\pi}) \pi^n}{P_{t+n}} = \frac{p_t^*(\bar{\pi}) \pi^n}{P_t \pi_{t+1} \pi_{t+2} \pi_{t+3} \ldots \pi_{t+n}}.
\]

Since \( P_t \) will increase in 70 percent of times (because in each period, the denominator is multiplied by the \( \pi_{t+n} \) which is less than the \( \bar{\pi} \) which numerator is multiplied

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⁶It is important to suppose that the distribution of inflation will be same through the time.

⁷Med is used from now on instead of Median
by) and increase in 30 percent of times. So in the long run the profit of the firm converges to zero\(^8\). this situation is depicted in Figure 5

\[ \text{Profit}_{t+n} \]

Figure 5: Since the unexpected inflation \(\pi_{t+n} - \bar{\pi}\) is more likely to be less than zero, the real price will increase through the time and the Profit\(_{t+n}\) converges to zero

4 Multiplying by Med as a Solution

In the Yun (1996) and Ascarì and Ropele (2009) the index multiplier is assumed to be the expected inflation \(E[\pi_{t+n}] = \pi_{t+n}\). But it works properly, only when we have symmetric distribution for \(\pi_{t+n}\) and \(Med(\pi_{t+n}) = E[\pi_{t+n}]\). If we consider the fact that inflation is positively skewed in the U.S. and almost all the other countries, the \(\bar{\pi}\) is no longer the proper index multiplier for the firms. One good candidate in this situation is \(Med(\pi_{t+n})\). Using Med as an index multiplier leads to the stability of the real price around the optimum price which is set in the current (first) period. Although the Mean is better measure of central tendency of distribution with respect to the fact that it is weighted average, since the deviation from the expected inflation is important here, and not the inflation itself, this advantage of Mean has no application in the maximization problem and hence it seems that using the Med(\(\pi_{t+n}\)) is more useful. Using Med(\(\pi t + n\)) sets the profit in each period around the maximum level, so the aggregate profit will be bigger. So it is more plausible for the intermediate good producer to use Med(\(\pi_t\)) as an index multiplier instead of the

\(^{8}\)In fact the profit converges to zero anyway by assuming the \(\alpha \in [0,1]\) and discounting factor \(\beta\), but skewness affects the optimization problem since the \(\bar{\pi}_{t+n}\) does not solve the dynamic optimization problem any more and the profit may converges to zero faster when the firm uses \(\bar{\pi}_{t+n}\) in an economy with positively skewed distribution of inflation.
This implies that this model is nearer to the behavior of the profit maximizer firm and so, has stronger Micro-founded bases. In the next section we calibrate a model to use it for simulation purpose.

## 5 Calibration

Following Ascari and Ropele (2009), frictionless or desired markup is 10 percent in product market. The calibration is for the quarterly data, so by setting $\alpha = .75$ The prices re-optimize approximately each year (after four periods). The steady state value of labor ($N$) and Consumption ($C$) which is used in our model set at 1. The calibrated parameters, their values and definitions can be found in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\sigma_n$</td>
<td>Intertemporal rate of substitution of Labor</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Intertemporal rate of substitution of Consumption</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective rate of time preference</td>
<td>0.99</td>
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<tr>
<td>$\alpha$</td>
<td>probability of not re-optimizing</td>
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<tr>
<td>$\theta$</td>
<td>Elasticity of substitution between the intermediate goods</td>
<td>11</td>
</tr>
<tr>
<td>$\varphi_i$</td>
<td>Coefficient of $i$ in the Taylor rule</td>
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</tr>
<tr>
<td>$\varphi_\pi$</td>
<td>Coefficient of $\pi$ in the Taylor rule</td>
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</tr>
<tr>
<td>$\varphi_c$</td>
<td>Coefficient of $c$ in the Taylor rule</td>
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</tr>
<tr>
<td>$\epsilon$</td>
<td>degree of indexation</td>
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</tr>
<tr>
<td>$\omega_\pi$</td>
<td>coefficient of AR(1) for the cost push shock</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 1: The values are calibrated for the united states postwar data

Like Schmitt-Groh and Uribe. (2004) and ? we consider the cost push shock here. The process which describes the evolution of the cost push shock assumed to be $AR(1)$ as follows:

$$z_\pi = \omega_\pi \cdot z_\pi + u_t$$  \hspace{1cm} (15)

In the above formula $u_t$ is assumed to be an i.i.d process with mean zero and standard error one. The nominal interest rate is %2.2 at steady state and marginal cost which is interpreted to real wage in this model is 0.9.
6 Simulating the Firm’s Profit

If the intermediate good producer indexes its price each year by multiplying initial price by inflation mean, the resulted price necessarily deviates from the optimal price when the economy experiences a non-symmetric shock which affects the prices and subsequently, inflation. This is the idea behind the simulation here. We simulate the economy in its steady state, hence as long as there is no shock hits this economy, using average inflation as index multiplier guarantees the profit. But when there is a shock, mean can be a better instrument for the firm. We simulate the firm’s profit instead of calculating the profit exactly. The reason is that, to solve the regular infinite horizon maximization problem like what we had in Calvo pricing procedure, one should use the substitution method which makes it possible to find a value for considering variable. To substitute each period’s by the forthcoming periods, the polynomial inside the summation sign should be constant in terms of forthcoming inflations. But here we assume that inflation is subject to change in the next periods so the usual method does not work here anymore. Indeed, we have infinite dimensional system of equations. That is the reason we simulate the profits here. knowing that net present value of profit at the period $t + 1$, converges to zero, we simulate profit for 10 periods after the current period. The simulated profits which are provided in Table 5 support the claim that the intermediate firm betters off using $\text{Med}(\pi)$ instead of $\bar{\pi}$ as an index multiplier. In the table a unit cost-push shock hits the economy in period 1 and the profits are calculated for 10 periods for two cases of using $\bar{\pi}$ and $\text{Med}(\pi)$ as index multiplier.

<table>
<thead>
<tr>
<th>t</th>
<th>$\bar{\pi}$ = Med($\pi$)</th>
<th>$\bar{\pi}$ = $\bar{\pi}$</th>
<th>\text{Difference between two Scenarios}</th>
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<td>189*</td>
<td>248</td>
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<td>822</td>
<td>576</td>
<td>246</td>
</tr>
<tr>
<td>t=10</td>
<td>612</td>
<td>382</td>
<td>229</td>
</tr>
</tbody>
</table>

Table 2: The profit of intermediate good producer until 10 periods after a unit cost-push shock hits the economy in two scenarios of indexing the initial price by $\text{Med}(\pi)$ and $\bar{\pi}$. In the simulation the $\epsilon = 0.5$ and $\alpha = 0.75$ | * The Values are just used here in order of comparison

When there is a cost push shock, using the $\text{Med}(\pi)$ provides more profit comparing to the $\bar{\pi}$. In the Table 6 we change . So using the $\text{Med}(\pi)$ increases the profit in the economy which is hit regularly by the shocks and as a result has the skewed distribution of inflation. Considering the importance of “Profit Maximization” assumption in the micro founded macro models, and the fact that multiplying the price in each period by $\text{Med}(\pi)$ increases the profit of the intermediate firm, it is reasonable for the intermediate firm to use $\text{Med}(\pi)$ instead of $\bar{\pi}$.
7 Conclusion

As we suggested in the first part of the paper, using $Med(\pi)$ as an index multiplier can maximize the firm’s benefit through the time.

We assume that inflation is not normally distributed and this assumption is based on the fact that inflation is positively skewed in almost all G7 countries. This asymmetry in distribution of historic inflation together with another asymmetry which is rooted in intermediate good producer’s profit function, leads to the unavailability of firms in keeping the price in its optimal level in upcoming periods (when they index their prices in each period by $\bar{\pi}$). Using the $Med(\pi)$ eliminates the systematic error in the model and is necessary for keeping the assumption that intermediate good producers are profit maximizer. Using the $Med(\pi)$ can minimize the error in inflation forecast of intermediate good producers. Because the firms are profit maximizers, they should avoid the loss from inaccurate forecasts.

Using the simple new Keynesian DSGE model we showed that multiplying the price in each period, by $Med(\pi)$ instead of $\bar{\pi}$ gets the considerable better results and in our model decreases the difference between the realized price of the intermediate good producer and the optimum price.

Finally the simulation results support the claim of the paper that indexing the initial price by $Med(\pi)$ provides more profit for the intermediate good producer which wants to maximize its profit by setting the initial price and index multiplier.

In summary, this paper has three contributions. Firstly, the fact that index multiplier can be set by intermediate good producer, in the Calvo pricing procedure is neglected in the literature and this paper is the first attempt to set the index multiplier endogenously. Secondly, this paper considers the inflation skewness which is the common property of the inflation distribution in almost all G7 countries and particularly United States. Thirdly, The $Med(\pi)$ is suggested here as a better index multiplier and it is shown by the simulation that it can produce better results comparing to the $\bar{\pi}$ and provides higher level of profit for the intermediate good producer when the economy is hit by shock.
References


8 Appendix
Figure 6: Response to the unit cost push shock under the standard Taylor Rule with $\phi_i = 1.2$, $\phi_c = .5$ and $\phi_{ps} = 1.5$