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Abstract: In this paper, we analyze the impacts of joint energy and output prices uncertainties on the inputs demands in a mean-variance framework. We find that an increase in expected output price will surely cause the risk averse firm to increase the inputs' demand, while an increase in expected energy price will surely cause the risk averse firm to decrease the demand for energy and increase the demand for non-risky inputs. Further, increasing the variance of energy price will necessarily cause the risk averse firm to decrease the demands for the non-risky inputs. Furthermore, we investigate the two cases with only uncertain energy price and only uncertain output price. In the case with only uncertain energy price, we find that the uncertain energy price has no impact on the demands for the non-risky inputs.

Keywords: Price Uncertainty; Mean-Variance; Energy price, Risk

1 Introduction

There are very few studies examine multiple sources of energy uncertainty. Examples of such studies in the agricultural sector include Alghalith (2007, 2010b), Kunmbhakar (2002), Nazlioglu and Soytas (2011), Nazlioglu, et al. (2013), and Du, et al. (2011). On the other hand, Broadstock, et al. (2012), Arouri, et al. (2012) and Li, et al. (2012) study the impact of oil shocks on the energy related stocks. Alghalith (2008) models energy price uncertainty in the U.S. manufacturing sector and estimates the impact of energy price uncertainty on the manufacturing output. Assuming the manufacturing output price to be uncertain, Alghalith (2010a) tests for the correlation between the energy price shocks and manufacturing price shocks and estimates the impact of the correlation on the manufacturing output.

On the other hand, Tobin (1958), Wong (2006), Meyer (1987), Wong and Ma (2008), and Eichner and Wagener (2009) showed that, under some conditions, the expected utility decision problem can be transformed into the mean (μ)-standard deviation (σ) framework. This approach has been widely used in literature, see, for example, Battermann et al (2002) and Broll et al (2006). Recently, Alghalith, et al. (2012) present a stochastic factor model with an additive background risk and present a dynamic model of simultaneous (correlated) multiplicative background risk and additive background risk. Guo, et al. (2013) study the impact of background risk on the indifference curve for risk averters, risk seekers, and risk-neutral investors. In addition, Guo, et al. (2013a) investigate the impact of multiplicative background risk on an investor's portfolio choice in a mean-variance framework.

In this paper, we extend their work by analyzing the impact of joint energy price and output price uncertainties on the demands for energy and the other non-risky inputs. We allow the dependence between energy price and output price and consider the effect of the covariance between these two random variables on the demands for inputs. By using this model setting, we find that the concepts of elasticities, decreasing absolute risk aversion (DARA) and variance vulnerability play important roles in the comparative statics analysis. Further, we also consider some special cases of our model. That is, the situation with only uncertain energy price and that involving only uncertain output price. In these two special cases, clearer and intuitive results are obtained.

2 The model

We first follow Alghalith (2010a) to assume the firm's random profit to be

$$\tilde{\Pi} = \tilde{p}F(\mathbf{x}) - \sum_{i=1}^{n-1} p_i x_i - \tilde{p}_n x_n, \tag{2.1}$$

where $\mathbf{x} = (x_1, \cdots, x_{n-1}, x_n)$ is a vector of inputs, p_i $(i = 1, \cdots, n-1)$ is a non-random input price, $F(\mathbf{x})$ is a neoclassical production function with $\partial F/\partial x_j = F_j > 0$ for $j = 1, \cdots, n$, $\tilde{p}_n$ is the price of energy, and $\tilde{p}$ is the price of output. In this paper, we assume both the price of energy, $\tilde{p}_n$, and the price of output, $\tilde{p}$, to be uncertain and random.

The objective of the firm is to maximize the expected value of a von Neumann-Morgenstern utility function of profit $U(\tilde{\Pi})$, defined on the profit, $\tilde{\Pi}$. The firm is risk-averse so that $U'(\tilde{\Pi}) > 0$ and $U''(\tilde{\Pi}) < 0$ for any $\tilde{\Pi} > 0$. In addition, we assume that the firm will maximize the expected utility of the profit stated in (2.1) such that

$$\max_{x_1, \cdots, x_n} EU\left(\tilde{p}F(\mathbf{x}) - \sum_{i}^{n-1} p_i x_i - \tilde{p}_n x_n\right), \tag{2.2}$$

where E denotes the expectation operator and all the terms are defined in (2.1).

In this paper we model risk preferences in a mean-variance framework (μ, σ) (see, e.g., Meyer, 1987) which infers that (i) the expected utility EU stated in (2.2) can be represented by a two-parameter function $V(\mu, \sigma)$ defined over mean μ and standard deviation σ of the underlying random variable; and (ii) the preference function V possesses the following properties: $\partial V(\mu, \sigma)/\partial \mu = V_\mu > 0$, $\partial^2 V(\mu, \sigma)/\partial \mu^2 = V_{\mu\mu} < 0$,

$\partial V(\mu,\sigma)/\partial\sigma = V_\sigma < 0$, $\sigma > 0$ and $V_\sigma(\mu,0) = 0$. We assume that $\partial^2 V(\mu,\sigma)/\partial\mu\partial\sigma$ is positive, $\partial^2 V(\mu,\sigma)/\partial\sigma^2$ exists and V is a strictly concave function. The indifference curves are convex in (σ,μ)-space.[1]

Using the (μ,σ) preferences, the decision problem of the firm maximizing the expected utility of the profit as stated in (2.2) is equivalent to the following problem:

$$\max_{x_1,\cdots,x_n} V(\mu_\Pi,\sigma_\Pi), \tag{2.3}$$

where $\mu_\Pi = E(\tilde{\Pi})$, $\sigma_\Pi = \sqrt{E(\tilde{\Pi} - E(\tilde{\Pi}))^2} > 0$, and all the terms are defined in (2.1) with

$$\begin{aligned} \mu_\Pi &= \mu_p F(\mathbf{x}) - \sum_i^{n-1} p_i x_i - \mu_{p_n} x_n \ , \\ \sigma_\Pi &= \sqrt{\sigma_p^2 F^2(\mathbf{x}) + \sigma_{p_n}^2 x_n^2 - 2F(\mathbf{x}) x_n \sigma_{p,p_n}} \ . \end{aligned}$$

We note that the slope S of the investor's indifference curve in the (σ,μ)-space at (σ,μ) is the marginal rate of substitution between risk, σ, and expected return of profit, μ. Lajeri and Nielsen (2000) and Ormiston and Schlee (2001) identify S as the two-parameter analogue of the Arrow-Pratt concept of absolute risk aversion. Eichner and Wagener (2003) investigate properties of S further. The slope of an indifference curve in $\mu - \sigma$ space is positive. Risk aversion implies that the indifference curves are upward sloping. Therefore, S can be interpreted as a measure of risk aversion within the mean-standard deviation approach. We also note that because comparisons of risk aversion are determined only from the family of risks in (2.3), risk aversion can be measured in terms of standard deviation and mean, and thus, it can be measured by the slope S. Wagener (2003), and Eichner and Wagener (2009, 2012) carried out some comparative static analysis under uncertainty within the mean-standard deviation approach and the notation S is widely used in these analysis.

To develop the model, we first introduce the following notations for the related elas-

[1] See, for example, Battermann, Broll and Wahl (2002), Broll, Wahl and Wong (2006), Wong and Ma (2008), and Eichner and Wagener (2011) and the references therein for more information.

ticities:

$$\begin{aligned}
\varepsilon_{F,x_j} &= \frac{\partial F}{\partial x_j}\frac{x_j}{F} = \frac{F_j x_j}{F}, j = 1, \cdots, n; \\
\varepsilon_{\mu,x_j} &= \frac{\partial \mu_\Pi}{\partial x_j}\frac{x_j}{\mu_\Pi}, j = 1, \cdots, n; \\
\varepsilon_{\sigma,x_j} &= \frac{\partial \sigma_\Pi}{\partial x_j}\frac{x_j}{\sigma_\Pi}, j = 1, \cdots, n; \\
\varepsilon_{S,\mu} &= \frac{\partial S}{\partial \mu_\Pi}\frac{\mu_\Pi}{S}; \text{and} \quad \varepsilon_{S,\sigma} = -\frac{\partial S}{\partial \sigma_\Pi}\frac{\sigma_\Pi}{S}.
\end{aligned} \tag{2.4}$$

To proceed our analysis, we obtain the following first-order conditions:

$$\begin{aligned}
\Phi(x_n^*, \lambda) &\equiv \mu_p F_n^* - \mu_{p_n} - S^* \frac{\partial \sigma_\Pi}{\partial x_n^*} = 0; \\
\Psi(x_i^*, \lambda) &\equiv \mu_p F_i^* - p_i - S^* \frac{\partial \sigma_\Pi}{\partial x_i^*} = 0, i = 1, \cdots, n-1;
\end{aligned} \tag{2.5}$$

in which

$$\begin{aligned}
\frac{\partial \sigma_\Pi}{\partial x_n} &= \frac{\sigma_p^2 F F_n + \sigma_{p_n}^2 x_n - \sigma_{p,p_n}(F + x_n F_n)}{\sigma_\Pi}; \\
\frac{\partial \sigma_\Pi}{\partial x_i} &= \frac{\sigma_p^2 F F_i - \sigma_{p,p_n} x_n F_i}{\sigma_\Pi}, i = 1, \cdots, n-1;
\end{aligned}$$

and $\lambda = (\mu_p, \mu_{p_n}, \sigma_p, \sigma_{p_n}, \sigma_{p,p_n})$.

Furthermore, from equations (2.5), we have

$$\begin{aligned}
\frac{\partial \sigma_\Pi}{\partial x_n} &= \frac{\mu_p F_n - \mu_{p_n}}{S} = \frac{\partial \mu_\Pi / \partial x_n}{S}; \\
\frac{\partial \sigma_\Pi}{\partial x_i} &= \frac{\mu_p F_i - p_i}{S} = \frac{\partial \mu_\Pi / \partial x_i}{S}, i = 1, \cdots, n-1.
\end{aligned}$$

We are interested in deriving the optimal input demands responds to a changes in the parameters of the decision problems. In the following section, we provide complete characterizations of the comparative statics of $x_i^*(\lambda)$ and $x_n^*(\lambda)$ with respect to all components of λ.

3 The Impact of expected energy price and expected output price

Our first results deal with the comparative statics for changes in expected energy and output prices μ_{p_n} and μ_p, respectively, as stated in the following theorems for the impacts of expected energy and output prices:

Theorem 3.1 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in (2.3), we have*

1. *the sign of* $\partial x_j/\partial \mu_p, j = 1, \cdots, n$ *depends on the relative size of* ε_{F,x_j} *and* $\varepsilon_{S,\mu}\varepsilon_{\mu,x_j}$*;*
2. *the firm will increase inputs when the expected output price increases if and only if*
   (a) *the elasticity of production function with respect to input is greater than the product of the elasticity of risk aversion with respect to the mean of the final profit, and*
   (b) *the elasticity of the mean of the final profit with respect to the inputs; and*
3. *If* $S_\mu < 0$*, then* $\partial x_j/\partial \mu_p > 0$ *for* $j = 1, \cdots, n$*; that is, when the utility function is a decreasing absolute risk aversion (DARA), increasing expected output price will surely cause the risk averse firm to increase the inputs' demand.*

Theorem 3.2 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in (2.3), we have*

1. $\partial x_n/\partial \mu_{p_n} < 0$ *if and only if* $\varepsilon_{S,\mu}$ *is less than* $1/\varepsilon_{\mu,x_n}$*;*
2. *the firm will decrease the demand for energy when expected energy price increases if and only if the elasticity of risk aversion with respect to the mean of the final profit is less than one over the elasticity of mean of final profit with respect to the energy; and*
3. *if* $S_\mu < 0$*, then* $\partial x_n/\partial \mu_{p_n} < 0$*; that is, when the utility function is DARA, increasing expected energy price will surely cause the risk averse firm to decrease the demand for the energy.*

Theorem 3.3 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in (2.3), we have*

1. $\partial x_i/\partial \mu_{p_n} < 0$ *if and only if* $S_\mu < 0$*; and*
2. *the firm will reduce the non-risky inputs when the expected energy price increases if and only if the utility function is DARA.*

From the above theorems, we know the impact of expected output price on the inputs demands is complex. That is, it not only depends on the neoclassical production function $F(\cdot)$, but also relates to the marginal rate of substitution, S, between expected profit and profit's risk. Furthermore, the impact of expected energy price on the demand for energy depends on the relative size of the elasticity of risk aversion with respect to the mean of the final profit $\varepsilon_{S,\mu}$ and the inverse of the the elasticity of the mean of the final profit with respect to the energy $1/\varepsilon_{\mu,x_n}$. However, when the utility function is a decreasing absolute risk aversion (DARA), increasing expected output price will surely cause risk-averse firm to increase the inputs' demand, while increasing expected energy price will surely cause risk-averse firm to decrease the demand for the energy and increase the demand for non-risky inputs.

4 Some Special Cases

In this section, we consider two special cases of our model. First, we deal with the situation with only uncertain energy price. In this case, we can have $\sigma_p = \sigma_{p,p_n} = 0$ and $\sigma_\Pi = \sigma_{p_n} x_n$. We have the following observations for the impacts of expected energy and output prices as follows:

Theorem 4.1 *Under the model setup to maximize the expected utility of the profit $V(\mu_\Pi, \sigma_\Pi)$ stated in (2.3), we have*

1. $\partial x_n/\partial \mu_p > 0$ *if and only if* $S_\mu < x_n F_n/(F\sigma_\Pi)$,
2. *an increase in the expected output price will surely cause the risk-averse firm to increase the demand for non-risky input, and*
3. *if* $S_\mu < 0$, *then* $\partial x_n/\partial \mu_p > 0$; *that is, when the utility function is DARA, an increase in the expected output price will surely cause the risk-averse firm to increase the inputs' demand.*

Theorem 4.2 *Under the model setup to maximize the expected utility of the profit $V(\mu_\Pi, \sigma_\Pi)$ stated in (2.3), we have*

1. $\partial x_n/\partial \mu_{p_n} < 0$ *if and only if* $S_\mu < 1/\sigma_\Pi$;
2. *if* $S_\mu < 0, \partial x_n/\partial \mu_{p_n} < 0$; *that is, when the utility function is DARA, an increase in the expected energy price will surely cause the risk-averse firm to reduce the demand for energy; and*

3. $\partial x_i / \partial \mu_{p_n} \equiv 0$; *that is, an increase in the expected energy price has no effect on the demands for inputs with fixed prices.*

The proofs of Theorems 4.1 and 4.2 are simple and similar to arguments in Section 3. We omit the details.

Thus, from the above theorems, we know that when only the energy price is uncertain, increasing the expected output price will surely cause the risk averse firm to increase the demand for the non-risky inputs inputs. This is different from the result obtained under the situation with joint energy and output price uncertainties and it is intuitive. On the other hand, the expected energy price has no impact on the demands for the non-risky inputs.

Now we turn to the case with only uncertain output price. In this situation, we can have $\sigma_{p_n} = \sigma_{p,p_n} \equiv 0$ and $\sigma_\Pi = \sigma_p F$. We have the following observations for the impacts of expected energy and output prices:

Theorem 4.3 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in (2.3), we have*

1. $\partial x_j / \partial \mu_p > 0, j = 1, \cdots, n$ *if and only if* $S_\mu < 1/\sigma_\Pi$;

2. *the firm will increase inputs when the expected output price increases if and only if* S_μ *is less than the inverse of the standard deviation; and*

3. *if* $S_\mu < 0, \partial x_n / \partial \mu_p > 0$; *that is, when the utility function is DARA, increasing the expected output price will surely cause the risk averse firm to increase the inputs' demand.*

Theorem 4.4 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in (2.3), we have*

1. $\partial x_n / \partial \mu_{p_n} < 0$ *if and only if* $S_\mu < F/(x_n F_n \sigma_\Pi)$; *and*

2. *If* $S_\mu < 0, \partial x_n / \partial \mu_{p_n} < 0$; *that is, when the utility function is DARA, increasing the expected energy price will surely cause risk averse firm to decrease the demand for energy.*

Theorem 4.5 *Under the model setup to maximize the expected utility of the profit* $V(\mu_\Pi, \sigma_\Pi)$ *stated in (2.3), we have*

1. *$\partial x_i/\partial \mu_{p_n} < 0$ if and only if $S_\mu < 0$; and*

2. *the firm will decrease non-risky inputs when the expected energy price increases if and only if the utility function is DARA.*

Theorems 4.3 to 4.5 demonstrate that the concepts of decreasing absolute risk aversion (DARA) are important in describing the behaviours of the risk averse firm under price uncertainties.

5 An Empirical example

We use the U.S. natural gas monthly data data for the period March 2001- March 2010 (obtained from Henry Hub). We also adopt the method of Alghalith (2007) to generate corresponding data series for μ_{p_n}. Thereafter, applying the approach used in Alghalith (2010c), we could estimate the following comparative statics for each month (and we calculated the average values for the entire period)

$$\frac{\partial x_n}{\partial \mu_{p_n}} .$$

For March 2010, we get

$$\frac{\partial x_n}{\partial \mu_{p_n}} = 409229.7,$$

and obtain the average values to be

$$\frac{\partial x_n}{\partial \mu_{p_n}} = 459511.6504 .$$

We note that $\partial x_n/\partial \mu_{p_n} > 0$ which is consistent with our theoretical result. That is, an increase in the energy price does not necessarily reduce the energy demand.

6 Concluding remarks

As documented in the literature such as Alghalith (2008) and Alghalith (2010), the energy price is uncertain. Furthermore, the price of output can be random also. In this paper, we analyze the impacts of joint energy and output price uncertainties in a mean-variance framework. The concept of elasticity plays a central role in the analysis. However, when the utility function is DARA, clear observations can be obtained. That is, increasing the expected output price will surely cause the risk averse firm to increase the inputs' demand, while increasing the expected energy price will surely cause the risk averse firm to decrease the demand for energy and increase the demand for the non-risky inputs. Furthermore, if the firms's preferences exhibit variance vulnerability, increasing the variance of energy price will necessarily cause the risk averse firm to decrease the demand for the non-risky inputs. As for the impacts of the covariance of energy price and output price, the results are unclear and greatly depend on several elasticities.

In this paper, we also consider two special cases of our model. In the first case of only uncertain energy price, we can assert that increasing the expected output price will surely cause the risk averse firm to increase the demand for the non-risky inputs. Moreover, the uncertain energy price has no impact on the demands for non-risky inputs. These results are very different from the results obtained under the case of joint energy and output price uncertainties and they are intuitive. We also consider the case of only uncertain output price. Agian, the concepts of DARA and variance vulnerability are important in describing the behaviours of a risk aversion firm under multiple price uncertainties.

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