



Munich Personal RePEc Archive

# **RD, Competition and Growth with Human Capital Accumulation : A Comment**

Bianco, Dominique

CRP Henri Tudor, University of Nice-Sophia-Antipolis, GREDEG  
(CNRS)

9 October 2007

Online at <https://mpra.ub.uni-muenchen.de/5243/>  
MPRA Paper No. 5243, posted 10 Oct 2007 UTC

# R&D, Competition and Growth with Human Capital Accumulation : A Comment

9 october 2007

**Dominique Bianco**<sup>12</sup>

*CPR Henri Tudor,*

*University of Nice-Sophia-Antipolis, GREDEG (CNRS)*

**Abstract :** *This paper shows that the results of Bucci (2003) depend critically on the assumption that there are no difference between the intermediate goods share in final output, the returns of specialization and the degree of market power of monopolistic competitors. In this paper, we disentangle the market power parameter from the intermediate goods share in final output and the returns to specialization. The main result of this paper is that the competition has no effect on growth contrary to Bucci (2003). This result is explained by the fact that economic growth rate depends on the parameters describing preference and the human capital accumulation technology but is completely independent of competition and R&D activity.*

**Keywords :** Endogenous growth, Horizontal differentiation, Technological change, Imperfect competition

**JEL Classification :** D43, J24, L16, O31, O41

---

<sup>1</sup>E-mail : dominique.bianco@tudor.lu

2A, rue Kalchesbrück L-1852 Luxembourg-Kalchesbrück (Luxembourg)

<sup>2</sup>E-mail : dominique.bianco@gredeg.cnrs.fr  
250, Rue Albert Einstein 06560 Valbonne (France)

# 1 Introduction

Bucci (2003) studies the impact of competition in the intermediate goods sector on growth and the sectoral distribution of skills within an R&D based growth model with human capital accumulation. By combining in the simplest possible way the Lucas (1988) model of human capital accumulation with the R&D based model developed by Grossman and Helpman (1991), Bucci (2003) shows that the market power has a positive effect on growth.

Among the assumptions used by Bucci (2003) to derive this result is that there are no difference between the intermediate goods share in final output, the returns to specialization and the degree of market power of monopolistic competitors. This leads to the natural question whether making such a difference to the model changes its predictions. In this note, we show that including this difference into the model developed by Bucci (2003) eliminates the result mentioned above. Indeed, in this case, the imperfect competition has no effect on growth.

## 2 The model

The model developed is based on Bucci (2003).<sup>3</sup> The economy is structured by three sectors : final good sector, intermediate goods sector and R&D sector. The final output sector produces output that can be used for consumption using skilled labor and intermediate goods. These are available in  $n$  varieties and are produced by employing only human capital. The R&D sector creates the blueprints for new varieties of intermediate goods which are produced by employing skilled labor. These blueprints are sold to the intermediate goods sector. Unlike the traditional R&D based growth models, we assume that the supply of human capital may grow over time.

### 2.1 The final good sector

In this sector atomistic producers engage in perfect competition. The final good sector produces a composite good  $Y$  by using all the  $j$ th type of intermediate goods  $x_j$  and skilled labor  $H_Y$ .<sup>4</sup> Production is given by :

$$Y = AH_Y^{1-\lambda} n^{\gamma-\lambda(\frac{1}{\alpha}-1)} \left[ \int_0^n x_j^\alpha dj \right]^{\frac{\lambda}{\alpha}}, \quad (1)$$

---

<sup>3</sup>We use the notations of Bucci (2003) in order to have a direct comparison with his model.

<sup>4</sup>Time subscripts are omitted whenever there is no risk of ambiguity.

where  $\alpha, \lambda, \gamma \in [0, 1]$  and  $A$  are technological parameters. This production function allow us to disentangle the degree of market power of monopolistic competitors in the intermediate sector ( $\frac{1}{\alpha} - 1$ ), the intermediate goods share in final output ( $\lambda$ ) and the degree of returns from specialization ( $\gamma$ ).<sup>5</sup> In this sense, this model is a generalization of Bucci (2003) model.<sup>6</sup> If we normalize to one the price of the final good, the profit of the representative firm is given by :

$$\pi_Y = AH_Y^{1-\lambda} n^{\gamma-\lambda(\frac{1}{\alpha}-1)} \left[ \int_0^n x_j^\alpha dj \right]^{\frac{\lambda}{\alpha}} - \int_0^n p_j x_j dj - w_Y H_Y, \quad (2)$$

where  $w_Y$  is the wage rate in the final good sector and  $p_j$  is the price of the  $j$ th intermediate good. Under perfect competition in the final output market and the factor inputs markets, the representative firm chooses intermediate goods and labor in order to maximize its profit taking prices as given and subject to its technological constraint. The first order conditions are the followings :

$$\frac{\partial \pi_Y}{\partial x_j} = \lambda AH_Y^{1-\lambda} n^{\gamma-\lambda(\frac{1}{\alpha}-1)} x_j^{\alpha-1} \left[ \int_0^n x_j^\alpha dj \right]^{\frac{\lambda}{\alpha}-1} - p_j = 0, \quad (3)$$

$$\frac{\partial \pi_Y}{\partial H_Y} = (1 - \lambda) AH_Y^{-\lambda} n^{\gamma-\lambda(\frac{1}{\alpha}-1)} \left[ \int_0^n x_j^\alpha dj \right]^{\frac{\lambda}{\alpha}} - w_Y = 0. \quad (4)$$

Equation (3) is the inverse demand function for the firm that produces the  $j$ th intermediate good whereas equation (4) characterizes the demand function of skilled labor.

## 2.2 The intermediate goods sector

In the intermediate goods sector, producers engage in monopolistic competition. Each firm produces one horizontally differentiated intermediate good and have to buy a patented design before producing it. Following Grossman and Helpman (1991), Bucci (2003), Bucci (2005b) and Bucci (2005c), we assume that each local intermediate monopolist has access to the same technology employing only skilled labor  $h_j$  :

$$x_j = h_j. \quad (5)$$

---

<sup>5</sup>Benassy (1996) made a simple modification to the Dixit and Stiglitz (1977) model which clearly disentangles taste for variety and market power. At the same time, Benassy (1998) and de Groot and Nahuis (1998) show that the introduction of this modification in an endogenous growth model with expanding product variety *à la* Grossman and Helpman (1991) affects the welfare analysis.

<sup>6</sup>Indeed, we obtain the Bucci (2003) model by introducing the following constraints  $\lambda = \alpha, \gamma = 1 - \alpha$  in our model.

We suppose that firms behavior which produce intermediate goods is governed by the principle of profit maximization at given factor prices under a technological constraint. The profit function of firms is the following :

$$\pi_j = p_j x_j - w_j h_j, \quad (6)$$

where  $w_j$  is wage rate in the intermediate goods. Using the first order condition, we obtain the price of the  $j$ th intermediate good :

$$p_j = \frac{w_j}{\alpha}, \quad (7)$$

At the symmetric equilibrium, all the firms produce the same quantity of the intermediate good, face the same wage rate and by consequence fix the same price for their production. The price is equal to a constant mark up  $\frac{1}{\alpha} - 1$  over the marginal cost  $w$ . Defining by  $H_j = \int_0^n h_j dj$ , the total amount of skilled labor employed in the intermediate goods sector and under symmetry among intermediate goods producers, we can rewrite the equation (5) as follows :

$$x_j = \frac{H_j}{n}, \quad (8)$$

Finally, the profit function of the firm which produces the  $j$ th intermediate good is

$$\pi_j = A\lambda(1 - \alpha)n^{\gamma-1}H_j^\lambda H_y^{1-\lambda}. \quad (9)$$

### 2.3 The R&D sector

There are competitive research firms undertaking R&D. Following Bucci (2003), Bucci (2005b) and Bucci (2005c), we assume that new blueprints are produced using an amount of R&D skilled labor  $H_n$  :

$$\dot{n} = CH_n, \quad (10)$$

where  $C > 0$  represents the productivity of the R&D process. Because of the perfect competition in the R&D sector, we can obtain the real wage in this sector as a function of the profit flows associated to the latest intermediate in using the zero profit condition :

$$w_n H_n = \dot{n} V_n, \quad (11)$$

where  $w_n$  represents the real wage earned by R&D skilled labor.  $V_n$  is the real value of such a blueprint which is equal to :

$$V_n = \int_t^\infty \pi_j e^{-r(\tau-t)} d\tau, \tau > t, \quad (12)$$

where  $r$  is the real interest rate. Given  $V_n$ , the free entry condition leads to :

$$w_n = CV_n. \quad (13)$$

## 2.4 The consumer behavior

The demand side is characterized by the representative household who holds asset in the form of ownership claims on firm and chooses plans for consumption ( $c$ ), asset holdings ( $a$ ) and human capital ( $h$ ).<sup>7</sup> Following Lucas (1988), we assume that the household is endowed with one unit of time and optimally allocates a fraction  $u$  of this time endowment to productive activities (final good, intermediate goods and research production) and the remaining fraction  $(1-u)$  to non productive activities (education). Following Romer (1990), we assume that the utility function of this consumer is <sup>8</sup> :

$$U = \int_0^{\infty} e^{-\rho t} \frac{c^{1-\theta} - 1}{1-\theta} dt, \quad (14)$$

where  $c$  is private consumption,  $\rho > 0$  is the rate of pure time preference and  $\sigma = \frac{1}{\theta}$  is the intertemporal elasticity of substitution. The flow budget constraint for the household is :

$$\dot{a} = wuh + ra - c, \quad (15)$$

where  $w$  is the wage rate per unit of labor services. The human capital supply function is given by :

$$\dot{h} = \delta(1-u)h, \quad (16)$$

where  $\delta > 0$  is a parameter reflecting the productivity of the education technology. From the maximization program of the consumer,<sup>9</sup> the first order conditions are :

$$\lambda_1 = c^{-\theta} e^{-\rho t}, \quad (17)$$

$$-\dot{\lambda}_1 = \lambda_1 r, \quad (18)$$

$$-\dot{\lambda}_2 = \lambda_1 wu + \lambda_2 \delta(1-u), \quad (19)$$

$$\lambda_1 = \lambda_2 \frac{\delta}{w}. \quad (20)$$

Equation (17) gives the discounted marginal utility of consumption which satisfies the dynamic optimality condition in equation (18). Equation (20)

---

<sup>7</sup>Like Bucci (2003), Bucci (2005b) and Bucci (2005c), for the sake of simplicity, we assume that there is no population growth.

<sup>8</sup>This specification of the utility function is a generalization of the Bucci (2003), Bucci (2005b) and Bucci (2005c) models. Indeed, these authors use a utility function which is logarithmic.

<sup>9</sup>The control variables of this problem are  $c$  et  $u$  whereas  $a$  and  $h$  are the state variables.  $\lambda_1$  et  $\lambda_2$  denote the shadow price of the household's asset holdings and human capital stock.

gives the static optimality condition for the allocation of time. The marginal cost of an additional unit of skills devoted to working evolves optimally as in equation (19). Conditions (17) through (20) must satisfy the constraints (15 and 16), together with the transversality conditions :

$$\lim_{t \rightarrow \infty} \lambda_{1t} a_t = 0, \quad (21)$$

$$\lim_{t \rightarrow \infty} \lambda_{2t} h_t = 0. \quad (22)$$

### 3 The equilibrium and the steady state

In this section, we characterize the equilibrium and give some analytical characterization of a balanced growth path.

#### 3.1 The equilibrium

It is now possible to characterize the skilled labor market equilibrium in the economy considered. On this market, because of the homogeneity and the perfect mobility across sectors, the arbitrage ensures that the wage rate that is earned by salaries which work in the final good sector, intermediate goods sector or R&D sector is equal. As a result, the following three conditions must simultaneously be satisfied :

$$u^* H = H_Y + H_j + H_n, \quad (23)$$

$$w_j = w_y, \quad (24)$$

$$w_j = w_n. \quad (25)$$

Equation (23) is a resource constraint, saying that at any point in the time the sum of the skilled labor demands coming from each activity must be equal to the total available supply. Equation (24) and equation (25) state that the wage earned by one unit of skilled labor is to be the same irrespective of the sector where that unit of skilled labor is actually employed.

We can characterize the product market equilibrium in the economy considered. Indeed, on this market, the firms produce a final good which can be consumed. Consequently, the following condition must be satisfied :

$$Y = C. \quad (26)$$

Equation (26) is a resource constraint on the final good sector.

We can describe the capital market equilibrium in our economy. Because the

total value of the household's assets must be equal to the total value of firms, the following condition must be checked :

$$a = nV_n, \quad (27)$$

where  $V_n$  is given by the equation (12) and satisfies the following asset pricing equation :

$$\dot{V}_n = rV_n - \pi_j, \quad (28)$$

### 3.2 The steady state

At the steady state, all variables as  $Y$ ,  $c$ ,  $n$ ,  $a$ ,  $H$ ,<sup>10</sup>  $H_Y$ ,  $H_j$ ,  $H_n$  grow at a positive constant rate.

**Proposition 1** *If  $u$  is constant then all the other variables grow at strictly positive rates with*

$$g_H = g_{H_Y} = g_{H_j} = g_{H_n} = g_n, \quad (29)$$

$$g_Y = g_c = (\gamma + 1)g_n = (\gamma + 1)g_a, \quad (30)$$

**Proof.** *From the equilibrium on the skilled labor market, given by the equation (23), it easy to show that  $g_H = g_{H_Y} = g_{H_j} = g_{H_n}$  if  $u$  is constant. From the definition of the firm research process, given by the equation (10), we obtain that  $g_n = g_{H_n}$ . Now, if we combine the two last equations, we obtain the equation (29). From the equilibrium on the product market, given by the equation (26), it easy to find that  $g_Y = g_c$ . The equation (27) implies that  $g_a = g_n$ . By substituting equation (8) into equation (1), then by log-differentiating the equation (1), we obtain  $g_Y = (\gamma + 1)g_n$ . By combining the previous equations, we find the equation (30). ■*

---

<sup>10</sup>Given the assumptions on the size of the representative household and the population growth rate,  $h \equiv H$  which implies that we can use  $g_H$  instead of  $g_h$ .

Using the previous equations, we can demonstrate the following steady state equilibrium values for the relevant variables of the model<sup>11</sup> :

$$r = \delta + \frac{\gamma(\delta - \rho)}{\gamma(\theta - 1) + \theta}, \quad (31)$$

$$H_j = \frac{n\alpha\delta}{C(1 - \alpha)}, \quad (32)$$

$$H_Y = \frac{n\delta(\lambda - 1)}{C(\alpha - 1)\lambda}, \quad (33)$$

$$H_n = \frac{n(\delta - \rho)}{C(\gamma(\theta - 1) + \theta)}, \quad (34)$$

$$u^* = \frac{(\gamma + 1)\delta(\theta - 1) + \rho}{\delta(\gamma(\theta - 1) + \theta)}, \quad (35)$$

$$g_H = g_{H_Y} = g_{H_j} = g_{H_n} = g_n = \frac{\delta - \rho}{\gamma(\theta - 1) + \theta}, \quad (36)$$

$$g_Y = g_c = (\gamma + 1)g_n = (\gamma + 1)g_a = \frac{(\gamma + 1)(\delta - \rho)}{\gamma(\theta - 1) + \theta}. \quad (37)$$

According to the equation (31), the real interest rate ( $r$ ) is constant. Equation (32), (33) and (34) give the amount of skilled labor in each sector at the equilibrium. Equation (35) represents the optimal and constant fraction of the household's time endowment that it will decide to devote to work ( $u^*$ ) in equilibrium. Equation (36) states that the growth rate of human capital and the innovation activity are equal and depend on technological and preference parameters ( $\delta$ ,  $\gamma$ ,  $\theta$  and  $\rho$ ). Equation (37) shows that the growth rate is a function of technological and preference parameters ( $\delta$ ,  $\gamma$ ,  $\theta$  and  $\rho$ ).

## 4 The relationship between product market competition and growth

In this section, we study the long run relationship between competition and growth in the model presented above. Following most authors, we use the so-called Lerner Index to gauge the intensity of market power within a market. Such an index is defined by the ratio of price ( $P$ ) minus marginal cost ( $Cm$ ) over price. Using the definition of a mark up ( $Markup = \frac{P}{Cm}$ ) and Lerner Index ( $LernerIndex = \frac{P - Cm}{P}$ ), we can use (7) to define a proxy of

---

<sup>11</sup>Results (31) through (37) are demonstrated in the appendix.

competition as follows<sup>12</sup> :

$$(1 - LernerIndex) = \alpha, \quad (38)$$

We show that our simple generalization of Bucci (2003)'s model that consists in having the monopolistic mark-up in the intermediate goods sector, the intermediate goods share in the final output and the returns to specialization treated separately, in order to have a better measurement of competition, the competition has no impact on growth.

**Proposition 2** *The competition has no effect on growth for all positive values of  $\rho$ ,  $\eta$ ,  $L$  and  $\gamma$ ,  $\lambda \in [0, 1]$ .*

**Proof.** *The proof is obtained by differentiating (37) with respect to  $\alpha$  :*

$$\frac{\partial g_Y}{\partial \alpha} = 0. \quad (39)$$

■

This result is clearly explained by the fact that economic growth rate only depends on the parameters describing preference and the human capital accumulation technology but is completely independent of competition and RD activity.

## 5 Conclusion

In this paper, we presented a generalization of production function of Bucci (2003) which disentangles the monopolistic mark-up in the intermediate goods sector, the intermediate goods share in the final output and the returns to specialization in order to have a better measurement of competition. Our main finding is that the result of his model depends critically on the assumptions that there are no differences between these three parameters. Indeed, for all values of parameters except to  $\gamma = 1 - \alpha$ , we could remove the effect of competition on growth.

---

<sup>12</sup>This is the same measure of product market competition used by Bucci (2003), Aghion, Bloom, Blundell, Griffith, and Howitt (2005), Aghion and Griffith (2005), Aghion and Howitt (2005), Bucci (2005a) and Bucci (2005b), Bucci (2005c) and Bianco (2007), contrary to Bucci and Parello (2006) which link the competition to two components : the input shares in income and the parameter of substitution between intermediates.

## Appendix

In these appendix, we describe the way followed in order to obtain the main results of this paper (31 through 37). Consider the representative consumer's problem (equations (14) through (23) in the main text), whose the first order conditions are stated in equations (17) through (23) with consumer's constraints and transversality conditions, we have :

$$\lambda_1 = c^{-\theta} e^{-\rho t}, \quad (40)$$

$$-\dot{\lambda}_1 = \lambda_1 r, \quad (41)$$

$$-\dot{\lambda}_2 = \lambda_1 w u + \lambda_2 \delta (1 - u), \quad (42)$$

$$\lambda_1 = \lambda_2 \frac{\delta}{w}. \quad (43)$$

$$\dot{a} = r a + w u h - c, \quad (44)$$

$$\dot{h} = \delta (1 - u) h, \quad (45)$$

$$\lim_{t \rightarrow \infty} \lambda_{1t} a_t = 0, \quad (46)$$

$$\lim_{t \rightarrow \infty} \lambda_{2t} h_t = 0. \quad (47)$$

Combining equations (43) and (42), we obtain :

$$\frac{\dot{\lambda}_2}{\lambda_2} = -\delta. \quad (48)$$

From equation (41), we get :

$$\frac{\dot{\lambda}_1}{\lambda_1} = -r. \quad (49)$$

Equation (43) implies that :

$$\frac{\dot{\lambda}_1}{\lambda_1} = \frac{\dot{\lambda}_2}{\lambda_2} - g_w. \quad (50)$$

Combining equations (48), (49) and (50), we obtain :

$$r = \delta + g_w. \quad (51)$$

In the balanced growth path equilibrium, the growth rate of the wage accruing to human capital ( $g_w$ ) is constant (see later on this appendix). This implies that the real interest rate ( $r$ ) will be also constant. With a constant real interest rate and using the equation (9), the equation (12) becomes :

$$V_{nt} = A\lambda(1 - \alpha)B^\lambda \int_t^\infty n_\tau^{\gamma-1} H_{j\tau}^\lambda H_{Y\tau}^{1-\lambda} e^{-r(\tau-t)} d\tau, \tau > t. \quad (52)$$

In order to compute the market value of one unit of research output at time  $t$  ( $V_{nt}$ ) along the balanced growth path equilibrium, we use the following equations :

$$n_\tau = n_t e^{g_n t}, \quad (53)$$

$$H_{j\tau} = H_{jt} e^{g_{H_j} t}, \quad (54)$$

$$H_{Y\tau} = H_{Yt} e^{g_{H_Y} t}. \quad (55)$$

$$(56)$$

Inserting equations (53), (54) and (55) into equation (52), and after some calculations, we get :

$$V_{nt} = \frac{A\lambda(1-\alpha)n^{\gamma-1}(BH_j)^\lambda H_Y^{1-\lambda}}{r - (\gamma-1)g_n - (1-\lambda)g_{H_Y} - \lambda g_{H_j}}. \quad (57)$$

Such result is obtained under the assumption that  $r > (\gamma-1)g_n - (1-\lambda)g_{H_Y} - \lambda g_{H_j}$ . In a moment, we will demonstrate that this hypothesis (which assures that  $V_{nt}$  is positive for each  $t$ ) is always checked along the balanced growth path equilibrium. Given  $V_{nt}$  and making use of equation (13) in the main text, we get :

$$w_n = \frac{CA\lambda(1-\alpha)n^{\gamma-1}(BH_j)^\lambda H_y^{1-\lambda}}{(r - (\gamma-1)g_n - (1-\lambda)g_{H_Y} - \lambda g_{H_j})}. \quad (58)$$

From equations (4) and (8), we get the value of the wage rate accruing to human capital employed in the final good sector :

$$w_Y = (1-\lambda)AH_Y^{-\lambda}n^\gamma B^\lambda H_j^\lambda. \quad (59)$$

From equations (3), (7) and (8), we get the value of the wage rate accruing to human capital in the intermediate goods sector :

$$w_j = \alpha\lambda AH_Y^{1-\lambda}n^\gamma B^\lambda H_j^{\lambda-1}. \quad (60)$$

Combining equations (40) and (41), we are able to obtain the usual Euler equation, giving the optimal household's consumption path :

$$g_c = \frac{r - \rho}{\theta}. \quad (61)$$

From the equation above, we clearly see that  $r$  must be greater than  $\rho$  and in the same time  $\theta > 0$  in order to have  $g_c$  positive. From equation (27) and using equation (57), we get :

$$g_a = g_n + g_{V_n} = g_n + (\gamma-1)g_n + \lambda g_{H_j} + (1-\lambda)g_{H_Y}. \quad (62)$$

Using equations (29), we can rewrite the equation (62) as follows :

$$g_a = (\gamma + 1)g_n. \quad (63)$$

From equations (44) and (49), we have :

$$\frac{\dot{\lambda}_1}{\lambda_1} = -g_a + uw\frac{h}{a} - \frac{c}{a}. \quad (64)$$

Using equations (45) and (48), we get :

$$\frac{\dot{\lambda}_2}{\lambda_2} = -g_h - u\delta. \quad (65)$$

Equations (58), (59) and (60) together also imply that :

$$g_{w_n} = g_{w_j} = g_{w_Y} = g_w = \gamma g_n. \quad (66)$$

From equations (63), (64) (65) and (66), we obtain :

$$\frac{c}{a} = \delta u + uw\frac{h}{a}. \quad (67)$$

Using equation (63) and equation (66) and knowing that  $u$  is constant at the equilibrium, equation (67) leads to the conclusion that  $\frac{c}{a}$  is constant. In other words :

$$g_c = g_a = (\gamma + 1)g_n. \quad (68)$$

Plugging equation (68) into equation (61), we get :

$$r = \theta(\gamma + 1)g_n + \rho. \quad (69)$$

Equating equation (51) and equation (69) yields :

$$g_w = \delta - \rho - \theta(\gamma + 1)g_n. \quad (70)$$

Now, equating equation (66) and equation (70), we get the growth rate of  $n$  along the balanced growth path equilibrium :

$$g_n = \frac{\delta - \rho}{\theta(\gamma + 1) - \gamma}. \quad (71)$$

Given  $g_n$ , it is now possible to compute the real interest by using equations (51) and (66) :

$$r = \delta + \frac{\gamma(\delta - \rho)}{\gamma(\theta - 1) + \theta}, \quad (72)$$

Combining equations (26) and (68), we get the growth rate :

$$g_Y = g_c = (\gamma + 1)g_n = (\gamma + 1)g_a = \frac{(\gamma + 1)(\delta - \rho)}{\gamma(\theta - 1) + \theta}, \quad (73)$$

From the equation (29), we obtain the growth rate of human capital :

$$g_H = g_{H_Y} = g_{H_j} = g_{H_n} = g_n = \frac{\delta - \rho}{\gamma(\theta - 1) + \theta}, \quad (74)$$

Equating (58) and (59) by using (69) and (71), we get the skilled labor allocated in the final good sector :

$$H_Y = \frac{n\delta(\lambda - 1)}{C(\alpha - 1)\lambda}, \quad (75)$$

Equating (58) and (60) by using (69) and (71), we get the skilled labor allocated in the intermediate goods sector :

$$H_j = \frac{n\alpha\delta}{C(1 - \alpha)}, \quad (76)$$

Combining equations (10) and (71), we get the skilled labor allocated in the research good sector :

$$H_n = \frac{n(\delta - \rho)}{C(\gamma(\theta - 1) + \theta)}, \quad (77)$$

Combining equations (48), (65) and (74), we obtain the time spending in production :

$$u^* = \frac{(\gamma + 1)\delta(\theta - 1) + \rho}{\delta(\gamma(\theta - 1) + \theta)}.$$

## Références

- AGHION, P., N. BLOOM, R. BLUNDELL, R. GRIFFITH, AND P. HOWITT (2005) : “Competition and Innovation : An Inverted-U Relationship,” *Quarterly Journal of Economics*, 120(2), 701–28.
- AGHION, P., AND R. GRIFFITH (2005) : *Competition and Growth : Reconciling Theory and Evidence*. MIT Press, Cambridge and London.
- AGHION, P., AND P. HOWITT (2005) : “Growth with Quality-Improving Innovations : An Integrated Framework,” in *Handbook of Economic Growth*, vol. 1, pp. 67–110. Elsevier Science, Amsterdam, North Holland.

- BENASSY, J.-P. (1996) : “Taste for Variety and Optimum Production Patterns in Monopolistic Competition,” *Economics Letters*, 52, 41–47.
- (1998) : “Is There Always Too Little Research in Endogenous Growth with Expanding Product Variety?,” *European Economic Review*, 42, 61–69.
- BIANCO, D. (2007) : “An inverted-U Relationship between Product Market Competition and Growth in an Extended Romerian Model : A Comment,” MPRA Paper 4492, University Library of Munich, Germany, available at <http://ideas.repec.org/p/pramprapa/4492.html>.
- BUCCI, A. (2003) : “R&D, Imperfect Competition and Growth with Human Capital Accumulation,” *Scottish Journal of Political Economy*, 50(4), 417–439.
- (2005a) : “Inverted-U Relationship between Product Market Competition and Growth in an Extended Romerian Model,” *Rivista di Politica Economica*, 95, 177–205.
- (2005b) : “Monopoly Power in Human Capital based Growth,” *Acta Oeconomica*, 55(2), 121–50.
- (2005c) : “Product Market Competition, R&D Effort and Economic Growth,” UNIMI - Research Papers in Economics, Business, and Statistics 1011, Università degli Studi di Milano, available at <http://ideas.repec.org/p/bep/unimip/1011.html>.
- BUCCI, A., AND C. PARELLO (2006) : “Horizontal Innovation-Based Growth and Product Market Competition,” UNIMI - Research Papers in Economics, Business, and Statistics 1032, Università degli Studi di Milano.
- DE GROOT, H. L., AND R. NAHUIS (1998) : “Taste for Diversity and the Optimality of Economic Growth,” *Economics Letters*, 58, 293–95.
- DIXIT, A. K., AND J. E. STIGLITZ (1977) : “Monopolistic Competition and Optimum Product Diversity,” *American Economic Review*, 67(3), 297–308.
- GROSSMAN, G. M., AND E. HELPMAN (1991) : *Innovation and growth in the global economy*. MIT Press, Cambridge, Mass. and London.
- LUCAS, R. J. (1988) : “On the Mechanics of Economic Development,” *Journal of Monetary Economics*, 22(1), 3–42.
- ROMER, P. M. (1990) : “Endogenous Technological Change,” *Journal of Political Economy*, 98(5), S71–102.