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Abstract

Gârleanu et al. (RFS 2009) show that a demand pressure phenomenon exists in option markets due to limit to arbitrage. They assert that if arbitrage is perfect, option demand does not impact option price. In this note we show that there is a positive relation between the demand for a redundant option and the option price, which is related to the beliefs of constrained investors regarding future payoffs.

JEL classification: G11, G13
Keywords: option; demand pressure; credit constraint

1. Introduction

A large amount of literature has documented a price change for stocks added to various indexes (Shleifer (1986), Harris and Gurel (1986), and Biktimirov et al. (2004)). To explain stock price behavior around index changes, several hypotheses have been offered (Biktimirov et al. (2004)). According to the price pressure hypothesis, buying pressure causes a temporary increase in the stock price to induce investors to sell their shares. Harris and Gurel (1986) observe event-day stock price changes reversing twenty days later and interpret these findings as evidence of price pressure. Recently, Gârleanu et al. (2009), and Bollen and Whaley (2004) investigate the relation between the demand for options and the price of options. Gârleanu et al. develop a model where option market makers cannot hedge their inventories perfectly. They show that a demand pressure phenomenon exists in the derivative markets since a demand pressure in an option raises its price. The expensiveness of an option is then positively related to the option demand of non-market makers. Bollen and Whaley (2004) examine two alternative hypotheses for the relation between demand for options and implied volatilities; the limits to arbitrage hypothesis and the volatility information hypothesis. Investors with information about future volatility use options. Since their trade contains information, option prices are then affected. Bollen and Whaley report evidence that changes in implied volatility are related to net buying pressure and show that their results are consistent with the limits to the arbitrage hypothesis. Kang and Park (2008) investigate the Korean index option market to examine how the net buying pressure impacts the implied volatility of options. They argue that their results are not consistent either with the limit to arbitrage hypothesis or with the volatility information hypothesis but are best explained by the directional information hypothesis. If prices are expected to rise, informed traders buy call options and sell put options. Since these positions contain information, an increase in option price and a decrease in put price will result.

Gârleanu et al. (2009) assert that if arbitrage is perfect, option demand does not impact option price. This is because the stock price is considered as exogenous. This note contributes to this literature by showing the existence of a positive relationship between the demand for a redundant option and the equilibrium option price, which is related to the beliefs of constrained investors regarding future payoffs. If the chance of the high payoff rises, constrained investors would like to increase their holdings of the stock. Since they cannot borrow at the riskless rate to buy more stocks, they increase their demand for the option. Unconstrained investors increase their short position of the option and then increase their demand for the stock to put a perfect hedge. The increase in the aggregate demand for the stock that results induces an increase in the stock price, and consequently, an increase in the option price occurs due to the no-arbitrage opportunity.

The rest of the paper is organized as follows: Section 2 describes the model. In section 3 we provide an equilibrium analysis. Section 4 investigates the impact of demand pressure on option price. Section 5 concludes.

2. The model

We consider a single good and an exchange economy with one period (two dates, zero and one). The financial market is composed of three assets: a stock in fixed supply, a call option written on the stock which is in zero net supply and a riskless asset in perfectly elastic supply at a price of one and yielding a rate of return equal to zero. The prices of the stock and the option are respectively p_S and p_0. We denote $\tilde{v}$ the stock payoff, $\tilde{g} = Max(\tilde{v} - k, 0)$ the payoff of the option and k the exercise price. We normalize the supply of stocks to be one unit. Investors have prior beliefs regarding the distribution of the stock payoff. The formation of expectations is exogenous to the model.

In our model, investors are competitive and form a continuum with measure 1. These investors are either borrowing constrained or unconstrained. Investors of the first type ($i = 1$), in fraction N, have unlimited access to credit. Investors of the second type ($i = 2$), in fraction $1 - N$, cannot rely on borrowing to buy stocks. At $t = 0$ investors determine their portfolio. At $t = 1$ the uncertainty resolves and investors consume. Let x_S^i and x_0^i represent respectively the shares holding of the stock and option, $W^i(0)$ the first date wealth and $W^i(1)$ the final date wealth of investor i. Let U_1 and U_2 be the utility of an investor of the first type and second type respectively, and which are strictly increasing and strictly concave.

We assume that the stock payoff can take only two possible values v_H and v_L where ($0 < v_L < v_H$) and ($v_L < k < v_H$). For unconstrained investors the option contract is redundant. The condition of no-arbitrage opportunity requires that $v_L < p_S < v_H$ and

$$\begin{cases} p_0 = \omega_1 + \omega_2 p_S \\ \tilde{g} = \omega_1 + \omega_2 \tilde{v} \end{cases} \quad (1)$$

where $\omega_1 = -(v_H - k)v_L / (v_H - v_L) < 0$, $\omega_2 = (v_H - k)/(v_H - v_L) > 0$.

3. Equilibrium analysis

Let $\Gamma_i(x_S^i) = E_i U_i(W^i(0) + x_S^i(\tilde{v} - p_S))$. In the absence of the option contract and if the borrowing constraint does not exist, the demand for the stock of investor i, denoted by X_S^i, is solution of equation $\Gamma_i'(x_S^i) = 0$. The quantity X_S^i verifies

$$E_i[(\tilde{v} - p_S)U_i'(W^i(0) + X_S^i(\tilde{v} - p_S))] = 0 \quad (2)$$

We are interested in the situation where the borrowing constraint imposed on investors of type 2 is binding.

Assumption 1 We restrict the set of parameters describing the economy such that $X_S^2 > W^2(0)/p_S$ in equilibrium.

This assumption indicates that constrained investors are optimistic about the stock payoff.

The date-1 wealth of each investor is given by:

$$\tilde{W}_1^i = W^i(0) + x_S^i(\tilde{v} - p_S) + x_0^i(\tilde{g} - p_0)$$

By (1) we have:

$\tilde{W}_1^i = W^i(0) + (x_S^i + \omega_2 x_0^i)(\tilde{v} - p_S)$

For the unconstrained investor the option is redundant. His optimal demands for the stock and option verify:

$$x_S^1 + \omega_2 x_0^1 = X_S^1 \qquad (3)$$

For the constrained investor the program is to maximize expected utility with the constraint that $p_S x_S^2 + p_0 x_0^2 \le W_0^2$.

$MaxE_2 U_2(W^2(0) + x_S^2(\tilde{v} - p_S) + x_0^2(\tilde{g} - p_0))$

$p_S x_S^2 + p_0 x_0^2 \le W^2(0)$

The first order conditions are:

$$\begin{cases} E_2[(\tilde{v} - p_S)U_2'(\tilde{W}^2(1))] - \lambda p_S = 0 \\ E_2[\omega_2(\tilde{v} - p_S)U_2'(\tilde{W}^2(1))] - \lambda p_0 = 0 \\ \lambda\left(W^2(0) - p_S x_S^2 - p_0 x_0^2\right) = 0\,,\ \lambda \ge 0 \end{cases}$$

The first two equations imply that $\lambda\omega_2 p_S = \lambda p_0$. From (1) it comes that $\lambda = 0$. Hence $E_2[(\tilde{v} - p_S)U_2'(\tilde{W}^2(1))] = 0$. Consequently we obtain the following equation:

$$x_S^2 + \omega_2 x_0^2 = X_S^2 \qquad (4)$$

The condition of clearing on the option market is $Nx_0^1 + (1-N)x_0^2 = 0$. Using (3), (4) and the stock market-clearing condition we get:

$$NX_S^1(p_S) + (1-N)X_S^2(p_S) = 1$$

Proposition 1 In equilibrium and under assumption 1, constrained investors hold a long position in the option.

Proof. Using (1) and (4) we have

$p_S x_S^2 + p_0 x_0^2 = p_S X_S^2 + \omega_1 x_0^2$

The budget constraint yields

$$x_0^2 \ge (W^2(0) - p_S X_S^2)/\omega_1 \qquad (5)$$

The result follows from the assumption 1. □

In equilibrium, a constrained investor holds a long position in the option such that (5) is verified and completes her portfolio by a quantity of stocks that satisfies equation (4). She has an infinite number of possibilities for the composition of her optimal portfolio, which lead to the same equilibrium prices for the stock and option. As an example she could not hold the riskless asset so the quantities of stocks and options in her portfolio are such that $x_S^2 + \omega_2 x_0^2 = X_S^2$ and $p_S x_S^2 + p_0 x_0^2 = W^2(0)$.

We consider a second restriction on the parameters of the economy.

Assumption 2 The demand functions $X_S^1(.)$ and $X_S^2(.)$ are strictly decreasing in the stock price.

This hypothesis guarantees, among others things, that the equilibrium stock price is unique.

4. A demand pressure analysis

Let's examine first how the unconstrained demand function for the stock X_S^2 of constrained investors change when the degree of their optimism, identified by the probability γ_2 they assign to the high state v_H, increases.

Lemma The unconstrained demand function X_S^2 of constrained investors increases when their prediction about the chance of the occurrence of the high state is more favorable.

Proof. Differentiating (2) with respect to the probability γ_2 we get:

$$\frac{\partial X_S^2}{\partial \gamma_2} = -\frac{(v_H - p_S)U_2'(W_0^2 + X_S^2(v_H - p_S)) + (p_S - v_L)U_2'(W^2(0) + X_S^2(v_L - p_S))}{\gamma_2(v_H - p_S)^2 U_2''(W_0^2 + X_S^2(v_H - p_S)) + (1-\gamma_2)(v_L - p_S)^2 U_2''(W^2(0) + X_S^2(v_L - p_S))}$$

It is clear that the sign of $\partial X_S^2/\gamma_2$ is positive for values of stock price that verify the condition of no-arbitrage opportunity ($v_L < p_S < v_H$). □

As stated in condition (5), the option holding of unconstrained investors is not uniquely determined. We assume that this condition is binding. The option holding of unconstrained investor is then $x_0^2 = (W^2(0) - p_S X_S^2)/\omega_1$. As constrained investors become more optimistic, both the option price and their option holdings change.

Proposition 2 Option trading volume and option price are jointly dependent on the degree of optimism of constrained investors regarding stock payoffs; the more they are optimistic, the greater the option trading volume and option price.

Proof. Consider an increase in the probability that constrained investors assign to the high state, with γ_2 the initial value and γ_2' the new value ($\gamma_2' > \gamma_2$). Let p_S be the initial equilibrium stock price and p_S' the new stock price. It comes from preceding analysis that

$$NX_S^1(p_S) + (1-N)X_S^2(p_S, \gamma_2) = 1 \qquad (6)$$

$$NX_S^1(p_S') + (1-N)X_S^2(p_S', \gamma_2') = 1 \qquad (7)$$

We deduce from the Lemma that stock prices increase when constrained investors are more optimistic, all other things equal. Hence $p_S' > p_S$. By assumption 2 it comes that $X_S^1(p_S') < X_S^1(p_S)$. It follows from equations (6) and (7) that $X_S^2(p_S', \gamma_2') > X_S^2(p_S, \gamma_2)$. Hence $p_S' X_S^2(p_S', \gamma_2') > p_S X_S^2(p_S, \gamma_2)$. We deduce easily that the option holding of constrained investors rises. On the other hand, since the stock price increases, by the no-arbitrage condition (1), the equilibrium option price must increase. □

As in Gârleanu et al. (2009), in our framework, option market traders are not informationally motivated. However, while their paper is agnostic about the source of non-market markers option demand, we have the result that option demand of constrained investors varies as their belief about stock payoff changes. Furthermore, the positive relation between option demand and option price is not due to limits to arbitrage. Gârleanu et al. assert that if arbitrage is perfect, option demand does not impact option price. This is because the stock price is considered as exogenous. The result of Proposition 2 demonstrates that the net buying pressure hypothesis holds for a redundant option market when the stock price is affected by supply and demand.

The result of Proposition 2 is very close to the empirical finding of Kang and Park (2008). They showed that option trading in the KOSPI200 index option market is due to informed traders who exploit their private information concerning the future underlying asset price movements. If the underlying asset price is expected to rise, informed traders buy call options which results in an increase in the option price. In contrast, our paper assumes that investors have subjective prior probabilities for the stock payoff[1]; no reevaluation of prior probabilities based on prices is made.

5. Conclusion

This paper contributes to the recent literature on the net buying pressure on the option market. We show that the net buying pressure hypothesis holds even though the option is redundant. Option demand and option price are positively related and jointly dependent on the degree of optimism of constrained investors. The more they are optimistic about the chance of occurrence of the high state, the greater the option trading volume and option price.

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[1] Whether investors trade because of different prior probabilities or different information, see Varian (2000).