Free Entry and Social Inefficiency under Co-opetition

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Abstract

We investigate the social desirability of free entry in the co-opetition model in which firms compete in a homogeneous product market while sharing common property resources that affect consumers' willingness to pay for products. Our findings show that free entry leads to socially excessive or insufficient market entry in the case of non-commitment co-opetition, depending on the magnitude of "business stealing" and "common property" effects of entry. On the other hand, in the case of pre-commitment co-opetition, free entry leads to excess entry and a decline in common property resources. Interestingly, in this case, the excess entry results of Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) hold even when there are no entry (set-up) costs. These results have important policy implications for entry regulation.

Keywords Excess entry; Free entry; Co-opetition; Entry regulations; Common property resource

JEL Code L13; D43; L51

1 Introduction

In many industries, firms compete for market share while cooperating to manage common property resources that affect the market size or consumers' willingness to pay for products. This simultaneous competition and cooperation is called "co-opetition" (Brandenburger and Nalebuff 1996). For example, mobile phone companies, tourist sites, and food courts share common property resources such as base-station antennas, historic ruins in natural environments, and common dining areas, respectively. The quality of the common property resources affects the market size and/or consumers' willingness to pay for products or services. In addition, the high quality of such resources generates non-excludable benefits for all firms in the industry. Therefore, each firm's investment in common property resources (e.g., erecting additional base-station antennas, preserving historic ruins and natural environment, and maintaining a hygienic eating environment) creates public-good benefits that all the firms can enjoy.[1] Another example of such co-opetitive behavior is generic advertising for various commodities such as beverages, fruit, and

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[1] A mobile phone company's investment to expand its service ares has a positive external effect on the profits of rival mobile phone carriers because the investment increases the availability of calls between different mobile carriers. The investment also benefits consumers and increases their willingness to pay for phone services.

meat and dairy products. Improving product image generates non-excludable benefits for all producers providing the same products. Firms' investment in product safety is also considered as a co-opetive behavior when they share an industry's collective reputation for product safety. In the aforementioned examples, firms share the product image as a common property resource and sometimes voluntarily contribute to improving it.[2]

Is free entry into such co-opetitive industries desirable from a social welfare perspective? To answer this question, we have to identify the following two effects of free entry on social welfare: One is the well-known *business stealing effect* of entry, which creates production inefficiency in the presence of scale economies (Mankiw and Whinston 1986, Suzumura and Kiyono 1987) and generally leads to socially excessive entry. Another is the effect of entry on the amount of common property resources, which we call the *common property effect* of entry. On the one hand, an increase in the number of firms may increase the total investment in common property resources. In this case, free entry may result in socially insufficient entry because firms do not consider the positive external effect of their investment in common property resources on other firms. On the other hand, an increase in the number of firms may also exacerbate the under-provision of common property resources, which may lead to a tragedy-of-the-commons situation. For instance, an increase in the number of tourism firms can deteriorate the quality of tourist attractions (such as wild life and historic ruins), which can eventually destroy tourism itself. As Puppim de Oliveira (2003) indicates, locations such as Acapulco in Mexico, the French Rivera and Mallorca and Torremolinos in Spain have faced tourism-related environmental issues. In these cases, the common property effect contributes to socially excessive entry. This conjecture leads us to the question of whether the government should regulate or encourage firm entry into the co-opetitive industry.

In this study, we formulate a simple model of co-opetition with endogenous entry to present a welfare analysis of free entry equilibrium. In particular, we consider whether the number of firms that can enter a co-opetitive market is excessive or insufficient from the viewpoint of social welfare. We distinguish between two types of co-opetitive investment in common property resources: pre-commitment and non-commitment investments. In the case of non-commitment investment, firms choose their output and investment in the same stage; we call this game a "simultaneous co-opetition game." In the case of pre-commitment investment, firms determine their investment before they choose their output; we call this game a "sequential co-opetition game." In both game, firms' entry decisions are made in the first stage. The difference between non-commitment and pre-commitment investments reflects the difference in reversibility and persistency in investments. When investment has long-term impacts and is difficult to reverse (e.g., renovating historic buildings in a tourist area and increasing base-station antennas for mobile phone networks), it has strategic commitment value, which can be described by a sequential co-opetition game. On the other hand, when investment has short-term impacts and is easy to reverse (e.g., providing generic advertising in daily newspapers and cleaning shopping malls or food courts), it has no strategic commitment value, which can be described as a simultaneous co-opetition game.

[2]Other examples of co-opetition include development of open-source software and rent-seeking or lobbying for permission to sell product to certain groups (e.g., lobbying for the relaxation of regulations against the sale of tobacco or alcohol products to under-age people, of specific medicines to a mass of people, and of financial products to inexperienced consumers).

We show that free entry into a co-opetitive industry is socially excessive or insufficient depends on the relative strength of business stealing and common property effects of entry. The former, as has been indicated by previous studies such as Mankiw and Whinston (1986) and Suzumura and Kiyono (1987), is based on the fact that an entrant firm does not take into account its negative impact (externality) on other firms' profits. Therefore, when firms face fixed entry (set-up) costs, the business stealing effect leads to the excess entry of firms in the market owing to socially wasteful replication of entry costs. The latter effect is novel and depends on the effect of entry on the total amount (or quality) of common property resources. An increase in the number of entrants increases the incentive to "free ride" on other firms' investments in common property resources (public goods). However, the total amount (or quality) of common property resources may increase because of a rise in the number of entrants. If that is the case, then such entry will generate positive external effects on other firms. Because private firms do not take the positive externality into account when deciding whether to enter a market, the common property effect leads to insufficient entry. On the other hand, if market entry results in a decline in the total amount (or quality) of common property resources, then such entry will create negative external effects on other firms. In this case, a negative common property effect leads to excess entry.

We find that in the simultaneous co-opetition game, an increase in the number of firms increases the total investment in common property resources while reducing individual investment per firm. Thus, the business stealing and common property effects function in opposite directions. In other words, whether free entry is socially excessive or insufficient depends on the relative magnitude of the two effects. In particular, by providing two concrete examples that assume linear and constant elasticity demand, we show that free entry is more likely to result in socially insufficient entry when initial market size as well as investment and production cost are smaller and/or the demand is more elastic.

However, the business stealing and common property effects work in the same directions in the sequential co-opetition game, in which investment has a commitment value. The important aspect in this case is that the total investment in common property resources is decreased by an increase in the number of entrants. This is in contrast to the result of the simultaneous co-opetition game, because when investment has a commitment value, an increase in firm's investments induces rival firms to respond more aggressively by increasing their output in the subsequent stage. Therefore, this pre-commitment effect of investment reduces the incentive to invest in common property resources. Because an increase in the number of rival firms strengthens the pre-commitment effect, the sequential co-opetition game gives rise to a negative common property effect of entry. As a result, excess entry holds in sequential co-opetition. Interestingly, we show that the excess entry results hold for the sequential co-opetition game even when there are no entry (set-up) costs due to the negative common property effect.

Our results enrich the established excess entry theorem in theoretical industrial organization literature.[3] The theorem shows that in a Cournot model with homogenous products, free entry is socially excessive when firms incur fixed entry costs.[4] Our results from the simultaneous co-opetition game suggest that free entry may lead to socially insufficient entry when firms share

[3]See Mankiw and Whinston (1986), Suzumura and Kiyono (1987), von Weizsacker (1980) and Perry (1984).

[4]Berry and Waldfogel (1999) empirically examine the problem of excess entry into U.S. commercial radio broadcasting and estimate the welfare loss due to excess entry.

common property resources that affect market size or consumers' willingness to pay for products. Furthermore, we also show that excess entry occurs in the sequential co-opetition game even when there are no entry costs.

In the existing literature, the excess entry theorem has been extended in various directions. For example, Konishi et al. (1990) extend the traditional Cournot model with free entry to a general equilibrium model and explore Pareto-improving tax-subsidy policies. Incorporating strategic cost-reducing R&D activities into the Cournot model with free entry, Okuno-Fujiwara and Suzumura (1993) show that the existence of R&D investment strengthens the tendency of excess entry in a free-entry equilibrium.[5] There is a critical difference between R&D investment in their study and investment in common property resources in our study: in their study, investment generates private benefits for the investing firm, while in our study, investment generates public benefits for all firms.

Previous studies have found that free entry can result in socially insufficient entry (e.g., Spence 1976, Dixit and Stiglitz 1977, Kühn and Vives 1999, and Ghosh and Saha 2007). Ghosh and Morita (2007) consider a vertical relationship between industries in a homogeneous Cournot model and show that free entry in the upstream sector can lead to socially insufficient entry. The driving force behind their insufficient entry result is that entry in the upstream sector has a positive external effect on the downstream sector's profit. On the other hand, the driving force behind our insufficient entry result is that entry may have a positive external effect on other firms' profits through changes in the quality of common property resources. Incorporating constant elasticity demand into a standard Salop (1979) spatial framework, Gu and Wenzel (2009) show that the excess entry theorem does not hold when the price elasticity of demand is large. Although their study differs from ours in several respects, the conclusions are similar: insufficient entry occurs when price elasticity of demand is large. Therefore, the degree of price elasticity of demand may serve as a guideline for entry regulation policy.[6]

This study is organized as follows. Section 2 presents the basic structure of the model. Section 3 considers a simultaneous co-opetition game, investigates the properties of free-entry equilibrium, and compares it with the second-best solution. In addition, it presents two examples that specify the functional form of demand (linear and constant elasticity demands) as well as the cost functions to provide more concrete results. Section 4 investigates the same for a sequential co-opetition game. Section 5 discusses some policy implications and a possible extension of the analysis. Section 6 concludes the study.

2 Basic Framework

Consider n firms producing a homogenous good. The firms compete in their output in a market while investing in common property resources that serve as a public good for all the competing

[5]Haruna and Goel (2011) also consider the problem of excess entry in the presence of cost-reducing R&D with spillovers and show that whether free entry is socially excessive or insufficient depends on the degree of research spillovers.

[6]Matsumura and Okamura (2006) show that the equilibrium number of firms can be either excessive or insufficient in a spatial price discrimination model. Mukherjee and Mukherjee (2008) and Mukherjee (2012) theoretically show that free entry can result in socially insufficient entry in the presence of technology licensing or market leaders.

firms. Profits of firm i ($i = 1, \cdots, n$) are given by

$$\pi_i = P(Q,Z) \cdot q_i - C(q_i) - D(z_i) - K, \tag{1}$$

where $P(Q,Z)$ is the market price (or inverse demand) of the product, $q_i \geq 0$ is firm i's output, $Q \equiv \sum_{i=1}^{n} q_i$ is the industry output, $z_i \geq 0$ is firm i's investment (or individual contribution to common property resources), $Z \equiv \sum_{i=1}^{n} z_i$ is the total investment (or quality of common property resources), $C(q_i)$ is the cost function for production, $D(z_i)$ is the cost function for investment, and $K \geq 0$ is the fixed entry (set-up) cost. The inverse demand $P(Q,Z)$ has the properties of $P_Q < 0$, $P_Z > 0$, and $P_{ZZ} \leq 0$. The second and third properties indicate that increasing the total investment increases consumers' willingness to pay, but by a non-increasing rate. In addition, the production cost function $C(\cdot)$ has the properties of $C' > 0$ and $C'' \geq 0$, while the investment cost function $D(\cdot)$ has those of $D' > 0$ and $D'' > 0$.

We consider two types of co-opetitive behavior of firms: simultaneous and sequential co-opetition. In simultaneous co-opetition, firms' investments have no commitment value and are modeled to be simultaneously decided with output. Therefore, it is modeled as a two-stage game: in the first stage, firms make entry decisions and the number of firms in the industry is endogenously decided; in the second stage, each firm non-cooperatively decides its investment and output. On the other hand, in sequential co-opetition, firms' investments are committed and are modeled to be decided before choosing output. Therefore, it is modeled as a three-stage game: in the first stage, firms make entry decisions; in the second stage, each firm non-cooperatively decides its investment; and in the last stage, each firm engages in Cournot competition. Within the above framework, we derive the number of firms in a free-entry equilibrium and compare it with the socially optimal number of firms.

To obtain clear and intuitive results, we employ specific functional forms and consider two types of demands. One is linear demand, expressed by

$$P(Q,Z) = (a + Z) - b\,Q, \tag{2}$$

where a and b are positive constants. Obviously, $P_Q = -b < 0$, $P_Z = 1 > 0$, and $P_{ZZ} = 0$, which satisfy our earlier assumptions. The other type of demand is constant elasticity demand, expressed by

$$P(Q,Z) = \left(\frac{a + Z}{Q} \right)^{1/\epsilon}, \tag{3}$$

where a is positive constant and ϵ is price elasticity of demand. We further assume $\epsilon \geq 1$ to satisfy $P_{ZZ} \leq 0$. Furthermore, we employ the constant marginal cost of production and the quadratic investment cost function, respectively, as

$$C(q_i) = c\,q_i, \quad \text{and} \quad D(z_i) = (d/2)(z_i)^2,$$

where $c > 0$ and $d > 0$.

3 Free entry under simultaneous co-opetition

In this section, we consider a simultaneous co-opetition in which firms' investments have no commitment value. This situation can be modeled by a procedure in which each simultaneously firm decides its output and investment.

3.1 Production and investment decisions

This game can be solved by backward induction. The first-order conditions for profit maximization are

$$P_Q \cdot q_i + P - C' = 0, \tag{4}$$
$$P_Z \cdot q_i - D' = 0. \tag{5}$$

We assume the existence of a symmetric Nash equilibrium[7] and denote the symmetric Nash equilibrium output and investment per firm as $\hat{q}(n)$ and $\hat{z}(n)$ and the total output and investment as $\hat{Q}(n) = n\hat{q}(n)$ and $\hat{Z}(n) = n\hat{z}(n)$. The comparative statics with respect to the number of firms n show that $d\hat{q}/dn < 0$, $d\hat{z}/dn < 0$, $d\hat{Q}/dn > 0$, and $d\hat{Z}/dn > 0$ are likely to hold in general.[8] In the following, we confirm that they hold for linear and constant elasticity demand cases.

■ Linear demand case

In the linear demand case, we obtain the symmetric equilibrium of the second stage as

$$\hat{q} = \frac{(a-c)d}{\hat{\Delta}}, \quad \hat{z} = \frac{a-c}{\hat{\Delta}}, \tag{6}$$

where the determinant $\hat{\Delta} = bd(1+n) - n > 0$ by assumption, which imply that $bd > 1$ and $d\hat{\Delta}/dn > 0$. Then, we have

$$\frac{d\hat{q}}{dn} = -\frac{(a-c)(bd-1)d}{\hat{\Delta}^2} < 0, \quad \frac{d\hat{z}}{dn} = -\frac{(a-d)(bd-1)}{\hat{\Delta}^2} < 0,$$
$$\frac{d\hat{Q}}{dn} = \frac{(a-c)bd^2}{\hat{\Delta}^2} > 0, \quad \frac{d\hat{Z}}{dn} = \frac{(a-c)bd}{\hat{\Delta}^2} > 0,$$

which indicates that as the number of firms increases, individual output and investment decrease, whereas the total output and investments increase. Furthermore, we have

$$\lim_{n\to\infty} \hat{q} = \lim_{n\to\infty} \hat{z} = 0, \ \lim_{n\to\infty} \hat{Q} = \frac{(a-c)d}{bd-1} > 0, \ \lim_{n\to\infty} \hat{Z} = \frac{a-c}{bd-1} > 0,$$

which implies that as the number of firms approaches infinity, individual output and investment converge to zero, while the total output and investment converge to positive and finite values.

[7]For the existence and uniqueness of a symmetric Nash equilibrium, we must assume $(1+n)P_Q + QP_{QQ} < 0$ (see Vives 1999) and the Hessian matrix to be negative definite.

[8]In a symmetric (second-stage) equilibrium, from (4) and (5), we have the following comparative static results:

$$\frac{d\hat{Q}}{dn} = \frac{1}{\Delta}\Big\{\hat{q}(D'' - P_{ZZ}\hat{Q})(C'' - P_Q) + (P_{QZ}\hat{Q} + nP_Z)(\hat{z}D'' - D')\Big\}$$
$$\frac{d\hat{Z}}{dn} = \frac{1}{\Delta}\Big\{\hat{q}(P_{QZ}\hat{Q} + P_Z)(C'' - P_Q) + \big[P_{QQ}\hat{Q} + (n+1)P_Q - C''\big](\hat{z}D'' - D')\Big\},$$

where the determinant is

$$\Delta \equiv \Big[P_{QQ}\hat{Q} + (n+1)P_Q - C''\Big](P_{ZZ}\hat{Q} - D'') - (P_{QZ}\hat{Q} + nP_Z)(P_{QZ}\hat{Q} + P_Z) > 0.$$

Furthermore, we find that $d\hat{Q}/dn > 0$ and $d\hat{Z}/dn > 0$ hold for (a) $P_{QZ}\hat{Q} + P_Z > 0$ and (b) the absolute value of $(\hat{z}D'' - D')$ is not significantly large. The condition (a) holds naturally because it only requires the marginal profit of production to be the increasing function of $\bar{z}$. The condition (b) requires the curvature of the investment cost function to not be significantly large or small.

■ **Constant elasticity demand case**

In the constant elasticity demand case, we obtain the symmetric equilibrium of the second stage as

$$\hat{q} = \frac{\Lambda^{\epsilon}(a\,d\,\epsilon + \Lambda^{\epsilon-1})}{n\,d\,\epsilon},\ \ \hat{z} = \frac{\Lambda^{\epsilon-1}}{n\,d\,\epsilon}, \tag{7}$$

where

$$\Lambda \equiv \frac{n\epsilon - 1}{c\,n\,\epsilon} > 0,$$

and $d\Lambda/dn > 0$. We confirm that the determinant $\hat{\Delta}$ is positive because

$$\hat{\Delta} = \frac{c\,n\,d^2\,\Lambda^{1-2\epsilon}}{1 + a\,d\,\epsilon\Lambda^{1-\epsilon}} > 0.$$

Then, we have the following comparative statics:

$$\begin{aligned}
\frac{d\hat{q}}{dn} &= -\frac{\Lambda^{2\epsilon-1}\left\{(n-2) + ad\left[(n-1)\,\epsilon - 1\right]\Lambda^{1-\epsilon}\right\}}{n^2\,d(n\epsilon - 1)} < 0,\\
\frac{d\hat{z}}{dn} &= -\frac{(n-1)\Lambda^{\epsilon-1}}{n^2 d(n\epsilon - 1)} < 0,\\
\frac{d\hat{Q}}{dn} &= \frac{\Lambda^{2\epsilon-1}\left[(2\epsilon - 1) + ad\epsilon^2\Lambda^{1-\epsilon}\right]}{nd\epsilon(n\epsilon - 1)} > 0,\\
\frac{d\hat{Z}}{dn} &= \frac{(\epsilon - 1)\Lambda^{\epsilon-1}}{nd\epsilon(n\epsilon - 1)} > 0.
\end{aligned}$$

As the number of firms increases, individual output and investment decrease, whereas the total output and investment increase. In addition, we have

$$\lim_{n\to\infty} \hat{q} = \lim_{n\to\infty} \hat{z} = 0,\ \lim_{n\to\infty} \hat{Q} = \frac{c^{-2\epsilon}(c + ac^{\epsilon}d\epsilon)}{d\epsilon} > 0,\ \lim_{n\to\infty} \hat{Z} = \frac{c^{1-\epsilon}}{d\epsilon} > 0,$$

which implies that as the number of firms approaches infinity, individual output and investment converge to zero, while the total output and investment converge to positive and finite values.

3.2 Entry decisions and the second best

In the first stage, firms enter the market until their profits fall to zero. Therefore, the free-entry number of firms is defined as $\hat{n}_f$ such that

$$\hat{\pi}\left(\hat{n}_f\right) = P\left(\hat{Q}\left(\hat{n}_f\right), \hat{Z}\left(\hat{n}_f\right)\right)\hat{q}\left(\hat{n}_f\right) - C\left(\hat{q}\left(\hat{n}_f\right)\right) - D\left(\hat{z}\left(\hat{n}_f\right)\right) - K = 0. \tag{8}$$

We then consider the second-best problem for a social planner who can control the number of firms entering the market. Let $\widehat{W}(n)$ denote the total surplus as

$$\widehat{W}(n) \equiv \int_0^{\hat{Q}} P(s, \hat{Z})ds - nC(\hat{q}) - nD(\hat{z}) - nK.$$

Using (4) and (5), we have

$$\begin{aligned}
\widehat{W}'(n) &= P\left(\hat{q} + n\frac{d\hat{q}}{dn}\right) + P_Z\hat{Q}\left(\hat{z} + n\frac{d\hat{z}}{dn}\right) - C - nC'\frac{d\hat{q}}{dn} - D - nD'\frac{d\hat{z}}{dn} - K\\
&= \hat{\pi} - P_Q\hat{Q}\frac{d\hat{q}}{dn} + P_Z\hat{Q}\left[\frac{d\hat{z}}{dn}(n-1) + \hat{z}\right].
\end{aligned}$$

The social planner chooses the second-best number of firms $n = \hat{n}_{sb}$, which maximizes $\widehat{W}(n)$, implying

$$\widehat{W}'(\hat{n}_{sb}) = 0 \quad \text{if} \quad \hat{n}_{sb} > 1.$$

We assume that the second-order condition should be satisfied, i.e., $\widehat{W}''(n) < 0$. Considering $\hat{\pi}(\hat{n}_f) = 0$, we have

$$\widehat{W}'(\hat{n}_f) = \underbrace{-P_Q \hat{Q} \frac{d\hat{q}}{dn}}_{\text{business stealing}} + \underbrace{P_Z \hat{Q} \frac{d\hat{Z}_{-1}}{dn}}_{\text{common property}}, \tag{9}$$

where $\hat{Z}_{-1} \equiv (n-1)\hat{z}$. Thus, $\hat{n}_f > \hat{n}_{sb}$ holds when (9) is negative; in this case, free entry leads to excess entry. On the other hand, $\hat{n}_f < \hat{n}_{sb}$ holds when (9) is positive; in this case, free entry leads to insufficient entry.

The first term on the right-hand side of (9) is the business stealing effect of entry (Mankiw and Whinston 1986). Firms enter the market without taking into account the negative impact of their entry on their rival's profitability. As shown below, the term is usually negative. The second term represents the common property effect of entry. Firms do not take into account the positive impact of their investment (or contribution to common property resources) on their rival's profitability. As shown above ($d\hat{Z}/dn > 0$ holds for both linear and constant elasticity demand cases), the term is usually positive in this simultaneous co-opetition case. Therefore, we have the following proposition.

Proposition 1
In a simultaneous co-opetition game, free entry results in socially insufficient entry when $\widehat{W}'(\hat{n}_f) > 0$ and socially excessive entry when $\widehat{W}'(\hat{n}_f) < 0$. In particular, insufficient entry results hold when the common property effect dominates the business stealing effect of entry.

In the following, we clearly demonstrate the conditions under which the excess or insufficient entry theorem applies in linear and constant elasticity demand cases.

■ Linear demand case

From (6), the free-entry equilibrium number of firms, $\hat{n}_f$, satisfies

$$\hat{\pi}(\hat{n}_f) = \frac{(a-c)^2(2bd-1)d}{2\left[bd(1+\hat{n}_f) - \hat{n}_f\right]^2} - K = 0.$$

Because $\hat{\pi}(n)$ is strictly decreasing in n and $\lim_{n\to\infty} \hat{\pi} = -K$, we confirm that

$$\lim_{K\to 0} \hat{n}_f = \infty.$$

Therefore, the number of firms under free entry goes to infinity when there are no entry costs.

The socially optimal (second-best) number of firms, $\hat{n}_{sb}$, satisfies

$$\widehat{W}'(\hat{n}_{sb}) = \hat{\pi}(\hat{n}_{sb}) - \frac{(a-c)^2 d\hat{n}_{sb}}{\left[bd(1+\hat{n}_{sb}) - \hat{n}_{sb}\right]^3}\left(1 - 3bd + b^2d^2\right) = 0.$$

In addition, we have

$$\widehat{W}'(\hat{n}_{sb})\big|_{K=0} = \frac{(a-c)^2 d[\hat{n}_{sb}(3bd-1) + bd(2bd-1)]}{2\left[bd(1+\hat{n}_{sb}) - \hat{n}_{sb}\right]^3} > 0,$$

which implies that

$$\lim_{K\to 0} \hat{n}_{sb} = \infty.$$

Therefore, the second-best number of firms goes to infinity when there are no entry costs. Then, we have

$$\widehat{W}'(\hat{n}_f) = -\frac{(a-c)^2 d\hat{n}_f}{[bd(1+\hat{n}_f)-\hat{n}_f]^3}\left(1-3bd+b^2d^2\right) \lesseqgtr 0 \quad \Leftrightarrow \quad bd \gtreqless \frac{3+\sqrt{5}}{2}.$$

Thus, we have the following corollary.

Corollary 1 *Under linear demand, constant marginal production cost, and quadratic investment cost, the free-entry equilibrium of the simultaneous co-opetition game yields excessive (insufficient) entry for greater (smaller) investment costs and/or steeper (flatter) inverse demand.*

The greater (smaller) b and/or d, the more likely that free entry results in excess (insufficient) entry. This result is quite intuitive. When b is large (or demand is less elastic), the equilibrium price significantly decreases by firm entry, leading to a greater business stealing effect. When d is large (or investment is more costly), the total investment is less sensitive to firm entry, leading to a smaller common property effect.

We provide the following numerical examples: in the case of $a = 10,\ b = 1,\ c = 1$, and $K = 2$, we find that $\hat{n}_f \approx 9 < \hat{n}_{sb} \approx 12$ for $d = 2$, which corresponds to the insufficient entry theorem. On the other hand, we find that $\hat{n}_f \approx 6 > \hat{n}_{sb} \approx 4$ for $d = 6$, which corresponds to the excess entry theorem.

■ Constant elasticity demand case

From (13), the free-entry equilibrium number of firms, $\hat{n}_f$, satisfies

$$\hat{\pi}(\hat{n}_f) = \frac{c\Lambda^{2\epsilon-1}\left(1+2ad\epsilon\Lambda^{1-\epsilon}\right)}{2\hat{n}_f d\epsilon(n\epsilon-1)} - K = 0.$$

Because $\hat{\pi}(n)$ is strictly decreasing in n and $\lim_{n\to\infty}\hat{\pi} = -K$, we confirm

$$\lim_{K\to 0} \hat{n}_f = \infty.$$

Thus, the free-entry number of firms goes to infinity when there are no entry costs, as in the linear demand case.

The socially optimal number of firms, $\hat{n}_{sb}$, satisfies

$$\widehat{W}'(\hat{n}_{sb}) = \hat{\pi}(\hat{n}_{sb}) - \frac{c\Lambda^{\epsilon}\left\{ad\left(\hat{n}_{sb}\epsilon-1\right)\left[\epsilon\left(\hat{n}_{sb}-1\right)-1\right] - c\hat{n}_{sb}\left[\epsilon\left(\hat{n}_{sb}+1\right)-\hat{n}_{sb}\right]\Lambda^{\epsilon}\right\}}{(\hat{n}_{sb}\epsilon-1)^3 d\hat{n}_{sb}} = 0.$$

We also find that

$$\lim_{K\to 0} \hat{n}_{sb} = \infty,$$

which implies that the second-best number of firms goes to infinity as K approaches zero.

Then we have

$$\begin{aligned}\widehat{W}'(\hat{n}_f) \quad &= \quad -\frac{c\Lambda^{\epsilon}\left\{ad\left(\hat{n}_f\epsilon-1\right)\left[\epsilon\left(\hat{n}_f-1\right)-1\right] - c\hat{n}_f\left[\epsilon\left(\hat{n}_f+1\right)-\hat{n}_f\right]\Lambda^{\epsilon}\right\}}{(\hat{n}_f\epsilon-1)^3 d\hat{n}_f} \lesseqgtr 0 \\ &\Leftrightarrow \; d \gtreqless \frac{c\,\hat{n}_f[\epsilon(\hat{n}_f+1)-\hat{n}_f]\,\Lambda^{\epsilon}}{a(\hat{n}_f\epsilon-1)[\epsilon(\hat{n}_f-1)-1].}\end{aligned}$$

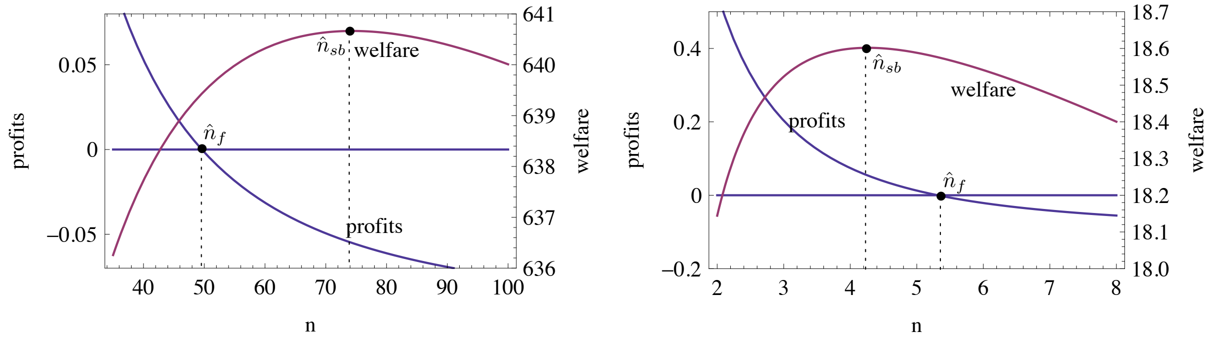

Figure 1: Equilibrium number of firms and the second-best: The case for $\epsilon = 3$ (left) and $\epsilon = 1.2$ (right).

Therefore, the excess (insufficient) entry theorem applies for larger (smaller) values of a, d and c. We cannot analytically derive the impact of ϵ on the sign of $\widehat{W}'(\hat{n}_f)$, but the numerical examples demonstrate that the excess entry theorem is more likely to hold for smaller value of ϵ, i.e., the case of less elastic demand.

Corollary 2 *Under constant elasticity demand, constant marginal production cost, and quadratic investment cost, the free-entry equilibrium of the simultaneous co-opetition game yields excessive (insufficient) entry for larger (smaller) market size, greater (smaller) investment and production costs, and smaller (greater) price elasticity of demand.*

In his simple Cournot model with endogenous entry, Varian (1995) demonstrates that excessive entry results hold for constant elasticity demand cases. Our result extends this by allowing firms co-opetitive investment and shows that free entry leads to either excessive or insufficient entry depending on the value of price elasticity and investment cost.

We provide the following numerical examples. In the case of $a = 2$, $c = 0.1$, $\epsilon = 2$, and $K = 0.1$, we find that $\hat{n}_f \approx 14$ and $\hat{n}_{sb} \approx 19$ for $d = 1$, and $\hat{n}_f \approx 10$ and $\hat{n}_{sb} \approx 8$ for $d = 8$. Therefore, free entry leads to excess (insufficient) entry when d is large (small). In the case of $a = 2$, $c = 0.1$, $d = 3$, and $K = 0.1$, we find that $\hat{n}_f \approx 49 < \hat{n}_{sb} \approx 74$ for $\epsilon = 3$, and $\hat{n}_f \approx 5 > \hat{n}_{sb} \approx 4$ for $\epsilon = 1.2$. Figure 1 depicts these situations. In each panel, profits and welfare in the second-stage equilibrium are depicted. The left (right) panel depicts the case in which price elasticity of demand is high (low), and indicates that insufficient (excess) entry occurs.

4 Free entry under sequential co-opetition

In this section, we consider a sequential co-opetition as a three-stage game. In the first stage, firms enter the market. In the second stage, they non-cooperatively decide their investment. Finally, in the third stage, they choose output in a Cournot fashion. In contrast with that in the simultaneous co-opetition game, investment in this game has a strategic nature in the sense that each firm chooses its investment taking into account its effect on market competition in the subsequent stage.

4.1 Production decisions

The game is solved by backward induction. In the third stage, firms choose their output and the first-order conditions are given by (4). Then, a symmetric Nash equilibrium output per firm is given by $\tilde{q}(n, Z)$ with $\partial\tilde{q}/\partial n < 0$ and $\partial\tilde{q}/\partial z_i = \partial\tilde{q}/\partial z_j > 0$, for $i \neq j$. In addition, the total output in a symmetric equilibrium is $\tilde{Q}(n, Z)$ with $\partial\tilde{Q}/\partial n > 0$ and $\partial\tilde{Q}/\partial z_i = \partial\tilde{Q}/\partial z_j > 0$, for $i \neq j$.[9]

4.2 Investment decisions

In the second stage, each firm chooses the amount of investment by solving the following maximization problem given other firms' investment $Z_{-i} \equiv \sum_{j\neq i} z_j$:

$$\max_{z_i} \pi_i(z_i, Z_{-i}) = P\left(\tilde{Q}, z_i + Z_{-i}\right)\tilde{q} - C(\tilde{q}) - D(z_i) - K.$$

Using (4), the first-order conditions are as follows:[10]

$$\frac{\partial \pi_i}{\partial z_i} = \left[P_Q \frac{\partial \tilde{q}}{\partial z_i}(n-1) + P_Z\right]\tilde{q} - D' = 0. \tag{10}$$

Comparing (5) and (10) clarifies the difference between investment choices in the simultaneous and sequential co-opetition games. In the latter, firms choose their investment with anticipation that their investment will make the rival aggressive (increase rival's output), as represented by the first term in parentheses in (10). This pre-commitment effect of investment reduces the incentive to invest. Therefore, *ceteris paribus*, firms' incentive to invest is lower in the sequential than in the simultaneous co-opetition game.

Solving (10) for all $i = 1, \cdots, n$, we derive the equilibrium investment in a symmetric subgame perfect Nash equilibrium in the second stage as denoted by $\bar{z}(n)$ and $\bar{Z}(n) \equiv n\bar{z}$. In addition, we denote the equilibrium output in a symmetric subgame perfect Nash equilibrium in the second stage as $\bar{q}(n) \equiv \tilde{q}(n, \bar{Z})$ and $\bar{Q}(n) \equiv \tilde{Q}(n, \bar{Z})$.

In general, the effects of entry on $\bar{q}$, $\bar{z}$, $\bar{Q}$, and $\bar{Z}$ are quite complex. Therefore, in the following, we provide the comparative static results for linear and constant elasticity demand cases.

■ Linear demand case

Specifying the inverse demand as (2), we obtain the third-stage equilibrium:

$$\tilde{q}(n, Z) = \frac{a - c + Z}{b(n+1)}, \quad \tilde{Q}(n, Z) = \frac{n(a - c + Z)}{b(n+1)}.$$

[9] We have the comparative static results:

$$\frac{\partial\tilde{q}}{\partial n} = -\frac{P_{QQ}\tilde{q}^2 + P_Q\tilde{q}}{P_{QQ}\tilde{Q} + (n+1)P_Q - C''} < 0, \quad \frac{\partial\tilde{q}}{\partial z_i} = \frac{\partial\tilde{q}}{\partial z_j} = -\frac{P_Z}{P_{QQ}\tilde{Q} + (n+1)P_Q - C''} > 0,$$
$$\frac{\partial\tilde{Q}}{\partial n} = \frac{(P_Q - C'')\tilde{q}}{P_{QQ}\tilde{Q} + (n+1)P_Q - C''} > 0, \quad \frac{\partial\tilde{Q}}{\partial z_i} = \frac{\partial\tilde{Q}}{\partial z_j} = -\frac{nP_Z}{P_{QQ}\tilde{Q} + (n+1)P_Q - C''} > 0.$$

[10] We assume that the second-order conditions are satisfied, i.e.,

$$\frac{\partial^2 \pi_i}{\partial z_i^2} = \left[P_Q \frac{\partial\tilde{q}}{\partial z_i}(n-1) + P_Z\right]\frac{\partial\tilde{q}}{\partial z_i} + \left[P_{QQ}\frac{\partial\tilde{Q}}{\partial\tilde{q}}\frac{\partial\tilde{q}}{\partial z_i}(n-1) + P_Q\frac{\partial^2\tilde{q}}{\partial z_i^2}(n-1) + P_{ZZ}\right]\tilde{q} - D'' < 0.$$

We can easily confirm $\partial\tilde{q}/\partial n < 0$, $\partial\tilde{q}/\partial z_i > 0$, $\partial\tilde{Q}/\partial n > 0$, and $\partial\tilde{Q}/\partial z_i = \partial\tilde{Q}/\partial z_j > 0$.

Solving the second-stage problem, we have the following reaction function:

$$z_i = R_i(Z_{-i}) \equiv \frac{2(a-c)}{(n+1)^2bd-2} + \frac{2}{(n+1)^2bd-2}Z_{-i},$$

which indicates that the investment choices are strategic complements. This is because one firm's investment increases total demand (or market size), which increases other firms' marginal profits of investment.

The second-stage equilibrium is characterized as

$$\bar{q}(n) = \frac{(a-c)(n+1)d}{\Theta}, \quad \bar{z}(n) = \frac{2(a-c)}{\Theta}, \tag{11}$$

where $\Theta \equiv (n+1)^2bd - 2n > 0$ from the stability of Nash equilibrium in the second stage. Therefore, we find that

$$\begin{aligned}
\frac{d\bar{q}}{dn} &= -\frac{(a-c)d[(n+1)^2bd-2]}{\Theta^2} < 0,\\
\frac{d\bar{z}}{dn} &= -\frac{4(a-c)[(n+1)bd-1]}{\Theta^2} < 0,\\
\frac{d\bar{Q}}{dn} &= \frac{(a-c)^2d[(n+1)^2bd-2n^2]}{\Theta^2} \gtreqless 0 \Leftrightarrow bd \gtreqless \frac{2n^2}{(n+1)^2},\\
\frac{d\bar{Z}}{dn} &= -\frac{2bd(a-c)(n^2-1)}{\Theta^2} < 0.
\end{aligned} \tag{12}$$

We should notice that $d\bar{Z}/dn < 0$ holds for any n, indicating that an increase in the number of firms actually decreases total as well as individual investment. However, as (12) indicates, whether the sign of $d\bar{Q}/dn$ is positive or negative depends on the value of b, d, and n. When b and/or d are small, an increase in the number of firms significantly decreases total investment ($\bar{Z}$) and thus total demand for the product. As described in Section 5, this property is particularly important when considering the effect of entry regulation on consumers.

Furthermore, we have

$$\lim_{n\to\infty} \bar{q} = \lim_{n\to\infty} \bar{z} = \lim_{n\to\infty} \bar{Z} = 0, \ \lim_{n\to\infty} \bar{Q} = \frac{a-c}{b} > 0,$$

which indicates that as the number of firms increases, the total output converges to the perfect competition outcome without investment activities because the total amount of common property resources converges to zero. This contrasts with the case of simultaneous co-opetition.

■ Constant elasticity demand case

Specifying the inverse demand as (3), we obtain the third-stage equilibrium:

$$\tilde{q}(n,Z) = \frac{(a+Z)\Lambda^{\epsilon}}{n}, \quad \tilde{Q}(n,Z) = (a+Z)\Lambda^{\epsilon},$$

where $\Lambda \equiv \frac{n\epsilon-1}{cn\epsilon} > 0$, as in the previous section. In addition, we obtain $\partial\tilde{q}/\partial n < 0$, $\partial\tilde{q}/\partial z_i > 0$, $\partial\tilde{Q}/\partial n > 0$, and $\partial\tilde{Q}/\partial z_i = \partial\tilde{Q}/\partial z_j > 0$.

Solving for the second-stage problem, we have

$$\bar{z} = \frac{c\Lambda^{\epsilon}}{(n\epsilon - 1)nd}, \quad \bar{q} = \frac{a\Lambda^{\epsilon}}{n} + \frac{c\Lambda^{2\epsilon}}{(n\epsilon - 1)dn}. \tag{13}$$

The comparative static yields the following:

$$\begin{aligned}
\frac{d\bar{q}}{dn} &= -\frac{\Lambda^{2\epsilon-1}\left[\{2(n-1)\epsilon - 1\} + adn\epsilon\{\epsilon(n-1)-1\}\Lambda^{1-\epsilon}\right]}{dn^2\epsilon(n\epsilon-1)} < 0,\\
\frac{d\bar{z}}{dn} &= -\frac{[\epsilon(2n-1)-1]\Lambda^{\epsilon-1}}{dn^2\epsilon(n\epsilon-1)} < 0,\\
\frac{d\bar{Q}}{dn} &= \frac{\epsilon\Lambda^{2\epsilon-1}\left[adn\epsilon\Lambda^{1-\epsilon} - (n-2)c\right]}{dn^2\epsilon(n\epsilon-1)} \gtreqless 0,\\
\frac{d\bar{Z}}{dn} &= -\frac{(n-1)\Lambda^{\epsilon-1}}{dn^2(n\epsilon-1)} < 0.
\end{aligned} \tag{14}$$

We find that $d\bar{Z}/dn < 0$ also holds under constant elasticity demand, indicating that an increase in the number of firms decreases total investment. The sign of (14) is ambiguous, but it is more likely to be negative when ϵ is smaller, c is larger, and/or d is smaller.[11]

In addition, we have

$$\lim_{n\to\infty}\bar{q} = \lim_{n\to\infty}\bar{z} = \lim_{n\to\infty}\bar{Z} = 0, \ \lim_{n\to\infty}\bar{Q} = ac^{-\epsilon} > 0,$$

Therefore, we find that as the number of firms increases, the total output converges to the perfect competition outcome without investment activities and the total investment converges to zero, as in the linear demand case.

4.3 Entry decisions and the second best

In the first stage, firms enter the market until their profits fall to zero. Therefore, the free entry number of firms is defined as $\bar{n}_f$ such that

$$\bar{\pi}(\bar{n}_f) = P\left(\bar{Q}(\bar{n}_f), \bar{Z}(\bar{n}_f)\right)\cdot\bar{q}(\bar{n}_f) - C(\bar{q}(\bar{n}_f)) - D(\bar{z}(\bar{n}_f)) - K = 0$$

We then consider the second-best problem for a social planner who can control the number of firms entering the market. Let $\overline{W}(n)$ denote the total surplus as

$$\overline{W}(n) \equiv \int_0^{\bar{Q}} P(s,\bar{Z})ds - nC(\bar{q}) - nD(\bar{z}) - nK.$$

Then, we have

$$\begin{aligned}
\overline{W}'(n) &= P\left(\bar{q} + n\frac{d\bar{q}}{dn}\right) + P_Z\bar{Q}\left(\bar{z} + n\frac{d\bar{z}}{dn}\right) - C - nC'\frac{d\bar{q}}{dn} - D - nD'\frac{d\bar{z}}{dn} - K\\
&= \bar{\pi} - P_Q\bar{Q}\left[\frac{d\bar{q}}{dn} + \frac{\partial\tilde{q}}{\partial z}\frac{d\bar{z}}{dn}(n-1)\right] + P_Z\bar{Q}\left[\frac{d\bar{z}}{dn}(n-1) + \bar{z}\right].
\end{aligned}$$

The social planner chooses $n = \bar{n}_{sb}$, which maximizes $\overline{W}(n)$, implying

$$\overline{W}'(n)|_{n=\bar{n}_{sb}} = 0 \quad \text{if} \quad \bar{n}_{sb} > 1.$$

[11] For example, we have $d\bar{Q}/dn < 0$ when $\epsilon = 1$, $c = 10$, $a = 2$, $d = 4$ and $n = 12$.

We assume that the second-order condition is satisfied ($\overline{W}'' < 0$). Thus, we have

$$\overline{W}'(\bar{n}_f) = \underbrace{-P_Q\bar{Q}\left[\frac{d\bar{q}}{dn} + \frac{\partial\tilde{Q}_{-1}}{\partial z}\frac{d\bar{z}}{dn}\right]}_{\text{business stealing}} + \underbrace{P_Z\bar{Q}\frac{d\bar{Z}_{-1}}{dn}}_{\text{common property}}. \tag{15}$$

The first term is the business stealing effect of entry, and its sign is negative. Private firms consider neither the negative direct impact of their entry on rivals' outputs (represented by $d\bar{q}/dn < 0$) nor the negative indirect impact through the change in rivals' investments (represented by $(d\tilde{Q}_{-1}/dz)(d\bar{z}/dn) < 0$). The second term is the common property effect of entry. Differing from the case of non-commitment investment, the sign of this effect is negative when $d\bar{Z}/dn < 0$ holds in the second stage. Private firms do not take into account the negative impact of their entry on their rivals' profitability through the decrease in total amount of investment. Then, we have the following proposition.

Proposition 2
In a sequential co-opetition game, free entry more likely to result in socially excessive entry and the depletion of common property resources.

This proposition contrasts with the result of the simultaneous co-opetition case in Proposition 1. In the sequential co-opetition case, each firm chooses its investment with anticipation that its investment increases not only its own output but also its rivals' output in the subsequent stage owing to the pre-commitment effect of investment, as shown in (10). Therefore, each firm's investment is strategically chosen to be smaller than that in the case of simultaneous co-opetition. In addition, as the number of firms increases, the strategic effect is strengthened. As a result, the total amount of common property resources becomes a decreasing function of the number of firms. In general, the total provision of voluntarily provided public goods is an increasing function of the number of players, while the individual contribution to public goods is a decreasing function. However, in our case, the total amount of common property resource is also decreasing function of the number of firms. This is because when n increases, there are two channels to reduce firms' incentive to invest: (1) firms tend to free ride on others' contributions and (2) individual outputs become small, thus reducing the marginal profits of investment.

In the following, we confirm the proposition for the linear and constant elasticity demand cases and obtain a strong result indicating that excess entry results hold for the sequential co-opetition game even when there are no entry costs.

■ Linear demand case

From (11), the free-entry equilibrium number of firms, $\bar{n}_f$, satisfies

$$\bar{\pi}(\bar{n}_f) = \frac{(a-c)\left[(\bar{n}_f+1)^2bd - 2\right]d}{\Theta^2} - K = 0.$$

On the basis of the fact that $\bar{\pi}(n)$ is strictly decreasing in n and $lim_{n\to\infty}\bar{\pi} = -K$, we confirm

$$\lim_{K\to 0}\bar{n}_f = \infty,$$

which indicates that the free-entry number of firms goes to infinity when there are no entry costs. After some tedious manipulation, we find that the second-best number of firms, $\bar{n}_{sb}$, satisfies

$$\overline{W}'(\bar{n}_{sb}) = \bar{\pi}(\bar{n}_{sb}) - \frac{(a-c)^2 \left[2bd\left(\bar{n}_{sb}^3 + \bar{n}_{sb}^2 - 6\bar{n}_{sb} - 6\right) + (\bar{n}_{sb}+1)^3 b^2 d^2 + 8\right] nd}{\Theta^3} = 0.$$

In addition, we have

$$\overline{W}'(n)\big|_{K=0} = \frac{(a-c)^2 d}{\Theta^3}\left[b^2 d^2 (n+1)^3 - 2bd(n^4 + 2n^3 - 3n^2 - 3n + 1) - 4n\right] = 0$$

when

$$bd = \frac{n^3 + n^2 - 4n + 1 + \sqrt{n^6 + 2n^5 - 7n^4 - 6n^3 + 22n^2 - 4n + 1}}{(n+1)^2}, \tag{16}$$

and $W''(n) < 0$. Therefore, we find that

$$\lim_{K \to 0} \bar{n}_{sb} = n^*,$$

such that n^* satisfies the condition of (16). In other words, the second-best number of firms is positive and finite even when there are no entry costs.

Then, we find that

$$\overline{W}'(\bar{n}_f) = -\frac{(a-c)^2 \left[2bd\left(\bar{n}_f^3 + \bar{n}_f^2 - 6\bar{n}_f - 6\right) + (\bar{n}_f+1)^3 b^2 d^2 + 8\right] \bar{n}_f d}{\Theta^3} < 0,$$

which indicates that free entry necessarily results in excessive entry.

Corollary 3 *Under linear demand, constant marginal production cost, and quadratic investment cost, the free-entry equilibrium of the sequential co-opetition game yields excessive entry. Furthermore, this result holds even when there are no entry costs.*

We provide the following numerical examples. In the case of $a = 10$, $b = 1$, $c = 1$, $d = 2$, and $K = 2$, then $\bar{n}_f \approx 6 > \bar{n}_{sb} \approx 2$. In the case of $a = 10$, $b = 1$, $c = 1$, $d = 8$, and $K = 0$, then $\bar{n}_f = \infty > \bar{n}_{sb} \approx 5$, which clearly shows that excess entry property holds even when $K = 0$.

■ Constant elasticity demand case

From (13), the free-entry equilibrium number of firms, $\bar{n}_f$, satisfies

$$\bar{\pi}(\bar{n}_f) = \frac{c\Lambda^{2\epsilon}\left[c(2\bar{n}_f - 1) + 2ad\bar{n}_f(\bar{n}_f\epsilon - 1)\Lambda^{-\epsilon}\right]}{2d\bar{n}_f^2(\bar{n}_f\epsilon - 1)^2} - K = 0.$$

On the basis of the fact that $\bar{\pi}(n)$ is strictly decreasing in n and $lim_{n\to\infty}\bar{\pi} = -K$, we confirm

$$\lim_{K \to 0} \bar{n}_f = \infty,$$

which indicates that the free-entry number of firms goes to infinity when there are no entry costs. After some tedious manipulation, we find that the second-best number of firms, $\bar{n}_{sb}$, satisfies

$$\overline{W}'(\bar{n}_{sb}) = \bar{\pi}(\bar{n}_{sb}) - \frac{c\Lambda^{2\epsilon}}{(n\epsilon - 1)^3 d\bar{n}_{sb}^2}\left[\begin{array}{l} ad\bar{n}_{sb}\Lambda^{-\epsilon}\left(\bar{n}_{sb}\epsilon - 1\right)\{\epsilon(\bar{n}_{sb} - 1) - 1\} \\ \quad + c\{\bar{n}_{sb}\epsilon(\bar{n}_{sb}^2 - 4) + \bar{n}_{sb}(\bar{n}_{sb}\epsilon - 1) + 1 + \epsilon\} \end{array}\right] = 0.$$

Then, we find that

$$\overline{W}'(\bar{n}_f) = -\frac{c\Lambda^{2\epsilon}}{(\bar{n}_f\epsilon - 1)^3 d\bar{n}_f^2}\left[\begin{array}{l} ad\bar{n}_f\Lambda^{-\epsilon}(\bar{n}_f\epsilon - 1)\{\epsilon(\bar{n}_f - 1) - 1\} \\ \quad + c\{\bar{n}_f\epsilon(\bar{n}_f^2 - 4) + \bar{n}_f(\bar{n}_f\epsilon - 1) + 1 + \epsilon\} \end{array}\right] < 0,$$

which indicates that free entry necessarily results in excessive entry.

Corollary 4 *Under constant elasticity demand, constant marginal production cost, and quadratic investment cost, the free-entry equilibrium of the sequential co-opetition game yields excessive entry. Furthermore, this result holds even when there are no entry costs.*

In this sequential co-opetition game, as Corollaries 3 and 4 indicate, the excess entry theorem applies even when there are no entry (set-up) costs for entrants. The results reflect that as the number of firms increases, the total amount of socially beneficial investment reduces independent of the existence of fixed entry costs.

In the constant elasticity demand case, the second-best number of firms when $K = 0$ cannot be analytically derived; therefore, we provide numerical examples to show that excess entry property holds even when $K = 0$. In the case of $a = 20$, $c = 0.1$, $d = 4$, and $\epsilon = 2$, $\bar{n}_f \approx 31 > \bar{n}_{sb} \approx 7$ holds for $K = 0.1$ and $\bar{n}_f \approx \infty > \bar{n}_{sb} \approx 9$ holds for $K = 0$.

5 Discussion

In this section, we discuss policy implications of our results for entry regulation. Then, we briefly consider a case of investment cooperation wherein firms cooperatively invest in common property resources to maximize producers' surplus (industry profits) but remain rivals in the market.

The standard excess-entry theorem presented by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) reveals that the entry of new firms into a Cournot oligopoly market reduces the output of other incumbent firms and yields a socially wasteful duplication of entry (set-up) costs. In other words, entry regulation may improve social welfare by preventing the duplication of entry costs, although it reduces consumers' surplus as a result of higher prices. Therefore, the amount of entry costs is used as a measure of whether entry regulation can improve welfare.[12]

However, our study presents results different from those of previous studies in terms of effects of entry regulation on social welfare and consumers' surplus. First, our results show that the magnitude of entry costs may not be a appropriate justification for entry regulations if the industry shares common property resources that affect market demand. One possibility is that a free-entry equilibrium leads to socially insufficient entry even when firms when firms incur large entry costs. Our Proposition 1 and Corollaries 1 and 2 imply that free-entry equilibrium may yield socially insufficient entry under a simultaneous co-opetition game even when the entry cost is large. This situation occurs because entry may improve (increases) the quality of (the investment for) common property resources. In this case, the government can improve welfare as well as consumers' surplus by encouraging market entry, e.g., by subsidizing entry. Another

[12]For welfare evaluation of entry regulation, Kim (1997) considers the strategic behavior of firms and the government and shows that entry regulation to prevent excess entry induces the incumbent to behave strategically against the government. As a result, the final outcome is socially suboptimal compared with that in the case without government intervention.

possibility is that a free-entry equilibrium leads to socially excessive entry even when there are no entry costs. Our Proposition 2 and Corollaries 3 and 4 imply that excess-entry results hold under a sequential co-opetition game even when there are no entry costs. The total amount of common property resources decreases by entry into the industry because the entry significantly reduces each firm's incentives to invest as a result of the pre-commitment effect. In this case, the government should regulate entry into the industry to prevent the depletion of the common property resources.

Second, our results imply that the effect of entry regulation on consumers' surplus depends on the types of investment for common property resources. When the excess entry result holds in a free-entry equilibrium of the simultaneous co-opetition game, entry regulation improves social welfare but reduces consumers' surplus because it reduces total consumption by shrinking the consumers' willingness to pay for the product (it shifts the demand inward). On the other hand, in a free-entry equilibrium of the sequential co-opetition game, the equilibrium necessarily leads to excess entry and entry regulation improves social welfare. In addition, the regulation may also improve consumers' surplus because it increases both the total amount of common property resources ($d\bar{Z}/dn < 0$) and total consumption (see (12) and (14)). In this case, the entry regulation alleviates the tragedy-of-the-commons problem and benefits both producers and consumers.

Finally, we briefly mention an extension of our model: cooperation in investment for common property resources. In some industries, firms cooperatively decide their investment (or minimum effort) for common property resources. One concrete example is the voluntary creation of safety standards in Japanese long-route bus companies. By cooperatively establishing stricter safety standard in bus services (e.g., reducing work hours for bus driver and installing safety systems such as collision mitigation brake systems, pre-crash safety systems, and event data recorders), the bus industry can improve consumers' safety concerns for long-route bus services, thereby increasing consumers' willingness to pay for the services. We can examine the situation by extending our sequential co-opetition game. Consider that firms cooperatively choose their investment to maximize the industry profits in the second stage. Then, as shown in the Appendix, an additional entry into the industry (an increase in the number of firms) necessarily increases the total investment; as a result, the quality of common property resources from such investment cooperation solves the free-rider problem of investment. Therefore, under certain conditions, free-entry equilibrium in the sequential co-opetition game with investment cooperation leads to insufficient entry. This implies that even if there are some entry costs and investment has commitment value, encouraging entry into the industry may improve social welfare when the industry can cooperatively manage its commonly shared resources.

6 Concluding Remarks

In many industries, firms share common property resources that affect the consumers' willingness to pay for products. This study investigates whether free entry leads to socially excessive or insufficient entry in a co-opetitive model in which firms simultaneously compete and cooperate. We explore two approaches to modeling firms' co-opetitive behavior: simultaneous and sequential co-opetition. We find that in the former, in which firms simultaneously decide their investments and output, free entry leads to insufficient or excessive entry depending on the relative magnitude of the business stealing and common property effects of entry. In particular, free entry is more likely to result in socially insufficient entry when production and investment costs are smaller and/or price elasticity is greater. On the other hand, in the latter, in which firms can use investment as a commitment, free entry leads to excess entry owing to the negative common property effect. Interestingly, this excessive entry result holds even when there are no entry (set-up) costs. These findings contribute to the literature on excess entry property in oligopoly markets.

Appendix

In this Appendix, we consider a case of investment cooperation in which firms invest cooperatively in common property resources to maximize industry profits but remain rivals in Cournot competition. In particular, we examine the social desirability of free entry in an extended sequential co-opetition game in which firms make entry decisions in the first stage, determine the level of investment required from all firms in the second stage, and compete in a Cournot fashion in the third stage.

The third stage equilibrium is the same as that derived in Section 4.1, so the Nash equilibrium output per firm is $\tilde{q}(n, Z)$ and total output $\tilde{Q}(n, Z)$. In the second stage, the level of investment ($z = Z/n$) is determined to maximize the joint profits:

$$\max_{z} \sum_{i=1}^{n} \pi_i = P(\tilde{Q}, Z)\tilde{Q} - nC(\tilde{q}) - nD(z) - nK.$$

Arranging the first-order condition by using (4), we have

$$\left[P_Q \frac{\partial \tilde{q}}{\partial z}(n-1) + nP_Z\right]\tilde{q} - D' = 0, \tag{17}$$

which implies that firms can internalize positive external effects of investment for market expansion (i.e., nP_Z in the first bracket). As a result, the level of cooperative investment is greater than that of non-cooperative investment.

In what follows, we solve a model for a linear demand case and show that free entry may lead to socially excessive or insufficient entry. Using (17), we characterize the second-stage equilibrium as

$$\breve{q}(n) = \frac{(a-c)(n+1)d}{\Psi}, \quad \breve{z}(n) = \frac{2n(a-c)}{\Psi},$$

where $\Psi \equiv (n+1)^2bd - 2n^2 > 0$ (the inequality follows from the second-order condition). Because $\Psi < \Theta$, from (11), we have $\breve{q}(n) > \bar{q}(n)$ and $\breve{z}(n) > \bar{z}(n)$: given that the number of firms is fixed, the individual output and investment under the cooperative investment case is greater than that under the non-cooperative investment case. Then, we have the following comparative statics:

$$\frac{d\breve{Q}}{dn} = \frac{(a-c)d\left[(n+1)^2bd + 2n^2\right]}{\Psi^2} > 0, \quad \frac{d\breve{Z}}{dn} = \frac{(a-c)(n+1)bdn}{\Psi^2} > 0.$$

In contrast to the non-cooperative investment case analyzed in Section 4, an additional entry in this case increases the total investment and total output because entry does not increase the incentive to free ride on investments in common property resources. In addition, we have

$$\lim_{n\to\infty} \breve{q} = \lim_{n\to\infty} \breve{z} = 0, \ \lim_{n\to\infty} \breve{Q} = \frac{(a-c)d}{bd-2} > 0, \ \lim_{n\to\infty} \breve{Z} = \frac{a-c}{bd-2} > 0.$$

Note that these limiting values are similar to those in the case of the simultaneous co-opetition. The total investment converges to positive and finite values as the number of firms approaches infinity.

In the first stage, firms enter the market until their profits fall to zero. The free-entry number of firms is defined as $\breve{n}_f$ such that $\breve{\pi}(\breve{n}_f) = 0$. The associated level of welfare is defined by $\breve{W}(\breve{n}_f)$. Then, we have

$$\breve{W}'(\breve{n}_f) = -\frac{\breve{n}_f(a-c)^2 d\left[b^2d^2\left(\breve{n}_f+1\right)^3 - 2bd\left(\breve{n}_f+1\right)\left(5\breve{n}_f+2\right)\breve{n}_f + 8\breve{n}_f^3\right]}{\left[bd(\breve{n}_f+1)^2 - 2\breve{n}_f^2\right]^3},$$

where

$$\breve{W}'(\breve{n}_f) \lesseqgtr \quad \text{if} \quad bd \gtreqless \frac{\breve{n}_f\sqrt{17\breve{n}_f^2+12\breve{n}_f+4}+5\breve{n}_f^2+2\breve{n}_f}{\left(\breve{n}_f+1\right)^2}.$$

This indicates that the greater (smaller) the value of b and/or d, the more likely free entry would be to lead to excess (insufficient) entry. In other words, even when investment has commitment value, investment cooperation may lead to insufficient entry.

References

Berry, S.T., Waldfogel, J. (1999) Free Entry and Social Inefficiency in Radio Broadcasting. *RAND Journal of Economics* 30, 397-420.

Brandenburger, A.M., Nalebuff, B.J. (1996) *Co-opetition*, Doubleday.

Dixit, A.K., Stiglitz, J.E. (1977) Monopolistic Competition and Optimum Product Diversity. *American Economic Review* 67, 297-308.

Ghosh, A., Morita, H. (2007) Free Entry and Social Efficiency under Vertical Oligopoly. *RAND Journal of Economics* 38, 541-554.

Ghosh, A., Saha, S. (2007) Excess Entry in the Absence of Scale Economies. *Economic Theory* 30, 575-586.

Gu, Y., Wenzel, T. (2009) A Note on the Excess Entry Theorem in Spatial Models with Elastic Demand. *International Journal of Industrial Organization* 27, 567-571.

Haruna, S., Goel, R.K. (2011) R&D, Free Entry, and Social Inefficiency. *Economics of Innovation and New Technology* 20, 89-101.

Kim, J. (1997) Inefficiency of Subgame Optimal Entry Regulation. *RAND Journal of Economics* 28, 25-36.

Konishi, H., Okuno-Fujiwara, M., Suzumura, K. (1990) Oligopolistic Competition and Economic Welfare. *Journal of Public Economics* 42, 67-88.

Kühn, K., Vives, X. (1999) Excess Entry, Vertical Integration, and Welfare. *RAND Journal of Economics* 30, 575-603.

Mankiw, N.G., Whinston, M.D. (1986) Free Entry and Social Inefficiency. *RAND Journal of Economics* 17, 48-58.

Matsumura, T., Okamura, M. (2006) Equilibrium Number of Firms and Economic Welfare in a Spatial Price Discrimination Model. *Economics Letters* 90, 396-401.

Mukherjee, A. (2012) Social Efficiency of Entry with Market Leaders. *Journal of Economics & Management Strategy* 21, 431-444.

Mukherjee, A., Mukherjee, S. (2008) Excess-Entry Theorem: The Implications of Licensing. *Manchester School* 76, 675-689.

Okuno-Fujiwara, M., Suzumura, K. (1993) Symmetric Cournot Oligopoly and Economic Welfare: A Synthesis. *Economic Theory* 3, 43-59.

Perry, M.K. (1984) Scale Economies, Imperfect Competition, and Public Policy. *Journal of Industrial Economics* 32, 313-330.

Puppim de Oliveira, J.A. (2003) Governmental Responses to Tourism Development: Three Brazilian Case Studies. *Tourism Management* 24, 97-110.

Salop, S.C. (1979) Monopolistic Competition with Outside Goods. *Bell Journal of Economics* 10, 141-156.

Spence, A.M. (1976) Product Selection, Fixed Costs, and Monopolistic Competition. *Review of Economic Studies* 43, 217-235.

Suzumura, K., Kiyono, K. (1987) Entry Barriers and Economic Welfare. *Review of Economic Studies* 54, 157-167.

Varian, H.R. (1995) Entry and Cost Reduction. *Japan and the World Economy* 7, 399-410.

Vives, X. (1999) *Oligopoly Pricing: Old Ideas and New Tools*, Cambridge, MA: MIT Press.

Von Weizsacker, C.C. (1980) A Welfare Analysis of Barriers to Entry. *Bell Journal of Economics* 11, 399-420.