Say's Law: A Rigorous Restatement

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Abstract

Say's Law has passed through various conceptual frameworks. As the next logical step, this paper provides a rigorous restatement in structural axiomatic terms. The main reason is that previous attempts have been methodologically unsatisfactory. Standard economics rests on behavioral assumptions that are expressed as axioms. Axioms are indispensable to build up a theory that epitomizes formal and material consistency. The crucial flaw of the standard approach is that human behavior does not lend itself to axiomatization. Small wonder that the accustomed attempt to explain how the economy works met with scant success. This battered also the discussion about Say's Law.

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1 Conceptual frameworks

> Finally, the long history of controversies over Say's Law sheds light on the enormous difficulties involved when even intelligent thinkers with honesty and goodwill try to understand each other's theories without clearly defined terms and without a clear sense of the conceptual framework of the opposing views. (Sowell, 2008, p. 5)

Ignoring for the moment the finer points of the argument, the message of Say's Law has been that if a recession occurred the cause could not possibly be a failure of demand. The theoretical underpinning of Say's Law, though, has always been open to various interpretations and serious critique. Keynes's redefinition in particular became influential. In the course of time the incoherent interpretations have been sorted out and Say's Law has been restated using the concepts and tools of general equilibrium theory (Becker and Baumol, 1952), (Shoul, 1957), (Baumol, 1977), (Blaug, 1997), (Kates, 1997), (Baumol, 1999), (Sowell, 2008).

Say's Law has passed through different conceptual frameworks. As the next logical step, this paper provides a rigorous restatement in structural axiomatic terms. The main reason is that previous attempts have been methodologically unsatisfactory.

Standard economics rests on behavioral assumptions that are formally expressed as axioms (Debreu, 1959; Arrow and Hahn, 1991; McKenzie, 2008). Axioms are indispensable to build up a theory that epitomizes formal and material consistency. The crucial flaw of the standard approach is that human behavior does not lend itself to axiomatization. Small wonder then that the accustomed attempt to explain how the economy works met with scant success (Ackerman and Nadal, 2004; Quiggin, 2010). This battered also the discussion about Say's Law.

Conceptual consequence demands to discard the subjective-behavioral axioms and to take objective-structural axioms as the formal point of departure. Since the axioms determine the whole theoretical superstructure their selection makes or breaks a theory. Standard economics made the wrong choice.

For an entirely new beginning Section 2 provides the formal foundations with the set of three structural axioms. These represent the most elementary economic configuration. In Section 3 the implications about the market clearing price and real supply changes are made explicit. The question is answered how the consumption economy can absorb real supply shocks such that the system is reproducible in principle. In Section 4 the Profit Law is derived. In Section 5 the stock of money, the average stock of transaction money, and the quantity of money is derived. With all necessary elements in their proper places it is then possible, in Section 6, to simulate product market clearing, labor market clearing, and long run budget balancing in Say's economy. Section 7 concludes.

2 Pure, simple, measurable

> ... at no stage of scientific development do we begin without something in the nature of a theory, ... (Popper, 1960, p. 134)

2.1 Axioms

The formal foundations of theoretical economics must be nonbehavioral and epitomize the interdependence of the real and nominal variables that constitutes the monetary economy. The actual monetary economy is neither a barter nor an exchange economy but a circular flow economy.

The first three structural axioms relate to income, production, and expenditure in a period of arbitrary length. The period length is conveniently assumed to be the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product. Axiomatization is about ascertaining the *minimum* number of premises. Three suffice for the beginning.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N. Nothing is implied at this stage about who owns the shares.

$$Y = WL + DN \quad |t \tag{1}$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment, no foreign trade, and no government.

The economic content of the structural axioms is plain. The sole point to mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. This distinction makes all the difference between good or bad economics.

2.2 Definitions

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (4) wage income Y_W and distributed profit Y_D is defined:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{4}$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

We define the sales ratio as:

$$\rho_X \equiv \frac{X}{O} \quad |t. \tag{5}$$

A sales ratio $\rho_X = 1$ indicates that the quantity bought/sold X and the quantity produced O are equal or, in other words, that the product market is cleared.

We define the expenditure ratio as:

$$\rho_E \equiv \frac{C}{Y} \quad |t. \tag{6}$$

An expenditure ratio $\rho_E = 1$ indicates that consumption expenditures C are equal to total income Y, in other words, that the household sector's budget is balanced.

3 Reproducible in principle

> Could the capitalist system absorb the constant increases in productive capacity without breakdown from limits inherent in the system? Say's Equality supplied an affirmative answer to this question: with flexible prices, the system is forever tending to full employment, full-capacity equilibrium. The classical economists never established this proposition with any rigor ... (Blaug, 1997, p. 233)

3.1 The market clearing price

From (3), (5), and (6) follows the price as dependent variable:

$$P = \frac{\rho_E}{\rho_X} \frac{W}{R} \left(1 + \frac{DN}{WL}\right) \quad |t. \tag{7}$$

Under the condition of market clearing follows:

$$P = \rho_E \frac{W}{R}(1+\rho_D)$$
$$\text{if} \quad \rho_X = 1 \quad \text{and with} \quad \rho_D \equiv \frac{DN}{WL} \quad |t. \tag{8}$$

This is the general structural axiomatic law of supply and demand for the pure consumption economy with one firm. Supply is represented by R and L, demand by ρ_E and indirectly by the income distribution as determined by D, N, W, L. In a nutshell, the price equation states that the market clearing price is ultimately determined by the expenditure ratio, unit wage costs, and the income distribution. Note that the quantity of money is *not* among the determinants. The structural axiomatic price formula is testable in principle.

Under the additional conditions of budget balancing and zero distributed profit then follows:

$$P = \frac{W}{R}$$
$$\text{if} \quad \rho_E = 1, \rho_D = 0, \rho_X = 1 \quad |t. \tag{9}$$

The market clearing price is equal to unit wage costs if the expenditure ratio is unity and distributed profit is zero. In this elementary case, profit per unit is zero and by consequence total profit is zero. All changes of the wage rate and the productivity affect the market clearing price in the period under consideration. We refer to this formal property as *conditional* price flexibility because (9) involves no assumption about human behavior, only the purely formal condition $\rho_X = 1$.

With (9) the real wage $\frac{W}{P}$ is *uno actu* given; it is under the enumerated conditions invariably equal to the hourly output R. Hence labor gets the whole product. Since profit is zero at all employment levels it makes no difference from the business sector's perspective whether full employment obtains or not. Under the rule of conditional price flexibility changes of the wage rate do not affect the real wage. Sticky wages are no issue at all, conditional product price flexibility is sufficient.

It is worth to recall that we do not speculate about human behavior. The equality of price and unit wage costs in (9) is an objective systemic property under the enumerated conditions.

3.2 Real supply changes (I)

Output in period t increases according to the 2nd axiom with increasing productivity and labor input. We first consider productivity changes in isolation.

Under the condition of market clearing, budget balancing and zero distributed profit the price is given by (9) and this means that the price falls with increasing productivity and vice versa.

We leave open the question of why the productivity changes. It is obvious, however, that a productivity increase is not due to more capital input because in the most elementary case of the pure consumption economy there is no investment. Hence productivity changes have either an organizational or an exogenous cause.

Whatever the causes of productivity changes, with conditional price flexibility the whole output can always be absorbed by the household sector, i.e. $X = O$ holds throughout. If there is an upper limit for the quantity the households wish to buy in a given period then they should adapt their labor input accordingly. Part of the productivity increase is then realized as leisure. From this follows that productivity increases of any magnitude do not lead to an oversupply of the consumption good. In brief, if the households do not want more consumption goods, they work less if productivity increases. Any partitioning of the productivity effect between consumption goods and leisure is possible.

In one of its numerous variants Say's Law asserts that productivity improvements need not, as a matter of principle, lead to an oversupply of consumption goods. This conclusion is confirmed by the structural axiomatic analysis provided the enumerated conditions, product price flexibility in particular, are met.

So far the argument has been a matter of pure deductive logic. Outside mathematics this does not suffice. In the second step it has to be established whether the enumerated conditions are met in the real world. If the price is not reduced as conditional price flexibility demands with (9) then a productivity increase results in momentarily unsold stocks and this in turn may lead to a reduction of labor input. Say's Law implicitly asserts that the business sector *never* reacts in this way. This proposition is testable in principle with regard to the past and undecidable with regard to the future.

Say's Law is, like all laws of standard economics, not a law in the strict sense because there is no such thing as a law of behavior. Human behavior is neither necessary nor random but contingent.

> ... the prevailing tendency to call propositions of pure theory "laws" is misleading and inappropriate, ... (Hutchison, 1960, p. 63)

The term Say's Law would be justified if it could be demonstrated that the agents' behavior is, without any exception, such that the enumerated conditions are exactly met. This question of material consistency is entirely different from logical consistency. There is no way around it: a law requires both. Until material consistency is established it would be more appropriate to speak of Say's Hypothesis.

3.3 Real supply changes (II)

Next we consider the case of a growing labor input at constant productivity. From the 2nd axiom follows that real supply increases. From the 3rd and 1st axiom in combination with definition (6) follows for nominal demand:

$$\begin{array}{c} C = \rho_E WL \\ \text{if} \quad \rho_D = 0 \\ \text{vs real supply} \\ O = RL \end{array} \quad |t. \tag{10}$$

Real supply O and nominal demand C are not independent but move in step. According to (9) employment variations have no effect on the market clearing price as long as wage rate and productivity remain unchanged. It can be said, then, that real supply creates a nominal demand. Because labor input features on the supply side as well as on the demand side it is inadmissible to paint *independent* supply and demand schedules. The inept demand-supply-equilibrium construct has to be replaced.

The equality of price and unit wage costs implies zero profit at all levels of employment. With respect to profit the business sector is therefore indifferent between unemployment and full employment. In other words, the business sector has no good reason not to realize full employment but it has, by the same token, no good reason either to move from unemployment to full employment. According to the principle of inertia the business sector simply stays where it is. With increasing labor supply due to population growth this means growing unemployment.

To break the deadlock, the principle of inertia has to be replaced by the principle of expansion. It says that the business sector expands employment at zero profit as long as there is labor supply available given the actual working conditions including the real wage. There is no need to speculate further about this indispensable expansion bias. Whether it is due to animal spirits, market forces, competition, hubris, compassion with the unemployed, greed, or the invisible hand is not of major importance at the moment. Whatever the motive for employment expansion, it cannot depend on profit or the real wage. Profit is zero and the real wage is equal to the productivity on all levels of employment. Wage rate changes lead, according to (9), only to price changes in the same direction.

It is important to discriminate between real supply changes that are due to productivity changes and those that are due to employment changes. The former have an effect on the market clearing price, the latter not. In the real world both kinds of supply shocks intermingle.

With regard to the interpretations of Say's Law it can be resumed that (a) there are no intrinsic limits to an employment expansion, and (b), increasing employment does not result in a real oversupply under the enumerated conditions. To what extent these conditions hold is the crucial empirical question.

4 Profit

> For whatever that elusive concept really is, the insatiable search for profit is still the fundamental driving force of a capitalist economy. (Bernstein, 1953, p. 407)

Total profit consists of monetary and nonmonetary profit. Here we are at first concerned with monetary profit. Nonmonetary profit is treated at length in (2012).

The business sector's monetary profit/loss in period t is defined with (11) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditure C – and costs – here identical with wage income Y_W:

$$Q_m \equiv C - Y_W \quad |t. \tag{11}$$

Because of (3) and (4) this is identical with:

$$Q_m \equiv PX - WL \quad |t. \tag{12}$$

This form is well-known from the theory of the firm. From (11) and (1) finally follows:

$$Q_m \equiv C - Y + Y_D \quad |t \tag{13}$$

or, using the definitions (5) and (6),

$$Q_m \equiv \left(\rho_E - \frac{1}{1+\rho_D}\right) Y \quad |t. \tag{14}$$

The four equations are formally equivalent and show profit under different perspectives. Eq. (14) tells us that total monetary profit is zero if $\rho_E = 1$ and $\rho_D = 0$. This corresponds to the equality of price and unit wage costs in (9). Profit or loss depends on the expenditure and distributed profit ratio and nothing else. Whether the agents maximize profit or not is irrelevant. Eq. (14) entails no behavioral assumptions and is testable in principle.

Until this day, neither orthodox nor heterodox economics has defined profit correctly (for details see 2013a). The *scientific* content of *both* approaches is therefore somewhere between negligible and nil.

Once profit has come into existence for the first time (that is: logically – a historical account is an entirely different matter) the business sector has the option to distribute or to retain it. This in turn has an effect on profit. This effect is captured by (13) but it is invisible in (11). Both equations, though, are formally equivalent.

Retained profit Q_{re} is defined for the business sector as a whole as the difference between profit and distributed profit in period t:

$$Q_{re} \equiv Q_m - Y_D \quad \Rightarrow \quad Q_{re} \equiv C - Y \quad |t. \tag{15}$$

Retained profit is, due to (13), equal to the difference of consumption expenditures and total income.

The household sector's monetary saving is given as the difference of income and consumption expenditures (for nonmonetary saving see 2012):

$$S_m \equiv Y - C \quad |t. \tag{16}$$

In combination with (15) follows:

$$Q_{re} \equiv -S_m \quad |t. \tag{17}$$

Monetary saving and retained profit always move in opposite directions. This is the Special Complementarity. It says that the complementary notion to saving is negative retained profit; positive retained profit is the complementary of dissaving. There is no such thing as an equality of saving and investment in the consumption economy, nor, for that matter, in the investment economy (for details see 2013c).

If distributed profit is zero then follows as a corollary of (17):

$$\begin{aligned} &Q_m = -S_m \\ &\qquad\qquad |t. \\ &\text{if} \quad Y_D = 0 \end{aligned} \tag{18}$$

Profit is zero in the limiting case of zero distributed profit and zero saving. Otherwise profit is equal to dissaving, loss is equal to saving.

Say's Law has always been discussed with reference to supply and demand but not with reference to profit. The implicit assumption has been that all firms can always earn a somewhat unspecified normal profit. This, however, cannot be taken for granted. One of the many foibles of Say's Law is that it is incomplete. The first level condition for a reproducible consumption economy is that profit in each firm must be greater or equal zero which implies $Q_m \geq 0$ which in turn implies $\rho_E \geq 1 \vee \rho_D \geq 0$. The second level condition is more ambitious and demands that the profit ratio is equal in all firms. Neither condition has been explicitly stated or rigorously proved by Say himself or the subsequent commentators.

The next of the many foibles has been the treatment of money. Let us put the matter now right with the help of the structural axioms.

5 Money and credit

> The production process (supply) generates the income necessary for the demand for these products. Money, therefore, is only an intermediary ... (Baumol, 1999, p. 197)

5.1 Stock of money

Money follows consistently from the given axiom set (for details see 2011a). If income is higher than consumption expenditures the household sector's stock of money increases. The change in period t is defined as:

$$\Delta\bar{\mathsf{M}}_{\mathbf{H}} \doteq Y - C \doteq Y\,(1-\rho_E) \quad |t. \tag{19}$$

The alternative identity sign $\doteq$ indicates that the definition refers to the monetary sphere. There is no change of stock if the expenditure ratio is unity.

The stock of money $\bar{\mathsf{M}}_{\mathbf{H}}$ at the end of an arbitrary number of periods $\bar{t}$ is defined as the numerical integral of the previous changes of the stock plus the initial endowment:

$$\bar{\mathsf{M}}_{\mathbf{H}t} \equiv \sum_{t=1}^{t} \Delta\bar{\mathsf{M}}_{\mathbf{H}t} + \bar{\mathsf{M}}_{\mathbf{H}0}. \tag{20}$$

The interrelation between the expenditure ratio and the households sector's stock of money, is then given by:

$$\bar{\mathsf{M}}_{\mathbf{H}t} \equiv \sum_{t=1}^{t} Y_t\,(1-\rho_{Et}) \quad \text{if} \quad \bar{\mathsf{M}}_{\mathbf{H}0} = 0. \tag{21}$$

Formally, the expenditure ratio takes the role of the first derivative.

The changes in the stock of money as seen from the business sector are symmetrical to those of the household sector:

$$\Delta\bar{\mathsf{M}}_{\mathbf{B}} \doteq C - Y \quad |t. \tag{22}$$

The business sector's stock of money at the end of an arbitrary number of periods is accordingly given by:

$$\bar{\mathsf{M}}_{\mathbf{B}t} \equiv \sum_{t=1}^{t} \Delta\bar{\mathsf{M}}_{\mathbf{B}t} + \bar{\mathsf{M}}_{\mathbf{B}0}. \tag{23}$$

The development of the stock of money follows without further assumptions from the axioms and is determined by variations of the elementary variables P, X, W and L.

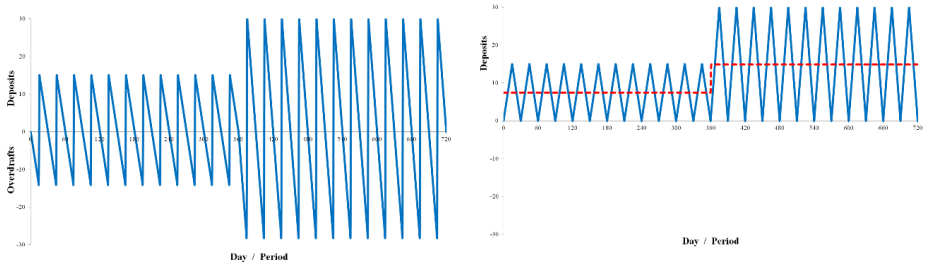
In order to reduce the monetary phenomena to the essentials it is supposed that all financial transactions are carried out without costs by the central bank. The stock of money then takes the form of current deposits or current overdrafts. Initial endowments can be set to zero. Then, if the household sector owns current deposits according to (21) the current overdrafts of the business sector are of equal amount according to (23) and vice versa if the business sector owns current deposits. Money and credit are symmetrical; the stock of money of each sector can be either positive or negative. The current assets and liabilities of the central bank are equal by construction. From its perspective the quantity of money at the end of an arbitrary number of periods is given by the absolute value either from (21) or (23):

$$\bar{\mathsf{M}}_t \equiv \left| \sum_{t=1}^{t} \Delta \bar{\mathsf{M}}_t \right| \quad \text{if} \quad \bar{\mathsf{M}}_0 = 0. \tag{24}$$

While the stock of money can be either positive or negative the quantity of money is always positive. It is assumed at first that the central bank plays an *accommodative* role and simply supports the autonomous market transactions between the household and the business sector. For the time being, money is the *dependent* variable.

5.2 Transaction money

By sequencing the initially given period length of one year into months the idealized transaction pattern that is displayed in Figure 1a results.



(a) Transaction pattern over two periods **(b)** Average stock of transaction money $\hat{\mathsf{M}}_T$

Figure 1: Household sector's transaction pattern for different nominal incomes in two periods

It is assumed that the monthly income $\frac{Y}{12}$ is paid out at mid-month. In the first half of the month the daily spending of $\frac{Y}{360}$ increases the current overdrafts of the households. At mid-month the households change to the positive side and have current deposits of $\frac{Y}{24}$ at their disposal. This amount reduces continuously towards the end of the month. This pattern is exactly repeated over the rest of the year. At the end of each sub-period, and therefore also at the end of the year, both the stock

of money and the quantity of money is zero. Money is present and absent depending on the time frame of observation.

In period 2 the wage rate, the dividend (if initially >0) and the price is doubled. Since no cash balances are carried forward from one period to the next, there results *no* real balance effect provided the doubling takes place exactly at the beginning of period 2.

From the perspective of the central bank it is a matter of indifference whether the household or the business sector owns current deposits. Therefore, the pattern of Figure 1a translates into the average amount of current deposits in Figure 1b. This average stock of transaction money depends on income according to the transaction equation

$$\hat{\mathsf{M}}_T \equiv \kappa Y \quad |t \tag{25}$$

which resembles Pigou's Cambridge equation; the underlying theory, though, is thereby not adopted. For the regular transaction pattern that is here assumed as a idealization the index is $\frac{1}{48}$. Different transaction patterns are characterized by different numerical values of the transaction pattern index.

By taking (25), (5) and (6) together one gets the explicit transaction equation for the limiting case of market clearing and budget balancing:

$$\text{(i) } \hat{\mathsf{M}}_T \equiv \kappa \frac{\rho_X}{\rho_E} RLP \quad \text{(ii) } \frac{\hat{\mathsf{M}}_T}{P} = \kappa O \quad \text{if} \quad \rho_X = 1, \rho_E = 1 \quad |t. \tag{26}$$

We are now in the position to substantiate the notion of accommodation as a money-growth formula. According to (i) the central bank enables the average stock of transaction money to expand or contract with the development of productivity, employment, and price. In other words, the real average stock of transaction money, which is a statistical artifact and not a physical stock, is proportional to output (ii) if the transaction index is given and if the ratios ρ_E and ρ_X are unity. Under these *initial* conditions money is endogenous and neutral in the structural axiomatic context. Money emerges from *autonomous* market transactions and has three aspects: stock of money ($\bar{\mathsf{M}}_{\mathbf{H}}$, $\bar{\mathsf{M}}_{\mathbf{B}}$), quantity of money (here $\bar{\mathsf{M}} = 0$ at period start and end because of $\rho_E = 1$) and average stock of transaction money (here $\hat{\mathsf{M}}_T > 0$).

To start with, money is a pure transaction medium. The quantity of money is not exogenously determined, as the commonplace quantity theory has it, but emerges from the autonomous transactions between the household and the business sector (for details see 2011a; 2011b). Money assumes the role of a store of value as soon as the expenditure ratio is different from unity.

6 Variations of the expenditure ratio

> But the General Theory was not a discourse on the history of economics. It had a deadly serious intent, which was to overturn the judgement associated with Say's Law. Where classical economists had denied the possibility of demand failure, Keynes set about trying to demonstrate that demand failure was the single most important cause of recession and unemployment. In this he was wildly successful. (Kates, 1997, p. 192)

In the foregoing we have found that Say's Law can be restated in structural axiomatic terms and that it holds under the enumerated conditions. The crucial condition is an expenditure ratio of unity, or in other words, that the households spend in a given period their whole income. This condition is acceptable as an analytical limiting case but it is, obviously, never met in the real world. Time now to generalize and to get closer to reality.

6.1 Randomness, evolution, simulation

The period values of the axiomatic variables are formally connected by the familiar growth equation, which is added to the structural set as the 4th axiom.

$$Z_t = Z_{t-1}\left(1+\dddot{Z}_t\right) \tag{27}$$

The path of the representative variable Z_t is then determined by the initial value Z_0 and the rates of change $\dddot{Z}_t$ for each period:

$$Z_t = Z_0\,(1+\dddot{Z}_1)\,(1+\dddot{Z}_2)\ldots(1+\dddot{Z}_t) = Z_0\prod_{t=1}^{t}(1+\dddot{Z}_t)\,. \tag{28}$$

Eq. (28) describes the path of a variable with the *rates of change* as unknowns. These unknowns are in need of determination and explanation. Without going deeper into any concrete details we can say that the rates are subject to what J. S. Mill called the 'plurality of causes and the intermixture of effects' (2006a, Ch. X). The observer sees at first pure chance at work. This, however, leaves the possibility open that one may find regularities at a deeper level. Randomness may turn out to be a superficial explanation. However, the chief merits of the random hypothesis are simplicity and testability, therefore it has to be applied first – for good methodological reasons (Kreuzenkamp and McAleer, 1995).

It is assumed now for a start that the elementary axiomatic variables vary at random. This produces an evolving economy. The respective probability distributions are given in general form by:

$$\begin{array}{c} Pr\{l_W \leq W \leq u_W\} \\ Pr\{l_P \leq P \leq u_P\} \\ Pr\{l_R \leq R \leq u_R\} \\ Pr\{l_X \leq X \leq u_X\} \quad |t. \\ Pr\{l_L \leq L \leq u_L\} \\ Pr\{l_D \leq D \leq u_D\} \\ Pr\{l_N \leq N \leq u_N\} \end{array} \tag{29}$$

The upper (u) and lower (l) boundaries of the respective distributions are, for the time being, symmetrical around zero, for example $\pm 3\%$. This produces an economy that over a longer time span neither grows nor shrinks.

The four axioms combined with (29) formally constitute a simulation. For the actual simulation the random variates for each period are taken from the worksheet random number generator and are then appropriately adapted. The assumed probability distributions can at any time be replaced by distributions that have been observed over a reasonable time span. There is, though, no need at this early stage to discus the merits and demerits of different probability distributions.

The four axioms combined with the random distributions produce at every run an outcome like that shown in Figure 2.

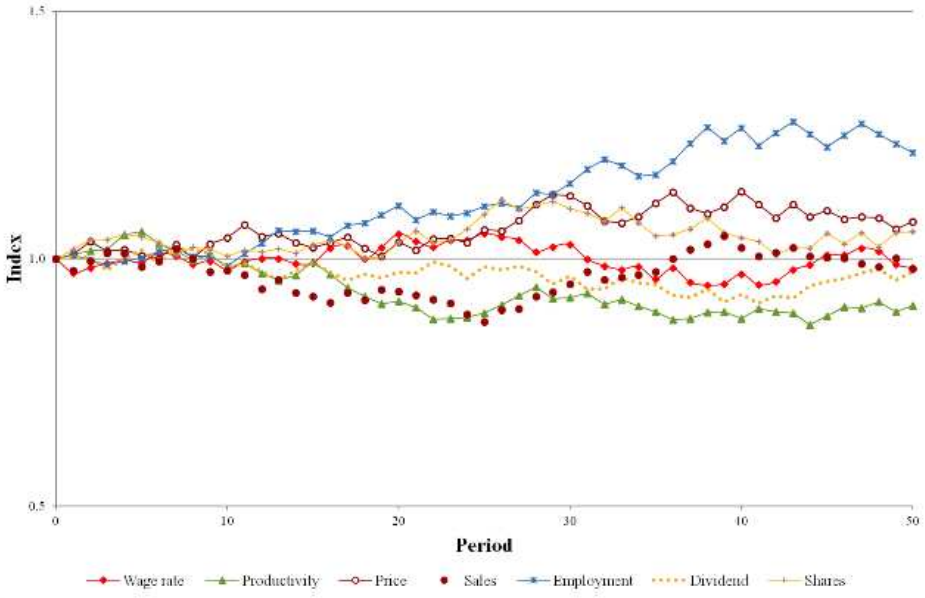


Figure 2: The evolving consumption economy consists initially of entirely independent random paths of the seven elementary axiomatic variables (shown here) and the paths of composed variables

A simulation is a mathematical object just like a system of equations – with the decisive advantage that change and chance can be formally represented in a quite

natural manner. With a system of equations one is formally locked in the blind alley of simultaneous equilibrium.

Given the paths of the elementary variables, the development of the composed variables is also determined. From the random paths of employment L and wage rate W follows the path of wage income Y_W according to (4). Likewise follows from the paths of dividend D and number of shares N the path of distributed profit Y_D. From the 1st axiom then follows the path of total income Y.

The path of monetary profit is *uno actu* determined. Profit depends on price P, sales X, wage rate W and employment L as defined with (12). The profit path follows from the random variations of four independent elementary variables and the structure of the pure consumption economy which is given with the axiom set.

It is decisive that the central bank accommodates all transactions between the household and the business sector. More specifically, that it expands credit when the households or firms apply for it. Otherwise the expansion and contraction of credit on the one side and money on the other cannot follow a pure random path. This condition keeps the central bank as an independent agent for a while out of the picture (for details see 2011a; 2011b).

The four structural axioms and the probability distributions (29) constitute the *minimum* of premises. Given the essentials, the simulation delivers the concrete values of all variables for all future periods under the condition that no events beyond the symmetric random changes interfere. There are no interdependencies between the paths; the evolution of the economy is open and only subject to statistical laws. If there are interdependencies they have to be explicitly added to the formal core.

The substitution of historical rates of change for the random change vectors delivers an exact picture of the past interactions of all variables. The simulation becomes a faithful description.

6.2 Say's economy

Say's economy follows from the pure random economy of Figure 2 by imposing the conditions of product market clearing, budget balancing, conditional price flexibility and self-referential labor market clearing.

Figure 3 shows a consumption economy with the following properties: wage rate, productivity and labor supply vary at random. Self-referential labor market clearing is established with (30) to which we will turn in a minute. The paths of output and sales are identical, that is, the market is cleared in each period. The paths of income and consumption expenditures are also identical, that is, the budget is balanced in each period. The market clearing price is determined by supply and demand as given by (9). According to (14) profit for the economy as a whole is zero in each period because the expenditure ratio is unity and the distributed profit ratio is zero. With this, Say's economy is fully specified. It should be noted in passing that

Walras's original economy was characterized by 'ni bénéfice ni perte', that is, by zero profit. Clearly, a zero profit economy is an analytical limiting case.

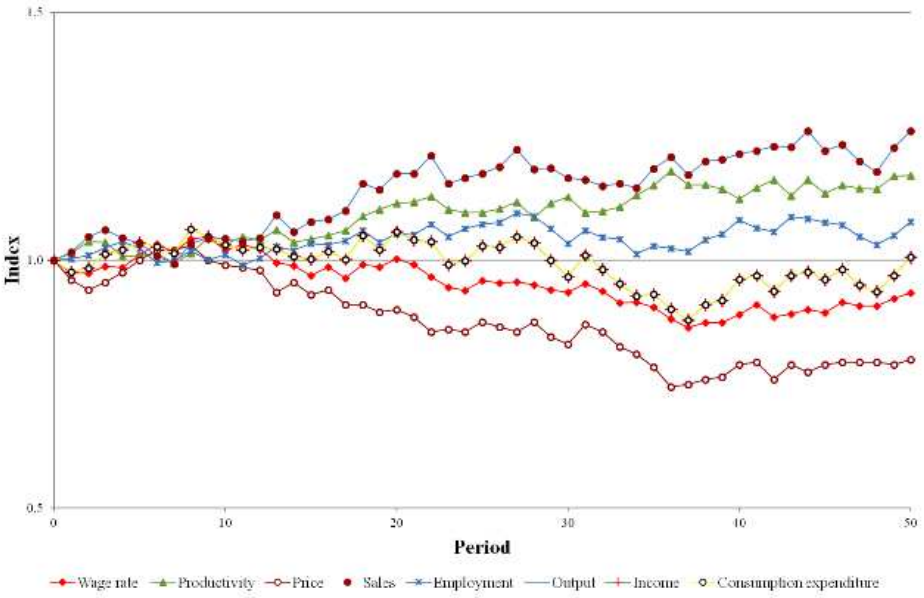


Figure 3: Say's zero profit economy with product market clearing, budget balancing, conditional price flexibility and self-referential labor market clearing

Within the time frame of observation the simulated economy is deflationary. The reason is that the wage rate falls and the productivity increases. Since increasing employment does not compensate the falling wage rate income and consumption expenditures decline. Output and sales on the other hand increase.

Let us assume that the household sector's labor supply L^{θ} increases due to exogenous population growth. What is now needed is a drive on the side of the business sector to expand labor input L, otherwise we are left with growing unemployment. The directed random changes that adapt labor input to labor supply are formally given by:

$$\begin{aligned} \underbrace{\{-1,0,1\}_t}_{\text{direction}} &= sgn\left(L_{t-1} - L^{\theta}_{t-1}\right) \\ \ddot{L}_t &= \underbrace{\{-1,0,1\}_t}_{\text{direction}} \underbrace{Pr\{0 \leq \ddot{L} \leq u_L\}_t}_{\text{magnitude}}. \end{aligned} \tag{30}$$

The upper half of (30) says that the sign, i.e. the direction of change in period t, depends on the difference between actual employment and labor supply in the previous period. In the case of unemployment the sign is positive, that is, the business sector increases labor input, and vice versa in the case of over-employment.

The lower half combines the direction with a random rate of change. In combination, the two halves define a rather elementary self-stabilizing feedback. The adaptation speed depends on the magnitude of the possible rates of change as defined by the probability distribution.

Since profit and the real wage do not depend on employment they cannot be used to steer the labor market towards full employment. Full employment does not coincide with an optimum of the business sector because there is none. The employment adaptation can therefore only depend on the state of the labor market itself, in other words, the adaptation is self-referential. In behavioral terms this means that the business sector expands employment whenever there is supply available and vice versa. This is what (30) conveys.

Figure 4 resumes the development of the labor market. Actual employment tracks the random path of labor supply quite closely. The deviation from full employment is at most 5 percent in both directions. There is no dependency between the wage rate and employment. The 3-dimensional diagram fully replaces the accustomed 2-dimensional representation of the labor market that consists of supply-schedule–demand-schedule–equilibrium. The latter lacks the time dimension and is therefore useless.

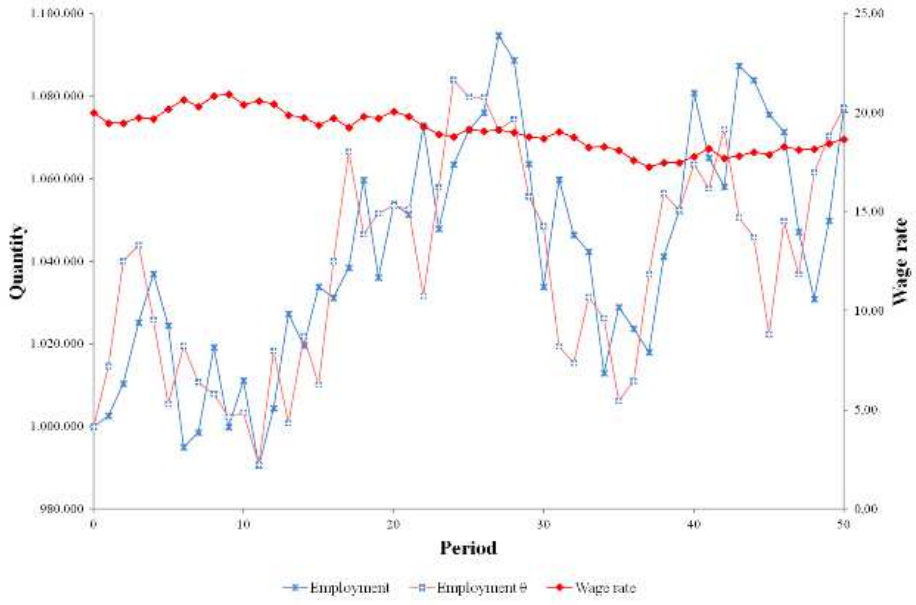


Figure 4: Self-referential labor market clearing

In the most elementary version of Say's economy profit for the business sector as a whole is zero. If the business sector is composed of more than one firm then the firms must all have the same unit wage costs. Otherwise the firms that are above average make a loss and those below average make a profit such that losses and profits sum up to zero. This is not a stable situation because part of the loss making

firms will eventually drop out. Therefore, with more than one firm the additional condition must be met that each individual firm's profit is zero. This is an analytical limiting case. The realistic alternative is that total profit is greater than zero. This in turn requires according to (14) an expenditure ratio greater unity and/or a distributed profit ratio greater zero (for details see 2013b).

Say's economy is reproducible for an indefinite time if the enumerated conditions are met. A big if, indeed.

6.3 Begging the question

If the axioms capture reality and if the enumerated conditions are realized then it is logically impossible that a glut occurs. This would be the correct formulation of the most popular version of Say's Law in structural axiomatic terms. There is, however, a seemingly petty semantic variation in Say's Law that deserves attention:

(a) If conditions a, b, c, ... are met no glut can ever occur.

(b) Because conditions a, b, c, ... are met no glut can ever occur. Alternatively: in a market economy (conditions a, b, c, ... are always met, therefore) no glut can ever occur.

Statement (a) is the result of deductive reasoning. Statement (b) involves an empirical corroboration of the conditions. It is inadmissible to verbally switch from (a) to (b) and to pretend that the conditions are met. The conditional form is correct, the affirmative form is false at least as long as there exists no independent proof that the conditions factually apply. No such proof is known with regard to Say's Law. Version (b) is a petitio principii (cf. Mill, 2006b, pp. 819-827), that is, in takes into the premises what has to be proved.

> Research is in fact a continuous discussion of the consistency of theories: formal consistency insofar as the discussion relates to the logical cohesion of what is asserted in joint theories; material consistency insofar as the agreement of observations with theories is concerned. (Klant, 1994, p. 31)

To concede the logical validity of Say's Law does not amount to confirming the truth of the law. To establish logical consistency is one half of the task. As long as the other half, material consistency, is missing it is an abuse of the scientific jargon to speak of a law. It almost goes without saying that the empirical basis of Say's Law has never been satisfactorily established. Note well that this is by no means an argument against the deductive method but against the proto-scientific status of economics.

Methodologically, standard economics is a petitio principii. The young discipline of economic methodology, completely absorbed by observing how economists are doing what they claim is normal science in the sense of Kuhn, has not yet advanced to this insight. Economic methodology itself urgently needs a paradigm shift.

6.4 Budget balancing in the course of time

Next, all conditions of Say's economy apply except budget balancing. The expenditure ratio is no longer fixated at unity. Its random rate of change is given by:

$$\ddot{\rho}_E = Pr\{l \leq \ddot{\rho}_E \leq u\} \quad |t. \tag{31}$$

It is assumed that the upper and lower boundary is symmetrical around zero, e.g. $\pm 5\%$. The expenditure ratio in each period is then given by:

$$\rho_{Et} = 1 + \ddot{\rho}_{Et} \tag{32}$$

The expenditure ratio varies randomly around unity. From this results a simple relation between income and consumption expenditure:

$$C = \rho_E Y \quad |t. \tag{33}$$

This is formally similar to the definition of the expenditure ratio (6). In fact it is a corollary if the expenditure ratio is given. Yet since this ratio changes in each period according to (32) the relationship between income and consumption expenditure holds only for one period and is not stable over time. Nominal demand is an indirect random variable. However, the expenditure ratio is defined such that it is to be expected that aggregate consumption expenditures become equal to aggregate incomes in the course of time. How this works as can be gleaned from Figure 5.

From (8) follows for the market clearing price:

$$\begin{gathered} P = \rho_E \frac{W}{R} \\ \text{if} \quad \rho_X = 1, \rho_D = 0 \quad |t. \end{gathered} \tag{34}$$

The market clearing price depends on unit wage costs and the expenditure ratio.

From (14) follows as a corollary for monetary profit:

$$\begin{gathered} Q_m = (\rho_E - 1) Y \\ \text{if} \quad \rho_D = 0 \quad |t. \end{gathered} \tag{35}$$

Monetary profit/loss depends also on the expenditure ratio.

Eq. (19), which is reproduced here, finally states that the change of the stock of money, too, depends on the expenditure ratio:

$$\Delta\bar{\mathrm{M}}_{\mathbf{H}} \equiv Y\,(1-\rho_E) \quad |t. \tag{36}$$

Price, profit and the change of the household sector's stock of money are all related via the expenditure ratio. A ratio greater than unity means dissaving and raises the market clearing price, boosts profit, and lowers the household sector's stock of money. The business sector's stock of money increases according to (22). The inverse happens if the households save. Price and profit are down but the stock of money increases. Figure 5 shows the interrelations between income, consumption expenditure and the stock of money.

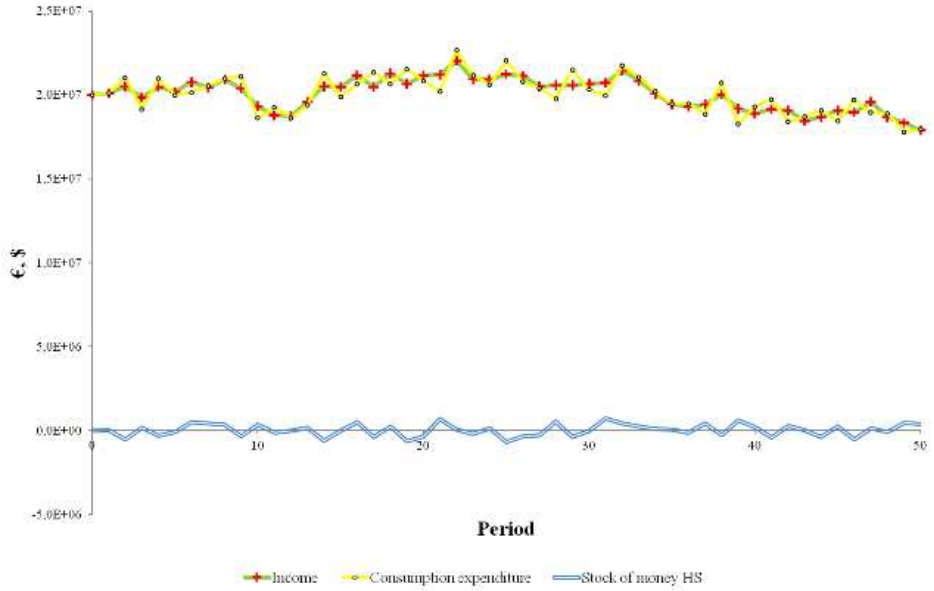


Figure 5: The difference between income and consumption expenditure in successive periods produces the variations of the households sector's stock of money

Income and consumption expenditure move close together but are never equal. They are related via the random expenditure ratio in (33). The difference between the two variables changes the stock of money according to (36). Due to the symmetric variations around unity as defined by (32) a cycle emerges. If cumulated income is equal to cumulated expenditure the stock of money is zero, that is, the cycle crosses the abscissa. This happens within relatively short time intervals. The observed alteration between saving and dissaving can be interpreted as an expression of the households' time preference. It can be said, then, that the consumption pattern is *optimal* over time. This behavioral property, though, has no palpable consequence of any sort.

Some other point is indeed crucial. According to (18) saving means loss and dissaving means profit. In the course of time profits and losses cancel out for the business sector as a whole. They do not necessarily cancel out for the individual firm. There is an asymmetry. In the saving phase with a loss for the business sector as a whole the probability is high that some firms go bankrupt. This reduces employment. But in the dissaving phase with positive overall profit these firms do not spring back into existence. And it takes some time to replace them by new firms according to (30). This asymmetry makes itself felt as unemployment. Even if the fall of nominal demand is temporary and swiftly reversed, as the classics argued, the asymmetry between the tear-down and the build-up of firms and workplaces causes prolonged unemployment.

The next crucial point is that it is by no means certain that the saving/dissaving cycle is short. If the saving phase last longer or the fall of nominal demand goes deep then the cycle may cause severe unemployment. Whether the cycle is relatively short or long is an empirical question. Even if there is a reliable tendency of a periodic equalization of cumulated income and consumption expenditure, as the classics argued, Keynes's critique of Say's Law is justified. The time scale matters. The temporary saving phase may simply be too long. Everything depends on how long firms can cope with low profits or even losses. This question cannot be settled with speculations about human behavior or the logical impossibility of secular stagnation. Equilibrium theory, in particular, has no mandate to say anything about the short or the long run because it lacks the dimension of time.

If nominal demand is cyclic then it makes perfect sense for an independent government agency to take up credit and to enter the market with additional nominal demand in the saving phase and to take the temporarily redundant part of the current output into stock. This, needless to stress, works only if the output consists of durables. In the dissaving phase the additional supply is brought to market, the inventory is liquidated, and the credit redeemed. These counter-cyclical measures stabilize the price, prevent both losses and profits and keep employment stable. They are in full *accordance* with the households' optimal consumption plans and cannot be rejected with a reference to Say's Law. Seen from the structural axiomatic viewpoint there is not such a deep antagonism between Say and Keynes. All depends on the actual length of 'temporary' and this is not a question for pure theory. If the saving/dissaving cycle is indeed short Say's conclusion is acceptable if the enumerated conditions are met, otherwise Keynes is right. This is not to say that the underlying theories are formally correct. In fact they are not and this is why they had to be restated rigorously in structural axiomatic terms in the first place.

Say's Law in structural axiomatic terms asserts that a reproducible monetary economy with full employment and price stability is *logically* possible (see also 2013b).

7 Uphold the law

> Keynes described Say's Law as fallacious. The law of markets was devised to demonstrate that theories based on failure of effective demand were themselves fallacious. (Kates, 1998, p. 217)

The quote summarizes what the discussion between orthodoxy and heterodoxy regularly is about: *both* sides have no serviceable frame of concepts. This guarantees the inconclusive filibuster of political economics as we know it and this dysfunction is regularly excused with a reference to the immense complexity of the subject matter. This amounts to an admission of scientific incompetence. The long standing discussion about Say's Law has followed this ineffective pattern.

What is necessary is (a) a set of axioms that represents the *objective* features of the monetary economy, and (b), a correct profit theory. It should be obvious that it is impossible to explain the economy we happen to live in without a correct profit theory. Standard economics lacks this prerequisite since Adam Smith and this goes some way to explaining its conspicuous scientific queerness.

Structural axiomatic economics enumerates exhaustively the conditions that must be met in order to make Say's Law applicable. No claim is made that these conditions obtain in the market economy. This empirical question is left open for another occasion.

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