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Financial Development, Economic Growth and R&D Cyclical Movement

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This paper builds up an endogenous growth model à la Aghion and Howitt (1992) and Boucekkine et al (2005). We assume that R&D firms use only investment good as input, instead of final good as hypothesized in the above two models. We show that investment price will be a negative function of aggregate quality index; and thus decline over time. In this model, subsidy on R&D has growth-enhancing effect. Moreover, this model predicts unambiguously that R&D is procyclical.

Keywords: *Endogeneous growth model, real business cycle, research and development*

1. Introduction

The notion that increasing R&D expenditure[2] as the engine of growth, is the key implication of most endogenous growth models. However, subsidy on R&D fails to enhance growth in these models, nor to obtain social optimum. The reason is that production of intermediate good is below optimum due to monopoly pricing. Only subsidy on intermediate good or the final good can stimulate enough demand to correct the market inefficiency. Our first task is revising the current endogenous growth model such that subsidy has direct impact on R&D outlay.

In fact, there is no lack of evidence of positive subsidy effect in the form of tax credit on R&D. Mansfield (1986) is one of those earliest studies of effectiveness of tax credit on R&D[3]. He examines the tax incentive programs in U.S, Canada and Sweden. His study suggests that R&D expenditure is effectively enhanced; moreover, the increased R&D expenditure outweights the lost government revenue. Aghion and Howitt (1992) try to resolve the problem by assuming that the research sector uses capital as input. But they have to make the assumption that capital and final good being perfect substitute. This paper takes on an alternative approach. We assume the existence of two final good sectors - the investment and non-durable good sector. The sole input to R&D sector is investment good; thus subsidy has direct impact on research expenditure.

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[2] No matter in the form of labor devoted t R&D as considered in Aghion and Howitt (1992), or consumption good as portrayed in Barro and Sala-i-Marti'n, X (1994).
[3] On the other hand, government corruption can drastically reduce the spillover effect of foreign direct investment on innovation (see Lau and Yang (2013)).

On the other hand, Aghion and Saint-Paul (1993), Canton and Uhlig (1996) construct models that contend that R&D is counter-cyclical. A reason why recessions might have a positive impact on productivity lies in the following opportunity cost argument: productivity-improving activities such as reorganization or training often take place at the expense of directly productive activities. Because the return to the latter is lower in recessions due to lower demand for the manufactured good, the opportunity cost in terms of forgone profits of reorganization activities will be lower in recessions than in expansions. The idea was first formalized by Hall (1991), he writes

"Measured output may be low during (recession) periods but the time spend reorganizing pays off in its contribution to future productivity."

Nonetheless, the U.S experience clearly favors a procyclical R&D movement (For instance, Fung (2012)). One plausible reason is that those models fail to capture the idea that *fund raising for research is less costly during expansion.* This can be explained in the King and Levine (1993) model by either lower agency cost or higher probability finding a capable researcher. King and Levine (1993) present cross-country evidence consistent with Schumpeter's view that the financial system can promote economic growth, using data on 80 countries over the 1960-1989 period. All four measures of financial development - namely, ratio of the size of the formal financial intermediary sector to GDP, the importance of banks relative to central bank, the percentage of credit issued to private individuals, and the ratio of credit issued to private firms to GDP - indicate positive correlation with growth rate. King and Levine (1993) provide a theoretical justification for the positive relation. They argue that the effective R&D cost in Aghion and Howitt (1992) model should be higher. Because of adverse selection, it incurs an agency cost to identify capable researchers. The agency cost is a measure of financial development and lower agency cost encourages R&D expenditure. A lower agency cost during economic boom can increase the probability of raising sufficient capital to finance investment projects. Still, the result is ambiguous, since a positive productivity shock raises labor wages, thus increasing R&D labor cost. We demonstrate in the paper that, with the assumption that investment good being the single input in research, R&D is unambiguously procyclical.

This paper is organized as follows. Section 1 sets up the general equilibrium model; section 2 analyzes the balanced growth path; section 3 evaluates the effect of subsidy on research outlay and the cyclical movement of R&D.

2. The Model

This paper combines the endogenous growth model with embodied technical change `*a* la Boucekkine et al (2005). There are two final good markets - namely investment good and the non-durable good. Technology in non-durable sector is Cobb-Douglas on labor and aggregate quality index. The non-durable good is allocated for consumption and production of intermediate goods. Technology in the investment sector is constant elasticity of substitution on a continuum of intermediate inputs, whose mass is normalized to be 1. Both final good sectors are competitive. The intermediate sector is monopolistic, using non-durable good as sole input. In the Boucekkine et al (2005) model, nondurable sector owns capital. They determine the dynamic path of capital by purchasing investment good. It predicts that investment prices decline over time, and thus accelerates capital use in the production of non-durable good. The rising demand for investment good increases profit for intermediate sector, and eventually the value of a new patent. In fact, it is consistent with Greenwood et al.(1997) and Young's (1995) findings that capital accumulation being the key source of growth. This paper assumes away capital. Unlike Boucekkine et al (2005) and Aghion and Howitt (1992), the research sector uses only investment good instead of final goods. With this setup, the investment price enters the research arbitrage equation and subsidy on R&D has growth effect.

2.1. Household

The economy is resided by many identical infinitely-lived consumers. Each of them has constant subjective discount rate, supplying labor inelastically with a mass of L_t, which grows at an exogenous rate of n. Capital is assumed away in this model. The asset evolution equation is given by:

$$\dot{a} = w - c - na$$

where a denotes wealth, w is wage and c is consumption. The consumers are risk-neutral, thus the rate of time preference coincides with the interest rate. The life-time utility is given by

$$U(c_t) = \int_0^\infty c_t e^{-\rho\tau} d\tau \tag{1}$$

where ρ is the subjective discount factor.

2.2. Investment

The investment good is produced by fabricating a continuum of differentiated intermediate goods, indexed by $j \in [0,1]$. Without loss of generality, the number of varieties is held constant[4].

$$I_t = \left(\int_0^1 x_{j,t}^\alpha dj \right)^{\frac{1}{\alpha}} \tag{2}$$

I_t is the investment good per capita; α is the cross-product elasticity of substitution, with compact support on [0,1]. It can be easily verified that the production has constant return to scale. The investment firms choose optimal price by solving the following minimization problem:

$$\min_{\{p_{j,t}\}} \int_0^1 p_{j,t} x_{j,t} dj$$

$$\left(\int_0^1 x_{j,t}^\alpha dj \right)^{\frac{1}{\alpha}} \geq \bar{x}$$

Using the fact that elasticity between investment and intermediate good being 1, the first order condition and equating supply and demand, the lagrangian multiplier can be verified to be P_t (the composite price index); while the optimal demand for intermediate good is given by:

[4] For the a treatment of increasing varieties, see Young (1995b).

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$$x_{j,t}^{D} = \left(\frac{p_{j,t}}{P_t}\right)^{\frac{1}{1-\alpha}} I_t \tag{3}$$

The demand price elasticity is the constant $\frac{1}{1-\alpha}$. Expanding investment sector directly benefits the intermediate sector. Notice that the elasticity with respect to investment good is one.

Substituting equation (3) into the production function, it turns out that the composite price index can be expressed as a function of intermediate good price.

$$P_t = \left(\int_0^1 p_{j,t}^{\frac{\alpha}{\alpha-1}} dj\right)^{\frac{\alpha-1}{\alpha}} \tag{4}$$

2.3. Intermediate Good

Technology in the intermediate sector is linear in its sole input - the non-durable good. For each sector, a monopolist possesses the patent of the leading-edge technology, imposing a monopoly price without competition from previous patent holder [5].

$$\max \Pi_{j,t} = (p_{j,t} - MC_t) x_{j,t} \tag{5}$$

subject to demand from the investment sector. MC_t is the nominal marginal cost. The first order condition sets the price

$$p_{j,t} = \frac{1}{\alpha} MC_t \tag{6}$$

The marginal cost is equal to the inverse of marginal productivity, which is given by the quality level of that sector- $q_{j,t}$. Thus

$$p_{j,t} = \frac{1}{\alpha q_{j,t}} \tag{7}$$

The mark-up $1/\alpha$ is constant and equal for all monopolists. From equations (6) and (7), we can compute the composite price index of investment good

$$P_t = \frac{1}{\alpha Q_t} \tag{8}$$

where

$$Q_t = \left(\int_0^1 q_{j,t}^{\frac{\alpha}{1-\alpha}} dj\right)^{\frac{\alpha}{1-\alpha}} \tag{9}$$

is a quality index of the inputs used in the production of the durable good. The price of the investment good is an inverse function of the aggregate quality of intermediate inputs.

[5] The Schumpeterian creative destruction process will be discussed in the next section.

In a stationary growth regime, Q_t must be growing at a positive constant rate, implying that the price of investment goods must be permanently declining - a fact consistent with the empirical finding, that a large portion of growth residual can be explained by capital accumulation. In the Greenwood et al. (1997) model, declining equipment price accelerates equipment capital accumulation, making capital accumulation a larger fraction of growth. Notice also that the price of any intermediate good relative to the price of the investment good is $p_{j,t}/P_t = Q_t/q_{j,t}$. It depends on its relative quality only. More efficient intermediate goods are sold at lowerr prices.

The aggregate amount of intermediate good can be derived from equations (3) and (9)

$$X_t = \int_0^1 \frac{x_{j,t}}{q_{j,t}} dj = \frac{I_t}{Q_t} \tag{10}$$

Hence, a reduced amount of final good is required to produce the same amount of investment, as the economy climbs up the quality ladder as depicted in the Grossman and Helpman model (1991).

2.4. Non-durable Good

The representative firm uses labor as the sole input and maximizes the discounted flow of profits subject to the technology constraint.

$$Y_t = Q_t^{\alpha} L_t^{1-\alpha} \tag{11}$$

The firm sets optimal labor input by equating marginal product of labor to wage rate

$$(1-\alpha) Q_t^{\alpha} L_t^{-\alpha} = w_t \tag{12}$$

From equation (8), the decline on investment prices is equal to the growth rate of the quality index, i.e. $\frac{\dot{P}}{P} = -\frac{\dot{Q}}{Q}$. As quality improves, demand for investment good increases over time. The rising investment level directly benefits R&D.

At equilibrium, per capita production of the non-durable good is allocated to consumption, C_t and as inputs in the production of intermediate sector, X_t. All variables are in per capita terms, formally,

$$C_t + X_t = Q_t^{\alpha} L_t^{1-\alpha} \tag{13}$$

We have assumed away capital. The reason is purely technical. Suppose the consumers or the non-durable good firms own the capital, then they have to purchase investment goods. If two sectors obtain investment good at the same time, the analysis will be intractable. There is one way to incorporate capital into the model. We can assume that the consumers own fixed amount of capital; there is no depreciation and the non-durable good sector rents it from the consumers. The balanced growth path equilibrium can be found in the appendix.

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2.5. Schumperterian R&D Process

We assume that the R&D sector is competitive. There are many research firms doing independent research[6], but they "ride on the shoulder of the giant", in the sense that leading edge technology in the j th sector is common knowledge. Once a firm makes a discovery, they will replace the existing monopoly and enjoy the monopoly profit until the next wave of innovation comes. Alternatively, we can assume that all research firms sign an agreement with the existing monopoly that the latter will pay a royalty to new technology [7] - the expected present discounted value of the patent.

We adopt the model `a la Barro and Sala-i-Marti'n, X (1994), that the marginal productivity factor of j th monopolist $q_{j,t} = q^{\kappa_{j,t}}$, where $\kappa_{j,t}$ represents the technology level along the quality ladder in sector j, i.e. the number of innovations up to time t. Each intermediate sector is monopolized by the holder of a patent to the latest generation of that good. Each innovation at date t in any sector j permits the innovator to start producing in sector j using the leading edge technology. Any innovation raises technology by a constant factor of $q > 1$ in the intermediate sector. However, the arrival rate of innovation remains random.

Competition is drastic in the sense that the existing firm will be replaced by the next innovator who can set price without restrictions from the incumbent. Extension to the non-drastic competition would not change the key implications of the model. The successful innovator destroys the surplus attributable to the previous generation of technology by rendering it obsolete, which is the Schumpeterian "creative destruction" (also known as the Arrow effect) in the growth literature.

Unlike Bouccekin et al.(2005), the R&D sector uses investment good as input in this model. Define $n_{\kappa_{j,t}}$ as the amount of investment input used in each firm, innovations arrive randomly with a Poisson arrival rate of

$$\eta_{\kappa_j} = \lambda n_{\kappa_{j,t}} \phi(\kappa_j) \tag{14}$$

where $\lambda > 0$ is a parameter indicating the productivity of research technology. We adopt the model `a la Barro and Sala-i-Marti'n, X (1994) model, that poisson arrival rate is a negative function of task complexity, meaning $\phi'() < 0$ and $\phi''() < 0$. It exhibits diminishing return and scale effect can be eliminated if there is any.

The instantaneous profits of the κ_{j+1} innovator, for all time $z \geq t$ until she will be displaced by the κ_{j+2} innovator is given by

$$\pi_{\kappa_{j+1,z}} = \left(p_{jz} - \frac{1}{q^{\kappa_{j+1}}} \right) x_{jz} = \frac{1-\alpha}{\alpha} \frac{I_z}{q^{\kappa_{j+1}}} \left(\frac{q^{\kappa_{j+1}}}{Q_z} \right)^{\frac{1}{1-\alpha}} \tag{15}$$

[6] Cabellero, R.J., and Jaffe, A.B.(1993) assume that all firms benefit from a common pool of leading edge technology in the economy. In the other word, spillover exists in the economy. But it will only affect the steady state growth rate, all comparative statics remain unchanged.
[7] One can even assume that research firms are owned by the intermediate monopolists.

The present value, which represents the prize for the κ_j th innovation, depends positively on the profit flow $\pi_{\kappa_{j+1}}$, and the duration of the monopoly for the inventor of rung j. It is given by

$$V_{\kappa_{j+1,t}} = \int_0^\infty \pi_{\kappa_{j+1,z}} e^{\int_t^z (r_s + \eta_{\kappa_{j+1,s}}) ds} dz \tag{16}$$

The arbitrage condition for a strictly positive amount of resources spent in $R\&D$ activities stipulates that the marginal cost of research should be equal to the expected present value of profits, that is:

$$\lambda\phi(\kappa_j) V_{\kappa_{j+1,t}} = P_t \tag{17}$$

The arbitrage condition implies

$$\frac{\dot{V}_{\kappa_{j+1,t}}}{V_{\kappa_{j+1,t}}} = \frac{\dot{P}}{P} \tag{18}$$

$\dot{V}_{\kappa_{j+1,t}}$ can be derived by differentiating equation (16) with respect to time[8], and it can be verified that

$$V_{\kappa_{j+1,t}} = \frac{1-\alpha}{\alpha} \frac{q^{(\kappa_j+1)\frac{\alpha}{1-\alpha}}}{r_t + \eta_{\kappa_j} - \frac{\dot{P}}{P}} Q_t^{\frac{1}{\alpha-1}} I_t \tag{19}$$

When the specification $\phi(\kappa) = q^{-(\kappa_j+1)\frac{\alpha}{1-\alpha}}$ is adopted, the Poisson arrival rate does not depend on the complexity of the research task. By the Law of Large Numbers, the average growth rate of Q_t is

$$\gamma_t = \tilde{q}\,\eta_t \tag{20}$$

where $\tilde{q} = \frac{\alpha}{1-\alpha}(q^{(\kappa_j+1)\frac{\alpha}{1-\alpha}} - 1)$. The arbitrage condition can be written as

$$P_t = \frac{1-\alpha}{\alpha} \frac{Q_t^{\frac{1}{\alpha-1}} I_t}{r_t + \frac{\gamma_t}{\tilde{q}} - \frac{\dot{P}}{P}} \tag{21}$$

3. Balanced Growth Path

[8] Simply apply the Leibnitz rule of differentiation.

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After some change of variables, the balanced growth path is characterized by the following system of equations:

$$r = \rho \tag{22}$$

$$1-\alpha\frac{Y}{L} = w \tag{23}$$

$$P = \frac{1-\alpha}{\alpha}\frac{i}{r+\frac{\gamma}{\tilde{q}}} \tag{24}$$

$$i = (1+\alpha)\gamma + (1-\alpha)n \tag{25}$$

where i denotes per capita investment good. The constancy of interest rate is the result of risk-neutral consumers' preference. The consumers do not require a higher return for steeper consumption profile. Equation (23) is simply the labor arbitrage condition. From the research arbitrage condition, we can easily verify that the level of research will be raised by a higher productivity of R&D (λ), and a larger size of innovation q. An increase in the size of innovation increases the marginal benefit by raising the size of the next interval's monopoly profits relative to this interval's productivity. An increase in arrival parameter decreases both the marginal cost and the marginal benefit of research. The latter effect is due to faster erosion of existing monopoly and lower expected monopoly profit. Comparison with social optimum follows the analysis of Aghion and Howitt (1992). The intertemporal spillover effect (private return being large than social rate) and the appropriability effect ($(1-\alpha)$ from the numerator of equation (24), which is distortion from monopoly) discourage R&D; while the business-stealing effect encourages it. The upshot is that if monopoly power (α close to zero) and innovation size is not too large, the business-stealing dominates. In this case, laissez-faire growth will be excessive.

4. Financial Development and Growth

4.1. On Subsidy

In most of the endogenous growth models, subsidy on R&D is ineffective, for instances Howitt and Aghion (1998), Barro and Sala-i-Marti'n, X (1994), and Bouccekine et al.(2005). The reason is that distortion comes from the monopolistic intermediate sector, subsidy on final or intermediate good can generate additional value to patent, and thus enhancing R&D outlay. However, subsidy on R&D investment has direct impact in this model. The revised arbitrage equation can be written as

$$(1-\beta_r)P_t = \frac{1-\alpha}{\alpha}\frac{Q_t^{\frac{1}{\alpha-1}}I_t}{r_t+\frac{\gamma_t}{\tilde{q}}-\frac{\dot{P}}{P}} \tag{26}$$

where β_r is the subsidy on R&D. The subsidy effectively lowers the unit research cost. This model is consistent with the notion that investment and R&D being complementary. More investment stimulates innovation by raising the equilibrium flow of profits, just as more innovation stimulates investment by raising the rate of productivity growth. Neither process could take place in the long run without the other. For without innovation, diminishing returns would choke off investment, and without investment, the rising cost would choke off innovation.

In a highly influential paper, Jones (1995) points out that the number of scientists and engineers engaged in R&D in the U.S and OECD economies have grown much faster; but there is no any tendency for economic growth to accelerate at comparable rate, rejecting the Schumpeterian approach. He argues that capital accumulation is the predominant factor underlying growth. In the current setup, labor is not used as inputs in delivering R&D templates. Rather than the growth of number of scientists, it is the growth of number of patents that matter.

4.2. R&D Procyclical or Counter-Cyclical?

King and Levine (1993) demonstrate how financial development accelerates growth. Let f be the agency cost of identifying capable researcher,[9] and φ be the fraction of capable researchers. The arbitrage equation is

$$(1+\frac{f}{\varphi}\omega)=\lambda+\frac{\gamma\tilde{\pi}}{r+\lambda n} \tag{27}$$

where n is labor input. The research arbitrage equation is essentially the same as Aghion and Howitt (1992). Profit depends on both labor and some exogenous parameters. The effective wage rate is increased by a factor of $\frac{f}{\varphi}$ due to adverse selection. Higher financial development is represented by a lower value of f.

Berthelemy and Varoudakis (1996) come up with the same conclusion by augmenting Aghion and Howitt (1992) model with a banking sector, which employs labor to raise loanable fund for R&D and intermediate sector. Although the arbitrage condition remains unchanged, the resources devoted to securing more savings deplete R&D. However, if only R&D sector demands for funding, then the intermediation cost will affect the arbitrage equation, and his result will be similar to King and Levine (1993). This paper also demonstrates a positive correlation between growth and development. Simply redefine the f as intermediation cost to raise investment fund, and φ as the probability of securing enough funding. The arbitrage equation can be written as

$$(1+\frac{f}{\varphi})P_t=\frac{1-\alpha}{\alpha}\frac{Q_t^{\frac{1}{\alpha-1}}I_t}{r_t+\frac{\gamma_t}{\tilde{q}}-\frac{\dot{P}}{P}} \tag{28}$$

The key implication is that *fund raising is less costly with higher degree of financial development.*

The Aghion and Howitt (1992) and Berthelemy and Varoudakis (1996) models, nonetheless, predict that R&D is counter-cyclical. An economic boom raise the productivity parameter, and thus wages, which retards R&D expenditure. However, agency cost can be

[9] Labor is assumed to be the only input in R&D.

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lower during expansion. For instance, more capable labor will enter the market for higher wages. Fung (2011) demonstrates how counter-cylcial agency cost is consistent with procyclical R&D expenditure by calibration. This paper unambiguously conclude that R&D is procyclical, because raising loanable fund is less costly at the times of expansion; and P_t is lower during economic boom.

5. Conclusion

This paper shows that by assuming R&D firms use only investment as input, instead of final good, subsidy on R&D has growth-enhancing effect. Moreover, this model predicts unambiguously that R&D is procyclcial. The model can be easily extended to study the relation between financial development and growth. However, financing in the R&D sector remains mysterious. To be specific, how can the independent research firms fare before being taken over by the monopoly? This calls for future research in this area.

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Appendix: Balanced Growth Path with Capital

Suppose that the consumers are infinitely lived as depicted in the standard Ramsey-Cass-Koopman model, with constant elasticity of substitution. The first order condition is given by.

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}(r_t - \rho) \tag{29}$$

Now, the non-durable good sector uses both labor and capital. We assume that initial stock of capital is fixed and large enough. The production function is still in Cobb-Douglas form.

$$Y_t = Q_t K_t^{\alpha} L_t^{1-\alpha} \tag{30}$$

The first order equations can be easily derived.

$$(1-\alpha) Q_t K_t^{\alpha} L_t^{-\alpha} = w_t \tag{31}$$

$$\alpha Q_t K_t^{\alpha-1} L_t^{1-\alpha} = r_t - \beta_k \tag{32}$$

where β_k is capital subsidy. The balanced growth path equilibrium will be the same as before, but the interest rate is no longer equal to ρ, and there will be a capital equilibrium condition. The balanced growth path is characterized by the following equation system.

$$\alpha k^{\alpha-1} l^{1-\alpha} = r - \beta_k \tag{33}$$

$$r = \sigma \frac{\alpha}{1-\alpha} \gamma + \rho \tag{34}$$

$$1 - \alpha \frac{Y}{L} = w \tag{35}$$

$$P = \frac{1-\alpha}{\alpha} \frac{i}{r + \frac{\gamma}{\tilde{q}}} \tag{36}$$

$$i = (1+\alpha)\gamma + (1-\alpha)n \tag{37}$$