Optimal Monetary Policy in the Sticky Information Model of Price Adjustment: Inflation Targeting or Price-Level Targeting?

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Abstract

I investigate optimal monetary policy in the sticky information model of price adjustment within a New Keynesian macroeconomic framework. The model is solved for optimal policy, and welfare implications of three alternative monetary policy regimes: unconstrained policy, price-level targeting and inflation targeting, are compared when there is a shock to the economy. The results for a cost-push shock illustrate that optimal policy depends on the degree of price stickiness and the persistence of the shock. Inflation targeting is the optimal policy if prices are flexible enough or the shock is persistent enough. However, for a demand shock, inflation targeting emerges as the best policy for all values of the price stickiness and the shock's persistence. When the volatility of nominal interest rate is taken into consideration, the results indicate that inflation targeting is the best policy, in the sense that it results in smaller welfare loss and volatility of nominal interest rate, if prices are sticky enough and the persistence of the shock is large enough. However, price-level targeting might be preferable to inflation targeting if prices are more flexible and the relative weight for the volatility of nominal interest rate is large.

JEL classification: E31; E37; E52

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1. Introduction

It is generally accepted that objective of any monetary policy is to achieve a low expected value of a discounted loss function. To be able to evaluate alternative policy rules, this loss function, which depends on the model's specification, needs to be specified. Currently, there is wide consensus on the central banks' objectives of keeping inflation close to zero and output close to its natural level. However, there is a considerable disagreement on the details of this general specification, such as what would be the relative weights of the output gap and inflation terms in the loss function and which price stabilization policy should be pursued: price-level targeting or inflation targeting.

This paper investigates optimal monetary policy for a closed economy within a New Keynesian macroeconomic framework with the sticky information model of price adjustment. This price setting model is proposed by Mankiw and Reis (2002). The main premise of their model is that information about macroeconomic conditions spreads slowly throughout the population; although prices are set every period, information collecting and processing take time. The model is solved for optimal policy, and welfare implications of three alternative monetary policy regimes: unconstrained policy, price-level targeting, and inflation targeting, are compared when there is a cost-push shock or a demand shock to the economy.[1] Unconstrained policy regime implies that the central bank does not have commitments or targets to manipulate the private-sector expectations. However, there is a target level for prices or inflation in price-level and inflation targeting regimes, and expectations can be manipulated if the commitment to the target is credible in these regimes.

Despite the common use of the term "targeting", there is no agreement on the meaning of it in the literature. In one terminology, such as in Svensson (1997, 2002) or Clarida, Gali and Gertler (1999), targeting means minimizing an objective function in which a target variable shows up. In the other, such as in McCallum and Nelson (1999), targeting means that the target variable is used in the feedback policy rule. In this study, targeting term is used in

[1] The cost-push shock appears in price adjustment equations, and represents everything other than the output gap that may affect the expected marginal costs. It causes model to generate inflation variation independent of the demand shocks, as observed in data.

the spirit of the first definition.

Targeting regimes have become the most common type of monetary policies in developed countries in recent years.[2] Inflation targeting has been adopted by many central banks for the conduct of monetary policy. Svensson and Woodford (2003) show that inflation targeting is an effective mean of maintaining low and stable inflation and inflation expectations, without negative consequences for the output gap. Price-level targeting, which may be thought of as an extreme version of inflation targeting, is also discussed in the literature, but has not received much support. Some problems, as mentioned in Clarida, Gali and Gertler (1999), might be that if the price level overshoots its target, the central bank may have to contract economic activity to return the price level to its target. So, there should be some inflation below the level implied by the price-level target to return the price level to its target. Under inflation targeting, bygones are bygones: overshooting of inflation in one year does not require an inflation level below the target in the following year.[3] Also, the net reduction in price uncertainty under a price-level targeting rule may be small relative to that obtained under an inflation targeting policy.[4] The rationale for targeting regimes might be to guarantee that monetary policy avoids mistakes easily by identifying a clear nominal anchor for the policy. There is voluminous literature on optimal policy and targeting regimes. However, there is no consensus on whether price-level targeting or inflation targeting is preferable.[5]

The analysis of this study shows that inflation targeting is generally the best policy in the sense that it results in smaller welfare loss and volatility of nominal interest rate for the sticky information model. Price-level targeting might be optimal only if prices are flexible enough. If only the loss function is considered and the volatility of nominal interest rate is not taken into

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[2] There is no agreement on how targeting regimes should be implemented, either by using "targeting rules", as studied in a number of papers by Svensson (1997, 2002, 2003), or by using some mechanical instrument rules, such as Taylor type rules. In those papers, Svensson proposes that inflation targeting is better described as a commitment to a "targeting rule" rather than following a mechanical instrument rule.

[3] The main difference between inflation and price-level targeting is that a base drift in the price level is allowed in inflation targeting.

[4] Rudebusch and Svensson (1999) show that monetary targeting or nominal GDP targeting is much more inefficient than inflation targeting, in the sense of inducing more variable inflation and output.

[5] Among others, Clarida, Gali and Gertler (1999, 2001, 2002) identifies optimal policy as inflation targeting. For an open economy, they obtain optimal policy as domestic inflation targeting under discretion and domestic price-level targeting under commitment. Gali and Monacelli (2002) find domestic inflation targeting is optimal for a small open economy. Ball, Mankiw and Reis (2005) find price-level targeting is the optimal policy while inflation targeting is suboptimal when the sticky information model of price setting is used.

consideration, inflation targeting generally emerges as the best policy for cost-push shocks and demand shocks. Price level targeting dominates only if prices are sticky enough and the shock is not very persistent when there is a cost-push shock to the economy. Optimal policy, in a similar framework, is also investigated by Ball, Mankiw and Reis (2005). They find price-level targeting to be the optimal policy while inflation targeting is suboptimal.[6] This implies that allowing base drift in the price level is not optimal in their analysis. Therefore, the findings in this study are in contrast to those in their study. They use a simple quantity equation to represent the demand side of their model, so the monetary policy instrument in their approach is money supply. They also do not solve their model explicitly to obtain the optimal money-supply rule; instead, they obtain reduced form for the price level and interpret the expected price level as the policy instrument. They first impose the policy constraints on the price adjustment equation, and then find the welfare losses. However, in this study, first a general equilibrium model is solved for optimal solution, and then the regime constraints are imposed on this solution to obtain the welfare implications of these regimes.

In a dynamic stochastic general equilibrium model, the price adjustment equation is the crucial part for optimal policy and dynamics of output and inflation, since it determines the specific relationship between the output gap and inflation. In these models, the specific form of the demand side is generally represented by an IS equation, which relates the nominal interest rate to output and inflation. Therefore, the nominal interest rate is generally used as the instrument of monetary policy in these models, which is now a common application in academia and monetary practice. Interest rate is sometimes taken as a variable to be stabilized along with the output gap and inflation. Therefore, volatility of interest rate, so the specific form of the IS equation, may be important to determine the optimal policy or compare alternative monetary policies.[7] Because of such considerations, an expectational IS equation, which is obtained from the optimization of households in the model, and relates

[6] Their exact finding is that the strict price-level targeting is optimal to aggregate demand or productivity shocks, and flexible price-level targeting (price level gradually returns to its target level) is optimal to cost-push shocks.

[7] Some theoretical and empirical advantages of interest-rate rules over the other rules including money-supply rules are illustrated in literature, such as see Bernanke and Mihov (1998), Clarida, Gali and Gertler (1999), Woodford (2003, Chp. 1, 2).

the interest rate to output and inflation, is used to represent the demand side of the economy in this study.

The rest of the paper is organized as follows. In the next section, the model is briefly outlined. In Section 3, utility approximation of welfare is described when the price adjustment mechanism is based on the sticky information model. The model is solved, and optimal policy is derived in Section 4. Welfare implications of the alternative policy regimes under optimal policy for a cost-push shock and a demand shock are investigated in sections 5 and 6, respectively. The results are given in section 7, and section 8 concludes.

2. The Model

2.1. Households and Firms

The model is a version of the standard New Keynesian dynamic general equilibrium model with price rigidities, which has been used extensively for theoretical analysis of monetary policy.[8] Households are assumed to be monopolistically competitive supplier of their labor to obtain a cost-push shock term in the model. The economy is closed and composed of a continuum of identical infinitely lived households indexed by $i \in [0, 1]$ and a continuum of firms indexed by $j \in [0, 1]$. Households supply labor, which is an imperfect substitute of other labors, purchase consumption goods, and hold bonds. Firms hire labor and specialize in the production of a single good that is an imperfect substitute of other goods. Since each firm and household has some monopoly power, the economy is the one having the monopolistic competitive markets similar to those studied in Dixit and Stiglitz (1977) or Blachard and Kiyotaki (1987).

Households and firms behave optimally and maximize their utility and profits, respectively. There is also a financial market in the economy in which households can trade in a range of securities that is large enough to completely cover all states of nature; that is, complete market is assumed and the households can insure themselves against idiosyncratic uncertainty.

[8] See Rotenberg and Woodford (1997), McCallum and Nelson (1999), Gali and Monacelli (2002), Arslan (2004).

Households derive utility from composite consumption goods and leisure, and the utility of household i in period t is given by:

$$U_{it} = \frac{C_{it}^{1-\sigma}}{1-\sigma} - \frac{N_{it}^{1+\varphi}}{1+\varphi} \ , \tag{1}$$

where C_{it} is a Dixit-Stiglitz type CES aggregator of composite consumption of household i and is defined over production C_{it}^j of firm j, and N_{it} is the household i's composite labor supply index and is defined over the labor demand N_{it}^j of firm j. These are defined as:

$$C_{it} = \left(\int_0^1 (C_{it}^j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \ , \tag{2}$$

$$N_{it} = \left(\int_0^1 (N_{it}^j)^{\frac{\eta_t-1}{\eta_t}} dj \right)^{\frac{\eta_t}{\eta_t-1}} \ . \tag{3}$$

The parameter of risk aversion σ and marginal disutility of labor φ are positive. The parameter ε is the elasticity of substitution among the goods, and η_t is the elasticity of labor demand, which follows some stochastic process, these two parameters are greater than one and same across households.

Each household i seeks to maximize the lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{it} \ , \tag{4}$$

subject to the intertemporal budget constraint, where $\beta < 1$ is the discount factor. The first order conditions are given by:[9]

$$\frac{W_t}{P_t} = \frac{\mu_t^w}{\mu} C_{it}^{\sigma} N_{it}^{\varphi} \ , \tag{5}$$

$$\beta I_t E_t \left\{ \left(\frac{C_{it+1}}{C_{it}} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \ , \tag{6}$$

where $\mu_t^w = \frac{\eta_t}{\eta_t-1}$ is the optimal wage markup, $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the constant price markup, and $E_t(Q_{t,t+1}) = I_t^{-1}$ is the price of a riskless one-period bond. The consumption of the household i can be obtained as:

$$c_{it} = E_t c_{it+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} + \ln \beta) \ , \tag{7}$$

[9]A more detailed solution of the model can be found in Arslan (2005)

where small characters represent the logarithm of those variables. $\pi_t = p_t - p_{t-1}$ is the inflation rate at period t, and $1/\sigma$ is the intertemporal elasticity of substitution of consumption.

Demand functions for consumption and labor index can also be obtained as:

$$C_{it}^j = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon} C_{it} \ , \tag{8}$$

$$N_{it}^j = \left(\frac{W_{it}}{W_t}\right)^{-\eta_t} N_{it} \ . \tag{9}$$

Each firm j produces its specialized product with a linear technology according to the production function:

$$Y_{jt} = A_t N_{jt} \ . \tag{10}$$

Output is only the function of labor input N_{jt} and aggregate productivity disturbance A_t. Firms hire labor, produce and sell their differentiated products in the monopolistically competitive market, and try to minimize their cost of production. Their cost minimization problem results in:

$$\gamma_t = \frac{2\mu_t^w - 1}{\mu_t^w} \frac{W_t/P_t}{A_t} \ , \tag{11}$$

where γ_t is equal to the firm's real marginal cost. Since technology is constant return to scale, and shocks are the same across firms, the real marginal cost γ_t is the same across firms.

In a monopolistically competitive model, it is assumed that each firm knows that its sales depend on the price of its product. When all purchases are made for private consumption, then the aggregate demand Y_t corresponds to the households total consumption index. Therefore, the demand function can be written from equation (8), in logarithms as:

$$y_{jt} = y_t - \varepsilon(p_{jt} - p_t) \ , \tag{12}$$

where y_{jt} is the log of output produced by firm j, p_{jt} is the log price charged for that product by firm j, y_t is the log aggregate output, and p_t is the price index for aggregate consumption.

Firms' optimal choice of price P_{jt}^* to maximize profit, given the above demand function, can be obtained as:

$$\frac{P_{jt}^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{W_t/P_t}{A_t} = \frac{\mu\mu_t^w}{2\mu_t^w - 1}\gamma_t \ . \tag{13}$$

This is the standard results in a monopolistic competitive market when all firms are able to adjust their price in every period; that is, each firm set its optimal price P_{jt}^* equal to a markup over its nominal marginal cost $P_t\gamma_t$. By using equation (5), the above expression can be rewritten as:

$$\frac{P_{jt}^*}{P_t} = \frac{\varepsilon}{\varepsilon-1}\,\frac{\mu_t^w}{\mu}\,\frac{C_{it}^\sigma N_{it}^\varphi}{A_t} = \mu_t^w\,\frac{C_{it}^\sigma N_{it}^\varphi}{A_t}\ . \tag{14}$$

Market clearing conditions require that labor demand and supply be equal, that is $N_{it} = N_{jt}$, and consumption should be equal to output, that is $C_{it} = C_t = Y_t$. Equation (14) can be written by using the production function in (10) and the demand function in (12) as:

$$p_{jt}^* = p_t + \frac{\sigma+\varphi}{1+\varepsilon\varphi}\,y_t - \frac{1+\varphi}{1+\varepsilon\varphi}\,a_t + \frac{\log\mu_t^w}{1+\varepsilon\varphi}\ . \tag{15}$$

When all firms can set their prices freely each period, that is when prices are flexible, all firms set the same price $p_{jt}^* = p_t$. The natural level of output y_t^N is defined as the level where prices are flexible and the wage markup is fixed at its steady state value μ^w. This setup means that there are no wage markup shocks, and variations in the natural level of output do not reflect the variations in the wage markup. Therefore, under flexible price equilibrium, the natural rate of output can be obtained from equation (15) as:

$$y_t^N = \frac{1+\varphi}{\sigma+\varphi}\,a_t - \frac{\log\mu^w}{\sigma+\varphi}\ . \tag{16}$$

If it is plugged into (15), the optimal price chosen by the adjusting firm canbe obtained as:

$$p_{jt}^* = p_t + \alpha(y_t - y_t^N) + u_t\ , \tag{17}$$

where α and u_t are defined as:

$$\alpha = \frac{\sigma+\varphi}{1+\varepsilon\varphi} < 1 \quad \text{and} \quad u_t = \frac{\log(\mu_t^w/\mu^w)}{1+\varepsilon\varphi}\ .$$

When equation (7) is aggregated over all households, It can be rewritten in terms of aggregate output index as:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} + \ln\beta)\ . \tag{18}$$

This is the *IS* equation of the model and when an exogenous AR(1) technology shock process $a_t = \rho_a a_{t-1} + \xi_t$ is assumed, it can be written in terms of output gap $x_t = y_t - y_t^N$ as:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + \nu_t \ , \tag{19}$$

where $\nu_t = -\dfrac{\log \beta}{\sigma} - \dfrac{(1-\rho_a)(1+\varphi)}{\sigma+\varphi}\, a_t$.

2.2. Price Setting

In the sticky information model of Mankiw and Reis (2002), prices are set every period, but information collecting and processing, that is optimal price computing, occur slowly over time. A randomly selected fraction $1-\theta$ of firms receive complete information about the state of the economy in each period, and adjust their prices according to this new information, while the remaining fraction θ of firms adjust their prices according to old information. The parameter θ measures the degree of price stickiness; a large one shows that few firms gets new information, so fewer firms adjust their prices, and the expected time between price adjustments will be longer. Firms that do not adjust their prices will adjust their output according to demand function of the market. When a firm j sets its price in period t, it will set it to its optimal expected price according to the last information it has at period $t-k$ as:

$$p_{jt}^k = E_{t-k} p_{jt}^* \ . \tag{20}$$

Since the new information arrives at the rate of $1-\theta$, the share of the firms that last adjusted their plan k periods ago will be $(1-\theta)\theta^k$. Therefore, the price index can be written as:

$$p_t = (1-\theta) \sum_{k=0}^{\infty} \theta^k E_{t-k} p_{jt}^* \ . \tag{21}$$

By using p_{jt}^* from equation (17), the sticky information price adjustment equation can be obtained in terms of prices as:

$$p_t = (1-\theta) \sum_{k=0}^{\infty} \theta^k E_{t-k}(p_t + \alpha x_t + u_t) \, , \tag{22}$$

and in terms of inflation as:

$$\pi_t = \phi \alpha \, x_t + \phi \, u_t + (1-\theta) \sum_{k=0}^{\infty} \theta^k E_{t-1-k}(\pi_t + \alpha \Delta x_t + \Delta u_t) \, , \tag{23}$$

where $\phi = (1-\theta)/\theta$ and u_t is the cost-push shock term that represents the changes in prices independent of demand shocks.

3. Welfare and Utility Approximation

3.1. Measuring the Welfare

Welfare is defined as the utility of the representative agent within the model. Such a utility optimization based welfare criterion is useful when comparing the consequences of alternative policy rules. Woodford (2001) shows that this approach can justify the traditional assumption of price stability, which assumes a quadratic loss function in some form. Also, a precise formulation of the appropriate loss function can be derived in this approach depending upon the model's assumption, especially upon the specification of the price adjustment.

Woodford (2001) shows that quadratic approximation to the utility function of households in a monopolistically competitive framework can be expressed as:

$$U_t = \frac{1}{2}\left[\lambda(x_t - x^*)^2 + var_i(p_i - p_t)\right] , \tag{24}$$

where x^* is the efficient level of output gap,[10] p_i is the price of differentiated goods, and p_t is the general price level. This representation is valid for any specification of price stickiness.[11]

In a New Keynesian framework with price adjustment based on the Calvo (1983) model, this utility function takes the familiar form of

$$U_t = \frac{1}{2}\left[\lambda(x_t - x^*)^2 + \pi_t^2\right] . \tag{25}$$

So, when x^* is assumed to be exogenous, then the objective function of the monetary policy can be expressed as:

$$max \ -\frac{1}{2}E_t\left\{\sum_{i=0}^{\infty}\beta^i(\lambda x_{t+i}^2 + \pi_{t+i}^2)\right\} , \tag{26}$$

where λ represent the relative weight on output gap. Since this objective function takes potential output as the target, it also implicitly takes zero as the target inflation. Much of the

[10]The level of output gap when there is no distortion such as due to taxes or market power.
[11]See Woodford (1999, 2001), Rotemberg and Woodford (1999).

literature a priori assumed the monetary policy objective as above. In such a representation the problem is what the relative weight of output and inflation losses should be. There is no specific answer to this, but there is widespread agreement that the primary objective of monetary policy should be to control inflation. Therefore, the weight of inflation loss should be much greater than that of output.

3.2. Welfare for the Sticky Information Model

The utility-based welfare criterion is the level of expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t \ , \tag{27}$$

approximated around the equilibrium point, where there are no real disturbances. U_t is the period utility function for any household and is given by equation (1). The period utility function U_t can be approximated as:[12]

$$U_t = -e^{(1-\sigma)\bar{y}_t} \frac{\sigma+\varphi}{2} \left[x_t^2 + \frac{\varepsilon^{-1}+\varphi}{\sigma+\varphi} \, var_j \left(y_{jt} - y_t \right) \right] + t.i.p. \ , \tag{28}$$

where x_t is the output gap, $\bar{y}_t$ is the equilibrium point around which the approximation takes place, and $t.i.p$ term represents the terms independent of policy.

The demand function given in equation (12) gives:

$$var_j \left(y_{jt} - y_t \right) = \varepsilon^2 var_j \left(p_{jt} - p_t \right) \ .$$

If this expression is used in equation (28), then utility function becomes:

$$U_t = -e^{(1-\sigma)\bar{y}_t} \frac{\sigma+\varphi}{2} \left[x_t^2 + \frac{\varepsilon(1+\varepsilon\varphi)}{\sigma+\varphi} \, var_j \left(p_{jt} - p_t \right) \right] + t.i.p. \tag{29}$$

This is the quadratic approximation to the period utility function and is valid for any model with monopolistic competition and price stickiness. It shows that policy should aim to stabilize the output gap and reduce price variability. However, the relation between price variability and stabilization of the general price level depends on the price adjustment mechanism. Ball, Mankiw, and Reis (2005) show that the cross-sectional price variability for the

[12] Details of the welfare approximation procedure are given in Appendix A, see also Woodford (2001,2002), Ball, Mankiw and Reis (2005).

sticky information model of price adjustment can be expressed as:

$$var_j\left(p_{jt}-p_t\right)=\sum_{i=1}^{\infty}\eta_i(p_t-E_{t-i}p_t)^2\ , \tag{30}$$

where $\eta_i=\frac{\theta^i(1-\theta)}{(1-\theta^i)(1-\theta^{i+1})}$. It demonstrates that the variance of relative prices depends upon the quadratic deviations of the price level from the levels expected at all past dates. Only unexpected components of the price level relative to past expectations affect the equilibrium price variability. This relationship can be rewritten in terms of inflation rate as:

$$var_j\left(p_{jt}-p_t\right)=\sum_{i=1}^{\infty}\eta_i\left(\sum_{l=0}^{i-1}(\pi_{t-l}-E_{t-i}\pi_{t-l})\right)^2\ , \tag{31}$$

Thus, the equilibrium price variability depends upon the sum of squared deviations of inflation rates from any past periods to any period t, and only unexpected components of the inflation rates at all previous periods are matters for the equilibrium price variability.

Therefore, the quadratic approximation to the utility of the representative agent, by ignoring *t.i.p.*, is given in terms of prices as:

$$U_t=-e^{(1-\sigma)\bar{y}_t}\frac{\varepsilon(1+\varepsilon\varphi)}{2}\left[\lambda x_t^2+\sum_{i=1}^{\infty}\eta_i(p_t-E_{t-i}p_t)^2\right]\ , \tag{32}$$

and in terms of inflation as:

$$U_t=-e^{(1-\sigma)\bar{y}_t}\frac{\varepsilon(1+\varepsilon\varphi)}{2}\left[\lambda x_t^2+\sum_{i=1}^{\infty}\eta_i\left(\sum_{l=0}^{i-1}(\pi_{t-l}-E_{t-i}\pi_{t-l})\right)^2\right]\ , \tag{33}$$

where $\lambda=\frac{\sigma+\varphi}{\varepsilon(1+\varepsilon\varphi)}$ is the relative weight of the output gap.

4. Optimal Policy

The sticky information Phillips curve proposed in Mankiw and Reis (2002) is given in terms of prices and inflation in equations (22) and (23), respectively. The IS curve of the model is given in equation (19) and can be rewritten in terms of prices as:

$$x_t=E_tx_{t+1}-\frac{1}{\sigma}(i_t-E_tp_{t+1}+p_t)+\nu_t\ . \tag{34}$$

Since the terms at the outside of the parenthesis in the period utility function (32) do not affect the optimization problem, they can be dropped. So, the intertemporal loss function can be written in terms of prices as:

$$-\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\lambda x_t^2+\sum_{i=1}^{\infty}\eta_i(p_t-E_{t-i}p_t)^2\right), \tag{35}$$

Optimal monetary policy will be choosing $x_t, p_t,$ and i_t, which maximizes the objective function (35) subject to the sticky information Phillips curve (22) and the IS curve (34). In the first step of the optimization, I maximize (35) subject to (22) and get a relationship between x_t and p_t, then optimum i_t implied by the IS equation (34) can be obtained.

The first step of this optimization problem yields the optimality condition, which relates x_t and p_t as:

$$x_t=-\Omega\sum_{i=1}^{\infty}\eta_i(p_t-E_{t-i}p_t)=-\Omega Np_t+\Omega\sum_{i=1}^{\infty}\eta_iE_{t-i}p_t\ , \tag{36}$$

where $\Omega=\frac{\alpha(1-\theta)}{\theta\lambda}$ and $N=\sum_{i=1}^{\infty}\eta_i$.

This optimality condition is valid for both discretionary and commitment cases of monetary policy. Discretionary policy implies that a central bank optimizes (35) period by period without any commitment, therefore it is not able to manipulate the private sector expectations. However, commitment to a policy implies the global optimization of (35) under a specific commitment, which affects private-sector expectations. In both cases, expectations are predetermined, and are taken as given; therefore, both of the period by period and global optimizations of (35) yield the same optimality condition given by equation (36).

In this study, I investigate optimal policy under discretion, in the sense that a central bank optimizes its objective function each period. However, this central bank can make some commitments to manipulate the expectations of the private sector. This approach also is in accord with today's modern central banking practice. Optimal policies for a cost-push shock and a demand shock are investigated under three policy regimes. First, unconstrained optimal monetary policy is investigated. A central bank optimizes without any commitments to manipulate private-sector expectations with this policy, so those expectations are free to change. In the other two policy regimes, the central bank makes commitments to affect the expectations, and targets some level of prices or inflation, so these regimes might be interpreted as the ones under commitment, although optimization takes place each period.[13]

5. Optimal Policy for a Cost-Push Shock

5.1. Unconstrained Policy with a Cost-Push Shock

Monetary policy does not have any announced target or credible commitment to manipulate private-sector expectations in this case. Therefore, the central bank maximizes the objective function (35) without any constraint on private-sector expectations, and they are free to change. The optimality condition for this regime is represented by equation (36). The sticky information Phillips curve (22) can be rewritten as:

$$p_t = (1-\theta)(p_t + \alpha x_t + u_t) + (1-\theta)\sum_{k=1}^{\infty}\theta^k E_{t-k}(p_t + \alpha x_t + u_t)\ . \tag{37}$$

If the optimality condition given by equation (36) is substituted into (37) to get rid of x_t, one can obtain:

$$\begin{aligned} p_t(1-(1-\theta)+\alpha(1-\theta)\Omega N) &= (1-\theta)u_t + \alpha(1-\theta)\Omega\sum_{i=1}^{\infty}\eta_i E_{t-i}p_t \\ &+ (1-\theta)\sum_{k=1}^{\infty}\theta^k E_{t-k}\left[p_t + \alpha\left(-\Omega N p_t + \Omega\sum_{i=1}^{\infty}\eta_i E_{t-i}p_t\right) + u_t\right]. \end{aligned} \tag{38}$$

It is also assumed that the cost-push shock term u_t is an AR(1) process and is given by $u_t = \rho u_{t-1} + \varepsilon_t$, where ε_t is the white noise. It can be expressed as an infinite moving average form as $u_t = \sum_{i=0}^{\infty}\rho^i\varepsilon_{t-i}$. If p_t is represented by $p_t = \sum_{i=0}^{\infty}\gamma_i\varepsilon_{t-i}$, equation (38) can be solved by the method of undetermined coefficients. To find p_t, the coefficients γ_i are obtained in terms of the correlation coefficient ρ of the cost-push shock. These serial correlation coefficients γ_k of prices for any k are obtained as:

$$\gamma_k = \frac{\Upsilon_k}{\Lambda - \Psi_k}, \tag{39}$$

[13] The central bank commits to a "targeting rule" in the terminology of Svensson, but implementation of this rule is discretionary.

where $\Upsilon_k = \rho^k \sum_{i=0}^{k} \theta^i$, $\quad \Lambda = (\theta + \alpha(1-\theta)\Omega N)/(1-\theta)$, and

$$\Psi_k = \alpha\Omega \sum_{i=1}^{k} \eta_i + (1 - \alpha\Omega N) \sum_{i=1}^{k} \theta^i + \alpha\Omega \Big(\sum_{i=1}^{k} \eta_i \Big) \Big(\sum_{i=1}^{k} \theta^i \Big).$$

In expression (39), both the numerator and denominator approach to zero when k approaches infinity. The convergence of the terms Υ_k and $\Lambda - \Psi_k$ depends on the values of ρ and θ, respectively. Therefore, an optimal solution for the price path depends on the convergence speed of these terms. If ρ is larger than a critical value for any given θ, then the numerator term Υ_k converges to zero more slowly than the denominator term $\Lambda - \Psi_k$; therefore, the solution to γ_k diverges, and a bounded solution cannot be obtained. A bounded solution requires that ρ is smaller than this critical value, which implies the faster convergence of Υ_k to zero than $\Lambda - \Psi_k$.

When a solution to the price path is found, the output gap can be obtained from the optimality condition represented in equation (36). By expressing the output gap as an infinite MA process, $x_t = \sum_{i=0}^{\infty} \beta_i \varepsilon_{t-i}$, and substituting it into (36), one can obtain:

$$\sum_{i=0}^{\infty} \beta_i \varepsilon_{t-i} = -\Omega N \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i} + \Omega \sum_{j=1}^{\infty} \sum_{i=j}^{\infty} \eta_j \gamma_i \varepsilon_{t-i} \, . \tag{40}$$

The coefficients β_k for any k can be obtained from this expression by the method of undetermined coefficients as:

$$\beta_k = -\Omega N \gamma_k + \Omega \gamma_k \sum_{i=1}^{k} \eta_i \; . \tag{41}$$

Optimal nominal interest rate i_t can be found from the IS equation:

$$i_t = \sigma(E_t x_{t+1} - x_t) + E_t p_{t+1} - p_t \; ,$$

by using the solutions to the output gap and prices. If i_t is expressed as $i_t = \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i}$, then the serial correlation coefficients ϕ_k for any k can be found by:

$$\phi_k = \sigma(\beta_{k+1} - \beta_k) + \gamma_{k+1} - \gamma_k \; . \tag{42}$$

Having the output gap and prices, the welfare loss of the model under unconstrained optimal policy can be calculated from equation (35) as:

$$Welfare\ Loss = -E_0 \sum_{t=0}^{\infty} \beta^t \left(\lambda x_t^2 + \sum_{i=1}^{\infty} \eta_i (p_t - E_{t-i} p_t)^2 \right) ,$$
$$= -\sum_{t=0}^{\infty} \beta^t \left[\lambda x_t^2 + \sum_{i=1}^{\infty} \eta_i \left(\sum_{k=0}^{i-1} \gamma_k \varepsilon_{t-k} \right)^2 \right]. \tag{43}$$

5.2. Price-level Targeting with a Cost-Push Shock

In this subsection, the optimal policy for a cost-push shock is investigated under price-level targeting regime. In this regime, monetary authority commits to keep the price level constant and takes actions to return the price level to its steady state value after a shock occurs. Since it is assumed that the central bank has a credible commitment to a targeting regime, it can manipulate the private-sector expectations. Therefore, the expectations about the price level under the price-level targeting regime satisfy:[14]

$$E_{t-i} p_t = 0 . \tag{44}$$

Under the price-level targeting, the optimality condition (36) reduces to:

$$x_t = -\Omega \sum_{i=1}^{\infty} \eta_i \, p_t = -\Omega N p_t . \tag{45}$$

So the price adjustment equation (22) becomes:

$$p_t = \frac{(1-\theta)}{\theta + (1-\theta)\alpha\Omega N} \left[u_t + \sum_{k=1}^{\infty} \theta^k \sum_{i=k}^{\infty} \rho^i \varepsilon_{t-i} \right]. \tag{46}$$

Let $p_t = \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i}$, then the coefficients γ_k for any k can be found by the method of undetermined coefficients from equation (46) and are given by:

$$\gamma_k = \frac{(1-\theta)}{\theta + (1-\theta)\alpha\Omega N} \, \rho^k \sum_{i=0}^{k} \theta^i . \tag{47}$$

The output gap can be obtained from equation (45) as:

$$x_t = \frac{-\Omega N(1-\theta)}{\theta + (1-\theta)\alpha\Omega N} \left[u_t + \sum_{k=1}^{\infty} \theta^k \sum_{i=k}^{\infty} \rho^i \varepsilon_{t-i} \right]. \tag{48}$$

[14]It is assumed here that the target price level is normalized to be 1.

If it is assumed that $x_t = \sum_{i=0}^{\infty} \beta_i \varepsilon_{t-i}$, then the coefficients β_k for any k can be found as:

$$\beta_k = \frac{-\Omega N(1-\theta)}{\theta + (1-\theta)\alpha\Omega N} \; \rho^k \sum_{i=0}^{k} \theta^k . \tag{49}$$

The nominal interest rate i_t can be obtained from the IS equation. If $i_t = \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i}$, then the coefficients ϕ_k for any k are given by:

$$\phi_k = \sigma(\beta_{k+1} - \beta_k) + \gamma_{k+1} - \gamma_k \; . \tag{50}$$

The welfare loss can be calculated after obtaining the output gap and prices. Therefore, the loss function given in equation (35) takes the following form under the price-level targeting

$$Welfare\ Loss = -\sum_{t=0}^{\infty} \beta^t \left(\lambda x_t^2 + \sum_{i=1}^{\infty} \eta_i \, p_t^2 \right) = -\sum_{t=0}^{\infty} \beta^t (\lambda x_t^2 + N p_t^2) \, . \tag{51}$$

5.3. Inflation Targeting with a Cost-Push Shock

Optimal policy under inflation targeting regime is investigated in this subsection. In this regime, the monetary authority commits to maintain a stable inflation level around zero. Any base drift in price level is allowed. Because of the central bank's credible commitment to the inflation target, the private-sector expectations about inflation are preset, and do not change when a temporary shock hits the economy. Therefore, expectations about the inflation under inflation targeting regime satisfy:

$$E_{t-i}\pi_t = 0 \; . \tag{52}$$

When the variable of interest is inflation, the intertemporal loss function given in equation (35) can be rewritten in terms of inflation as:

$$-\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda x_t^2 + \; \sum_{i=1}^{\infty} \eta_i \left(\sum_{l=0}^{i-1} (\pi_{t-l} - E_{t-i}\pi_{t-l}) \right)^2 \right] . \tag{53}$$

Optimality condition (36) can be written in terms of inflation by using equation (31) as:

$$x_t = -\Omega \sum_{i=1}^{\infty} \eta_i \sum_{k=0}^{i-1} (\pi_{t-k} - E_{t-i}\pi_{t-k}) \; .$$

Under inflation targeting, this expression reduces to:

$$x_t = -\Omega \sum_{i=1}^{\infty} \eta_i \sum_{k=0}^{i-1} \pi_{t-k} \; , \tag{54}$$

and the Phillips curve in equation (23) can be rewritten as:

$$\pi_t = \phi\alpha\left(-\Omega\sum_{i=1}^{\infty}\eta_i\sum_{k=0}^{i-1}\pi_{t-k}\right)+\phi\, u_t+(1-\theta)\sum_{k=0}^{\infty}\theta^k E_{t-1-k}\left(\Delta u_t-\alpha\Omega\sum_{k=1}^{\infty}\eta_k\,(\pi_t-\pi_{t-k}\right). \quad (55)$$

Let $\pi_t = \sum_{i=0}^{\infty}\gamma_i\varepsilon_{t-i}$, then equation (55) can be solved to obtain the inflation path by finding the coefficients γ_k for any k. These coefficients are obtained from the following expression:

$$\gamma_k(1+\alpha\phi\Omega N) = -\alpha\phi\Omega N\sum_{i=0}^{k-1}\gamma_i + \alpha\phi\Omega\sum_{i=0}^{k-1}\gamma_i\sum_{j=1}^{k-i}\eta_j + (1-\theta)\alpha\Omega N\sum_{i=0}^{k-1}\gamma_i\theta^{k-1-i}$$
$$-(1-\theta)\alpha\Omega\theta\sum_{i=0}^{k-2}\gamma_i\theta^{k-2-i}\sum_{j=1}^{k-1-i}\eta_j + \phi\rho^k + (\rho-1)\rho^{k-1}\sum_{i=0}^{k-1}\theta^i. \quad (56)$$

Once the inflation is obtained, the output gap can be found from equation (54). If $x_t = \sum_{i=0}^{\infty}\beta_i\varepsilon_{t-i}$, then the serial correlation coefficients β_k for any k are given by:

$$\beta_k = -\Omega\sum_{i=0}^{k}\gamma_k\left(N-\sum_{j=1}^{k-i}\eta_j\right). \quad (57)$$

The nominal interest rate i_t can be obtained from the IS equation, and if $i_t = \sum_{i=0}^{\infty}\phi_i\varepsilon_{t-i}$, then the coefficients ϕ_k for any k are given by:

$$\phi_k = \sigma(\beta_{k+1}-\beta_k)+\gamma_{k+1}\ . \quad (58)$$

Having inflation and the output gap, the welfare loss under the inflation targeting can be calculated from (53) as:

$$Welfare\ Loss = -\sum_{t=0}^{\infty}\beta^t\left[\lambda x_t^2 + \sum_{i=1}^{\infty}\eta_i\left(\sum_{k=0}^{i-1}\pi_{t-k}\right)^2\right]. \quad (59)$$

6. Optimal Policy for a Demand Shock

Disturbances in the demand side of the economy are represented by a random process ν_t in IS equation (34). These disturbances basically come from technological shocks as shown in equation (19). The random technological process in the economy is assumed to be $a_t = \rho_a a_{t-1} + \varepsilon_t$, and can be expressed as $a_t = \sum_{i=0}^{\infty}\rho_a^i\varepsilon_{t-i}$. If a demand shock occurs when the economy is in the steady state, a central bank can stabilize the economy under any regime by trying to neutralize the effect of the shock by changing the nominal interest rates.

When a demand shock occurs, it first affects the potential output as can be seen from equation (16); the output gap changes in opposite direction due to this change in potential output. In our framework, dynamic implications of a demand shock can be found by finding price paths for the above changes in the output gap by using the sticky information Phillips curve. The optimal output can be obtained by using the optimality condition in (36). After having prices and the output gap, optimal path for nominal interest rate can be found through the IS equation.

6.1. Unconstrained Policy with a Demand Shock

This is the same policy regime described above for the cost-push shock, except that a demand shock occurs when the economy is in the steady state. When a shock occurs, it changes the potential output given by equation (16), then the change in the output gap is given by:

$$x_t = -\frac{1+\varphi}{\sigma+\varphi}\, a_t = -\frac{1+\varphi}{\sigma+\varphi} \sum_{i=0}^{\infty} \rho_a^i \varepsilon_{t-i}\,. \tag{60}$$

When there is no supply shock, the sticky information Phillips curve given by equation (22) can be written as:

$$p_t = \frac{\alpha(1-\theta)}{\theta}\, x_t + \frac{1-\theta}{\theta} \sum_{k=1}^{\infty} \theta^k\, E_{t-k} p_t + \frac{\alpha(1-\theta)}{\theta} \sum_{k=1}^{\infty} \theta^k\, E_{t-k} x_t\,. \tag{61}$$

If the infinite moving average representation of prices, which is $p_t = \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i}$, and the output gap expression given in equation (60) are used, equation (61) can be rewritten as:

$$\sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i} = Z \sum_{i=0}^{\infty} \rho_a^i \varepsilon_{t-i} + \frac{1-\theta}{\theta} \sum_{k=1}^{\infty} \theta^k \sum_{i=k}^{\infty} \gamma_i \varepsilon_{t-i} + Z \sum_{k=1}^{\infty} \theta^k \sum_{i=k}^{\infty} \rho_a^i \varepsilon_{t-i}\,, \tag{62}$$

where $Z = -\frac{\alpha(1-\theta)}{\theta}\frac{1+\varphi}{\sigma+\varphi}$. Therefore, the serial correlation coefficients γ_k of prices for any k can be found by the method of undetermined coefficients as:

$$\gamma_k = Z \left(\frac{\rho_a}{\theta}\right)^k \sum_{i=0}^{k} \theta^i. \tag{63}$$

This expression implies a bounded solution only if $\rho_a < \theta$, otherwise the solution diverges. If $x_t = \sum_{i=0}^{\infty} \beta_i \varepsilon_{t-i}$, then the serial correlation coefficients β_k can be found by using the optimality condition in (36) as:

$$\beta_k = -\Omega N \gamma_k + \Omega \gamma_k \sum_{i=1}^{k} \eta_i \ . \tag{64}$$

The optimal nominal interest rate i_t can be found from the IS equation:

$$i_t = \sigma(E_t x_{t+1} - x_t) + E_t p_{t+1} - p_t + \sigma\, \nu_t \ . \tag{65}$$

The disturbance term ν_t in this IS equation results from the technological process a_t, and it is obtained from equation (19) as:

$$\nu_t = -Q\, a_t = -Q \sum_{i=0}^{\infty} \rho_a^i \varepsilon_{t-i} \ , \tag{66}$$

where $Q = \dfrac{(1-\rho_a)(1+\varphi)}{\sigma + \varphi}$.

If i_t is expressed as $i_t = \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i}$, then the serial correlation coefficients ϕ_k for any k are given by:

$$\phi_k = \sigma(\beta_{k+1} - \beta_k) + \gamma_{k+1} - \gamma_k - \sigma Q \rho_a^k \ . \tag{67}$$

The welfare loss of the model under unconstrained optimal policy for a demand shock can also be calculated from equation (35) after having prices and the output gap. The expression for the welfare loss is the same expression for the case of cost-push shock, and it is given by equation (43).

6.2. Price-level Targeting with a Demand Shock

This is also the same regime described above for the cost-push shock, where monetary authority commits to keep the price level unchanged. Therefore, in such a regime, the private-sector expectations take the form of $E_{t-i} p_t = 0$, and the same optimality condition given in equation (45) for the cost-push shock is still valid here.

In this case the price adjustment equation (22) becomes:

$$p_t = (1-\theta)(p_t + \alpha x_t) + \alpha(1-\theta) \sum_{k=1}^{\infty} \theta^k\, E_{t-k} x_t \,. \tag{68}$$

If the output gap expression in equation (60) and $p_t = \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i}$ are used, the above price equation in (68) can be written as:

$$\sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i} = Z \sum_{i=0}^{\infty} \rho_a^i \varepsilon_{t-i} + Z \sum_{k=1}^{\infty} \theta^k \sum_{i=k}^{\infty} \rho_a^i \varepsilon_{t-i} \,. \tag{69}$$

From this equation, the coefficients γ_k for any k can be found by the method of undetermined coefficients as:

$$\gamma_k = Z \, \rho_a^k \sum_{i=0}^{k} \theta^i \,. \tag{70}$$

After having coefficients γ_k, the serial correlation coefficients of the output gap and nominal interest rate, β_k and ϕ_k respectively, can be obtained as:

$$\beta_k = -\Omega \, N \, \gamma_k \,, \tag{71}$$

$$\phi_k = \sigma(\beta_{k+1} - \beta_k) + \gamma_{k+1} - \gamma_k - \sigma Q \rho_a^k \,. \tag{72}$$

The loss function takes the same form as the one for the case of the cost-push shock, and is given by equation (51).

6.3. Inflation Targeting with a Demand Shock

In this regime, expectations about the inflation satisfy $E_{t-i} = 0$ as explained above for the cost-push shock case, and the optimality condition is the same given in equation (54).

Under this monetary policy regime, the sticky information Phillips curve in equation (23) takes the form of

$$\pi_t = \phi\alpha \, x_t + (1-\theta) \sum_{k=0}^{\infty} \theta^k E_{t-1-k} \Delta x_t \,, \tag{73}$$

and Δx_t can be calculated by using equation (60) as:

$$\Delta x_t = -\frac{1+\varphi}{\sigma+\varphi} (a_t - a_{t-1}) = -\frac{1+\varphi}{\sigma+\varphi} \sum_{i=0}^{\infty} \rho_a^i (\varepsilon_{t-i} - \varepsilon_{t-1-i}) \,. \tag{74}$$

If inflation is expressed as $\pi_t = \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i}$, equation (73) can be rewritten by using the expressions for x_t and Δx_t in equations (60) and (74), respectively, as:

$$\sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i} = Z \sum_{i=0}^{\infty} \rho_a^i \varepsilon_{t-i} - Q \, Z \sum_{k=0}^{\infty} (\theta \rho_a)^k \varepsilon_{t-1-k} + Q \, Z \sum_{k=0}^{\infty} \theta^k \sum_{i=k+1}^{\infty} \rho_a^i (\varepsilon_{t-i} - \varepsilon_{t-1-i}) \,. \tag{75}$$

This equation can be solved to obtain a solution to inflation by finding the coefficients γ_k for any k, and this solution is given by:

$$\gamma_k = Z\,\rho_a^k - Q\,Z(\rho_a^{k-1} - \rho_a^k)\sum_{i=0}^{k-1}\theta^i\,. \tag{76}$$

Once the coefficients γ_k of inflation are obtained, the serial correlation coefficients of the output gap and nominal interest rate, β_k and ϕ_k respectively, can be obtained as:

$$\beta_k = -\Omega\sum_{i=0}^{k}\gamma_k\left(N - \sum_{j=1}^{k-i}\eta_j\right), \tag{77}$$

$$\phi_k = \sigma(\beta_{k+1} - \beta_k) + \gamma_{k+1} - \sigma Q\rho_a^k\,. \tag{78}$$

The welfare loss function can be calculated after having the output gap and inflation. Again, it takes the same form as the one for the case of the cost-push shock, and is given by equation (59).

7. Simulations and Results

Model parameters are calibrated as elasticity of substitution among goods, $\varepsilon = 6$; risk aversion factor, which is also the inverse of the intertemporal elasticity of substitution of consumption, $\sigma = 1$; marginal disutility of labor, $\varphi = 1.25$; and discount factor $\beta = 0.99$. In literature, especially there is no consensus on the value of parameter σ and there are different estimations on this parameter.[15] My calibration of the parameters σ and φ is the same as in Chari et al. (2000).[16]

7.1. Results for a Cost-push shock

The dynamic paths of the output gap, price level, inflation and nominal interest rate are given for $\rho = 0.8$ and $\rho = 0.4$ when $\theta = 0.85$ in Figure 1. The figure shows that price level converges back to its target level under unconstrained policy and price-level targeting

[15] See Arslan (2005) for a review of different estimation of σ.

[16] In Ball, Mankiw and Reis (2005), the parameters α given in equation (17) is taken as 0.1; with my calibration this parameter is around 0.25 in my study. They also make their analysis just for one value of parameter θ, which is 0.25.

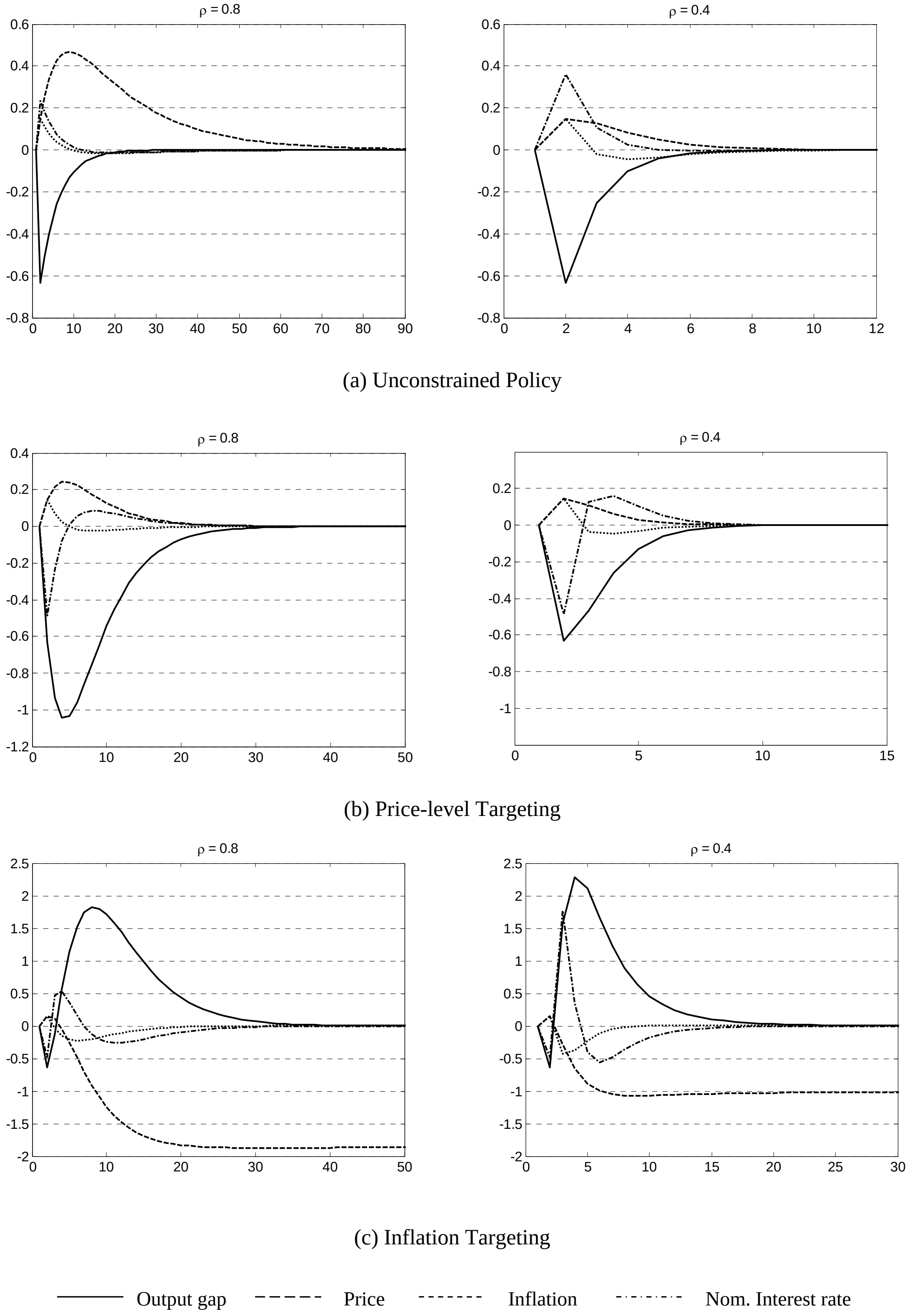


Figure 1: Optimal policy impulse responses to a unit cost-push shock when $\theta = 0.85$.

regimes, while there is a base drift in price level, and inflation returns to its target level under inflation targeting. The output gap decreases while price level and inflation increase at the beginning, and then they all return to their steady-state values. Inflation overshoots in all policy regimes and the sizes of these overshoots increase when the persistence of the shock decreases; however, output gap overshoots only in inflation targeting. It can also be seen that fluctuations in the nominal interest rate increase when the shock becomes less persistent. This figure also illustrates that the persistence of the variables depends on the persistence of the shock term. However, these impulse responses do not say much about which policy is better, so the welfare implications of these alternative policy regimes need to be investigated.

The welfare implications of the policy regimes are given in Figure 2. The panels show the sensitivity of welfare losses to the persistence of the shock for different values of θ in alternative policy regimes. Since there are no bounded solution for some values of θ and ρ, loss functions diverge for these cases as shown in the figure. Figure 2 illustrates that losses are similar in unconstrained policy and price-level targeting regimes, especially when prices are stickier. The loss in inflation targeting regime behaves similar to them when θ values are low, however it is steady and somewhat increasing function of ρ until it becomes very close to one for higher θ values. These graphs illustrate that the best policy depends on the values of the parameters θ and ρ, which are the degree of price stickiness and the persistence of the cost-push shock, respectively. Inflation targeting is the best policy when prices become more flexible, that is when θ is small. However, when prices become stickier, that is for large values of θ, best policy depends on the persistence of the shock, and unconstrained policy and price-level targeting dominate inflation targeting when the shock is not very persistent, that is ρ is less than some threshold value. Therefore, it can be concluded that inflation targeting is the best policy and dominates the other regimes in the sense that it results in smaller welfare loss when prices are more flexible or the shock is persistent. However, price-level targeting dominates inflation targeting when prices become stickier and shock is not very persistent.

In the framework here, a central bank tries to stabilize the economy with interest-rate instrument, and the loss function involves the output gap and price level (or inflation) terms without giving any attention to the variability of nominal interest rate. However, some

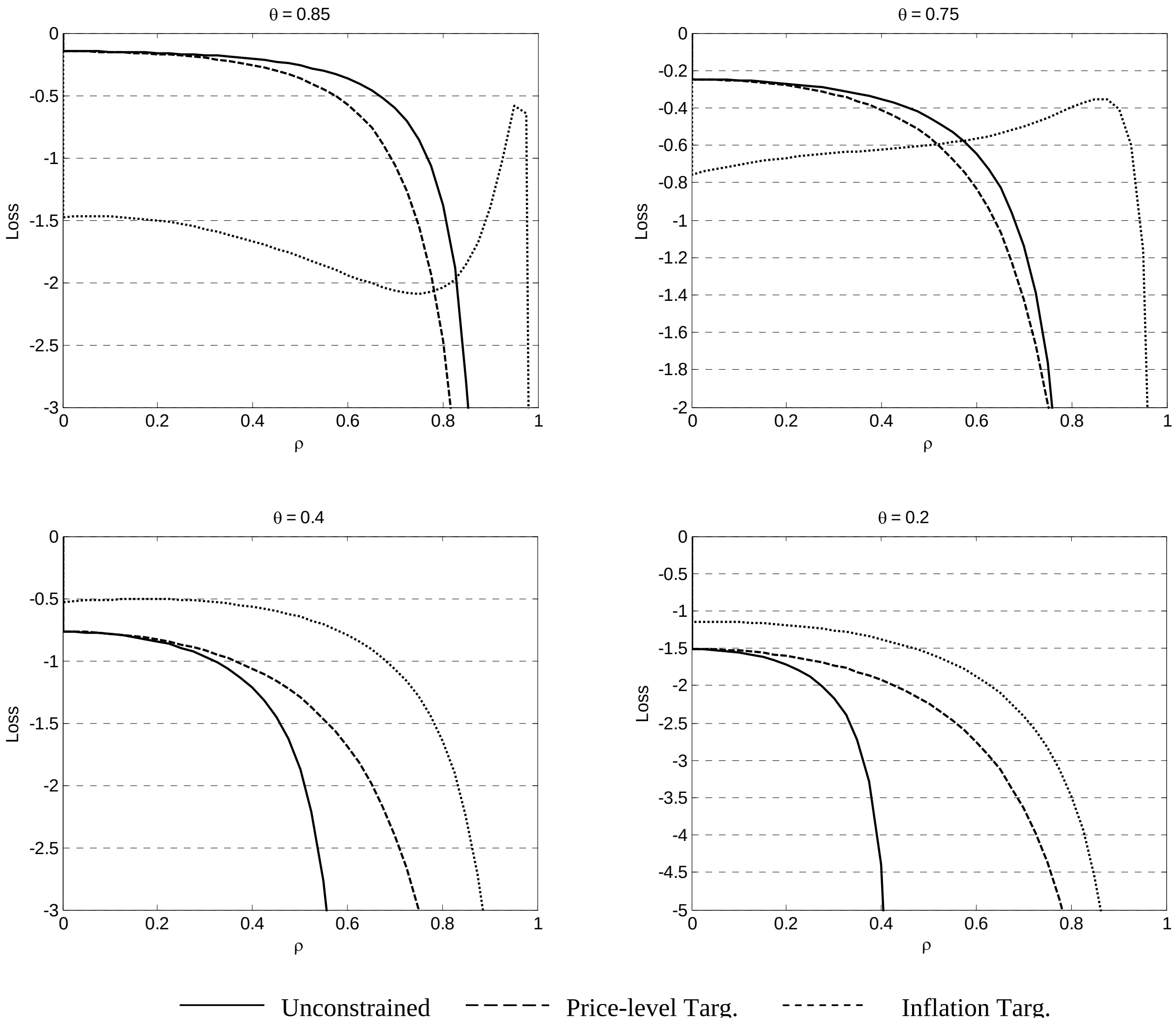


Figure 2: Sensitivity of welfare loss to the persistence of the cost-push shock.

empirical and theoretical literature take the interest rate as another variable to be stabilized. In such a case, either policy rule is updated by taking the interest rate smoothing into account, or a term is added to the loss function to penalize the deviations of interest rate from the target level.[17] Therefore, it makes sense also to investigate the variability of nominal interest rate in alternative policy regimes. Figure 1 shows the optimal nominal interest rate responses under these regimes. One can see from these responses that nominal interest rate volatility is largest in inflation targeting, while smallest in unconstrained policy regimes, and somewhere

[17] See, for example, Woodford (2003, Chp. 6).

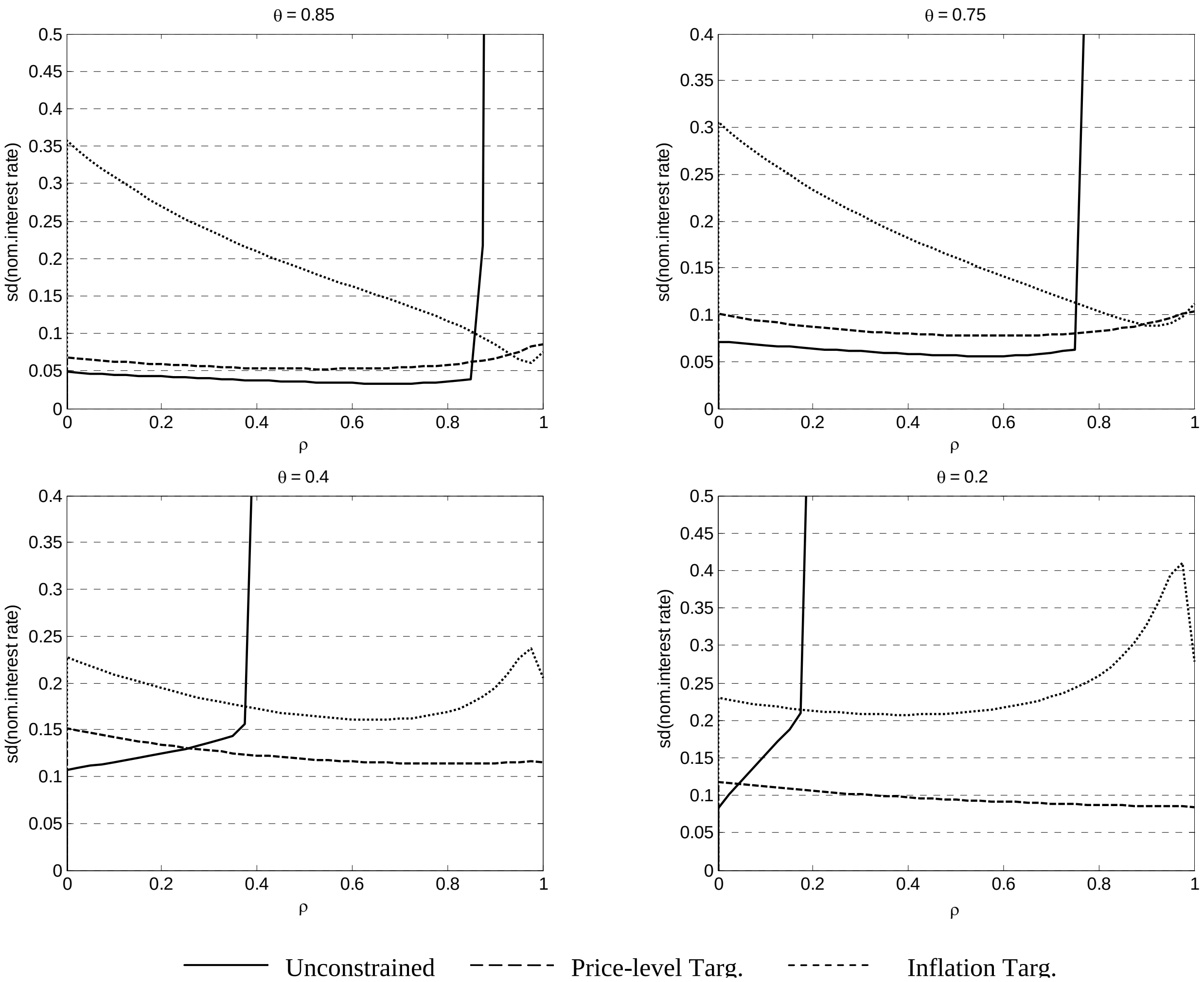


Figure 3: Standard deviations of optimal nominal interest rates for a cost-push shock.

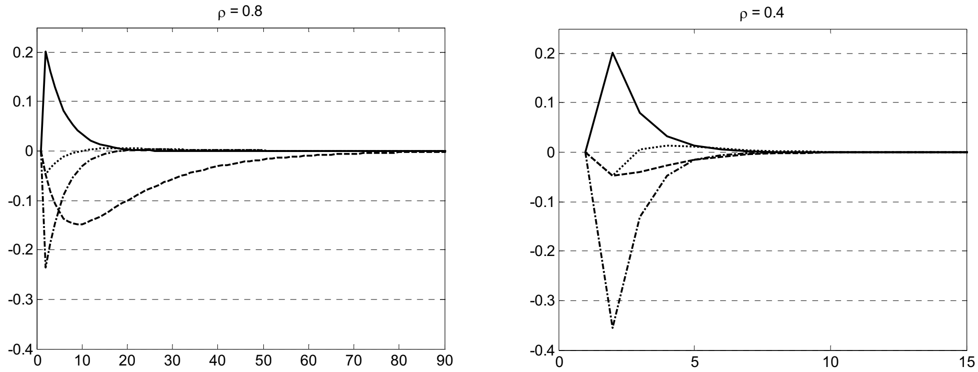
between these two in price-level targeting.

However, volatility of nominal interest rates in alternative policy regimes might be better illustrated when the standard deviations of optimal nominal interest rate across the policy regimes are drawn for different levels of price stickiness while ρ changes as in Figure 3. Since there are no bounded solutions for unconstrained policy regime for some parameter values, standard deviations of nominal interest rate diverge in these cases. As shown in top two panels of the figure, when prices are sticky, the lowest standard deviations are implied by unconstrained policy and price-level targeting regimes until the shock becomes very persistent. The standard deviation in inflation targeting regime decreases and approaches the others when ρ increases. However, when prices are not sticky enough as shown in the bottom two panels of the figure, price-level targeting has the lowest standard deviations. Therefore, if the volatility of the interest rate is included into the analysis, one may conclude that inflation targeting is the best policy in the sense that it results in smaller welfare loss and variability of nominal interest rate only when prices are sticky enough and the persistence of the cost-push shock is large and close to one. However, when prices are more flexible, price-level targeting might be preferable to the inflation targeting only if the relative weight of the stability of nominal interest rate is large.

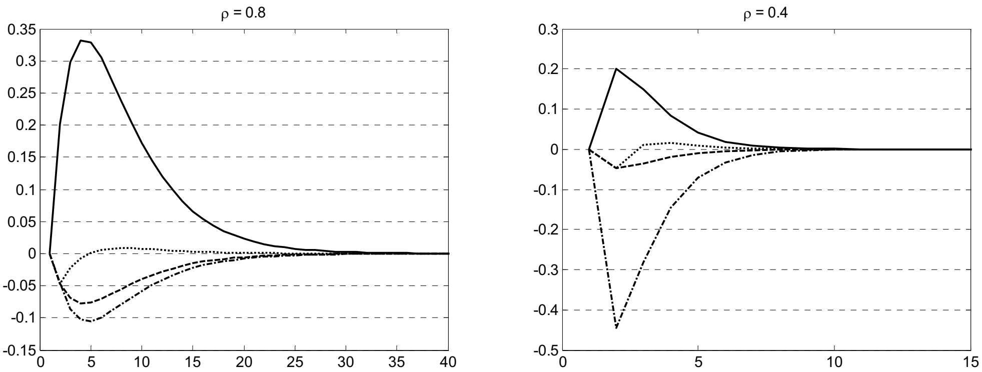
7.2. Results for a Demand shock

The dynamic paths of the output gap, price level, inflation and nominal interest rate are shown in Figure 4 when there is a positive demand shock to the economy. The figure shows that there is a base drift in price level under inflation targeting regime. The output gap increases and price level or inflation decreases after the shock, and then they all return to their steady-state values. Similar to the cost-push shock case, fluctuations in nominal interest rate increase when the shock becomes less persistent. Again, to be able to choose the best policy for a demand shock, the welfare implications of the alternative policy regimes need to be compared. The welfare losses under policy regimes are given in Figure 5. This figure shows that inflation targeting regime produces less welfare loss than the other two regimes for all values of ρ.

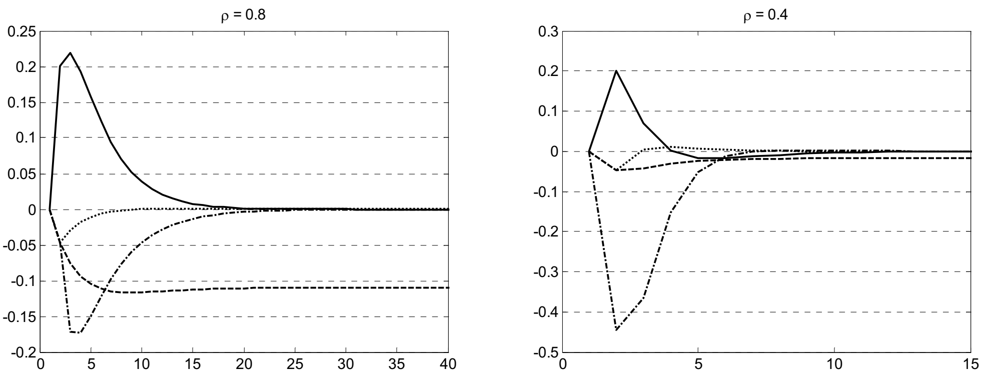
Similar to the case of the cost-push shock, the volatility of nominal interest rate in alternative policy regimes needs be investigated for a unit demand shock. Although Figure 4 illustrates the impulse responses of nominal interest rate, relative volatilities in alternative policy regimes might be better illustrated if the standard deviations of optimal nominal interest rate across the policy regimes are plotted for different levels of price stickiness while ρ changes as in Figure 6. Since there are no bounded solutions in unconstrained policy regime for some parameter values, the standard deviations of nominal interest rate diverge in these cases. As shown in the top two panels of the figure, when prices are sticky, the standard deviations implied by inflation targeting and price level targeting regimes are very small and close to each other and decrease when ρ increases. However, when prices are not sticky



(a) Unconstrained Policy



(b) Price-level Targeting



(c) Inflation Targeting

Output gap   Price   Inflation   Nom. Interest rate

Figure 4: Optimal policy impulse responses to a unit demand shock when $\theta = 0.85$.

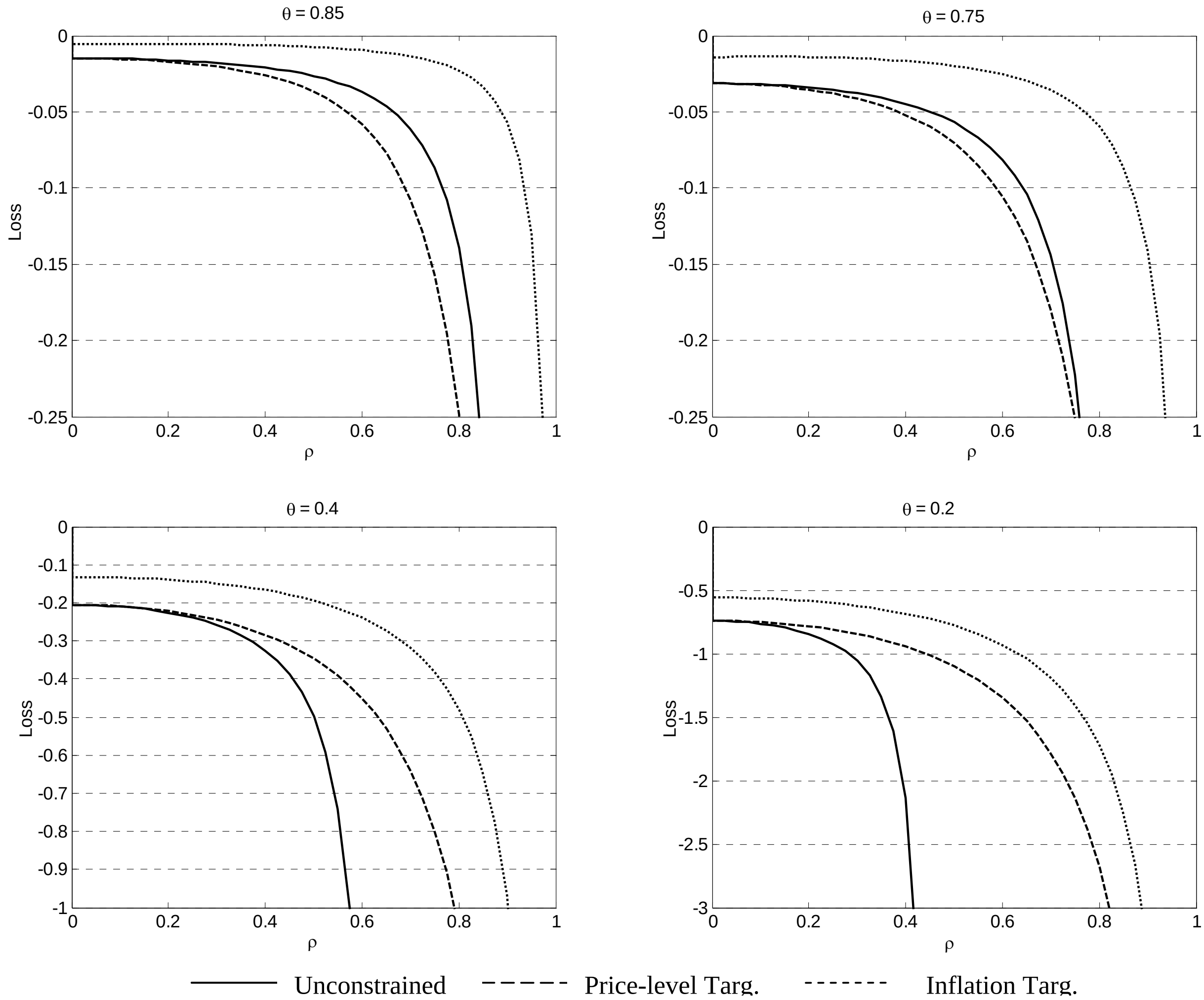


Figure 5: Sensitivity of welfare loss to the persistence of the demand shock.

enough as shown in the bottom two panels of the figure, price-level targeting has the lowest standard deviations.

Therefore, if the volatility of the interest rate is included into the analysis, it may be concluded that inflation targeting is the best policy in the sense that it results in smaller welfare loss and reasonable variability of nominal interest rate when prices are sticky enough. However, when the prices are not very sticky, inflation targeting might not be the best policy depending on the relative importance of the variability of nominal interest rate with respect to the variabilities of the output gap and prices. Because, in this case, variability of interest rate in price-level targeting is less than the one in inflation targeting, so price-level targeting

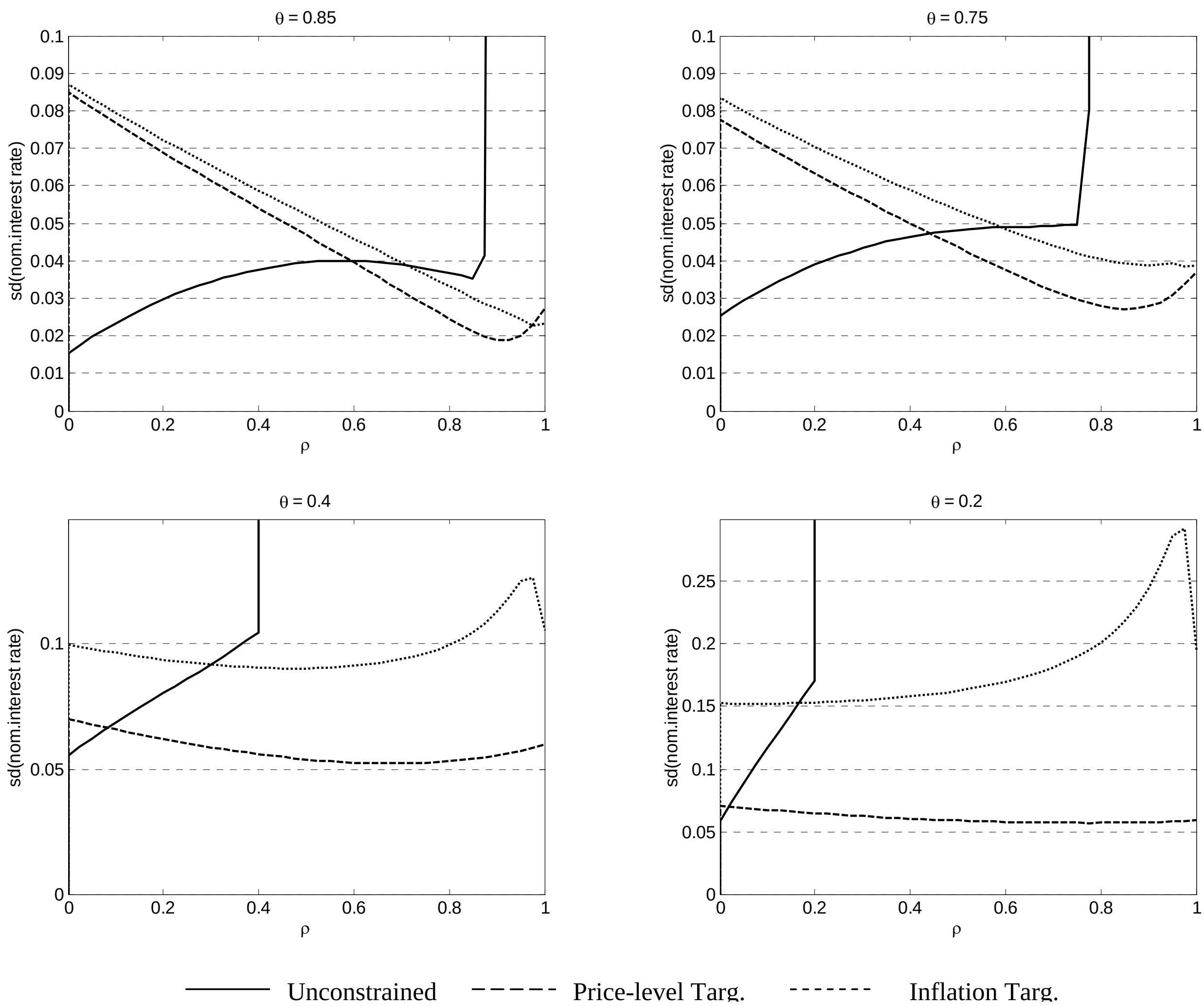


Figure 6: Standard deviations of optimal nominal interest rates for a demand shock.

might be preferable over inflation targeting regime.

8. Conclusions

In this study, optimal monetary policy for a closed economy within a New Keynesian macroeconomic model with the sticky information model of price adjustment is investigated. Specifically, the welfare implications of three alternative monetary policy regimes are compared for a unit cost-push shock and a demand shock. The first one is a discretionary policy regime in the sense that a central bank optimizes each period without any commitment or target to manipulate the private-sector expectation, and it is called unconstrained policy regime. The second policy regime is price-level targeting in which the central bank still optimizes each period, but this time it commits to a targeting rule that puts a specific target level for prices, so it is able to manipulate the private-sector expectations. The third one is inflation targeting in which central bank has a specific target level for the inflation rate rather than prices, so a base drift in price level is allowed. Optimal monetary policy with the sticky information Phillips curve is also studied by Ball, Mankiw and Reis (2005). This study differs from their study in solving the model for optimal policy and direct comparison of the welfare losses under the three specific policy regimes. Also, here an IS equation for the demand side is used, and therefore the nominal interest rate is taken as the policy instrument

The analysis of this study shows that inflation targeting generally is the best policy in the sense that it results in smaller welfare loss and volatility of nominal interest rate, which is in contrast with Ball, Mankiw and Reis's (2005) result. They find price-level targeting as the optimal policy, while inflation targeting is suboptimal. When the volatility of nominal interest rate is not taken into consideration, the results for a cost-push shock show that optimal policy depends on the degree of price stickiness and the persistence of the shocks. Inflation targeting is the optimal policy if prices are flexible enough or the shock is persistent enough. For a demand shock, inflation targeting emerges as the best policy for all values of the price stickiness and the shock's persistence. When the volatility of nominal interest rate is taken into consideration, the results indicate that again inflation targeting is generally the best policy for both the cost-push shocks and demand shocks, in the sense that it results in smaller welfare loss and volatility of nominal interest rate. However, price-level targeting might be preferable to inflation targeting only if prices are more flexible and the relative weight for the volatility of nominal interest rate is large enough.

References

[1] Arslan, M.M., 2005. Monetary Policy in a New Keynesian Framework with Stikcy Information Model of Price Adjustment. Mimeo, University of Houston.

[2] Ball, L., Mankiw, N.G., Reis, R., 2005. Monetary Policy for Inattentive Economies. Journal of Monetary Economics 52 (4), 703-725.

[3] Bernanke, B.S., Mihov, I., 1998. Measuring Monetary Policy. Quarterly Journal of Economics 113 (3), 869-902.

[4] Blanchard, O.J., Kiyotaki, N., 1987. Monopolistic Competition and the Effects of Aggregate Demand. American Economic Review 77 (4), 647-666.

[5] Calvo, G., 1983. Staggered Prices in a Utility Maximizing Framework. Journal of Monetary Economics 12 (3), 383-398.

[6] Chari, V.V., Kehoe, P.J., McGrattan, E.R., 2000. Stikcy Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem? Econometrica 68, 1151-1179.

[7] Clarida, R., Gali, J., Gertler, M., 1999. The Science of Monetary Policy: A New Keynesian Perspective. Journal of Economic Literature 37 (4), 1661-1707.

[8] Clarida, R., Gali, J., Gertler, M., 2001. Optimal Monetary Policy in Open versus Closed Economies: An Integrated Approach. American Economic Review 91 (2), 253-257.

[9] Clarida, R., Gali, J., Gertler, M., 2002. A Simple Framework for International Monetary Policy Analysis. NBER Working Paper, 8870.

[10] Dixit, A.K., Stiglitz, J.E., 1977. Monopolistic Competition and Optimum Product Diversity. American Economic Review 67 (3), 297-308.

[11] Gali, J., Monacelli, T., 2002. Monetary Policy and Exchange Rate Volatility in a Small Open Economy. Mimeo, Universitat Pompeu Fabra, Barcelona.

[12] Mankiw, N.G., Reis, R., 2002. Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve. Quarterly Journal of Economics 117 (4), 1295-1328.

[13] McCallum, B.T., Nelson, E., 1999. An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis. Journal of Money, Credit and Banking 31, 296-316.

[14] Rotemberg, J., Woodford, M., 1997. An Optimization Based Econometric Framework for the Evaluation of Monetary Policy. NBER Macroeconomics Annual 12, 297-346.

[15] Rotemberg, J., Woodford, M., 1999. Interest Rate Rules in an Estimated Sticky Price Model. NBER Working Paper, 6618.

[16] Rudebusch, G.D., Svensson, L.E., 1999. Eurosytem Monetary: Lessons from U.S. Data. NBER Working Paper, 7179.

[17] Svensson, L.E., 1997. Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets. European Economic Review 41, 1111-1146.

[18] Svensson, L.E., 2002. Inflation Targeting: Should It Be Modeled as an Instrument Rule or a Targeting Rule? NBER Working Paper, 8925.

[19] Svensson, L.E., 2003. What is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules. Journal of Economic Literature 41 (2), 426-478.

[20] Svensson, L.E., Woodford, M., 2003. Implementing Optimal Policy through Inflation-Forecast Targeting. In: B. S. Bernanke and M. Woodford (Ed.), Inflation Targeting. University of Chicago, Chicago.

[21] Woodford, M., 1999. Optimal Monetary Policy Inertia. NBER Working Paper, 7261.

[22] Woodford, M., 2001. Inflation Stabilization and Welfare. NBER Working Paper, 8071.

[23] Woodford, M., 2003. Interest and Prices. Princeton University Press, Princeton, NJ.

Appendix

A. Quadratic Approximation to Welfare

The welfare is defined as the total utility of households in the model and given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t \ , \tag{A.1}$$

where U_t is the period utility function for any households given in equation (1). This can be approximated around the equilibrium point where there are no real disturbances. Under the equilibrium with no real disturbances, real marginal cost is given by:[18]

$$\gamma_t = \frac{2\mu^w - 1}{\mu\mu^w} \equiv 1 - \Phi_y \ , \tag{A.2}$$

where $\Phi_y = \frac{\mu\mu^w - 2\mu^w + 1}{\mu\mu^w}$, μ is the constant markup resulting from firms' market power, and μ^w is the wage markup with no shocks resulting from households' labor market power. So in equation (A.2), I can interpret that Φ_y summarizes the all distortions in the equilibrium as a results of frictions in the markets. Since real marginal cost γ_t equals to one for efficient level of output Y^*, Φ_y can be taken very close to zero to obtain an approximation around the equilibrium, which is very close to efficient level. Such a situation can be obtained if it is assumed $\mu = \mu^w = 1$, then the equilibrium point from equation (16) will be:

$$\bar{y}_t = \frac{1 + \varphi}{\sigma + \varphi} \bar{a}_t \ . \tag{A.3}$$

Under equilibrium condition, and given the aggregate production function, the period utility function can be expressed as:

$$U_t = \frac{e^{(1-\sigma)y_t}}{1-\sigma} - \int_0^1 \frac{e^{(1+\varphi)(y_{jt} - a_t)}}{1+\varphi} dj \ . \tag{A.4}$$

The quadratic Taylor-series approximation of the first term on the right around the equilibrium value $\bar{y}_t$ is given by:

$$\begin{aligned} \frac{e^{(1-\sigma)y_t}}{1-\sigma} &= \frac{e^{(1-\sigma)\bar{y}_t}}{1-\sigma} + e^{(1-\sigma)\bar{y}_t}(y_t - \bar{y}_t) + \frac{1}{2}(1-\sigma)e^{(1-\sigma)\bar{y}_t}(y_t - \bar{y}_t)^2 \\ &= \frac{e^{(1-\sigma)\bar{y}_t}}{1-\sigma} + e^{(1-\sigma)\bar{y}_t}\left[(y_t - \bar{y}_t) + \frac{1}{2}(1-\sigma)(y_t - \bar{y}_t)^2\right]. \end{aligned} \tag{A.5}$$

[18] See equation (13) in the text.

The second term can be approximated as:

$$\int_0^1 \frac{e^{1+\varphi)(y_{jt}-a_t)}}{1+\varphi}\,dj = \int_0^1 \Bigg[\frac{e^{1+\varphi)(\bar{y}_t-\bar{a}_t)}}{1+\varphi} + e^{(1+\varphi)(\bar{y}_t-\bar{a}_t)}(y_{jt}-\bar{y}_t) - e^{(1+\varphi)(\bar{y}_t-\bar{a}_t)}(a_t-\bar{a}_t)$$
$$+\frac{1}{2}e^{(1+\varphi)(\bar{y}_t-\bar{a}_t)}(1+\varphi)(y_{jt}-\bar{y}_t)^2 + \frac{1}{2}e^{(1+\varphi)(\bar{y}_t-\bar{a}_t)}(a_t-\bar{a}_t)^2$$
$$- e^{(1+\varphi)(\bar{y}_t-\bar{a}_t)}(1+\varphi)(y_{jt}-\bar{y}_t)(a_t-\bar{a}_t)\Bigg]\,dj\ . \tag{A.6}$$

If $\hat{y}_t = y_t - \bar{y}_t,\ \hat{a}_t = a_t - \bar{a}_t,\ \hat{y}_{jt} = y_{jt} - \bar{y}_{jt}$, and $t.i.p.$ is the terms independent of policy, the period utility in (A.4) can be written as:

$$U_t = e^{(1-\sigma)\bar{y}_t}\big(\hat{y}_t + \frac{1}{2}(1-\sigma)\hat{y}_t^2\big) - e^{(1+\varphi)(\bar{y}_t-\bar{a}_t)}\int_0^1 \left[\hat{y}_{jt} + \frac{1+\varphi}{2}\hat{y}_{jt}^2 - (1+\varphi)\hat{y}_{jt}\hat{a}_t\right]dj + t.i.p.$$

$$U_t = e^{(1-\sigma)\bar{y}_t}\big(\hat{y}_t + \frac{1}{2}(1-\sigma)\hat{y}_t^2\big) - e^{(1+\varphi)(\bar{y}_t-\bar{a}_t)}\Bigg[E_j\hat{y}_{jt} + \frac{1+\varphi}{2}(var_j\hat{y}_{jt}$$
$$+(E_j\hat{y}_{jt})^2) - (1+\varphi)\hat{a}_t E_j\hat{y}_{jt}\Bigg] + t.i.p.\,, \tag{A.7}$$

where $E_j\hat{y}_{jt} = \int \hat{y}_{jt}\,dj$ and $var_j\hat{y}_{jt}$ are the mean value and the variance of $\hat{y}_{jt}$ across all goods j at date t, respectively. In the model, Dixit-Stiglitz index of aggregate demand can be written in terms of output as:

$$Y_t = \left(\int_0^1 (Y_{jt})^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.8}$$

This expression can be rewritten in log-linear form, and be approximated as a second order Taylor series expansion around $\bar{y}_t$ as:

$$\begin{aligned}
e^{\frac{\varepsilon-1}{\varepsilon}y_t} &= \int_0^1 e^{\frac{\varepsilon-1}{\varepsilon}y_{jt}}dj \\
&= \int_0^1 e^{\frac{\varepsilon-1}{\varepsilon}\bar{y}_t}dj + \int_0^1 \left[\frac{\varepsilon-1}{\varepsilon}e^{\frac{\varepsilon-1}{\varepsilon}\bar{y}_t}(y_{jt}-\bar{y}_t) + \frac{1}{2}\big(\frac{\varepsilon-1}{\varepsilon}\big)^2 e^{\frac{\varepsilon-1}{\varepsilon}\bar{y}_t}(y_{jt}-\bar{y}_t)^2\right]dj \\
&= e^{\frac{\varepsilon-1}{\varepsilon}\bar{y}_t} + \frac{\varepsilon-1}{\varepsilon}e^{\frac{\varepsilon-1}{\varepsilon}\bar{y}_t}\left[\int_0^1 \big(\hat{y}_{jt} + \frac{1}{2}\frac{\varepsilon-1}{\varepsilon}\hat{y}_{jt}^2\big)dj\right] \\
&= e^{\frac{\varepsilon-1}{\varepsilon}\bar{y}_t}\left[1 + \frac{\varepsilon-1}{\varepsilon}\big(E_j\hat{y}_{jt} + \frac{1}{2}\frac{\varepsilon-1}{\varepsilon}E_j\hat{y}_{jt}^2\big)\right].
\end{aligned} \tag{A.9}$$

Since $var_j\,\hat{y}_{jt} = E_j\,\hat{y}_{jt}^2 - (E_j\,\hat{y}_{jt})^2$, expression (A.9) can be written as:

$$\frac{\varepsilon-1}{\varepsilon}\hat{y}_t = \log\left[1 + \frac{\varepsilon-1}{\varepsilon}\bigg(E_j\hat{y}_{jt} + \frac{1}{2}\frac{\varepsilon-1}{\varepsilon}(var_j\,\hat{y}_{jt} + (E_j\,\hat{y}_{jt})^2)\bigg)\right]. \tag{A.10}$$

When y_t is close enough to $\bar{y}_t$, as expected, the above expression can be approximated as:

$$\hat{y}_t = E_j \hat{y}_{jt} + \frac{1}{2}\frac{\varepsilon - 1}{\varepsilon} var_j \hat{y}_{jt} \ . \tag{A.11}$$

If this expression is used in the period utility function (A.7) to eliminate the $E_j \, \hat{y}_{jt}$ terms, and then some small terms are ignored, one can obtain:

$$\begin{aligned} U_t = e^{(1-\sigma)\bar{y}_t} \big(\hat{y}_t + \frac{1}{2}(1-\sigma)\hat{y}_t^2\big) - e^{(1+\varphi)(\bar{y}_t - \bar{a}_t)} \\ \left[\hat{y}_t + \frac{1+\varphi}{2}\hat{y}_t^2 + \frac{\varepsilon^{-1}+\varphi}{2} var_j \, \hat{y}_{jt} - (1+\varphi)\hat{y}_t \hat{a}_t\right] + t.i.p. \end{aligned} \tag{A.12}$$

This equation can be rewritten by using equation (A.3) as:

$$U_t = -e^{(1-\sigma)\bar{y}_t} \frac{\sigma+\varphi}{2}\left[\hat{y}_t^2 - \frac{2(1+\varphi)}{\sigma+\varphi}\hat{a}_t \hat{y}_t + \frac{\varepsilon^{-1}+\varphi}{\sigma+\varphi} var_j \, \hat{y}_{jt}\right] + t.i.p. \tag{A.13}$$

It can be written from the natural rate expression and equation (A.3) that:

$$\hat{y}_t^N = y_t^N - \bar{y}_t = \frac{1+\varphi}{\sigma+\varphi} \, \hat{a}_t \ , \tag{A.14}$$

then by plugging this expression into (A.13), one can get:

$$U_t = -e^{(1-\sigma)\bar{y}_t} \frac{\sigma+\varphi}{2}\left[\hat{y}_t^2 - 2\hat{y}_t^N \hat{y}_t + \frac{\varepsilon^{-1}+\varphi}{\sigma+\varphi} var_j \, \hat{y}_{jt}\right] + t.i.p. \tag{A.15}$$

If $-(\hat{y}_t^N)^2$ and $\hat{y}_t$ (it is constant with respect to j) terms are added into the $t.i.p.$ and into the variance of the above equation, respectively, (A.15) can be rewritten as:

$$U_t = -e^{(1-\sigma)\bar{y}_t} \frac{\sigma+\varphi}{2}\left[(\hat{y}_t - \hat{y}_t^N)^2 + \frac{\varepsilon^{-1}+\varphi}{\sigma+\varphi} \ var_j \, (\hat{y}_{jt} - \hat{y}_t)\right] + t.i.p. \tag{A.16}$$

If the hat terms are expended by using their definition, the above expression can be written as:

$$U_t = -e^{(1-\sigma)\bar{y}_t} \frac{\sigma+\varphi}{2}\left[x_t^2 + \frac{\varepsilon^{-1}+\varphi}{\sigma+\varphi} \ var_j \, (y_{jt} - y_t)\right] + t.i.p. \tag{A.17}$$

This is equation (28) given in the text.