Forecasting with Factor Models: A Bayesian Model Averaging Perspective

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Abstract

We use Bayesian factor regression models to construct a financial conditions index (FCI) for the U.S. Within this context we develop Bayesian model averaging methods that allow the data to select which variables should be included in the FCI or not. We also examine the importance of different sources of instability in the factors, such as stochastic volatility and structural breaks. Our results indicate that ignoring structural breaks in the loadings can be quite costly in terms of the forecasting performance of the FCI. Additionally, Bayesian model averaging can improve in specific cases the performance of the FCI, by means of discarding irrelevant financial variables during the estimation of the factor.

Keywords: financial stress; stochastic search variable selection; early-warning system; forecasting

JEL Classification: C11, C32, C52, C53, C66

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1 Introduction

The recent financial crisis of 2007-2009 has raised new important issues for econometricians and applied economists who want to accurately measure financial shocks to the real economy. An important lesson learned is that financial developments, not necessarily driven by monetary policy actions or fundamentals, may have a strong impact to the general economy. Taking into account the globalization of trade of both physical goods and financial products, it might become very challenging for policy-makers in the future to mitigate the effects of the next financial crisis. Hence, in order to prevent panic measures, such as injecting large amounts of money into the economy, or lowering short-term interest rates to the zero lower bound, policy-makers should be proactive and closely monitor financial conditions.

Many authors have recognized the importance of measuring and monitoring financial conditions, and a large literature has revived recently on the issue of constructing financial conditions indexes (FCIs). These indexes contain information from "many" (depending on availability, and the application at hand) financial variables, and they act as early-warning systems to be used by policy-makers and other agents in the economy. Hatzius, Hooper, Mishkin, Schoenholtz and Watson (2010) extract a financial conditions index (FCI) from 45 quarterly financial variables using simple principal components analysis (PCA) methods. This is an impressive application due to the amount of relevant financial variables used to construct the FCI. Other notable studies in this literature include the works of Balakrishnan, Danninger, Elekdag and Tytell (2008), Brave and Butters (2011), English, Tsatsaronis and Zoli (2005), Gomez, Murcia and Zamudio (2011), and Matheson (2011), among others.

Nevertheless, all these post-crisis studies mentioned above rely on ex-post selection of relevant variables to be included in the final index, that is, variables are selected after having observed the characteristics and drivers of the global crisis. Thus, it is not surprising that most FCIs include, for instance, measures of the housing market conditions such as mortgage rates (since the crisis was initiated by the US housing market crisis), or the rate and issuance of securities such as commercial paper and asset backed securities (which the Fed bought abundantly with the quantitative easing programs of 11/2008 and 11/2010, respectively). Our aim in this paper is to develop econometric methods which allow the data to determine ex-ante which variables should be included in the FCI. Why such an elaborate econometric attempt is potentially important for an index of financial conditions? For the simple reason that the next financial crisis is unlikely to be similar to the last one, and it might be the case that, say, mortgage rates will be a very poor index of a future financial breakdown (and hence this variable should not be included in the FCI, in order to maximize the "relevant" information that the index carries).

Additionally, many papers in this literature rely on linear Gaussian factor models estimated using principal components or maximum likelihood. Admittedly one should not ignore the structural instabilities and nonlinearities which are evident in financial data. Ideally the factor methods used should account for instabilities such as structural breaks and stochastic volatility. Our aim is to construct an FCI which has the ability to adapt to the different states of the economy. For example, by allowing stochastic volatility in the errors, we can enhance the quality of the FCI by allowing its volatility to increase during turbulent times (e.g., recessions, bear markets, oil shocks).

In this paper we formally deal with both of these issues in an integrated Bayesian setting. We begin with the simple factor model which describes how financial variables load on an unobserved factor. We have monthly measurements from 1980 till mid-2011 on 28 financial variables which measure/proxy the financial situation, and from which we extract the unobserved factor (that is used as our financial conditions index). Then, we develop a Bayesian model averaging (BMA) prior that determines in a data-based way what variables among all those in our dataset should load on the FCI. BMA allows us to incorporate model uncertainty in a comprehensive framework: we estimate the posterior probabilities of inclusion of each of the 28 financial variables in the final factor/FCI. Next, we examine possible divergence from normality and nonlinearity by examining sequentially the performance of i) a nonparametric factor that relaxes the usual Gaussian assumption, ii) a factor with stochastic volatility that allows for time-variation in volatility, and iii) a factor with an unknown number of structural breaks in the loadings that can handle structural instabilities. The various factors extracted are used and assessed in an application where the target is to forecast the total industrial production index.

Our results indicate that nonlinearities, which are routinely ignored by the forecasting literature when using factor models, play a crucially important role in terms of extracting meaningful factors. Given our forecasting results, we find that allowing for stochastic volatility in the factor and idiosyncratic errors is quite vital for extracting a factor from "fast-moving" financial variables (i.e., our FCI). Additionally, allowing for structural breaks in the loadings also seems to be very important, given that the recent financial crisis is included in the sample. In fact, the Bayesian methodology that is used to estimate a factor model with an unknown number of breaks at unknown points in time identifies a single break occurring in 2006m1.

When it comes to Bayesian model averaging (BMA) the conclusions are mixed. While it is well understood that more data are not always better when estimating factor models (Bai and Ng, 2006), whether this observation applies to our settings or not depends on the factor model specification. For the simple Gaussian factor model and the structural breaks factor model, we find that BMA extracts a factor that has better predictive ability than an unrestricted factor obtained from each of the respective models. Nevertheless, forecasts deteriorate when we use BMA in the nonparametric factor model and the factor stochastic volatility model.

The next section summarizes in a compact way the basic factor model, the several extensions we use in order to deal with nonlinearities and instabilities, as well as the simple prior formulation for performing Bayesian model averaging. Then identification and estimation in each model are discussed. An empirical application follows, which involves forecasting the growth in the industrial production index by means of the FCIs constructed from the various models. Section 5 concludes and discusses the implications of our findings, which are not only relevant in the construction of FCIs, but also in the general literature that studies factor models for business cycle measurement, asset pricing, or measuring monetary policy, to name but a few.

2 Constructing FCIs in a Bayesian paradigm

2.1 The simple factor model

Let $x_{i,t}$ be the time series vector on financial variable $i = 1, ..., n$, observed for $t = 1, ..., T$. Following Hatzius et al. (2010) we construct a single Financial Conditions Index (FCI) based on the simple static factor model for the aggregate vector $x_t = (x_{1,t}, ..., x_{n,t})'$ of the form

$$x_t = \beta f_t + \varepsilon_t, \tag{1}$$

where $\beta = (\beta_1, \ldots, \beta_n)'$ are the factor coefficients, also called *loadings*, f_t is a single unobserved factor with $f_t \sim N(0,1)$, and ε_t is the innovation error with $\varepsilon_t \sim N(0, \Sigma)$. A typical assumption is that Σ is a diagonal covariance matrix with σ_i^2 on its diagonal. This allows an identified decomposition of our data x_t into a "common component" βf_t and idiosyncratic shocks $\varepsilon_{it} \sim N(0, \sigma_i^2)$. Specifically, the conditional covariance of the data x_t explained by the factor model is

$$\Omega = var(x_t|\beta, \Sigma) = \beta\beta' + \Sigma. \tag{2}$$

This model has been used extensively in the finance literature with attention to asset pricing models, see for instance Roll and Ross (1980) . A Bayesian implementation of this model using Markov Chain Monte Carlo methods, and further identification and model selection strategies are discussed in Lopes and West (2004).

2.2 A Bayesian Model Averaging prior

The first step in our analysis is to use Bayesian model averaging (BMA) to help us decide which of the n variables should be used when constructing the financial conditions index (FCI). To do that, we follow George and McCulloch (1993) and Korobilis (2008) and adopt a hierarchical prior on the coefficients β of the form

$$\beta_i|\gamma_i \sim (1-\gamma_i) N\left(0, \tau_{0i}^2\right) + \gamma_i N\left(0, \tau_{1i}^2\right) \tag{3}$$

$$\gamma_i \sim Bernoulli(\pi_0). \tag{4}$$

This prior has two levels of hierarchy. At the first level in eq. (3), with probability γ_i the prior for each individual element β_i of the loadings vector β is $N(0, \tau_{1i}^2)$, and with probability $1-\gamma_i$ the prior is $N(0,\tau_{0i}^2)$. The prior variances of each component, τ_{0i}^2 and τ_{1i}^2, are chosen to be some "small" (close to zero) and "large" constants respectively. Thus, when $\gamma_i = 0$, the prior for β_i is concentrated around zero, which has the implication that the i-th financial indicator is removed from the construction of the FCI. The second level of hierarchy in eq. (4) allows the parameters γ_i to have their own prior, so that their posterior is updated by the likelihood. In this case the data (likelihood) will determine which variables β_i will be restricted ($\gamma_i = 0$) or not ($\gamma_i = 1$).

In our application we use this prior for Bayesian model averaging [1], and we also calculate an unrestricted version using the typical Gaussian prior. The unrestricted model is a special case of the prior (3), which is achieved by fixing $\gamma_i = 1$ for all i (or equivalently, by setting the tight prior $\gamma_i \sim Bernoulli(1)$). In this case, we have $\beta_i \sim \gamma_i N(0,\tau_{1i}^2)$, where τ_{1i}^2 is "large enough" to guarantee that the likelihood dominates estimation β (non-informative prior).

2.3 Incorporating sources of instability and nonlinearities

When constructing an FCI, we also have to consider sources of instabilities and nonlinearities in both the variances of the factor and the innovation errors. Here we summarize the extensions that can be incorporated in a Bayesian factor model, which might possibly add more flexibility and increase the informational content of the FCI for forecasting economic activity.

1. **Nonparametric factor model**: The assumption that $f_t \sim N(0,1)$ in the Bayesian likelihood-based factor model is a quite restrictive one. In contrast, the popular principal components analysis (PCA) method allows the estimation of factors free from parametric assumptions. Hence, the first step towards achieving more flexibility in the construction of an FCI is to consider f_t to be nonparametric. That is, we assume that f_t is distributed as $F(f_t)$, where $F(\cdot)$ is a general unknown density function. This can be done using Dirichlet process as follows:

$$\begin{aligned} f_t &\sim F(f_t) \\ F &\sim Dirichlet(aF_0) \\ F_0 &\sim N(0,1). \end{aligned}$$

   For each $t = 1, ..., T$, denote by f_{-t} the set of $T-1$ factor vectors with f_t removed. A key feature of the Dirichlet process prior is that all complete conditionals for f_t (marginalizing over the uncertain F) are standard. In fact, they are given by

$$f_t | f_{-t} \sim a_{T-1} N(0,I) + (1-a_{T-1}) \sum_{r=1, r\neq t}^{T} \delta_{f_r}(f_t)$$

[1] See Section 3.2 for more details on how we estimate the BMA probabilities.

where $\delta_{f_r}(f_t)$ is the Dirac delta function that is degenerate at the point f_r and $a_{T-1} = a/(a+T-1)$. This model has been studied recently (with the addition of Bayesian model averaging) in Carvalho et al. (2008).

2. **Factor stochastic volatility model**: An obvious assumption to test is whether there is time-variation in the volatility of the factor, f_t, and the idiosyncratic errors, ε_t. When modelling financial data it is desirable to have a complete model for time-varying volatilities; see for instance the factor stochastic volatility model of Pitt and Shephard (1999). Following these authors, we assume that $f_t \sim N(0, h_t)$ and $\varepsilon_t \sim N(0, \Sigma_t)$, where $\Sigma_t = diag\left(\sigma^2_{1,t}, ..., \sigma^2_{n,t}\right)$. These volatilities follow geometric random walks of the form

$$\begin{aligned} \log h_t &= \log h_{t-1} + \rho_1 \zeta_t^h \\ \log \sigma_t^2 &= \log \sigma_t^2 + \rho_2 \zeta_t^{\sigma}, \end{aligned}$$

with $(\zeta_t^h, \zeta_t^{\sigma}) \sim N_{n+1}(0, I)$, where $\sigma_t^2 = (\sigma^2_{1t}, ..., \sigma^2_{nt})'$. See also Korobilis (2013a) for more details on factor models with stochastic volatility.

3. **Structural breaks factor model**: In order to capture the desirable feature that some variables might load more during normal periods, while others might load more during crises, we also estimate a factor model where we allow structural breaks to occur in the loadings. That is, the loadings matrix β is allowed to change value abruptly K times in-sample (i.e., there are $K+1$ different regimes that can occur). To do this, we generalize the simple factor model to the following case

$$x_t = \begin{cases} \beta_1 f_t + \varepsilon_t, & \text{if } s_t = 1 \\ \beta_2 f_t + \varepsilon_t, & \text{if } s_t = 2 \\ \vdots & \vdots \\ \beta_{K+1} f_t + \varepsilon_t, & \text{if } s_t = K+1 \end{cases}.$$

Here $s_t \in [1, ..., K+1]$ is a first order Markov process with block-diagonal transition matrix of the form

$$P = \begin{bmatrix} p_{11} & p_{12} & 0 & \cdots & 0 \\ 0 & p_{22} & p_{23} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ & & 0 & p_{KK} & p_{K,K+1} \\ 0 & \cdots & 0 & 0 & 1 \end{bmatrix}.$$

Note that this structural breaks factor model is a restricted form of the Markov switching factor model, since the transition probabilities matrix P imposes that we can only move from one regime to the next and never return to any previous regimes. Note that the assumption of structural instabilities in the loadings β does not affect variable selection negatively. In contrast, now we are able to select independently which financial variables should load on the Global FCI in each separate regime. Therefore, variable selection can now help determine whether the information from different variables should be added/removed from our FCI in different time periods (for instance normal periods as opposed to crises). Model averaging is implemented in a way similar to the algorithm of Jochmann et al. (2008) for structural breaks VAR models; see also Korobilis (2013c) and the Technical Appendix.

Note that unlike many applications, we do not necessarily want a standardized FCI (zero mean and variance one), nor do we require the loadings to be in the range $[-1, 1]$. Our final purpose is to examine the information content of these FCIs for forecasting output. Hence, interpretation and identification restrictions play no role here. We do however impose a statistical normalization restriction in order to make sure that the various FCIs we estimate are comparable. In all Bayesian models we restrict the last variable (ABS Issuance; see also the Data Appendix) to load on the factor with coefficient equal to 1 (we do this even when BMA is present, that is this variable always loads on the final FCI). This is a standard normalization restriction, which in a single factor model does not affect estimation[2].

2.4 Estimation, identification, and priors

We estimate all four models (simple normal factor model, plus the nonlinear extensions) using Markov Chain Monte Carlo methods. In particular, since the joint posterior of all unknown parameters in each model is intractable, we sample from the posterior of each parameter conditional on the remaining parameters. It turns out that for all models, these conditional posteriors come from distributions which are easy to sample from, such as the Gaussian and gamma densities. Exact details for the sampling schemes are given in the Technical Appendix. Here we provide a pseudo-algorithm for sampling from a factor model such as the simple factor model in equation (1). Given initial values for β and Σ, we follow the following three steps:

1. Sample f_t from $p\left(f_t|\beta, \Sigma, x\right)$
2. Sample β from $p\left(\beta|f_t, \Sigma, x\right)$
3. Sample Σ from $p\left(\Sigma|\beta, f_t, x\right)$.

It turns out that sampling from these conditional densities is equivalent to obtaining draws from the target joint posterior density $p\left(f_t, \beta, \Sigma|x_t\right)$ which is the density of the model parameters after we observe data x. For the nonlinear models additional steps are

[2]For k factors, $k > 1$, the loadings matrix should become lower triangular, in which case the ordering of the original variables can affect estimation; see Lopes and West (2004) for a complete discussion.

needed to sample from the conditional posteriors of additional parameters (for instance, the latent structural breaks indicator variable s_t), but exact expressions are easy to derive. Note that since our final purpose is forecasting, we do not compare models using in-sample fit criteria (Bayesian information criterion or marginal likelihoods). The ultimate purpose is to see which factors carry important information for forecasting economic activity, hence we rank factor models according to forecast error statistics.

Since we are using likelihood-based methods to extract factors, we need also to consider the issue of identification. As it was highlighted above, in practice the factor model implies a decomposition of our data of the form of equation (2). Hence, it is imperative to identify the matrix $\Psi = \beta\beta'$ from Ω, since there are infinite ways to do that. The restriction that Σ is diagonal (and that the errors ε_t are not correlated at all leads and lags with the factors f_t) helps towards the unique separation of Ω into these two matrices, since it implies that Ψ measures covariances/comovements in the data x_t, while Σ measures the variances of the idiosyncratic components/shocks ε_t. However further restrictions need to be imposed on β since there are many different ways to construct Ψ using these parameter vectors. For instance, for a matrix P such that $PP' = I$, we can see that $\Psi = \beta P (P\beta)' = \widetilde{\beta}\widetilde{\beta}'$ with $\widetilde{\beta} = \beta P$; see also Lopes and West (2004) for a discussion of such issues. When using nonlinear models the identification problem is even more pronounced, for instance in the stochastic volatility model we have

$$\Omega = var\left(x_t|\beta, h_t, \Sigma_t\right) = \beta h_t \beta' + \Sigma_t.$$

In this case additional identification restrictions are needed to separate β from h_t when estimating a unique covariance $\Psi = \beta h_t \beta'$.

In order to deal with these issues and maintain interpretability, we choose the same identification restrictions across all factor models, even if for some of these factor models identification could be achieved using milder restrictions. Hence, we impose that the issuance of asset backed securities loads in the FCI/factor with coefficient 1 at all instances (i.e. even when we estimate breaks in the loadings). This normalization restriction is sufficient to identify a unique factor for each type of model used, and that we can make direct comparisons between the outcome of the different factor models.

Finally, we need to define values for the prior parameters, and more specifically those associated with the loadings, β, and the covariance matrix Σ. For the BMA prior in equation (3) we choose $\tau_{0i}^2 = 0.001$ and $\tau_{1i}^2 = 1$ (see also our discussion and suggestions in that subsection). For the BMA probabilities we set the prior probability to be equal to $\pi_0 = 0.1$, i.e. a priori we expect only that 10% of the variables will be in the final, best model. Given that we have a rich dataset of 380 time series observations and only 28 variables from which to extract a single factor, these prior choices are not overly restrictive and the rich information in the likelihood dominates the prior (experimenting with the choice $\tau_{0i}^2 = 0.01$ and $\tau_{1i}^2 = 10$ and $\pi_0 = 0.5$ gives quite similar results). The unrestricted models are estimated as a special case of the restricted models for which $\pi_0 = 1$. This prior gives posterior values for γ_i which are always equal to one, hence the prior for β collapses to $\beta_i|\gamma_i = 1 \sim (1-1)\, N(0, 0.001) + 1N(0,1) \equiv N(0,1)$, i.e. a relatively uninformative Gaussian prior (note that our data are standardized so that the loadings are expected to be roughly in the support $[-1, 1]$).

3 Empirical Results

3.1 Data

We use a total of 28 financial variables measuring stock prices and volatilities, exchange rates, oil prices, and interest rate spreads. The Bloomberg financial conditions index, and the St. Louis Fed financial stress index are used as benchmarks for comparison with our FCIs. All data are measured for the period 1980m1-2011m8, although some of the series start a little bit later and we treat their missing values as zeros during the estimation of the factors. The Data Appendix provides more details on the nature and source of each series.

3.2 Bayesian model averaging in factor models

The first step in our analysis is to examine the properties of the factor restrictions imposed by Bayesian model averaging (BMA). The BMA probabilities can be calculated as the average of the posterior draws of all γ_i, $i = 1, ..., n$. That is, given S draws from the posterior simulator, the probability that a financial indicator x_{it} loads on the factor (FCI) f_t is given by

$$\overline{\pi}_{\beta_i} = \frac{1}{S} \sum_{s=1}^{S} \gamma_i^s,$$

where γ_i^s is the s-th draw of the parameter γ_i. Note that although $\gamma_i^s, s = 1, \ldots, S$ is a sequence of zeros and ones, its average, $\overline{\pi}_{\beta_i}$, is a number between 0 and 1 that can be interpreted as the "proportion of times x_{it} has been used to extract the factor f_t". That is if for 30% of the posterior draws γ_i^s was 1 (and 70% of the time $\gamma_i^s = 0$), then β_i has been restricted to be zero 70% of the time, while it was used to extract f_t only 30% of the time. Then the final extracted factor contains the effect of financial variable x_{it} with an average probability of $\overline{\pi}_{\beta_i} = 0.3$. This averaging scheme is quite popular in forecasting, since it reduces the two risks associated with using x_{it} in a single model: if x_{it} is removed completely from the final model, we ignore its 30% explanatory power. On the other hand, if x_{it} is always included in the final model, we overestimate its explanatory power, since we assume that it is included 100% of the time.

Tables C1 to C4 in the Appendix show estimates of the factor loadings for each factor model with (coefficients β^r) and without BMA (coefficients β^u). When BMA is present the associated averaging probabilities (coefficient π_{β_r}) for each element of the loadings is presented. Looking at these probabilities, there is strong evidence that BMA favours volatility variables (VIX, Merrill Lynch Volatility Index), the 2 year swap spread, S&P

500 returns, the Michigan house conditions survey, and the mortgage spread. There is strong evidence (probability equal to one) that these variables are important, irrespective of the factor model specification used.

Nevertheless, attention is needed when interpreting the results of the structural breaks factor model. The unrestricted version of this model shows that there is one structural break in the loadings circa 2006m1, using the full sample[3]. However, applying model averaging in the loadings matrix for the two subsamples shows that some variables which are important post 2006 were not important before 2006, and vice-versa. Therefore, we see from Table C4 that stock market volatility or swap spreads were not the main drivers of the FCI pre-2006. In contrast, variables such as the 2 & 10 year bond spread and the interest rate used in financing the purchases of new cars were very important pre-2006 but not post-2006. The mortgage spread and the Michigan surveys have very high probabilities throughout the sample period.

3.3 New financial conditions indexes for the U.S.

Figures 1 to 4 present estimates of the financial conditions indexes from the four Bayesian factor models with and without Bayesian model averaging. For comparison the estimated factor from the 28 series using principal component analysis is given in each figure. Note that all financial series used to extract factors have been standardized first, so that all changes in the FCI are in terms of standard deviations from a zero mean. The FCI is constructed in such a way that downward movements signify deterioration of financial conditions (for instance, increase in stock market volatility), while positive movements signify an improvement (for instance, increase in stock market returns). Hence, it is not surprising to observe that all factors agree that around 1987, 1998, 2003 and 2008-2009 financial stress has hit the U.S. economy.

The information contained in the factors varies with the model specification, as well as whether Bayesian model averaging is present or not. The linear factor model without BMA complies with the shape of the PCA estimates. However, adding BMA or nonlinearities in the factor model produces various patterns which carry different information. For instance the factors obtain with the additional assumption of stochastic variances does not pick up the slump caused by the recent financial crisis. This is because the volatility component of the factor and the idiosyncratic error has absorbed most of this shock (which is mainly a volatility shock), hence it is not reflected in the estimation of factor.

In order to get an idea how these factors differ from other FCIs closely monitored by the Fed, commercial banks and financial institutions, Figure 5 plots three FCIs from the Federal Reserve Banks of Kansas, St. Louis and Chicago, as well as the Bloomberg FCI.

[3] In the recursive forecasting exercise, given the uncertainty about the break date, the estimate of the break date will change as new data are added to the sample. For a further examination of this issue when forecasting with structural breaks models see Figure 4 of Pesaran, Pettenuzzo and Timmermann (2006) and the discussion therein.

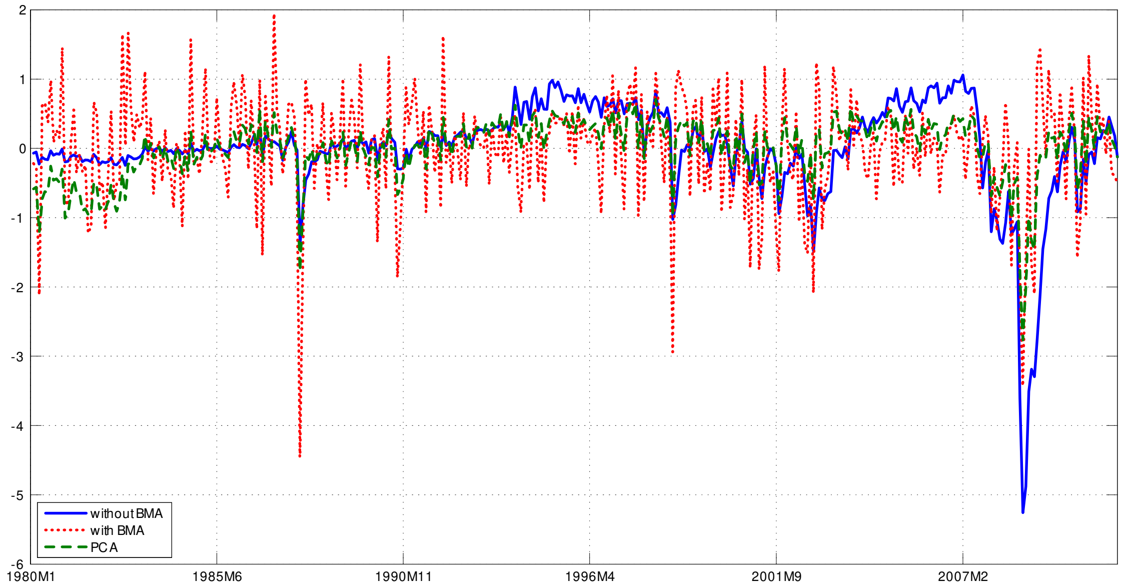


Figure 1: FCIs from the simple factor model with (dotted line) and without (solid line) Bayesian model averaging. The factor from principal components analysis (PCA) is given for comparison (dashed line).

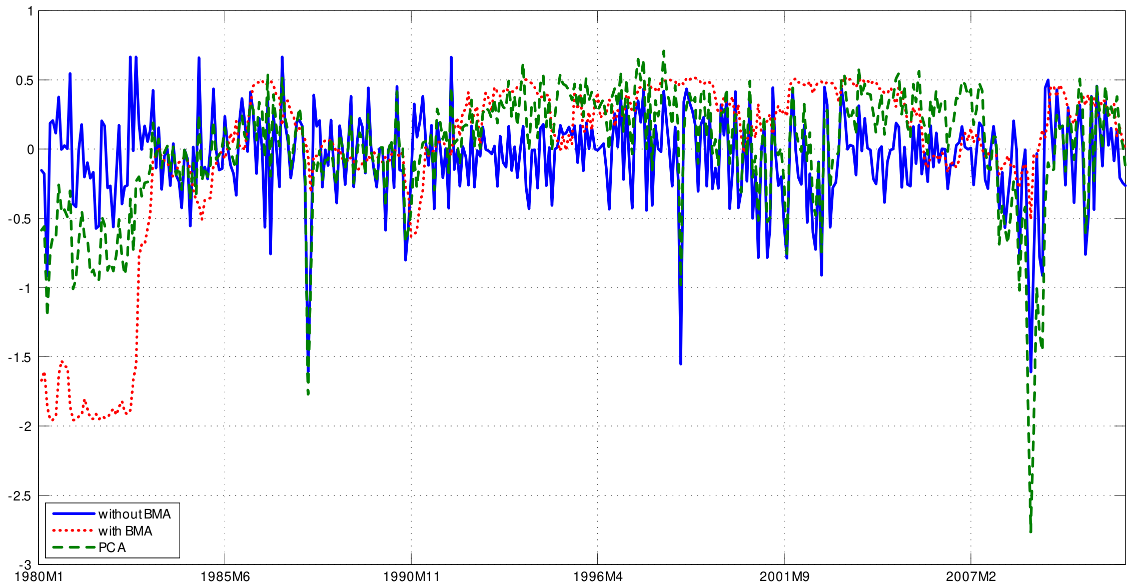


Figure 2: FCIs from the nonparametric factor model with (dotted line) and without (solid line) Bayesian model averaging. The factor from principal components analysis (PCA) is given for comparison (dashed line).

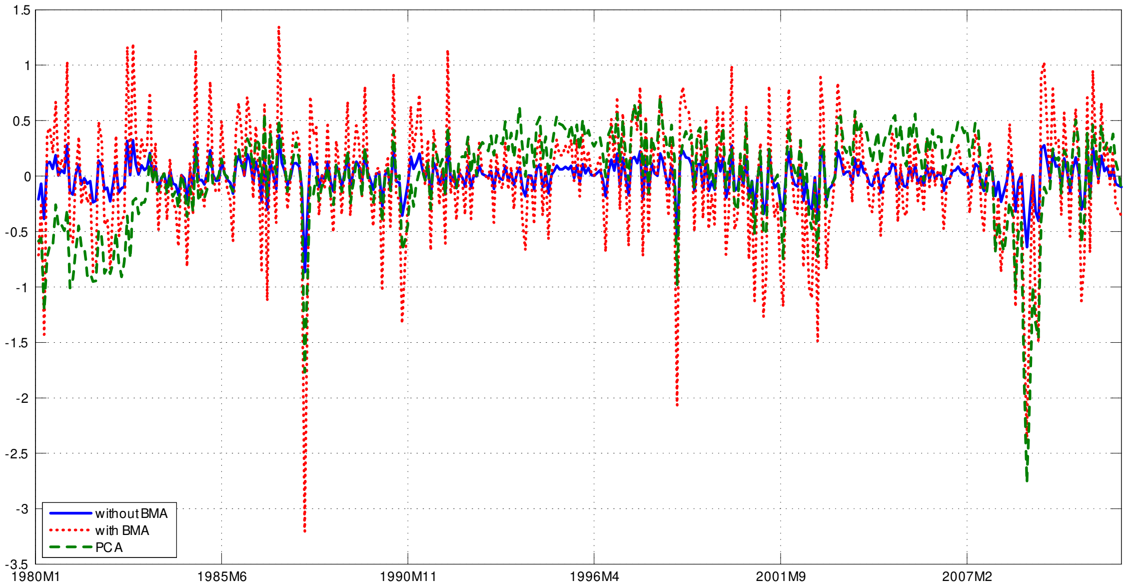


Figure 3: FCIs from the factor stochastic volatility model with (dotted line) and without (solid line) Bayesian model averaging. The factor from principal components analysis (PCA) is given for comparison (dashed line).

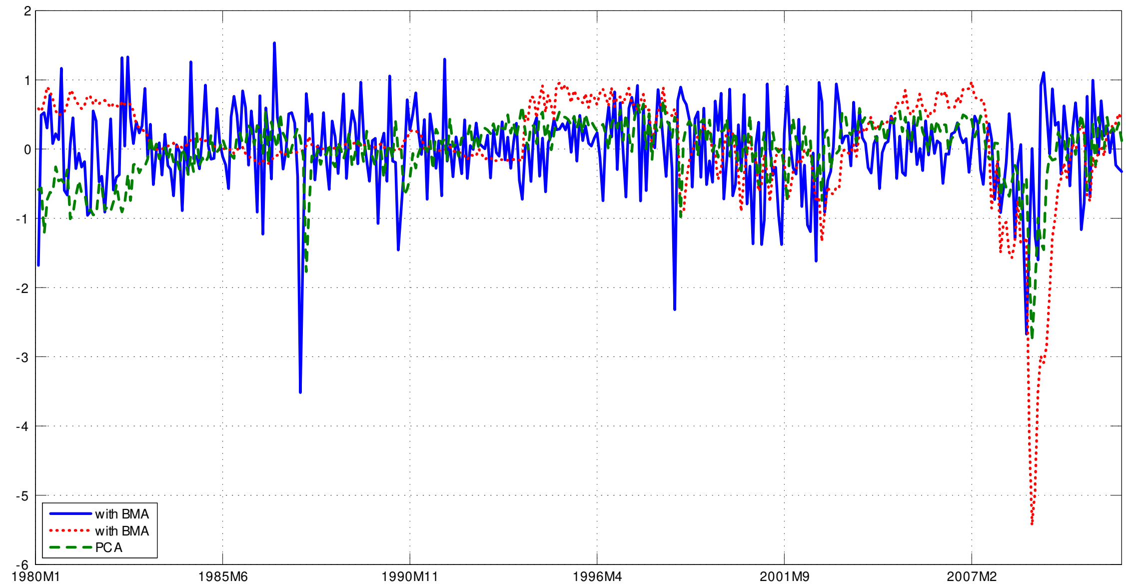
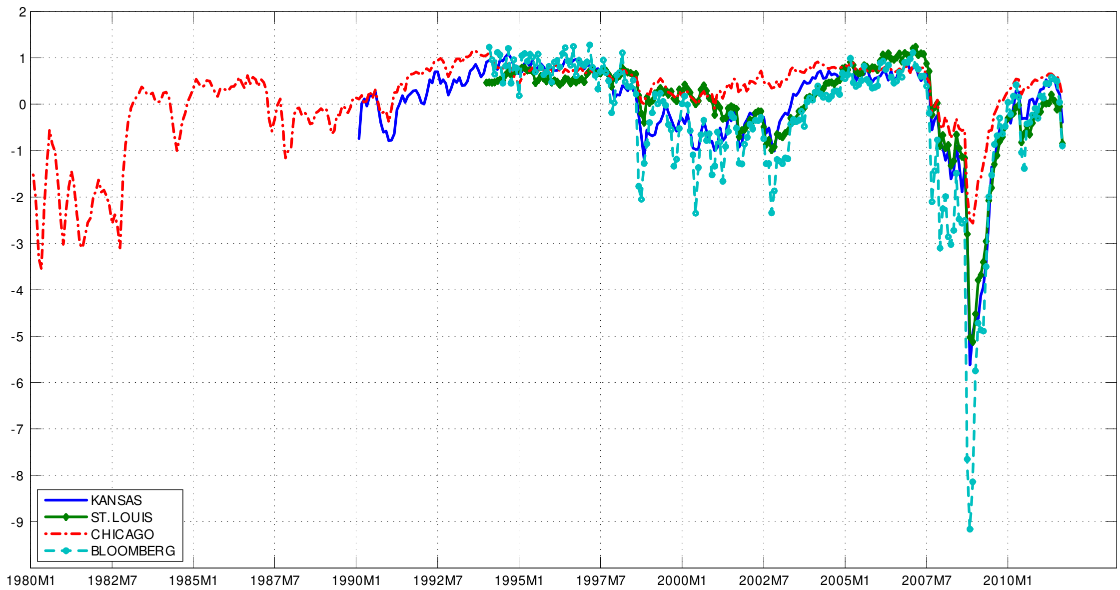


Figure 4: FCIs from the structural breaks factor model with (dotted line) and without (solid line) Bayesian model averaging. The factor from principal components analysis (PCA) is given for comparison (dashed line).

As it is evident from the graph, the St. Louis and Bloomberg FCIs span a much smaller sample than the other two indexes, which is why they are not considered as benchmarks in the forecast evaluation below.

The reader should note that these indexes are based on different datasets and different modelling assumptions, so in theory they are not directly comparable. However we can notice that the shape of these FCIs comply with our estimate from the Gaussian factor model, something which is expected since these organizations use similar linear and Gaussian factor models to extract their FCIs. Additionally, the magnitude of deterioration of financial conditions is similar to our estimates from the Gaussian factor model, that is, at the bottom of the crisis all FCIs agree that conditions deteriorated by 5 standard deviations (with the Bloomberg FCI estimating that this deterioration was around 9 standard deviations).



Graphs of the Kansas City, St. Louis and Chicago Fed FCIs, and the Bloomberg FCI.

3.4 Forecasting output using FCIs.

The variable we forecast is

$$y_{t+h} = (ip_{t+h} - ip_t) = 100 \times (\log (IP_{t+h}) - \log (IP_t)) .$$

were IP_t is the total industrial production index[4] measured over the period 1980m1 - 2011m8. Forecasts are implemented using a simple two step procedure:

[4]Data on industrial production are from the Federal Reserve Economic database (FRED) of St. Louis Fed, http://research.stlouisfed.org/fred2/.

1. Estimate each of the four factor models (normal, nonparametric, stochastic volatility, and structural breaks) using MCMC, with and without variable selection. Obtain and save the median of the posterior of the Financial Conditions Index, $\widehat{f}_t$.

2. At a second stage estimate the regression

$$y_{t+h} = \sum_{i=1}^{2} \varphi_i y_{t+1-i} + \alpha \widehat{f}_t + \eta_t \tag{5}$$

   where we use everywhere two lags of the dependent variable, as is the case with most macroeconomic applications. This regression gives forecast estimates $E_t(y_{t+h}) = \widehat{y}_{t+h|t}$ where $h > 0$ is the forecast horizon.

One could argue that the specification of the forecasting regression (5) is quite simple. While we could add more interesting features in this equation, such as structural breaks or BMA, our intention is to evaluate the performance of the several FCIs. Therefore, we keep a simple regression setting which is typically used in realistic situations (what an applied economist in business and industry would do). Additionally, adding features such as structural breaks might lead to a false ranking of the different factor models since good or bad forecasting performance will also rely on the presence and the number of breaks. By using a simple, constant coefficients dynamic regression we are able to understand better the exact contribution of each FCI over a benchmark regression which only includes lags of industrial production.

We generate forecasts from the four factor models with and without Bayesian model averaging (hence, eight forecasting models in total). For comparison, we also construct forecasts from the model in (5) where $\widehat{f}_t$ is replaced by i) a simple principal component analysis (PCA) estimate, ii) the Chicago Fed National Financial Conditions Index, and iii) the Kansas City Financial Stress Index. Lastly, the model with two lags of the dependent variable, and no exogenous predictors is used as a global benchmark for evaluation of the forecasts of all indexes.

Forecasts for the log industrial production index are recovered as

$$\log IP_{t+h|t} = \widehat{y}_{t+h|t} + ip_t.$$

In order to measure the forecasting performance of each model, we use the mean absolute forecast error (MAFE) and the mean squared forecast error (MSFE), which are defined respectively as

$$\begin{aligned} MAFE_{IP}^{h} &= \frac{1}{T-\tau_0-h+1} \sum_{t=\tau_0}^{T} \left| \widehat{y}_{t+h|t} - y_{t+h} \right| \\ MSFE_{IP}^{h} &= \frac{1}{T-\tau_0-h+1} \sum_{t=\tau_0}^{T} \left(\widehat{y}_{t+h|t} - y_{t+h} \right)^2 . \end{aligned}$$

In the results below, we present MSFEs and MAFEs for each factor model relative to the benchmark AR(2) model with no FCI.

Recall that the whole sample runs from 1980m1 to 2011m8, and the evaluation period for the forecasts is from 1994m1 to 2011m8$-h$. That means that in the formulas above $T = 2011\text{m}6$ and $\tau_0 = 1999\text{m}12$. Forecasts are computed recursively: we first estimate the models with the sample from 1980m1 to 1993m12, and forecasts are calculated for $h = 1, 3, 6$ and 12 months ahead. Then the observation for 1994m1 is added to the estimation sample and the forecasting exercise is repeated. This procedure continues until we have used all available data.

Tables 1 and 2 present MSFE and MAFE results, respectively. Entries in these tables are relative to the MSFE (MAFE) of the benchmark AR(2) model for industrial production without any FCI. Hence numbers higher (lower) than one show that the AR(2) model is doing better (worse) compared to each of the forecasting models using an FCI. In terms of MSFE we see that the principal component analysis (PCA) estimate of the FCI is performing relatively well. In fact, it is much better than the KANSAS and CHICAGO financial indexes. Nevertheless, the factor stochastic volatility and structural breaks factor models stand out. In particular, we find that the best model for all forecast horizons is the structural breaks FCI with the addition of BMA in the loadings. The worst MSFE forecasts, especially for longer horizons, come from the linear Gaussian factor. Assuming a nonparametric distribution of the factors helps improve the information that the FCI carries for forecasting industrial production. However, this improvement is not as large as allowing for nonlinearities. Lastly, notice how the unrestricted nonparametric factor model has similar forecasting performance to the PCA (this holds for $h = 1, 3, 6$ but not so much for $h = 12$). This should not be surprising given that the PCA factor is a "parameter-free" estimate.

In terms of the MAFE results in Table 2, the story is similar to the one highlighted above. The stochastic volatility model is still dominant, followed by the structural breaks factor model. The only exception is for $h = 1$ where the nonparametric factor model is the best performing model in terms of the absolute value of the forecast error. Hence, the results in these two tables support the story that simple principal components work well in general. However, we can improve the forecast performance by using nonlinear likelihood-based factor models.

Table 1: rMSFE results

		$h = 1$	$h = 3$	$h = 6$	$h = 12$
Bayesian FCIs					
Normal	no BMA	0.9507	0.8990	0.9305	1.0206
	$\tau_0 = 0.01$, $\tau_1 = 10$	0.9466	0.9111	0.9679	0.9847
Nonparametric	no BMA	0.9484	0.8478	0.8773	0.9329
	$\tau_0 = 0.01$, $\tau_1 = 10$	0.9550	0.8917	0.9130	0.9509
Stochastic Volatility	no BMA	0.9421	0.8053	0.8430	0.9092
	$\tau_0 = 0.01$, $\tau_1 = 10$	0.9726	0.8880	0.9235	0.9394
Structural Breaks	no BMA	0.9533	0.8155	0.8161	0.9360
	$\tau_0 = 0.01$, $\tau_1 = 10$	0.9252	0.8016	0.8228	0.9005
Benchmark FCIs					
PCA		0.9405	0.8407	0.8831	0.9886
CHICAGO		0.9291	0.8980	0.8980	0.9334
KANSAS		0.9306	0.9217	0.9365	0.9634

Note: Results are relative to the MSFE of an AR(2) model for Industrial Production

Table 2: rMAFE results

		$h = 1$	$h = 3$	$h = 6$	$h = 12$
Bayesian FCIs					
Normal	no BMA	0.9852	0.9834	0.9718	0.9980
	$\tau_0 = 0.01$, $\tau_1 = 10$	0.9806	0.9703	0.9837	0.9833
Nonparametric	no BMA	0.9534	0.9355	0.9328	0.9366
	$\tau_0 = 0.01$, $\tau_1 = 10$	0.9709	0.9636	0.9725	0.9811
Stochastic Volatility	no BMA	0.9660	0.9018	0.9282	0.9463
	$\tau_0 = 0.01$, $\tau_1 = 10$	0.9850	0.9500	0.9717	0.9653
Structural Breaks	no BMA	0.9651	0.9206	0.9223	0.9757
	$\tau_0 = 0.01$, $\tau_1 = 10$	0.9582	0.9309	0.9355	0.9424
Benchmark FCIs					
PCA		0.9715	0.9516	0.9495	0.9809
CHICAGO		0.9623	0.9796	0.9874	0.9877
KANSAS		0.9788	0.9792	0.9847	0.9853

Note: Results are relative to the MSFE of an AR(2) model for Industrial Production

Where the results seem to disagree is whether BMA is useful or not. While BMA improves factor forecasts from the Gaussian and structural breaks factor models, it appears to be quite harmful in the case of the factor stochastic volatility model. However, before making any conclusions that BMA is actually harmful, a note of caution is needed when interpreting these results. Bayesian model averaging is designed to reduce the mean squared error (MSE) in a factor model such as the one in equation (1), which is similar to what, say, model selection using an information criterion or a shrinkage estimator would do. This does not mean that the resulting estimated FCI, $\widehat{f}_t$, is necessarily more useful or more meaningful in economic terms than when not using Bayesian model averaging. If our final purpose was to forecast x_t in a factor model such as the one in equation

(1), experience in the vast BMA literature suggests that it is probable that we would be better-off using BMA. However, here we want to assess whether BMA can be helpful to extract an FCI that could be useful to monitor closely and use in forecasting future movements of the economy. Since our forecasting model is the one in equation (5), which is different from the models of the form in equation (1) where BMA is applied, it is expected that results might be mixed.

4 Conclusions

To be completed.....

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Appendices

A Posterior sampling in the factor models with stochastic search variable selection

In this appendix we give details of the Gibbs sampling schemes used to estimate the models in this paper.

A.1 Simple Factor Model

The simple factor model analysed in this appendix is of the form

$$x_{i,t} = \beta_i f_t + \varepsilon_{i,t},$$

with the assumption that $f_t \sim N(0,1)$ and $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$. The subscript i, $i = 1, ..., n$, is inserted to variables and parameters to denote that the multivariate factor model is equivalent to n-univariate regressions (due to the fact that the error covariance matrix Σ is diagonal with elements σ_i^2, i.e. $\Sigma = diag(\sigma_1^2, ..., \sigma_n^2)$). Using this notation we have $\beta = (\beta_1, ..., \beta_n)'$.

For $i = 1, ..., n$, the model selection/averaging prior we assign on β is

$$\beta_i \sim (1-\gamma_i) N\left(0, \tau_{0i}^2\right) + \gamma_i N\left(0, \tau_{1i}^2\right) \quad \text{(A.1)}$$

$$\gamma_i \sim Bernoulli(\pi_0) \quad \text{(A.2)}$$

The error variances are integrated out using a noninformative prior of the form

$$\sigma_i^2 \propto \frac{1}{\sigma_i^2}.$$

Given initial values for the vector γ_i and the choice of hyperparameters r_1, r_2, π_0, estimation of the unknown parameters is implemented by sampling from the following conditional densities:

1. Sample $f_t|-$ from

$$N\left(\left(1+\beta'\Sigma^{-1}\beta\right)^{-1}\beta\Sigma^{-1}x_t, \left(1+\beta'\Sigma^{-1}\beta\right)\right),$$

for $t = 1, ..., T$.

2. Sample $\gamma_i|-$ from

$$Bernoulli\left(\frac{u_{1i}}{u_{1i}+u_{2i}}\right),$$

where $u_{1i} = \pi_0\phi(0, \tau_{0i}^2)$ and $u_{2i} = (1-\pi_0)\phi(0, \tau_{1i}^2)$.

3. Sample $\beta_i|-$ for $i = 1, ..., n-1$[5]

$$N\left(\left((DD)^{-1} + \sigma_i^{-2} f'f\right)^{-1} \sigma_i^{-2} f x_i', \left((DD)^{-1} + \sigma_i^{-2} f'f\right)\right),$$

where $D = diag\,(d_1, ..., d_n)$ with $d_i = \begin{cases} \tau_{0i} & \text{, if } \gamma_i = 0 \\ \tau_{1i} & \text{, if } \gamma_i = 1 \end{cases}$.

A.2 Nonparametric Factor Model

In this model we drop the assumption that $f_t \sim N(0,1)$. Instead we assume a nonparametric density for the factors of the form $f_t \sim F(f_t)$. From a Bayesian point of view the density $F(\cdot)$ is approximated using infinite mixtures. The nonparametric Dirichlet process prior is of the form

$$\begin{aligned} f_t &\sim F(f_t) \\ F &\sim Dirichlet\,(aF_0) \\ F_0 &\sim N(0, I). \end{aligned}$$

Compared to the simple static factor model, we need to change step 1, which samples the factors, to step 1* presented below. First, split the factors into a mixture of C normal components. Then:

1* Sample $f_t|-$ from the mixture posterior

$$q_0 N\left(\overline{m}_t, \overline{M}\right) + \sum_{j=1}^{C} q_j N\left(\overline{v}_j, \overline{V}_j\right),$$

where $q_0 \propto aN(0, \beta\beta' + \Sigma)$ and $q_j \propto n_j N\left(\beta f_j^*, \Sigma\right)$ with n_j being the number of factor values f_t which belong to mixture component j[6]. That is, with probability q_0 we sample f_t from $N\left(\overline{m}_t, \overline{M}\right)$, and with probability q_j we sample f_t from $N\left(\overline{v}_j, \overline{V}_j\right)$. In the above equation it holds that $\overline{m}_t = \overline{M}^{-1}\beta'^{-1}x_t$, $\overline{M} = I + \beta'\Sigma^{-1}\beta$, and $\overline{v}_j = \overline{V}_j^{-1}\beta'\Sigma_t^{-1}\left(\sum_{t:f_t\in\{j\}} x_t\right)$, $\overline{V}_j = I + n_j\beta'\Sigma^{-1}\beta$, where the notation $\sum_{t:f_t\in\{j\}} x_t$ means "take the sum of all x_t for those observations t for which f_t belongs to mixture component j.

[5] For $i = n$, impose the identification condition $\beta_i = 1$.

[6] Note that for some components j, $j = 1, ..., C$, it might hold that $n_j = 0$, i.e. no factor value is assigned to them.

A.3 Factor Stochastic Volatility Model

In the factor stochastic volatility model the factor follows $f_t \sim N(0, h_{f,t})$ where

$$\log h_t = \log h_{t-1} + \gamma_1 \zeta_t^h$$

Sampling of this model requires to write the model in state-space form and sample the log volatilities using the Kalman filter and smoother. Among the many papers providing algorithms for stochastic volatility models, Pitt and Shephard (1999) provide a detailed MCMC scheme.

A.4 Structural Breaks Factor Model

In this case, we make the assumption that there is an unknown break in the loadings matrix β, which also affects how the factors are being sampled. First, we add the following steps to the Gibbs sampler of the simple factor model:

7 Sample $s_t|-$ using Chib's (1998) algorithm

8 Sample $p_{ii}|-$ from

$$Beta(\lambda_1 + T_i, \lambda_2 + 1),$$

where T_i are the number of observations in regime i.

Then, sampling of the β's and the factors has to be adapted slightly, and steps 1 and 6 of the simple factor model are replaced with the following steps

1'. Sample $f_t|-$ from

$$N\left(\left(1 + \beta_{s_t}' \Sigma^{-1} \beta_{s_t}\right)^{-1} \beta_{s_t}'^{-1} x_t, \left(1 + \beta_{s_t}' \Sigma^{-1} \beta_{s_t}\right)\right),$$

for $t = 1, ..., T$, where β_{s_t} is the value of the loadings in each of the $K+1$ regimes.

6'. Denote by β_{i,s_t} the i-th element of β_{s_t} where $i = 1, ..., n-1$. Sample $\beta_{i,s_t}|-$ from

$$N\left(\left((DD)^{-1} + \sigma_i^{-2} f_{t:s_t=j}' f_{t:s_t=j}\right)^{-1} \sigma_i^{-2} f_{t:s_t=j} x_{i,t:s_t=j}', \left((DD)^{-1} + \sigma_i^{-2} f_{t:s_t=j}' f_{t:s_t=j}\right)\right)$$

where the notation $f_{t:s_t=j}$ denotes the f_t for those time periods t for which it holds that $s_t = j$, $j = 1, ..., K+1$.

B Data sources and transformations

Table : Description of data and sources

No	Mnemonic	Description	Sample	Source
1	VIX	CBOE Volatility Index	1986m6	Bloomberg
2	VXO	CBOE S&P 100 Volatility Index	1986m6	Bloomberg
3	MOVE Index	Merrill Lynch One-Month Treasury Options Volatility Index	1986m6	Bloomberg
4	CCOINEW Index	Federal Reserve New Car Loans at Auto Finance Cos Avg Interest Rate	1980m1	Bloomberg
5	W5000 Index	Wilshire 5000 Index	1980m1	Bloomberg
6	CCMP Index	NASDAQ Composite Index	1980m1	Bloomberg
7	INDU Index	Dow Jones Industrial Average	1980m1	Bloomberg
8	S5HOME	S&P 500 Homebuilding Index	1989m9	Bloomberg
9	SPX	S&P 500 Index	1980m1	Bloomberg
10	USSP2 Curncy	USD SWAP SPREAD SEMI 2YR	1988m11	Bloomberg
11	USSP10 Curncy	USD SWAP SPREAD SEMI 10Y	1988m11	Bloomberg
12	AGGVNT2	U.S. Two-Year Agency Spread	1994m12	Bloomberg
13	AGGVNT10	U.S. Ten-Year Agency Spread	1994m12	Bloomberg
14	TED spread	LIBOR /3monthTbill	1980m1	Bloomberg
15	10yspread	10 year yield/3 month Tbill	1980m1	Bloomberg
16	2yspread	2 year yield/3 month Tbill	1980m1	Bloomberg
17	AAA/BAA spread	AAA/BAA	1980m1	FRED
18	Mortgage spread	30year mortgage rate/ 10 year bond yield	1980m1	FRED
19	High Yield	BofA Merrill Lynch US High Yield Master II Total Return Index Value	1993m3	FRED
20	MZM	MZM Money Stock	1980m1	FRED
21	OILPRICE	Spot Oil Price: West Texas Intermediate	1980m1	FRED
22	TWEXMMTH	Real Trade Weighted Exchange Index: Major Currencies	1980m1	FRED
23	Mich1	Michigan Survey: Good/Bad Conditions for Buying HH Goods Spread	1980m1	Michigan
24	Mich2	Michigan Survey: Good/Bad Conditions for Buying Houses Spread	1980m1	Michigan
25	Mich3	Michigan Survey: Good/Bad Conditions for Buying Autos Spread	1980m1	Michigan
26	SWAP	Securities Industry & Financial Markets Association Swap Index	1989m7	SIFMA
27	COM	Commercial Paper Rate/3-month Tbill	1980m1	FRED
28	ABS	ABS Issuance (Relative to 24Month MA)	1985m1	SIFMA
Benchmark FCIs				
	CHICAGO	Chicago Fed National Financial Conditions Index	1980m1	Bloomberg
	KANSAS	Kansas City Financial Stress Index	1990m1	Bloomberg

Note: Column "Sample" denotes the first observation for each series (last observation is, in all instances, 2011m8). All data are transformed to stationarity using first (log) differences when necessary. FRED is the Federal Reserve Economic Data, SIFMA stands for Securities Industry and Financial Markets Association.

C Posterior estimates of factor loadings, with and without Baysian model averaging

Table C1. Posterior means and st.d - simple factor model

No	β^u	std β^u	β^r	std β^r	$\overline{\pi}_{\beta^r}$
1	-0.967	0.045	-0.929	0.043	1.00
2	-0.966	0.045	-0.931	0.043	1.00
3	-0.805	0.050	-0.782	0.049	1.00
4	0.138	0.072	0.000	0.006	0.00
5	0.481	0.068	0.009	0.064	0.03
6	0.374	0.070	0.002	0.027	0.01
7	0.459	0.069	0.010	0.066	0.04
8	0.203	0.060	0.001	0.014	0.00
9	0.482	0.069	0.008	0.057	0.06
10	-0.657	0.052	-0.643	0.051	1.00
11	-0.055	0.061	0.000	0.005	0.00
12	0.269	0.060	0.001	0.018	0.01
13	0.394	0.048	0.003	0.036	0.02
14	0.314	0.050	0.002	0.024	0.01
15	-0.364	0.068	-0.002	0.026	0.01
16	-0.258	0.071	-0.001	0.010	0.01
17	-0.013	0.070	0.000	0.002	0.00
18	-0.833	0.059	-0.815	0.057	1.00
19	0.010	0.070	0.000	0.004	0.00
20	0.251	0.053	0.000	0.010	0.00
21	-0.206	0.070	0.000	0.008	0.01
22	0.224	0.070	0.001	0.018	0.01
23	-0.064	0.071	0.000	0.005	0.00
24	0.695	0.062	0.680	0.060	1.00
25	0.165	0.074	0.001	0.014	0.00
26	0.276	0.069	0.001	0.020	0.01
27	0.542	0.049	0.002	0.012	0.00
28	1.000	0.000	1.000	0.000	1.00

Note: β are the factor loadings, $\overline{\pi}_{\beta}$ are the averages of draws of the restriction indices. Subsript u (r) is for the unrestricted (restricted) model. The last element of β_u (and β_r) is set equal to 1 (std equal to 0) for identification reasons.

Table C2. Posterior means and st.d - nonparametric factor model

No	β^u	std β^u	β^r	std β^r	$\overline{\pi}_{\beta^r}$
1	-0.982	0.060	-0.972	0.079	1.00
2	-0.982	0.060	-0.973	0.079	1.00
3	-0.584	0.051	-0.591	0.062	1.00
4	0.130	0.056	0.010	0.039	0.07
5	0.418	0.058	0.427	0.060	1.00
6	0.332	0.056	0.342	0.059	1.00
7	0.404	0.057	0.417	0.061	1.00
8	0.166	0.047	0.128	0.086	0.75
9	0.410	0.056	0.422	0.061	1.00
10	-0.431	0.050	-0.437	0.056	1.00
11	-0.136	0.049	-0.041	0.071	0.29
12	0.110	0.048	0.009	0.033	0.08
13	0.160	0.041	0.144	0.065	0.88
14	0.103	0.041	0.004	0.022	0.04
15	-0.225	0.055	-0.233	0.060	1.00
16	-0.087	0.055	-0.002	0.016	0.03
17	-0.003	0.055	0.000	0.006	0.01
18	-0.458	0.057	-0.466	0.062	1.00
19	0.061	0.056	0.001	0.011	0.02
20	0.177	0.043	0.177	0.054	0.97
21	-0.157	0.057	-0.030	0.068	0.19
22	0.130	0.055	0.006	0.030	0.04
23	-0.005	0.056	0.000	0.005	0.01
24	0.293	0.056	0.300	0.059	1.00
25	0.030	0.057	0.000	0.006	0.01
26	0.057	0.056	0.001	0.011	0.02
27	0.312	0.044	0.023	0.056	0.07
28	1.000	0.000	1.000	0.000	1.00

Note: β are the factor loadings, $\overline{\pi}_{\beta}$ are the averages of draws of the restriction indices. Subsript u (r) is for the unrestricted (restricted) model. The last element of β_u (and β_r) is set equal to 1 (std equal to 0) for identification reasons.

Table C3. Posterior means and st.ds - factor stochastic volatility model

No	β^u	std β^u	β^r	std β^r	$\overline{\pi}_{\beta^r}$
1	-0.967	0.045	-0.929	0.043	1.00
2	-0.966	0.045	-0.931	0.043	1.00
3	-0.805	0.050	-0.782	0.049	1.00
4	0.138	0.072	0.000	0.006	0.00
5	0.481	0.068	0.009	0.064	0.03
6	0.374	0.070	0.002	0.027	0.01
7	0.459	0.069	0.010	0.066	0.04
8	0.203	0.060	0.001	0.014	0.00
9	0.482	0.069	0.008	0.057	0.06
10	-0.657	0.052	-0.643	0.051	1.00
11	-0.055	0.061	0.000	0.005	0.00
12	0.269	0.060	0.001	0.018	0.01
13	0.394	0.048	0.003	0.036	0.02
14	0.314	0.050	0.002	0.024	0.01
15	-0.364	0.068	-0.002	0.026	0.01
16	-0.258	0.071	-0.001	0.010	0.01
17	-0.013	0.070	0.000	0.002	0.00
18	-0.833	0.059	-0.815	0.057	1.00
19	0.010	0.070	0.000	0.004	0.00
20	0.251	0.053	0.000	0.010	0.00
21	-0.206	0.070	0.000	0.008	0.01
22	0.224	0.070	0.001	0.018	0.01
23	-0.064	0.071	0.000	0.005	0.00
24	0.695	0.062	0.680	0.060	1.00
25	0.165	0.074	0.001	0.014	0.00
26	0.276	0.069	0.001	0.020	0.01
27	0.431	0.062	0.001	0.022	0.01
28	1.000	0.000	1.000	0.000	1.00

Note: β are the factor loadings, $\overline{\pi}_{\beta}$ are the averages of draws of the restriction indices. Subsript u (r) is for the unrestricted (restricted) model. The last element of β_u (and β_r) is set equal to 1 (std equal to 0) for identification reasons.

Table C4. Posterior means (only) - structural breaks factor model

No	β_1^u	β_2^u	β_1^r	β_2^r	$\overline{\pi}_{\beta_1^r}$	$\overline{\pi}_{\beta_2^r}$
1	0.266	-0.898	0.089	-0.876	0.02	1.00
2	0.251	-0.910	0.085	-0.886	0.02	1.00
3	0.210	-0.772	0.062	-0.756	0.01	1.00
4	2.114	0.194	2.493	0.143	1.00	0.01
5	-0.019	0.427	0.001	0.348	0.01	0.34
6	0.002	0.331	0.007	0.222	0.01	0.05
7	-0.123	0.392	-0.020	0.292	0.01	0.18
8	0.028	0.207	0.007	0.148	0.01	0.01
9	-0.093	0.428	-0.012	0.348	0.01	0.68
10	0.270	-0.755	0.089	-0.730	0.02	1.00
11	0.208	-0.111	0.052	-0.077	0.02	0.00
12	0.205	0.231	0.048	0.173	0.01	0.01
13	0.000	0.382	-0.001	0.344	0.01	0.34
14	0.000	0.347	0.001	0.288	0.01	0.14
15	2.553	-0.330	2.983	-0.263	1.00	0.09
16	-1.134	-0.250	-1.333	-0.174	1.00	0.02
17	0.107	0.036	0.016	0.023	0.01	0.00
18	2.084	-0.802	2.429	-0.784	1.00	1.00
19	3.028	0.058	3.539	0.049	1.00	0.02
20	0.035	0.304	0.012	0.235	0.01	0.05
21	0.066	-0.218	0.015	-0.144	0.01	0.00
22	-0.020	0.249	-0.004	0.167	0.01	0.01
23	0.432	-0.085	0.112	-0.053	0.08	0.00
24	-2.193	0.673	-2.584	0.656	1.00	1.00
25	-3.343	0.095	-3.885	0.077	1.00	0.00
26	-2.910	0.223	-3.399	0.178	1.00	0.00
27	0.020	0.453	0.001	0.332	0.01	0.77
28	1.000	1.000	1.000	1.000	1.00	1.00

Note: β are the factor loadings, $\overline{\pi}_\beta$ are the averages of draws of the restriction indices. Subsript u (r) is for the unrestricted (restricted) model. The last element of β_u (and β_r) is set equal to 1 (std equal to 0) for identification reasons.