Explaining the logic of pure preference in a neurodynamic structure

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EXPLAINING THE LOGIC OF PURE PREFERENCE IN A NEURODYNAMIC STRUCTURE

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SUMMARY:

This paper uses Category Theory to integrate a nonlinear, nonhomogeneous ordinary differential equation system into an input/output representation in an attempt to capture the mechanism behind the formation of pure preference in humans. The model shows that the human brain belongs to the class of functions U ε $C^2(R^3, R)$. In addition, it shows that there exists an emerging factor, e, absent from Boolean logic, but is *sine qua non* for expressing a preference. The factor, e, may be associated with 'judgement' which, in turn, may neatly subsume 'consciousness', the arrival of new information, and cases of selection under risks and uncertainty.

KEYWORDS: Input/output, Dynamo, Universal Unfoldings, Emergent factor, Awareness, and Preference.

1-INTRODUCTION

Over the centuries, the study of the structure of human preference has engaged the attention of a very diverse group of scholars, namely, philosophers, psychologists, and economists. I single out these three groups because a significant portion of their respective literature is devoted to that topic. However, as regard the fundamental question: Why, from two suitably matched items, one comes to be preferred to the other? No intra or inter-group consensus has emerged to date. This paper will try to provide an answer, while focusing on the construct of the economists and on the problems to which it gives rise.

The pioneers of the economists' construct recognized quite early on the complexity involved, but in their eagerness to build a social science, they decided to ignore the 'brain', out of which nothing could have been directly observed and understood, to rely on Boolean logic to build a construct they thought would have allowed them to make a multiform and frontal attack on the slipperiness of the subject. Consequently, they approached it either as 'pure preference', or as some related rubrics, such as 'selection of alternatives under certainty, risks and uncertainty', 'decision theory', 'rational choice', etc., or as an offshoot of the latter, known as 'utility theory'. But, regardless of the way they approached the subject, numerous difficulties, to be discussed shortly, remain unresolved.

Many are those who see this as an impasse. In other disciplines (e.g. physics), the way to overcome an impasse is to make either a partial or complete break with the past. Should economics follow suit? I think so, but when and how will such a consensus ever be reached? No one can say at this juncture. At any rate, the purpose of this paper is to suggest a partial break that requires, as a first step, a closer attention to the topology of the structure that generates pure preference in humans. The paper starts with the 'observables', but will not follow the behavioristic route to conjectures. Instead, it will integrate three well-studied concepts that allow the introduction of time, an important missing element in my view. To do so, I will use Category Theory to merge a dynamic structure with an emerging property (analyzed by Beltrami, 1987) with an Input/Output structure (in the sense of Casti (1989)), and next rely on the Classification Theorem (due to Thom, 1975) to mathematically classify the brain and help analyze the results.

Admittedly, the brain is an exceedingly complex object, and I do not claim to have extensive knowledge about it, although, to date, no one has come up with a robust theory of the brain. However, no violence is done by comparing it to a dynamic input/output structure, because it is largely so, among other things. If, as I postulate, pure preference is indeed time-dependent, then there is a good chance that a differentiable mapping that takes sensory inputs and memory at time t_0 and delivers an output some time t_1 will increase our understanding. If further its results are cast in terms of a universal unfolding, they will provide still a better understanding of the formation of pure preference; furthermore, the 'universal unfoldings' identified by Thom comprise a very broad class of functions whose solutions are easy to interpret. Anyhow, this is the plan of action for peering into brains' functionalism, although, I should perhaps state right at the outset that because the analysis starts with observables, I am not restricting myself to the case of 'complete certainty'; for, in reality, I am aiming at a result that will be generic enough to subsume the concepts of risks and uncertainty.

2-PRELIMINARIES

Unable to observe brains' function, economists made a foray into Boolean logic to study preference as an axiomatic foundation on which to rest their theory of utility. That effort may be said to have started in earnest with Bernoulli (1738), passing through Savage (1954) and Arrow and Debreu (1954), to arrive finally at Debreu (1959) in which the construct receives its final form. But subsequent observed anomalies, such as 'preference reversal [(1)] obviously pointed to some underlying weakness in their representation, weakness whose nature has never been identified. It was therefore carried over to the utility apparatus in the forms of paradoxes (see, for example, the 'Allais Paradox'[(2)]), or surprises detected in experimental designs such as the Ultimatum Game [(3)]. To see how these difficulties may have arisen, let us turn to the underlying axioms.

Using economists' terminology and symbols, the construct may be succinctly represented as follows. There exists a binary relation, P, on a non-empty set X of commodity bundles whose elements are, say, x, y, z, etc., which may also be viewed as decision alternatives or outcomes of choice. To describe the P-relation, two main symbols, (xPy) and (xIy), are used throughout. The first says x is preferred to y and the other represents the relation of indifference. Following von Wright (1963), two axiomatic principles follow:

i) $(xPy) \rightarrow \sim (yPx),$

ii) $(xPy) \rightarrow (xPz) \vee (zPy).$

That is, by i) P is asymmetric, and by ii) if x is preferred to y, then x is either preferred to any third alternative z or z is preferred to y. If i) and ii) hold, the indifference relation is expressed as:

iii) $(xIy) = \sim (xPy)$ and $\sim (yPx).$

Alternatively, from i) and ii), economists claim that the notion of strict preference obtains, which means that x is strictly preferred to y if and only if x is as good as y and y is not as good as x. Then strict preference is supposed to be irreflexive and transitive, making the indifference relation reflexive, symmetric, and transitive. These latter assumptions are not supported by empirical tests. And, in addition, nuanced or restatements of i) can not obviate the problem arising from the assumption of completeness and transitivity in ii). To put it more succinctly, let us return to von Wright's representation above. That is, if i) and ii) hold, the P-relation is connected and transitive, and the I-relation is, therefore, reflexive, symmetric and transitive. But consider ii), while leaving aside for the moment the potential role of agents' 'state of mind' to focus on products differentiation in modern markets. Producers' obsession with product differentiation aims precisely at preventing the consumers' 'information set' from being complete. Hence, producers' behavior itself renders ii) too strong an assumption. Therefore, the P-relation is not connected, meaning the ordering is not complete, and hence the transitivity of the I-relation does not obtain. Even if asymmetry and transitivity are imposed outright on the P-relation, the ranking order remains partial rather than complete and preference equality can not be recovered in iii). Nevertheless, economists went around these difficulties by imposing the additional axiom of continuity, which in essence, requires that the agent behave consistently. Then they posit the following: there exists, given the properties of X and P, a utility function u: $X \rightarrow R$, where R is the set of real numbers, if, for (xPy), $u(x) \geq u(y)$ for the agent whose strict preference is just described; although we should also note in passing that the identity of the agent and time are unspecified. But when intransitive choices are observed, the agent is accused of being irrational. Other related but unanswered questions are: As aggregation follows, is the decider representative? If not, how about decision by majority rule? And are agents' preferences valid for all future times?

Criticisms abound, but roundabout efforts to circumvent them do not convince (see, Sonnenschein, 1965; Fisfburn, 1970; Loomes and Sugden, 1982, among others), as anomalies continue to surface. What I think is happening in the above construct is that agents are unable to comply due either the lack of or timely information and to what I term "emotional drift"; I will come back to the latter. Indeed, what is frequently observed is $y^*(t_0)$ as outcome at time t_0, and $y^{*\prime}(t_1)$ at t_1 later, or even $y^{**}(.)$, where $[y^*(.),\text{and } y^{*\prime}(.)]$ ε R_{++} and $y^{**}(.)$ ε R_{--} ; R_{++} and R_{--} are subsets of real numbers, defined as: $R_{++} = \{y \text{ ε } R \mid y > 0\}$, and R_{--} is its negative analog. In other words, y^* may vary but stays in R_{++}, or may jump to R_{--} upon a change in some parameter, absent from the economists' representation. It is just like observing (xPy) at t_o and (yPx) at t_1. Invoking nonconvex preferences here (see, Barthold and Hochman, 1988) explains nothing other than another accusation of irrationality. Such anomalies have led von Wright (1963) to conclude: "Pure preference is in reality a 'value judgement' and is, in addition, relative since it may vary over time." The very nature of the observables and the above comment by von Wright define in essence the central question for us, namely, what is the topology of the object (the brain) that produces these observables? But beforehand, let us see what other scientists have found in their studies of the brain.

3-ADDITIONAL REMARKS

As philosophers, psychologists, and economists are unable to arrive at a consensus, neuroscientists have joined in, because they happen to have at their disposal technologies that simply were not available previously. With the advent of computed tomography (CT) scans, positron emission tomography (PET) scans, magnetic resonance imaging (MRI), functional magnetic imaging (fMRI), magneto-encephalography (MEG), and trans-cranial stimulation (TMS), neuroscientists are able to observe the inside of the brains of deciders in a non intrusive manner. They do not all agree on explanations, but there is no disagreement on the following:

P1 Sequential changes in the permeability of nerve cell membrane to positive ions produce 'action potentials' up to about 100 millivolts via the system of axons and synapses, and flows . or signals that travel at speed up to 525 feet per second;

P2 starting with the concept of 'brain-assembly', researchers have identified permanent brain circuitries, associated with perception, memory, and qualia. The processing that goes on, from lower to higher regions, exists for all sensory information. In other words, extensive circuitries are made of specialized and permanent neurons;

P3 the permanent neurons are organized in neural networks consisting of layers of inter-connected processors via the system of synapses. They are, input, intermediate, and output processsors. Thus, input signals from different channels accumulate in the intermediary layer until a critical level is reached, then the cell fires an output signal;

P4 observed bottom-up and top-down effects have led many neuroscientists to conclude that an emergent brain property allows humans to interpret raw data, make decisions, and initiate actions.

P1 and P2 may be extended with an example. Photons bouncing on an incoming automobile, say, first reach the eyes of an observer, where they form a pattern. Photoreceptors in the retina transform light waves into neural signals that are next sent to the lateral geniculate nucleus in the thalamus and the visual cortex in the occipital lobe. There, the rough shape of the automobile is recognized. Sub-patterns are next sent to a higher region for color identification, then to higher regions for determining motion and direction, and still to other regions where the identity of the automobile is encoded. Similar processing stream exists for sounds, touch, and other sensory information. Therefore, what one sees, feels or believes depends on extensive circuitries arising out of experience that stands to interpret raw information [(4)]. Hence, P1and P2 refer to potentials that must be minimized, implying flows through electrical circuits. P3 leads to the notion of an input/output structure and the delay, τ, between inputs and awareness or consciousness; for more on this, see Carter (2000, ch.3). The larger is the input, the higher the potential, and above a certain threshold, consciousness occurs. Finally, P1 to P4 suggest that the brain operates as a dynamo.

Given these and the difficulties mentioned in Section 2, this paper proposes to tackle the problem with a dynamic input/output model in an attempt to uncover the nature of pure preference. The basic scheme is an integration of a dynamic model with emergent property in line with the previous work done by Beltrami (1987) in this regard into a modified version of Casti's (1989) representation. The analysis to follow will then rely on Thom's Classification Theorem on the general class of Universal Unfoldings. The advantage of doing so is that the dynamic structure is rescaled so that only three coefficients remain. Further, if relying on a dynamic input/output structure is intuitively appealing enough, it may not be so for universal unfoldings? Then, let me then remark at the outset that Thom's Theorem is of great theoretical and empirical significance in this kind of analyses, as it considers a broad class of mathematical objects in which any two members of the same class can be smoothly deformed into one another by means of an appropriate change of coordinates in the underlying space; all such coordinate changes are origin-preserving diffeomorphisms.

4-THE MODEL

For the sake of clarity, let me first precise a few concepts. Consider a general class of smooth functions:

(1) $U(y\ (t), \alpha)\ \varepsilon\ C^{\infty}\ (\mathsf{R}^{v+c}, \mathsf{R})$,

where U(.) is a potential, v is the corank of U(.), c is the codim of U(.), and y(.) and α are, respectively, sets of variables and parameters, but y(.) is scalar valued. Thom studies the class for which $v \leq 2$, codim ≤ 5; hopefully, our U(.) will fall within that range. More generally, however, for any such U(.), the equilibrium manifold, $S\ \varepsilon\ \mathsf{R}^{v+c}$, defined by $\partial U(.)/\partial y(.) = 0$, is smooth. Let the orthogonal projection, Þ, of S onto the plane $\alpha\ \varepsilon\ \mathsf{R}^{c+v}$, whose image under the projection is Ŝ, where the tangent to S is normal to the plane, gives the bifurcation set B, defined as:

(2) $$B = \{\alpha_c : \partial U(.)/\partial y(t) = 0,\ \det[\partial^2 U(.)/\partial y^2(.) = 0, \quad c = 1, 2, \ldots, 5\}.$$

Then S can be deformed locally into Ş in such a way that Ŝ is deformed into Š. This holds for small perturbations of U.

At first sight, the observables given in Section 2 suggest that the unknown U(.) is of the type, $U(y(.), \alpha)\ \varepsilon\ C^2\ (\mathsf{R}^3, \mathsf{R})$, called the cusp catastrophe, but that is only a first impression, because the observed behavior of the unknown U(.) does not quite obey the dictates of the ordinary cusp catastrophe. Ours must be some transformed Û(.) for the following reasons. The observables of $U(.)\ \varepsilon\ C^2\ (\mathsf{R}^3, \mathsf{R})$ are characterized by:

i) Outcomes, y*, y**, fall within a bounded interval u about the origin such that $\{y^*\ \varepsilon\ u\ \varepsilon\ \mathsf{R} \mid 0 < y^* \leq b\}$, $\forall t$, and $\{y^{**} \varepsilon\ u\ \varepsilon\ \mathsf{R} \mid -b \leq y^{**} < 0\}$, $\forall t$, where –b and b are bifurcation points;

ii) motion is uniform over u, except of course at bifurcation points. Hence a hysteresis cycle is observed.

Whereas what is observed in the present case is:

iii) y* depends on values of a specific parameter $-1 \leq e \leq 1$such that each change in e over the time interval $[t_0, t_1]$, where $t_1 > t_0$, yields unique values of $y^* \varepsilon\ u\ \varepsilon\ \mathsf{R}_{++}$, or unique values for $y^{**} \varepsilon\ u\ \varepsilon\ \mathsf{R}_{--}$over the same time interval;

iv) motion over u is not uniform. That is, y* may zigzag in R_{++}, or jump to y** in R_{--} upon a change in e over the time interval. Hence Û(.) is bimodal in the sense of Zeeman (1977).

Put differently, observations show that outputs, y* and y**, assume fixed values for a given value of the control parameter e. From there, it may safely be inferred that y* and y** arise from some Û (.) whose characteristics we want to investigate.

Beforehand, I will explain the input/output structure. It consists of a number of maps and sets. They are:

$\alpha = \{a_1, a_2, \ldots, a_n\}$, the set of stimuli,

$\ddot{Y} = \{y^*, y^{**}\}$, the set of observables or outcomes,

Y = the state space, or the set of flows $\{y_1(t), y_2(t), e\}$,

$\Psi: \alpha \rightarrow \ddot{Y}$, $\theta_1: \alpha \rightarrow Y$, $d: Y \rightarrow \square_1$, $h_1: \square_1 \rightarrow \ddot{Y}$, $h_2: \ddot{Y} \rightarrow x$, and $\theta_2: Y \rightarrow x$.

Ψ maps input into output; θ_1 is an 'into' map of input stimuli into state space; d and h_1 are 'one-to-one' and 'onto' mappings of flows into equilibrium values; h_2, and θ_2 are one-to-one and onto maps from $\hat{U}(.)$ to U(.) as shown in Figure 1.

For the time being, I only note that maps θ_1, d and h_1 are necessary to prove:

Proposition 1: The observables y, y*', y**, etc. are points on a bimodal manifold arising out of a dynamic structure consisting of 2 functions and a stochastic control variable.*

Figure 1:The Input/output Structure

$$\begin{array}{ccc} \alpha & \Psi & \ddot{Y} \\ & h_1 & \\ \theta_1 & \square & h_2 \\ & d & \\ Y & \theta_2 & x \end{array}$$

While $\theta_2 = h_2 \circ (h_1 \circ d)$ is necessary to prove:

Corollary 1: There exists at least one transformation of the manifold of Proposition 1 to $\hat{U}(x, \alpha, \beta) \varepsilon C^2(R^3, R)$.

Let me remark that in this paper, I suppose the control factor, e, to be a dynamo effect, but it could arise from other causes. For example, it could arise out of an electromotive force created by the sodium and potassium ions down the axons. Or the output impulse in the brain may in reality be an electromagnetic wave. Only further research can provide a definitive answer. For the time being though, I will proceed on the assumption that it is due to a dynamo effect. As alluded to in Section 3, a dynamic structure that is capable of exhibiting emergent property, e, (which in turn may be associated with judgement, additional information, or even emotions from the limbic systems of human, here termed emotional drift) is the rescaled nonlinear, nonhomogeneous ordinary differential equation system of Beltrami (1987), inserted in Figure 1as map, d, where:

$$d = \{(y_1, y_2, \hat{y}, t) | \mathring{y}_1 = y_2\hat{y} - a_1y_1 \text{ and } \mathring{y}_2 = (1 - y_1\hat{y}) - a_2y_2, \forall y\}.$$

For tractability, the differentiable mapping, d, may be written as:

$$(3) \qquad d: \; (-a_1 y_1 + \hat{y}\, y_2) \rightarrow \mathring{y}_1 \; ,$$

$$(1 - y_1 \hat{y} - a_2 y_2) \rightarrow \dot{y}_2$$

where $\hat{y} = (y_1 + e)$, and the dot refers to differentiation with respect to time. In equilibrium, $\dot{y}_1 = \dot{y}_2 = 0$, substituting, rearranging, and applying map $h_1:[a_1 \square_1 /(\square_2 + e)] \rightarrow \square_2 = y^*$ in (3) give:

(4) $$(h_1 \circ d): Y \rightarrow y^* ;$$

the symbol, $\circ$, stands for composition and recalling that $e \,\varepsilon\, \mathrm{R} \,|\, (-1 \leq e < 0 \;;\; 0 < e \leq 1)$; $a_1 a_2 \neq 1$. Upon the application of h_1, the resulting the equilibrium equation is:

(5) $$(y^* + e)^2 y^* = e + (1 - a_1 a_2)\, y^* .$$

I now denote the cubic term on the left-hand side of (5) as m(y*) and the linear one on the right as g(y*). Then the equilibrium can now be read graphically at the intersection of m(y*) and g(y*) in Figures 2 and 3 for different values of e.

Figure 2: $e = 1$; $a_1 a_2 = 3/4$

Figure 3: $e = -1$; $a_1 a_2 = 3/4$

m(y*)

m(y*)

g(y*)

y**

y*

y*

g(y*)

As shown, for a given value of $a_1 a_2 = 3/4$ and $e = 1$, the cubic term in (5) has critical points at $-1/3$ and -1. The unique equilibrium is $y^* = 1/2$, shown in Figure 2. When $e = -1$ in Figure 3, the cubic term admits critical points respectively at 1/3 and 1 and the unique equilibrium is now at $y^{**} = -1/2$. Thus, as e goes from 1 to -1, both curves abruptly shift, yielding a new equilibrium in the second quadrant. Hence e is the stochastic parameter locating equilibria on the bimodal manifold, proving Proposition 1.

In other words, there are two equilibrium surfaces, and the cubic term is the loci of equilibria, $\forall y$. However, if e were to take a zero value, the cubic term would vanish according to (5). This would be in conformity with P4, in the sense that for $e = 0$, equilibria are simultaneously at $y^* = \pm\, (1 - a_1 a_2)^{1/2}$; that is, two real numbers whose sum is zero. From there, we conclude that without awareness, there can be no pure preference; but I will return to this later after dwelling a little more on the possible cause of the emergent factor, e.

The emergent property, e, in this framework, arises naturally, I believe, as a torque applied to an electrical potential, or as the sums of components of directional vectors are crossed. In other words, equation (3) defines a vector field on the unknown Û and the operator d crosses two orthonormal vectors, y_1 and y_2, yielding e. A natural system that does that is a dynamo, hence the reason for this approach.

How can this physics explanation be cast into a biological one? The emergent factor, e, arises from neuronal flows from the brain stem to the cerebral cortex and its related circuits in response to input stimuli or memory. Neuroscientists call these circuits ‘brain assemblies. When these assemblies are aroused, e is not zero. Put differently, the level of consciousness depends inversely on the degree of synaptic inhibition.

Turning now to Figure 1, the fact that map θ_1 is only ‘into’ shows that the system is indeed an input/output system, as input signals can only be recovered from memory. Large populations of neurons or assemblies act in synch to represent and form memory. Subsets of these populations in the hippocampus, for example, respond to different aspects of a given input. Then, one may wish to consider memory as a valid input channel in biological systems. This would be quite correct, but here we have already included memory in the specialized neurons living in state space, Y; at any rate, with no external inputs whatsoever, the system described in Figure 1 can exist only for a short interval Δt. With external stimuli, the diagram commutes only on the right-hand side, as θ_2 has a unique inverse.

This result seems to imply that the Boolean logic assumed by economists might not be applicable to human preference formation. As shown above, $y^* = f(e)$, is the missing element and it might very well explain many an anomaly. Then, it also follows that there exists an optimal e. This can be verified by finding the slope of that relation in equilibrium. Total differentiation of (5) gives:

(6) $dy^*/de = (1- 2ey^*- 2y^{*2}) / [3y^{*2} + 4ey^* + e^2 – (1- a_1a_2)] \gtreqless 0.$

For $a_1a_2 = 3/4$, and $1 \geq e > 1/4$, $dy^*/de < 0$, meaning that y can be increased as e is decreased; but for $e \leq 1/4$, $dy^*/de > 0$. Then, for: $a_1a_2 = 3/4$, $e^* = 1/4$, $dy^*/de^* > 0$, but the constrained solution to (5) is: $y^*= 0.5768$….I will return to this shortly. But, already, the structure of $y^*= f(e)$ (Figure 4) shows that there exists a limit to attempts at manipulating preferences.

Figure 4: The Structure of $y^* = f(e^*)$

y^*

½

e*

- ¼ ¼

- ½

Thom (1983, 107-08) has argued that more often than not " An elementary catastrophe is embedded in a larger system with time variable." We can verify that assertion by first applying h_2 to (5), where h_2: $y^* \rightarrow [x^* - (2/3\ e)]$, and that will identify the family of unfoldings to which $\hat{U}$ belongs. The application gives:

(7) $- x^{*3} + (1 - a_1\ a_2\ - e^2/3)\ x^* + [\ e/3\ (1 + 2a_1\ a_2\ - 2e^3/\ 27)] = 0.$

Letting the splitting factor, $\alpha_1 = (1 - a_1a_2 - e^2/3)$ and the normal factor, $\alpha_2 = [e/3\ [1 + 2a_1a_2) - 2e^2/9]$, then (7) becomes:

(8) $$- x^{*3} + \alpha_1 x^* + \alpha_2 = \partial\hat{U}(.)/\ \partial x^* = 0;$$

since (8) is the gradient of $\hat{U}(x, \alpha_1, \alpha_2)$, then $\hat{U}(.)\ \varepsilon\ C^2\ (R^3, R)$, proving Corollary 1.

Now if (8) is equivalent to (5), then the optimal solution to (8) should equal y* via the inverse of h_2. The solution to (8) is $x^* = 0.7434\ldots$. Then:

$$y^* \approx x^* \text{ or } y^* + 2e/3 = 0.5768\ldots + 1/6 = 0.7434\ldots$$

Hence, (8) and (5) are equivalent, vindicating Thom (1983).

Now, one may ask what the usefulness of (8) or (7) is? To answer this question, consider the gradient of the standard cusp catastrophe,

(9) $$\text{grad}\ (U(x, \beta_1, \beta_2) = -x^3 + \beta_1 x + \beta_2 = 0.$$

An immediate consequence is that:

(10) $$[(\nabla U(.))(\partial x(t)/\partial t] = \|\nabla U\|^2$$

is negative on some open subset, Ω, at x* where it is zero. There can be no closed orbits, x* minimizes U on Ω, ∇U is zero at x*, and therefore, the function $V(x) = U(x) - U(x^*)$ is Liapunov on Ω. We can now compare U(.) and $\hat{U}(.)$. In the case of U(.), both β_1 and β_2 are non zero constants. Then for higher values of both, we have a unique equilibrium. For low β_2 and high β_1 values, there are two stable equilibria, one in R_{++} and the other in R_{--}. But, when we substitute α_1 and α_2 given above, we have $\hat{U}(.)$, written as:

(11) $$x^3 = \alpha_2 + \alpha_1 x.$$

Then for:

e = 1, there is a unique equilibrium, $x^* \varepsilon\ R_{++}$, $\forall x$;
e = -1, there is a unique equilibrium, $x^{**}\ \varepsilon\ R_{--}$, $\forall x$
e = 0, equilibria are at x* =1/2, -1/2, and 0; the zero value is unstable.

This then confirms what was alluded to above, namely, no preference can be expressed if e = 0. Observe further that, for a given value of the product of the coupling parameters, a_1a_2, the Jacobian of $\hat{U}(.)$ is:

$$(12) \qquad (J) = \begin{matrix} \partial^2\hat{U}/\partial x^{*2} \\ \partial^2\hat{U}/\partial\alpha_1 \\ \partial^2\hat{U}/\partial\alpha_2 \end{matrix} = \begin{matrix} 3x^2 - \alpha_1 \\ - x \\ - 1 \end{matrix} .$$

Smooth changes in equilibria occur on the set defined by $\partial^2\hat{U}/\partial x^{*2} = \partial\hat{U}/\partial x^* = 0$. Hence (8) and (12) can be used to eliminate x^*. With α_1, α_2, and a nonzero e, the geometrical structure of B (or the cusp curve on the projected plane), given at the beginning of Section 4, is:

$$(13) \qquad (1- a_1a_2 – e^2/3)^3 = 27/4 \, [(1 + 2a_1a_2)e/3 – 2e^3/27]^2.$$

Since for U, $\beta_1^3 = 27\beta_2^2/4$, equation (13) gives the cusp curve $\hat{U}(.)$ and suffices to show the difference between U and $\hat{U}$.

Equation (7) is instructive in another way. We have two conditions to guide us: i) $dy^*/de \gtreqless 0$, and ii) $dx^*/de \gtreqless 1$. From (5): $a_1a_2 = 3/4$, $e = 1/4$, $y^* = 0.5768…$, $dy^*/de > 0$; from (7):$a_1a_2 = 3/4$, $e =1/4$, $x^* = 0.7434….$, $dx^*/de < 1$. But at $e = 1/3$, $dx^*/de = 1$. This indicates that we should examine the solution to (5) at $e = 1/3$ as well. Then for $a_1a_2 = 3/4$, $e = 1/3$, $y^* = 0.5768….$ and $dy^*/de > 0$, but non constrained $dy^*/de = 0$ at $e = 1/4$, $y^* =0.5930…$, and $dy^*/de = 0$ at $e = 1/3$, $y^* = 0.5598….$ This tells us that the non constrained $y^* = f(e)$ picks at a higher value of y^* than that shown in Figure 4, but also falls off more rapidly as well. From (5), we can then conclude the following: the best solution to (5) for $a_1a_2 = 3/4$, is $e^* = 1/4$, and $y^* = 0.5768….$In other words, y^* can not be increased beyond 0.5768… even if we move to $e = 1/3$.This last result has implication for advertising, as I will explain in the next section.

5-DISCUSSION

The nature of pure preference in humans is not well-known either to philosophers or scientists. However, there is a consensus to the effect that there can be no pure preference in the absence of something that scientists call consciousness. According to Carter (2000), over the last ten years, some 30,000 papers have been published on the topic. In the words of one scientist, all these papers slither around the subject, discuss various neural correlates, but little about consciousness. In other words, consciousness has not yielded to scientific investigation. David Chalmers (2002, p.50) puts it this way: “Consciousness poses the most baffling problem in the science of the mind. There is nothing that we know more intimately, . . ., but there is nothing that is harder to explain.” Here the answer is clear; it is due to the emerging stochastic factor.

This is a good place to emphasize that brain activity ceases only at death, but here our approach provides a simple explanation of the difference between sleep and wakefulness. During sleep, the limbic system remains active, but flows to the cerebral cortex is negligible. The subject is not conscious because e is close to zero. Our approach also explains how anesthetics work That is, by simply reducing the size of neuronal assemblies; they reduce the value of e below the threshold necessary for consciousness.

Conventional psychologists concentrate on building *intuitive* models of the mind. Some philosophers have traditionally dealt with the mind as though it is unconnected with the brain, while others (the monists) regard consciousness and the whole universe as one. Economists, for their part, ignore the brain to rely on Boolean logic; but their approach is unable to account for the observed pathologies in the behavior of economic agents.

This paper proposes a unifying approach that incorporates recent discoveries in neuroscience into a dynamic input/output structure that seems to account for the missing element, namely, an emergent property that can be associated with judgement (consciousness for some) without which no preference can be expressed. Consequently, the monists may associate the element, e, to the kinds of emergent property observed in the unfolding of the laws of nature, except gravity of course. Physicists, e. g., Sir Roger Penrose, who suggest that consciousness may be the interface between the fundamental quantum world of information and the classical world, may choose to view 'e' as an 'information field'. Finally, economists who are in dearest need for a more robust setup may be able to rescue the demand side of their traditional model. For, if indeed the human brain operates like a dynamo, then individual preference may be indexed to a real number for each good or alternative, that is, e_{ij}, or agent i's preference for good or alternative j. Next, set of indices may safely be established for comparables alternatives. In fact, e_{ij} may remain unchanged for quite some time, as in the case of an agent's attraction to the color red, as long as there is no change in e; this would appear as fixed preference. Further, once a set of indices is on hand, the P-relation could really account for observations, in the sense that the observed hiccups discussed in the Introduction would not be viewed as pathologies. For example, preference reversal may be explained by a jump; or the subject may be thinking of the consequence of his or her choice. The indifference relation may be explained by $e = 0$ in equation (7); the result of the Ultimatum Game may be explained by an emotional understanding of fairness, followed by a flip in e_i. This is to say that the rationality of the economic agent would become mental-state and time dependent, as suggested long ago by Elster, 1998).

Other features of the model are that the factor, e, is a missed bag, for, it can be associated with judgement or new information that perturbs e, or even cause e to change polarity. In the event that it is associated with new information, then, Figures 2 to Figure 4 would confirm what I have discussed elsewhere (Dominique, 1988), namely, there is little if any increasing returns to advertising. In fact, a closer look at (6) and Figure 4 shows none. Hence, advertising blitz or excessive propaganda might be a total waste, because for $1/4 \leq e \leq 1/3$, preferences can not be increased further; that is, one cannot get more than 57.68 percent of what goes in, depending of course on the value of a_1a_2. This model also shows that a flip in e due to new information is a convincing explanation to speculative bubbles, which remains to date one of the most poorly understood and mysterious economic anomalies. Finally, if e is viewed as 'judgement', then our results can easily accommodate the concept of risks and uncertainty.

NOTES

(1) Preference reversal is an experimentally observed phenomenon in which participants are asked to choose between two suitably matched alternatives and to state the lowest amount of money they would be willing to accept in exchange for the selected alternative. The surprise comes when they place the lowest amount on the chosen one. The phenomenon was first observed by Litchenstein and Slovic (1971).

(2) The Allais Paradox deals with examples that are not compatible with the conclusions of Bernoulli-based de-seigns. For more, see the entry: 'Allais Paradox' in the *New Pelgrave Dictionary of Economics* (1087, 80-82.

(3) This experiment involves two players, A and B. A, say, is given a certain sum, y, and he or she must give a certain percentage, k, of his choosing to B. If B accepts ky, then A keeps (1 – k)y. However, if B refuses ky as she deems k too low, both players end up with nothing. The experiment shows that if k is set below 20 percent, the offer is rejected. This formulation is given in Thaler (2000).

(4) Many of these findings are discussed in the popular press. See, for example, *The New York Times*, June 17, 2006 and Nov. 22, 2006; see also *US & World Report* of Oct.18, 2006.

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