Long-run Consumption Risk and Asset Allocation under Recursive Utility and Rational Inattention

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The University of Hong Kong, University of Virginia

8 July 2013

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Abstract

We study the portfolio decision of a household with limited information-processing capacity (rational inattention or RI) in a setting with recursive utility. We find that rational inattention combined with a preference for early resolution of uncertainty could lead to a significant drop in the share of portfolios held in risky assets, even when the departure from standard expected utility with rational expectations is small. In addition, we show that the equilibrium equity premium increases with the degree of inattention because inattentive investors with recursive utility face greater long-run risk and thus require higher compensation in equilibrium. Our results are robust to the presence of correlation between the equity return and the RI-induced noise and the presence of non-tradable labor income.

JEL Classification Numbers: *D53, D81, G11.*

Keywords: *Rational Inattention, Recursive Utility, Portfolio Choice, Asset Pricing.*

*We thank Pok-sang Lam (the Editor) and two anonymous referees for many constructive comments and suggestions. We are also grateful for useful suggestions and comments from Hengjie Ai, Michael Haliassos, Winfried Koeniger, Jonathan Parker, Chris Sims, and Wei Xiong, as well as seminar and conference participants at European University Institute, University of Warwick, University of Hong Kong, Utah State University, the North American Summer Meetings of the Econometric Society, and the SED conference for helpful comments and suggestions. Luo thanks the Hong Kong General Research Fund (GRF) and HKU seed funding program for basic research for financial support.

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1 Introduction

The canonical optimal consumption-portfolio choice models implicitly assume that consumers and investors have *unlimited* information-processing capacity and thus can observe the state variable(s) without errors; consequently, they can adjust their optimal plans instantaneously and completely to innovations to equity returns. However, plenty of evidence exists that ordinary people only have *limited* information-processing capacity and face many competing demands for their attention. As a result, agents react to the innovations slowly and incompletely because the channel along which information flows – the Shannon channel – cannot carry an infinite amount of information. In Sims (2003), this type of information-processing limitation is termed "Rational Inattention"(henceforth, RI). In the RI framework, *entropy* is used to measure the uncertainty of a random variable, and the reduction in the entropy is used to measure information flow.[1] For finite Shannon channel capacity, the reduction in entropy is bounded above; as capacity becomes infinitely large, the RI model converges to the standard full-information rational expectations (RE) model.[2]

Luo (2010) applies the RI hypothesis in the intertemporal portfolio choice model with time separable preferences in the vein of Merton (1969) and shows that RI alters the optimal choice of portfolio as well as the joint behavior of aggregate consumption and asset returns. In particular, limited information-processing capacity leads to smaller shares of risky assets. However, to generate the observed share and realistic joint dynamics of aggregate consumption and asset returns, the degree of attention must be as low as 10 percent (the corresponding Shannon capacity is 0.08 bits of information); this number means that only 10 percent of the uncertainty is removed in each period upon receiving a new signal about the aggregate shock to the equity return. Since we cannot estimate the degree of average inattention directly (that is, without a model), it is difficult to determine whether this limit is empirically reasonable. Indirect measurements of capacity uncover significantly higher channel capacity; we discuss them explicitly later in the paper.[3]

[1]Entropy of a random variable X with density $p(X)$ is defined as $E[\log(p(X))]$. Cover and Thomas (1991) is a standard introduction to information theory and the notion of entropy.

[2]There are a number of papers that study decisions within the LQ-RI framework: Sims (2003, 2006), Adam (2005), Luo (2008, 2010), Maćkowiak, and Wiederholt (2009), and Luo and Young (2010a,b).

[3]The effect of RI on consumption growth and asset prices in the standard expected utility framework has been examined in Luo and Young (2010a). That paper showed that an agent with incomplete information-processing ability will require a higher return to hold a risky asset because RI introduces (i) higher volatility into consumption and (ii) positive autocorrelation into consumption growth. In addition, Luo and Young (2010b) examine how the risk-sensitive preferences, a special case of Epstein-Zin recursive utility, affects consumption, precautionary savings, and the welfare of inattentive agents.

The preferences used in Luo (2010) are known to entangle two distinct aspects of preferences. Risk aversion measures the distaste for marginal utility variation across states of the world, while the elasticity of intertemporal substitution measures the distaste for deterministic variation of consumption across time; with expected utility these two attitudes are controlled by a single parameter such that if risk aversion increases the elasticity of intertemporal substitution must fall. The result in Luo (2010) shows that RI interacts with this parameter in a way that raises the apparent risk aversion (lowers the apparent intertemporal substitution elasticity) of the investor; however, it is unclear which aspect of preferences is actually being altered. As a result, interpretation of the results is ambiguous. Here, we develop an RI-Portfolio choice model within the recursive utility (RU) framework and use it to examine the effects of RI and RU on long-run consumption risk and optimal asset allocation. Specifically, we adopt preferences from the class studied by Kreps and Porteus (1978) and Epstein and Zin (1989), where risk aversion and intertemporal substitution are disentangled. These preferences also break indifference to the timing of the resolution of uncertainty, an aspect of preferences that plays an important role in determining the demand for risky assets (see Backus, Routledge, and Zin 2007). Indeed, it turns out that this aspect of preferences is key.

For tractability reasons we are confined to small deviations away from the standard class of preferences. However, we find that even a small deviation from unlimited information-processing capacity will lead to large changes in portfolio allocation if investors prefer early resolution of uncertainty. The intuition for this result lies in the long-term risk that equities pose: with rational inattention, uncertainty about the value of the equity return (and therefore the marginal utility of consumption) is not resolved for (infinitely) many periods. This postponement of information is distasteful to agents who prefer early resolution of uncertainty, causing them to prefer an asset with an even and certain intertemporal payoff (the risk-free asset); in the standard time-separable expected utility framework, agents must be indifferent to the timing of the resolution of uncertainty, preventing the model in Luo (2010) from producing significant effects without very low channel capacity. Due to the nature of the accumulation of uncertainty, even small deviations from indifference (again, in the direction of preference for early resolution) combined with small deviations from complete information-processing leads to large declines in optimal risky asset shares. Thus, we provide a theory for why agents hold such a small share of risky assets without requiring extreme values for preference parameters.

This result is based on the fact that RI introduces positive autocorrelation into consumption growth, i.e., consumption under RI reacts gradually to the wealth shock.[4] Here we show that

[4]Reis (2006) showed that inattentiveness due to costly planning could lead to slow adjustment of aggregate consumption to income shocks. The main difference between the implications of RI and Reis' inattentiveness

this effect may be amplified by a preference for early resolution of uncertainty and can become quite large, even when the deviation from indifference is arbitrarily small. Around the expected utility setting with unitary intertemporal elasticity of substitution and relative risk aversion, what matters for the size of this effect is the relative size of the deviation in IES from 1 as compared to the size of the deviation from relative risk aversion of 1.

To explore the equilibrium asset pricing implications of RU and RI, we consider a simple exchange economy in the vein of Lucas (1978) using the optimal consumption and portfolio rules. Specifically, we assume that in equilibrium the representative agent receives an endowment, which equals optimal consumption obtained in the consumption-portfolio choice model, and can trade two assets: a risky asset entitling the consumer to the endowment and a riskless asset with zero net supply. Using the optimal consumption and portfolio rules and the market-clearing condition, we find that how the interaction of RU and RI significantly increase the equilibrium equity premium and also improve the joint behavior of aggregate consumption and the equity return.

We consider two important extensions. First, we permit correlation between the equity return and the RI-induced noise.[5] We find that the sign of the correlation affects the long-run consumption and optimal asset allocation. Specifically, a negative correlation will further reduce the optimal share invested in the risky asset. We then present the results of adding nontradable labor income into the model, generating a hedging demand for risky equities. We find that our results survive essentially unchanged – rational inattention combined with a preference of early resolution of uncertainty still decreases the share of risky assets in the portfolio for small deviations around standard log preferences. In addition, we find that the importance of the hedging demand for equities is increasing in the degree of rational inattention. As agents become more constrained, they suffer more from uncertainty about consumption; thus, they are more interested in holding equities if they negatively covary with the labor income shock and less interested if they positively covary. Given that the data support a small correlation between individual wage income and aggregate stock returns (Heaton and Lucas 2000), our results survive this extension intact.

Our model is closely related to van Nieuwerburgh and Veldkamp (2010) and Mondria (2010). van Nieuwerburgh and Veldkamp (2010) discuss the relationship between information acquisition, the preference for early resolution of uncertainty, and portfolio choice in a static model broken into

for consumption behavior is that in the inattentiveness economy individuals adjust consumption *infrequently but completely* once they choose to adjust and aggregate consumption stickiness comes from aggregating across all individuals, whereas individuals under RI adjust their optimal consumption plans *frequently but incompletely* and aggregate consumption stickiness comes from individuals' incomplete consumption adjustments.

[5] This assumption generalizes the iid noise assumption used in Sims (2003) and Luo (2010).

three periods. Specifically, they find that information acquisition help resolves the uncertainty surrounding asset payoffs; consequently, an investor may prefer early resolution of uncertainty either because he has Epstein-Zin preferences or because he can use the early information to adjust his portfolio. In other words, van Nieuwerburgh and Veldkamp (2010) focuses on the static portfolio under-diversification problem with information acquisition, while we focus on the dynamic aspect of the interaction between incomplete information and recursive preferences. Mondria (2010) also considers two-period portfolio choice model with correlated risky assets in which investors choose the composition of their attention subject to an information flow constraint. He shows that there is an equilibrium in which all investors choose to observe a linear combination of these asset payoffs as a private signal. In contrast, the mechanism of our model is based on the effects of the interplay of the preference for early resolution of uncertainty and finite capacity on the dynamic response of consumption to the shock to the equity return that determines the long-run consumption risk; in our model, the preference for early resolution of uncertainty amplifies the role of finite information-processing capacity in generating greater long-run risk.

This paper is organized as follows. Section 2 presents and reviews an otherwise standard two-asset portfolio choice model with recursive utility. Section 3 solves an RI version of the RU model and examines the implications of the interactions of RI, the separation of risk aversion and intertemporal substitution, and the discount factor for the optimal portfolio rule and the equilibrium equity premium. Section 4 discusses two extensions: the presence of the correlation between the equity return and the noise and the introduction of nontradable labor income. Section 5 concludes and discusses the extension of the results to non-LQ environments. Appendices contain the proofs and derivations that are omitted from the main text.

2 A Stylized Portfolio Choice Model with Rational Inattention and Recursive Utility

In this section, we present and discuss a standard portfolio choice model within a recursive utility framework. Following the log-linear approximation method proposed by Campbell (1993), Viceira (2001), and Campbell and Viceira (1999, 2002), we then incorporate rational inattention into the standard model and solve it explicitly after considering the long-run consumption risk facing the investors. Another major advantage of the log-linearization approach is that we can obtain a quadratic expected loss function by approximating the original value function from the nonlinear problem when relative risk aversion is close to 1 and thus can justify Gaussian posterior uncertainty under RI. We then discuss the interplay between RI, risk aversion, and intertemporal substitution for portfolio choice and asset pricing.

2.1 Specification and Solution of the Standard Recursive Utility Model of Portfolio Choice

Before setting up and solving the portfolio choice model with RI, it is helpful to present the standard portfolio choice model first and then discuss how to introduce RI in this framework. Here we consider a simple intertemporal model of portfolio choice with a continuum of identical investors. Following Epstein and Zin (1989), Giovannini and Weil (1989), and Campbell and Viceira (1999), suppose that investors maximize a recursive utility function U_t by choosing consumption and asset holdings,

$$U_t = \left\{ (1-\beta) C_t^{1-1/\sigma} + \beta \left(E_t \left[U_{t+1} \right] \right)^{(1-1/\sigma)/(1-\gamma)} \right\}^{\frac{1-\gamma}{1-1/\sigma}}, \tag{1}$$

where C_t represents individual's consumption at time t, β is the discount factor, γ is the coefficient of relative risk aversion over wealth gambles (CRRA), and σ is the elasticity of intertemporal substitution.[6] Let $\rho = (1-\gamma) / (1-1/\sigma)$; if $\rho > 1$, the household has a preference for early resolution of uncertainty.

We assume that the investment opportunity set is constant and contains only two assets: asset e is risky, with one-period log (continuously compounded) return $r_{e,t+1}$, while the other asset f is riskless with constant log return given by r_f. We refer to asset e as the market portfolio of equities, and to asset f as the riskless bond. $r_{e,t+1}$ has expected return μ, $\mu - r_f$ is the equity premium, and $r_{e,t+1}$ has an unexpected component u_{t+1} with $\text{var}\left[u_{t+1}\right] = \omega^2$.[7]

The intertemporal budget constraint for the investor is

$$A_{t+1} = R_{p,t+1} \left(A_t - C_t \right) \tag{2}$$

where A_{t+1} is the individual's financial wealth (the value of financial assets carried over from period t at the beginning of period $t+1$), $A_t - C_t$ is current period savings, and $R_{p,t+1}$ is the one-period gross return on savings given by

$$R_{p,t+1} = \alpha_t \left(R_{e,t+1} - R_f \right) + R_f \tag{3}$$

[6] When $\gamma = \sigma^{-1}$, $\rho = 1$ and the recursive utility reduces to the standard time-separable power utility with RRA γ and intertemporal elasticity γ^{-1}. When $\gamma = \sigma = 1$ the objective function is the time-separable log utility function.

[7] Under unlimited information-processing capacity two-fund separation theorems imply that this investment opportunity set is sufficient. All agents would choose the same portfolio of multiple risky assets; differences in preferences would manifest themselves only in terms of the share allocated to this risky portfolio versus the riskless asset. We believe, but have not proven, that this result would go through under rational inattention as well.

where $R_{e,t+1} = \exp(r_{e,t+1})$, $R_f = \exp(r_f)$, and $\alpha_t = \alpha$ is the proportion of savings invested in the risky asset.[8] As in Campbell (1993), we can derive an approximate expression for the log return on wealth:

$$r_{p,t+1} = \alpha(r_{e,t+1} - r_f) + r_f + \frac{1}{2}\alpha(1-\alpha)\omega^2. \tag{4}$$

Given the above model specification, it is well known that this simple discrete-time model can not be solved analytically. We therefore follow the log-linearization method proposed in Campbell (1993), Viceira (2001), and Campbell and Viceira (2002) to obtain a closed-form solution to an approximation of this problem.[9] Specifically, the original intertemporal budget constraint, (2), can be written in log-linear form:

$$\Delta a_{t+1} = \left(1 - \frac{1}{\phi}\right)(c_t - a_t) + \psi + r^p_{t+1}, \tag{5}$$

where $c - a = E[c_t - a_t]$ is the unconditional (log of) consumption's share of financial wealth, $\phi = 1 - \exp(c-a)$, $\psi = \log(\phi) - (1 - 1/\phi)\log(1-\phi)$, and lowercase letters denote logs. Note that the approximation, (5), holds exactly in our model because the consumption-wealth ratio in the model with iid returns is constant.[10] As shown in Viceira (2001), the assumptions on the preference and the investment opportunity set ensure that along the optimal path, financial wealth (A_t), savings ($A_t - C_t$), and consumption (C_t) are strictly positive. Because the marginal utility of consumption approaches ∞ as consumption approaches zero, the investor chooses consumption-savings and portfolio rules that ensure strictly positive consumption next period. Thus, we must have $A_{t+1} > 0$ and $A_t - C_t > 0$, so that the log of these objects is well-defined (note that the intertemporal budget constraint implies that $A_t - C_t > 0$ is a necessary condition for next period's financial wealth to be positive). The optimal consumption and portfolio rules are then

$$c_t = b_0 + a_t, \tag{6}$$

$$\alpha = \frac{\mu - r_f + 0.5\omega^2}{(\rho/\sigma + 1 - \rho)\,\omega^2}, \tag{7}$$

[8]Given iid equity returns and a power utility function, α_t will be constant over time.

[9]This method proceeds as follows. First, both the flow budget constraint and the consumption Euler equations are log-approximated around the steady state. The Euler equations are log-approximated by a second-order Taylor expansion so that the second-moment are included; these terms are constant and thus the resulting equation is log-linear. Second, the optimal consumption and portfolio choices that satisfy these log-linearized equations are chosen as log-linear functions of the state. Finally, the coefficients of these optimal decision rules are pinned down using the method of undetermined coefficients.

[10]Campbell (1993) and Campbell and Viceira (1999) have shown that the approximation is exact when the consumption-wealth ratio is constant over time, and becomes less accurate when the ratio becomes more volatile.

where $b_0 = \log\left(1-\beta^{\sigma}\left(E_t\left[R_{p,t+1}^{1-\gamma}\right]\right)^{\frac{\sigma-1}{1-\gamma}}\right)$ and $\frac{\rho}{\sigma}+1-\rho=\gamma$.[11] Note that in the full-information RE model, $b_0 = \log(1-\phi)$ and $\phi = \beta^{\sigma}$ when γ is close 1.

Two aspects of preferences play a role in determining the portfolio share α: intertemporal substitution, measured by σ, and the preference for the timing of the resolution of uncertainty, measured by ρ. A household who is highly intolerant of intertemporal variation in consumption will have a high share of risky assets. If $\sigma < 1$, a household who prefers earlier resolution of uncertainty (larger ρ) will have a lower share of risky assets. Using the identity this statement is equivalent to noting that larger ρ means larger γ for fixed σ, so that more risk aversion also implies lower share of risky assets. Thus, as noted in Epstein and Zin (1989), risk aversion and intertemporal substitution, while disentangled from each other, are entwined with the preference for the timing of uncertainty resolution.

We choose to focus on the temporal resolution aspect of preferences, rather than risk aversion, for two reasons. First, results in Backus, Routledge, and Zin (2007) show a household with infinite risk aversion and infinite intertemporal elasticity actually holds almost entirely risky assets, and the opposite household (risk neutral with zero intertemporal elasticity) holds almost none (when risks are shared efficiently, at least). The second household prefers early resolution of uncertainty, a preference that cannot be expressed within the expected utility framework, and thus prefers paths of consumption that are smooth, while the first household prefers paths of *utility* that are smooth. Holding equities makes consumption risky, but not future utility, and therefore the risk-neutral agent will avoid them. Second, it will turn out that rational inattention will have a strong effect when combined with a preference regarding the timing of the resolution of uncertainty, independent of the values of risk aversion and intertemporal elasticity; specifically, our model will improve upon the standard model by reducing the portfolio share of risky assets if the representative investor has a preference for early resolution.

3 RI-Recursive Utility Model

Given the optimal consumption and portfolio rules derived in the last section, it is straightforward to show that the value function under full-information about the state, $V(W_t)$, corresponding to the recursive utility model (1), takes the following form:

$$V(A_t) = \Phi A_t^{1-\gamma}, \tag{8}$$

[11] Note that a unitary marginal propensity to consume and a constant optimal fraction invested in the risky asset are valid not only for CRRA expected utility but also for Epstein-Zin recursive utility when the return to equity is iid. See Appendices in Giovannini and Weil (1989) and Campbell and Viceira (1999) for detailed deviations.

where Φ is a coefficient determined by the model parameters. Now we assume that the agents cannot observe the true state. In this case, we approximate the value function around the perceived value for the true state variable ($a_t = \ln A_t$), $\widehat{a}_t$, as follows

$$\begin{aligned} V &= \Phi \exp\left((1-\gamma)\, a_t\right) \\ &\cong \widehat{V} + \Phi\left[(1-\gamma)\exp\left((1-\gamma)\,\widehat{a}_t\right)(a_t - \widehat{a}_t) + \frac{1}{2}(1-\gamma)^2 \exp\left((1-\gamma)\,\widehat{a}_t\right)(a_t - \widehat{a}_t)^2\right], \end{aligned}$$

where $\widehat{V} = \Phi \exp\left((1-\gamma)\,\widehat{a}_t\right)$. Note that this Taylor approximation is accurate when $1-\gamma$ is close to 0.[12] Therefore, minimizing the expected welfare loss due to imperfect observations, $E_t\left[(a_t - \widehat{a}_t)^2\right]$, is equivalent to minimizing

$$\min_{\widehat{a}_t} E_t\left[(a_t - \widehat{a}_t)^2\right]. \tag{9}$$

In other words, when the approximation is accurate, the best estimate of the true state, $\widehat{a}_t$, should be its conditional mean based on the available information, $E_t[a_t]$. In the next subsection, we use the RI hypothesis proposed by Sims (2003) to rationalize the imperfect-state-observation setting.

3.1 Introducing RI

Following Sims (2003), we introduce rational inattention (RI) into the otherwise standard intertemporal portfolio choice model by assuming consumers/investors face information-processing constraints and have only finite Shannon channel capacity to observe the state of the world. Specifically, we use the concept of entropy from information theory to characterize the uncertainty about a random variable; the reduction in entropy is thus a natural measure of information flow. Formally, entropy is defined as the expectation of the negative of the log of the density function, $-E\left[\log\left(f(X)\right)\right]$. For example, the entropy of a discrete distribution with equal weight on two points is simply $E\left[\log_2\left(f(X)\right)\right] = -0.5\log_2(0.5) - 0.5\log_2(0.5) = 1$, and the unit of information contained in this distributed is one "bit".[13] In this case, an agent can remove all uncertainty about X if the capacity devoted to monitoring X is $\kappa = 1$ bit.

With finite capacity $\kappa \in (0, \infty)$, the true state a (a continuous variable) cannot be observed without error; thus the information set at time $t+1$, $mathcalI_{t+1}$, is generated by the entire history of noisy signals $\left\{a_j^*\right\}_{j=0}^{t+1}$. Following Sims (2003), we assume in this paper that the noisy signal takes the additive form: $a_{t+1}^* = a_{t+1} + \xi_{t+1}$, where ξ_{t+1} is the endogenous noise caused by finite

[12] In our numerical exercises below, we restrict γ to be less than 1.01. When $\gamma = 1.01$ the omitted third-order expansion, $(a - \widehat{a})^3$, is less than 1% of the size of the second-order expansion, $(a - \widehat{a})^2$.

[13] For alternative bases for the logarithm, the unit of information differs: with natural log the unit of information is the 'nat' and with base 10 it is a 'hartley.'

capacity. We further assume that ξ_{t+1} is an iid idiosyncratic shock and is independent of the fundamental shock. Note that the reason that the RI-induced noise is idiosyncratic is that the endogenous noise arises from the consumer's own internal information-processing constraint. Investors with finite channel capacity will choose a new signal $a_{t+1}^* \in mathcalI_{t+1} = \left\{a_1^*, a_2^*, \cdots, a_{t+1}^*\right\}$ that reduces the uncertainty about the state variable a_{t+1} as much as possible. Formally, this idea can be described by the information constraint

$$\mathcal{H}\left(a_{t+1}|\mathcal{I}_t\right) - \mathcal{H}\left(a_{t+1}|\mathcal{I}_{t+1}\right) = \kappa, \tag{10}$$

where κ is the investor's information channel capacity, $\mathcal{H}\left(a_{t+1}|\mathcal{I}_t\right)$ denotes the entropy of the state *prior to* observing the new signal at $t+1$, and $\mathcal{H}\left(a_{t+1}|\mathcal{I}_{t+1}\right)$ is the entropy *after* observing the new signal. κ imposes an upper bound on the amount of information – that is, the change in the entropy – that can be transmitted in any given period. We assume that the noise ξ_{t+1} is Gaussian. Finally, following the literature, we suppose that the *ex ante* a_{t+1} is a Gaussian random variable. As shown in Sims (2003), the optimal posterior distribution for a_{t+1} will also be Gaussian given the quadratic loss function specification, (9). Furthermore, we assume that all individuals in the model economy have the same channel capacity; hence the average capacity in the economy is equal to individual capacity.[14] In this case the effective state variable is not the traditional state variable (the wealth level a_t), but rather the so-called *information state*: the distribution of the state variable a_t conditional on the information set available at time t, $\mathcal{I}_t$.

As noted earlier, ex post Gaussian uncertainty is optimal:

$$a_{t+1}|\mathcal{I}_{t+1} \sim N\left(\widehat{a}_{t+1}, \Sigma_{t+1}\right), \tag{11}$$

where $\widehat{a}_{t+1} = E\left[a_{t+1}|\mathcal{I}_{t+1}\right]$ and $\Sigma_{t+1} = \text{var}\left[a_{t+1}|\mathcal{I}_{t+1}\right]$ are the conditional mean and variance of a_{t+1}, respectively. The information constraint (10) can thus be reduced to

$$\frac{1}{2}\left(\log\left(\Psi_t\right) - \log\left(\Sigma_{t+1}\right)\right) = \kappa \tag{12}$$

where $\Sigma_{t+1} = \text{var}_{t+1}\left[a_{t+1}\right]$ and $\Psi_t = \text{var}_t\left[a_{t+1}\right]$ are the posterior and prior variance, respectively. Given a finite transmission capacity of κ bits per time unit, the optimizing consumer chooses a signal that reduces the conditional variance by $\frac{1}{2}\left(\log\left(\Psi_t\right) - \log\left(\Sigma_{t+1}\right)\right)$.[15] In the univariate state

[14] Assuming that channel capacity follows some distribution in the cross-section complicates the problem when aggregating, but would not change the main findings.

[15] Note that given Σ_t, choosing Σ_{t+1} is equivalent to choosing the noise $\text{var}\left[\xi_t\right]$, since the usual updating formula for the variance of a Gaussian distribution is

$$\Sigma_{t+1} = \Psi_t - \Psi_t\left(\Psi_t + \text{var}\left[\xi_t\right]\right)^{-1}\Psi_t$$

where Ψ_t is the ex ante variance of the state and is a function of Σ_t.

case this information constraint completes the characterization of the optimization problem and everything can be solved analytically.[16]

The intertemporal budget constraint (5) then implies that

$$E_t\left[a_{t+1}\right] = E_t\left[r_{p,t+1}\right] + \psi + \widehat{a}_t, \tag{13}$$

$$\mathrm{var}_t\left[a_{t+1}\right] = \mathrm{var}_t\left[r_{p,t+1}\right] + \left(\frac{1}{\phi}\right)^2 \Sigma_t. \tag{14}$$

Substituting (13) into (12) yields

$$\kappa = \frac{1}{2}\left[\log\left(\mathrm{var}_t\left(r_{p,t+1}\right) + \left(\frac{1}{\phi}\right)^2 \Sigma_t\right) - \log\left(\Sigma_{t+1}\right)\right], \tag{15}$$

which has a unique steady state $\Sigma = \frac{\mathrm{var}_t[r_{p,t+1}]}{\exp(2\kappa)-(1/\phi)^2}$ with $\mathrm{var}_t\left[r_{p,t+1}\right] = \alpha^2\omega^2$. Note that here ϕ is close to β as both γ and σ are close to 1.

The separation principle applies to this problem.[17] Hence, the optimal consumption rule is

$$c_t = b_0 + \widehat{a}_t \tag{16}$$

and the perceived state $\widehat{a}_t$ evolves according to the following equation:

$$\widehat{a}_{t+1} = \frac{1}{\phi}\widehat{a}_t + \left(1 - \frac{1}{\phi}\right) c_t^* + \psi + \eta_{t+1}, \tag{17}$$

where η_{t+1} is the innovation to the perceived state:

$$\eta_{t+1} = \theta\left(r_{p,t+1} + \xi_{t+1}\right) + \frac{\theta}{\phi}\left(a_t - \widehat{a}_t\right), \tag{18}$$

$a_t - \widehat{a}_t$ is the estimation error:

$$a_t - \widehat{a}_t = \frac{\left(1-\theta\right) r_{p,t+1}}{1 - \left(\left(1-\theta\right)/\phi\right) \cdot L} - \frac{\theta \xi_t}{1 - \left(\left(1-\theta\right)/\phi\right) \cdot L}. \tag{19}$$

$\theta = 1 - 1/\exp\left(2\kappa\right)$ is the optimal weight on a new observation, ξ_{t+1} is the iid Gaussian noise with $E\left[\xi_{t+1}\right] = 0$ and $\mathrm{var}\left[\xi_{t+1}\right] = \Sigma/\theta$, and $a_{t+1}^* = a_{t+1} + \xi_{t+1}$ is the observed signal. (See Online Appendix 6.1.)

Before moving on, we want to comment briefly on the decision rule of an agent with rational inattention. An agent with RI chooses a joint distribution of states and controls, subject to the

[16]With more than one state variable, there is an additional constraint that requires the difference between the prior and the posterior variance-covariance matrices be positive semidefinite; the resulting optimal posterior cannot be characterized analytically, and generally poses significant numerical challenges as well. See Sims (2003) for some examples.

[17]This principle says that under the LQ assumption optimal control and state estimation can be decoupled.

information-processing constraint and some fixed prior distribution over the state; with $\kappa = \infty$ this distribution is degenerate, but with $\kappa < \infty$ it is generally nontrivial. The noise terms ξ_t can be viewed in the following manner: the investor instructs nature to choose consumption in the current period from a certain joint distribution of consumption and current and future permanent income, and then nature selects at random from that distribution (conditioned on the true current permanent income that the agent cannot observe). Thus, an observed signal about future permanent income a_{t+1}^* is equivalent to making the signal current consumption.

We make the following assumption.

Assumption 1:

$$2\kappa > \log(1/\phi). \tag{20}$$

Equation (20) ensures that agents have sufficient information-processing ability to "zero out" the unstable root in the Euler equation. It will also ensure that certain infinite sums converge. Note that using the definition of θ we can write this restriction as $1 - \theta < \phi^2 < \phi$; the second inequality arises because $\phi < 1$. (Note that $\phi = \beta^\sigma$ when γ is close to 1.) Note that along the optimal path, financial wealth (A_t), savings $(A_t - C_t)$, perceived financial wealth $\left(\widehat{A}_t = \exp(\widehat{a}_t)\right)$, and consumption (C_t) are strictly positive. Given that $\lim_{C_t \to 0} u'(C_t) = \infty$, the investor chooses optimal consumption-savings and portfolio rules to ensure strictly positive consumption next period; that is, we must have $A_{t+1} > 0$ and $A_t - C_t > 0$ $\left(\text{i.e., } A_t - (1-\beta)\widehat{A}_t > 0\right)$, to guarantee that the logarithm of these objects is well-defined.

The following example is illustrative. An inattentive investor does not have perfect information about his banking account. He knows that he has about \$1000 in the account but he does not know the *exact* amount (say \$1010.00). He has already made a decision to purchase a sofa in a furniture store; when he uses his debit card to check out, he finds that the price of the sofa (say \$1099.99) exceeds the amount of money in his account. He must then choose a less expensive sofa (say \$999) such that consumption is always less than his wealth. In effect, the consumer constrains nature from choosing points from the joint distribution that imply negative consumption at any future date.

Combining (5), (16), and (17) gives the expression for individual consumption growth:

$$\Delta c_{t+1} = \theta \left\{ \frac{\alpha u_{t+1}}{1 - ((1-\theta)/\phi) \cdot L} + \left[\xi_{t+1} - \frac{(\theta/\phi)\,\xi_t}{1 - ((1-\theta)/\phi) \cdot L} \right] \right\}, \tag{21}$$

where L is the lag operator.[18] Note that all the above dynamics for consumption, perceived state, and the change in consumption are not the final solutions because the optimal share invested in stock market α has yet to be determined. To determine the optimal allocation in risky assets,

[18] Note that $\phi = \beta^\sigma$ when γ is close to 1.

we have to use an intertemporal optimality condition. However, the standard Euler equation is not suitable for determining the optimal asset allocation in the RI economy because consumption adjusts slowly and incompletely, making the relevant intertemporal condition one that equates the marginal utility of consumption today to the covariance between marginal utility and the asset return *arbitrarily far* into the future; that is, it is the "long-run Euler equation" that determines optimal consumption/savings plans. We now turn to deriving this equation.

3.2 Long-run Risk and the Demand for the Risky Asset

Bansal and Yaron (2004), Hansen, Heaton, and Li (2006), Parker (2001, 2003) and Parker and Julliard (2005) argue that long-term risk is a better measure of the true risk of the stock market if consumption reacts with delay to changes in wealth; the contemporaneous covariance of consumption and wealth understates the risk of equity.[19] Long-term consumption risk is the appropriate measure for the RI model.

Following Parker (2001, 2003), we define the long-term consumption risk as the covariance of asset returns and consumption growth over the period of the return and many subsequent periods. Because the RI model predicts that consumption reacts to the innovations to asset returns gradually and incompletely, it can rationalize the conclusion in Parker (2001, 2003) that consumption risk is long term instead of contemporaneous. Given the above analytical solution for consumption growth, it is straightforward to calculate the ultimate consumption risk in the RI model. Specifically, when agents behave optimally but only have finite channel capacity, we have the following equality for the risky asset e and the risk free asset f:

$$E_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\left(R_{e,t+1}-R_f\right)\right]=0, \tag{22}$$

where $U_{i,t}$ for any t denotes the derivative of the aggregate function with respect to its ith argument evaluated at $(C_t, E_t\left[U_{t+1}\right])$.[20] Note that with time additive expected utility, the discount factor

[19] Bansal and Yaron (2004) also document that consumption and dividend growth rates contain a long-run component. An adverse change in the long-run component will lower asset prices and thus makes holding equity very risky for investors.

[20] This long-term Euler equation can be obtained by combining the standard Euler equation for the excess return

$$E_t\left[U_{1,t+1}\left(R_{e,t+1}-R_f\right)\right]=0$$

with the Euler equation for the riskless asset between $t+1$ and $t+1+S$,

$$U_{1,t+1}=E_{t+1}\left[\left(\beta_{t+1}\cdots\beta_{t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\right], \tag{23}$$

where $\beta_{t+1+j}=U_{2,t+1+j}$, for $j=0,\cdots,S$. In other words, the equality can be obtained by using $S+1$ period consumption growth to price a multiperiod return formed by investing in equity for one period and then transforming to the risk free asset for the next S periods. See Appendix 6.2 for detailed derivations.

$U_{2,t+1+j}$ is constant and equal to β. (22) implies that the expected excess return can be written as

$$E_t\left[R_{e,t+1}-R_f\right]=-\frac{\operatorname{cov}_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S},R_{e,t+1}-R_f\right]}{E_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\right]},$$

so that

$$\mu-r_f+\frac{1}{2}\omega^2=\operatorname{cov}_t\left[\frac{\rho}{\sigma}\left(\sum_{j=0}^{S}\Delta c_{t+1+j}\right)+(1-\rho)\left(\sum_{j=0}^{S}r_{p,t+1+j}\right),u_{t+1}\right],\tag{24}$$

where we have used $\gamma\simeq 1$, $c_{t+1+S}-c_t=\sum_{j=0}^{S}\Delta c_{t+1+j}$, and Δc_{t+1+j} as given by (21). Furthermore, since the horizon S over which consumption responds completely to income shocks under RI is infinite, the right hand side of (24) can be written as

$$\lim_{S\to\infty}\left\{\sum_{j=0}^{S}\operatorname{cov}_t\left[\frac{\rho}{\sigma}\Delta c_{t+1+j}+(1-\rho)\left(\sum_{j=0}^{S}r_{p,t+1+j}\right),u_{t+1}\right]\right\}=\alpha\left(\frac{\rho}{\sigma}\varsigma+1-\rho\right)\omega^2.\tag{25}$$

ς is the ultimate consumption risk measuring the accumulated effect of the equity shock to consumption under RI:

$$\varsigma=\theta\sum_{i=0}^{\infty}\left(\frac{1-\theta}{\phi}\right)^i=\frac{\theta}{1-(1-\theta)/\phi}>1\tag{26}$$

when Assumption 1 holds.

3.3 Optimal Consumption and Asset Allocation

Combining Equations (16), (24), with (25) gives us optimal consumption and portfolio rules under RI. The following proposition gives a complete characterization of the model's solution for optimal consumption and portfolio choice.

Proposition 1 *Suppose that γ is close to 1 and Assumption 1 is satisfied. The optimal share invested in the risky asset is*

$$\alpha^*=\left(\frac{\rho}{\sigma}\varsigma+1-\rho\right)^{-1}\frac{\mu-r^f+0.5\omega^2}{\gamma\omega^2}.\tag{27}$$

The consumption function is

$$c_t^*=\log(1-\phi)+\widehat{a}_t,\tag{28}$$

actual wealth evolves according to

$$a_{t+1}=\frac{1}{\phi}a_t+\left(1-\frac{1}{\phi}\right)c_t^*+\psi+\left[\alpha^*\left(r_{t+1}^e-r^f\right)+r^f+\frac{1}{2}\alpha^*\left(1-\alpha^*\right)\omega^2\right],\tag{29}$$

and estimated wealth $\widehat{a}_t$ is characterized by the following Kalman filtering equation

$$\widehat{a}_{t+1} = \frac{1}{\phi}\widehat{a}_t + \left(1 - \frac{1}{\phi}\right) c_t^* + \psi + \eta_{t+1}, \tag{30}$$

where η_{t+1} is defined in (18), $\psi = \log(\phi) - (1 - 1/\phi)\log(1-\phi)$, $\theta = 1 - \exp(-2\kappa)$ is the optimal weight on a new observation, ξ_t is an iid *idiosyncratic noise shock with $\omega_\xi^2 = \mathrm{var}[\xi_{t+1}] = \Sigma/\theta$, and $\Sigma = \frac{\alpha^{*2}\omega^2}{\exp(2\kappa)-(1/\phi)^2}$ is the steady state conditional variance. The change in individual consumption is*

$$\Delta c_{t+1}^* = \theta \left\{ \frac{\alpha^* u_{t+1}}{1 - ((1-\theta)/\phi) \cdot L} + \left[\xi_{t+1} - \frac{(\theta/\phi)\,\xi_t}{1 - ((1-\theta)/\phi) \cdot L} \right] \right\}. \tag{31}$$

Proof. *The proof is straightforward.* ■

The proposition clearly shows that optimal consumption and portfolio rules are *interdependent* under RI. Expression (27) shows that although the optimal fraction of savings invested in the risky asset is proportional to the risk premium ($\mu - r^f + 0.5\omega^2$), the reciprocal of both the coefficient of relative risk aversion (γ), and the variance of the unexpected component in the risky asset (ω^2), as predicted by the standard Merton solution, it also depends on the interaction of RI and RU measured by $\frac{\rho}{\sigma}\varsigma + 1 - \rho$. We now examine how the interplay of RI and the preference for the timing of uncertainty resolution affects the long-term consumption risk and the optimal share invested in the risky asset. Denote $\frac{\rho}{\sigma}\varsigma + 1 - \rho$ in (27) the long-run consumption risk, and rewrite it as

$$\frac{\rho}{\sigma}\varsigma + 1 - \rho = \gamma + \Gamma, \tag{32}$$

where

$$\Gamma = \frac{\gamma - 1}{1 - \sigma}(\varsigma - 1) \tag{33}$$

measures how the interaction of recursive utility $\left(\frac{\gamma-1}{1-\sigma}\right)$ and the long-run impact of the equity return on consumption under RI (ς) affect the risk facing the inattentive investors. Expression (32) clearly shows that both risk aversion (γ) and Γ determine the optimal share invested in the risky asset. Specifically, suppose that investors prefer early resolution of uncertainty: $\gamma > \sigma$; even a small deviation from infinite information-processing capacity due to RI will generate large increases in long-run consumption risk and then reduce the demand for the risky asset.[21] In fact, it is not the scale of the deviation from $\sigma = \gamma = 1$ that matters, but the relative size of the deviations from $\sigma = 1$ and $\gamma = 1$.

Figures 1 and 2 illustrate how RI affects the long-run consumption risk Γ when σ is close to 1 (here we set it to be 0.99999); following Viceira 2001 and Luo 2010, we set $\beta = 0.91$. The

[21] That is, θ is very close to 100% and therefore ς is only slightly greater than 1.

figures show that the interaction of RI and RU can significantly increase the long-run consumption risk facing the investors. In particular, it is obvious that even if θ is high (so that investors can process nearly all the information about the equity return), the long-run consumption risk is still non-trivial. For example, when $\gamma = 1.01$ and $\theta = 0.9$ (i.e., 90 percent of the uncertainty about the equity return can be removed upon receiving the new signal), $\Gamma = 11$; if θ is reduced to 0.8, $\Gamma = 25$. That is, a small difference between risk aversion γ and intertemporal substitution σ has a significant impact on optimal portfolio rule.

Note that Expression (27) can be rewritten as

$$\alpha^* = \frac{\mu - r^f + 0.5\omega^2}{\widetilde{\gamma}\omega^2}, \tag{34}$$

where $\widetilde{\gamma} = \gamma \left(\frac{\rho}{\sigma}\varsigma + 1 - \rho\right)$ is the *effective* coefficient of relative risk aversion.[22] When $\theta = 1$, $\varsigma = 1$ and optimal portfolio choice (27) under RI reduces to (7) in the standard RU case, which we have discussed previously. Similarly, when $\rho = 1$ (27) reduces to the optimal solution in the expected utility model discussed in Luo (2010). Later we will show that $\widetilde{\gamma}$ could be significantly greater than the true coefficient of relative risk aversion (γ). In other words, even if the true γ is close to 1 as assumed at the beginning of this section, the effective risk aversion that matters for the optimal asset allocation is $\gamma + \Gamma$, which will be greater than 1 if the capacity is low and $(\gamma - 1)$ is greater than $(1 - \sigma)$ (indeed, it can be a lot larger even for small deviations from $\gamma = \sigma = 1$). Therefore, both the degree of attention (θ) and the discount factor (β) amount to an increase in the effective coefficient of relative risk aversion. Holding β constant, the larger the degree of attention, the less the ultimate consumption risk. As a result, investors with low attention will choose to invest less in the risky asset. For example, with RI, a 1 percent negative shock in investors' financial wealth would affect their consumption more than that predicted by the standard RE model. Therefore, investors with finite capacity are less willing to invest in the risky asset.[23]

As argued in Campbell and Viceira (2002), the effective investment horizon of investors can be measured by the discount factor β. In the standard full-information RE portfolio choice model (such as Merton 1969), the investment horizon measured by β is *irrelevant* for investors who have power utility functions, have only financial wealth, and face constant investment opportunities. In contrast, it is clear from (26) and (27) that the investment horizon measured by β does matter for optimal asset allocations under RU and RI because it affects the valuation of long-term consumption risk. Expression (27) shows that the higher the value of β (the longer the

[22] By effective, we mean that if we observed a household's behavior and interpreted it as coming from an individual with unlimited information-processing ability, $\widetilde{\gamma}$ would be our estimate of the risk aversion coefficient.

[23] Luo (2010) shows that with heterogeneous channel capacity the standard RI model would predict some agents would not participate in the equity market at all. It is clear that the same result would obtain with recursive utility.

investment horizon), the higher the fraction of financial wealth invested in the risky asset. Figure 3 illustrates how the investment horizon affects the long-run consumption risk Γ when $\gamma = 1.01$, $\sigma = 0.99999$, $\theta = 0.8$, and $\beta = 0.91$. The figure shows that the investment horizon can significantly affect the long-run consumption risk facing the investors. For example, when $\beta = 0.91$, $\Gamma = 25$; if β is increased to 0.93, $\Gamma = 19$. That is, a small reduction in the discount factor has a significant effect on long-run consumption risk and the optimal portfolio share when combined with RI.

Given RRA (γ), IES (σ), and β, we can calibrate θ using the share of wealth held in risky assets. Specifically, we start with the annualized US quarterly data in Campbell (2003), and assume that $\omega = 0.16$, $\pi = \mu - r^f = 0.06$, $\beta = 0.91$, $\sigma = 0.99999$, and $\gamma = 1.001$. We then calibrate θ to match the observed $\alpha = 0.22$ estimated in Section 5.1 of Gabaix and Laibson (1999) to obtain

$$\alpha^* = \left[\gamma + \frac{\gamma - 1}{1 - \sigma}(\varsigma - 1)\right]^{-1} \frac{\pi + 0.5\omega^2}{\gamma\omega^2} = 0.22, \tag{35}$$

which means that $\theta = 0.48$.[24] That is, approximately 48 percent of the uncertainty is removed upon receiving a new signal about the equity return. Note that if $\gamma = 1$, the RE version of the model generates a highly unrealistic share invested in the stock market: $\alpha = \frac{\pi + 0.5\omega^2}{\omega^2} = 2.84$. To match the observed fraction in the US economy (0.22), γ must be set to 13.

Equation (31) shows that individual consumption under RI reacts not only to fundamental shocks (u_{t+1}) but also to the endogenous noise (ξ_{t+1}) induced by finite capacity. The endogenous noise can be regarded as a type of "consumption shock" or "demand shock". In the intertemporal consumption literature, some transitory consumption shocks are often used to make the model fit the data better. Under RI, the idiosyncratic noise due to RI provides a theory for these transitory consumption movements. Furthermore, Equation (31) also makes it clear that consumption growth adjusts slowly and incompletely to the innovations to asset returns but reacts quickly to the idiosyncratic noise.

Using (31), we can obtain the stochastic properties of the joint dynamics of consumption and the equity return. The following proposition summarizes the major stochastic properties of consumption and the equity return.

Proposition 2 *Given finite capacity κ (i.e., θ) and optimal portfolio choice α^*, the volatility of consumption growth is*

$$\mathrm{var}\left[\Delta c_t^*\right] = \frac{\theta \alpha^{*2}}{1 - (1 - \theta)/\phi^2} \omega^2, \tag{36}$$

[24] Gabaix and Laibson (2001) assume that all capital is stock market capital and that capital income accounts for 1/3 of total income.

the relative volatility of consumption growth to the equity return is

$$\mu = \frac{\operatorname{sd}\left[\Delta c_t^*\right]}{\operatorname{sd}\left[u_t\right]} = \sqrt{\frac{\theta}{1-(1-\theta)/\phi^2}}\alpha^*, \tag{37}$$

the first-order autocorrelation of consumption growth is

$$\rho_{\Delta c(1)} = \operatorname{corr}\left[\Delta c_t^*, \Delta c_{t+1}^*\right] = 0, \tag{38}$$

and the contemporaneous correlation between consumption growth and the equity return is

$$\operatorname{corr}\left[\Delta c_{t+1}^*, u_{t+1}\right] = \sqrt{\theta\left(1-(1-\theta)/\phi^2\right)}. \tag{39}$$

Proof. See Online Appendix 6.3. ■

Expression (37) shows that RI affects the relative volatility of consumption growth to the equity return via two channels: (i) $\frac{\theta}{1-(1-\theta)/\phi^2}$ and (ii) α^*. Holding the optimal share invested in the risky asset α^* fixed, RI increases the relative volatility of consumption growth via the first channel because $\partial\left(\frac{\theta\alpha^{*2}}{1-(1-\theta)/\phi^2}\right)/\partial\theta < 0$. (31) indicates that RI has two effects on the volatility of Δc: the gradual response to a fundamental shock and the presence of the RI-induced noise shocks. The former effect reduces consumption volatility, whereas the latter one increases it; the net effect is that RI increases the volatility of consumption growth holding α^* fixed. Furthermore, as shown above, RI reduces α^* as it increases the long-run consumption risk via the interaction with the RU preference, which tends to reduce the volatility of consumption growth as households switch to safer portfolios. Figure 4 illustrates how RI affects the relative volatility of consumption to the equity return for different values of β in the RU model; for the parameters selected RI reduces the volatility of consumption growth in the presence of optimal portfolio choice.

Expression (38) means that there is no persistence in consumption growth under RI. The intuition of this result is as follows. Both MA(∞) terms in (31) affect consumption persistence under RI. Specifically, in the absence of the endogenous noises, the gradual response to the shock to the equity return due to RI leads to positive persistence in consumption growth: $\rho_{\Delta c(1)} = \frac{\theta(1-\theta)}{\phi} > 0$. (See Appendix 6.3.) The presence of the noises generate negative persistence in consumption growth, exactly offsetting the positive effect of the gradual response to the fundamental shock under RI.

Expression (39) shows that RI reduces the contemporaneous correlation between consumption growth and the equity return because $\partial \operatorname{corr}\left(\Delta c_{t+1}^*, u_{t+1}\right)/\partial\theta > 0$. Figure 5 illustrates the effects of RI on the correlation when $\beta = 0.91$. It clearly shows that the correlation between consumption growth and the equity return is increasing with the degree of attention (θ).

If the model economy consists of a continuum of consumers with identical capacity, we need to consider how to aggregate the decision rules across all consumers facing the idiosyncratic noise shock. Sun (2006) presents an exact law of large numbers for this type of economic models and then characterizes the cancellation of individual risk via aggregation. In this model, we adopt this law of large numbers (LLN) and assume that the initial cross-sectional distribution of the noise shock is its stationary distribution. Provided that we construct the space of agents and the probability space appropriately, all idiosyncratic noises are canceled out and aggregate noise is zero. Specifically, after aggregating over all consumers, we obtain the expressions for changes in aggregate consumption:

$$\Delta c_{t+1}^{*}=\frac{\theta\alpha^{*}u_{t+1}}{1-\left(\left(1-\theta\right)/\phi\right)\cdot L}. \tag{40}$$

where the iid idiosyncratic noises in the expressions for individual consumption dynamics have been canceled out. The following proposition summarizes the results of the joint dynamics of aggregate consumption and the equity return:

Proposition 3 *Given finite capacity κ (i.e., θ) and optimal portfolio choice α^*, the relative volatility of consumption growth to the equity return is*

$$\mu=\frac{\mathrm{sd}\left[\Delta c_t^{*}\right]}{\mathrm{sd}\left[u_t\right]}=\sqrt{\frac{\theta^2}{1-\left(1-\theta\right)/\phi^2}}\alpha^{*}, \tag{41}$$

the first-order autocorrelation of consumption growth is

$$\rho_{\Delta c(1)}=\mathrm{corr}\left[\Delta c_t^{*},\Delta c_{t+1}^{*}\right]=\frac{\theta\left(1-\theta\right)}{\phi}, \tag{42}$$

and the contemporaneous correlation between consumption growth and the equity return is

$$\mathrm{corr}\left[\Delta c_{t+1}^{*},u_{t+1}\right]=\sqrt{\left(1-\left(1-\theta\right)/\phi^2\right)}. \tag{43}$$

Proof. See Online Appendix 6.3. ■

3.4 Channel Capacity

Our required channel capacity ($\theta = 0.48$ or $\kappa = 0.33$ nats) may seem low; 1 nat of information transmitted is definitely well below the total information-processing ability of human beings.[25] However, it is not implausible for little capacity to be allocated to the portfolio decision because individuals also face many other competing demands on their attention. For an extreme case, a young worker who accumulates balances in his 401 (k) retirement savings account might pay no attention to the behavior of the stock market until he retires. In addition, in our model for simplicity we only consider an aggregate shock from the equity return, while in reality consumers/investors

[25] See Landauer (1986) for an estimate.

face substantial idiosyncratic shocks (in particular labor income shocks) that we do not model in this paper; Sims (2010) contains a more extensive discussion of low information-processing limits in the context of economic models.

As we noted in the Introduction, there are some existing estimation and calibration results in the literature, albeit of an indirect nature. For example, Adam (2005) found $\theta = 0.4$ based on the response of aggregate output to monetary policy shocks; Luo (2008) found that if $\theta = 0.5$, the otherwise standard permanent income model can generate realistic relative volatility of consumption to labor income; Luo and Young (2009) found that setting $\theta = 0.57$ allows a otherwise standard RBC model to match the post-war US consumption/output volatility. Finally, Melosi (2009) uses a model of firm rational inattention (similar to Maćkowiak and Wiederholt 2009) and estimates it to match the dynamics of output and inflation, obtaining $\theta = 0.66$. Thus, it seems that somewhere between 0.4 and 0.7 is a reasonable range, and our number lies right in the middle of this interval while the one required in Luo (2010) is much lower.

3.5 Implications for Equilibrium Asset Pricing

According to the standard consumption-based capital asset pricing theory (CCAPM), the expected excess return on any risky portfolio over the risk-free interest rate is determined by the covariance of the excess return with contemporaneous consumption growth and the coefficient of relative risk aversion. Given the observed low contemporaneous covariance between equity returns and contemporaneous consumption growth, the standard CCAPM theory predicts that equities are not very risky. Consequently, to generate the observed high equity premium (measured by the difference between the average real stock return and the average short-term real interest rate), the coefficient of relative risk aversion must be very high. Given that $\omega = 0.16$, $\pi = \mu - r^f = 0.06$, and $\text{cov}\left[\Delta c^*_{t+1}, u_{t+1}\right] = 6 \times 10^{-4}$ (annualized US quarterly data from Campbell 2003), to generate the observed equity premium we need a risk aversion coefficient of $\gamma = 100$.

To explore the equilibrium asset pricing implications of the optimal consumption and portfolio rules under RU and RI derived in Section 3.3, we now consider a simple exchange economy in the vein of Lucas (1978) and Hansen (1987). Specifically, we assume that the representative agent receives an endowment, which equals consumption in equilibrium and can trade two assets in the economy: a risky asset entitling the consumer to the dividend (i.e., endowment) and a riskless asset (an inside bond, i.e., in equilibrium its net supply is 0). The returns to the assets then adjust to support a no-trade equilibrium. Using the optimal consumption and portfolio rules under RU and RI derived in the above partial equilibrium model, we can then explore how the interaction of RU and RI affects the equilibrium equity premium. The following is the definition of the RU-RI equilibrium in our model economy:

Definition 4 *The RU-RI equilibrium consists of (i) the portfolio rule α^*, (27), (ii) the consumption rule c^*, (28), and (iii) the perceived state ($\widehat{s}$) evolution equation, (30) such that simultaneously,*

(1) Markets clear in each period: c^* is just the endowment and $\alpha^* = 1$;

(2) The consumer solves for α^* and c^* using the RU-RI model specified in Sections 3.1-3.3.

The following proposition summarizes the implications of the interaction of RU and RI for the equity premium in the general equilibrium defined above:

Proposition 5 *Given finite capacity κ (i.e., θ), the equilibrium equity premium, π, is given by:*

$$\pi = (\gamma + \Gamma)\,\omega^2, \tag{44}$$

where $\Gamma = \frac{\gamma-1}{1-\sigma}(\varsigma - 1)$ and $\varsigma = \frac{\theta}{1-(1-\theta)/\beta} > 1$.

Proof. (44) can be obtained by setting α^* in (27) to be 1. ■

It is clear from (44) that the interaction between RI and RU induces a higher equity premium because risk aversion and intertemporal substitution are disentangled and the accumulated effect of the innovation to the equity on consumption $\varsigma = \frac{\theta}{1-(1-\theta)/\phi} > 1$. The intuition behind this result is that for inattentive investors the uncertainty about consumption changes induced by changes in the equity return takes many periods to be resolved and this postponement is distasteful for these investors who prefer early uncertainty resolution; consequently, they require higher risk compensation in equilibrium.

Figures 1 and 2 can be used again to illustrate how RI affects the equity premium in equilibrium via increasing the long-run consumption risk Γ when $\beta = 0.91$ and both γ and σ are close to 1. Using the same example in the portfolio choice problem, when $\gamma = 1.01$ and $\theta = 0.9$, $\Gamma = 11$, which means that the required equity premium would be increased by 11 times; when θ is smaller Γ is larger, as we showed earlier, so the required return must be larger. That is, a small difference between risk aversion γ and intertemporal substitution σ can have a significant impact on the equilibrium equity return if agents have limited attention.

Table 1 reports how RI affects the joint behavior of aggregate consumption and the equity return and the equilibrium equity premium in the RU model. There are two interesting observations in the table. First, inattention governed by low θ can significantly increase the equilibrium equity return by interacting with the preference for early uncertainty resolution. For example, when $\theta = 25\%$, $\gamma = 1.01$, and $\sigma = 0.998$, the equilibrium equity premium is about 7.3%. Second, lowering attention can simultaneously improve the joint dynamics of aggregate consumption and the equity premium. Specifically, RI can (i) reduce the relative volatility of consumption growth to the equity return, (ii) generate positive autocorrelation of consumption growth, and (iii) reduce the contemporaneous correlation between consumption growth and the equity return. From the table, it is clear that it is difficult to generate the observed relative volatility of consumption growth in the equilibrium model. The reason is that we are considering a pure exchange economy where the share invested in the risky asset is 100% in equilibrium, which significantly increases consumption volatility. In addition, from (41)-(43) and (44), we can see that the value of EIS only affects the equilibrium equity premium and does not affect the consumption.

3.6 Comparison of Portfolio Choice and Asset Pricing under Alternative Hypotheses

3.6.1 Model Uncertainty and Robustness

Robust control and filtering emerged in the engineering literature in the 1970s, and was introduced into economics and further developed by Hansen, Sargent, and others. A simple version of robust optimal control considers such a question: How to make decisions when the agent does not know the probability model that generates the data? The agent with the preference for robustness considers a range of models, and makes decisions that maximize utility given the worst possible model. The work of Uppal and Wang (2003) and Maenhout (2004) explores how model uncertainty due to a preference for robustness affects optimal portfolio choice. In particular, Maenhout (2004) shows that robustness leads to environment-specific effective risk aversion and thus significantly reduces the demand for the risky asset. In addition, after calibrating the robustness parameter, he finds that robustness increases the equilibrium equity premium.[26] In his model, the optimal portfolio rule is

$$\alpha = \frac{\pi}{(\gamma + \vartheta)\,\omega^2},$$

where ϑ measures the degree of robustness and $\gamma + \vartheta$ is the effective coefficient of relative risk aversion. Compared with the portfolio rule derived in our RU-RI model, it is clear that although both of these two specifications, model uncertainty due to robustness and state uncertainty due to inattention, can reduce the optimal share invested in the risky asset, the mechanisms to generate low allocation in the risky asset are distinct: In the former, the aversion to model uncertainty increases the effective degree of risk aversion, and thus reduces the optimal allocation in the equity,

[26] Cecchetti, Lam, and Mark (2000) and Abel (2002) examine how exogenously distorting subjective beliefs can help resolve the equity premium puzzle and the risk-free rate puzzle; robust control distorts beliefs in exactly the right manner.

whereas in the latter the interaction of rational inattention and a preference for early resolution of uncertainty strengthens long-run consumption risk and thus reduce the optimal share in the equity.

3.6.2 Infrequent Adjustment

Another closely related hypothesis about informational frictions is the infrequent adjustment specification. (See Lynch 1996; Gabaix and Laibson 1999; Abel, Eberly, Panageas, 2007; and Nechio 2012 for discussions of the implications of infrequent adjustment in consumption on portfolio choice or/and asset pricing.) Among these models, Gabaix and Laibson (1999)'s 6D bias model is most related to our work. Particularly, the key difference between Gabaix and Laibson's infrequent-adjustment model and our RI model is that in their model, investors adjust their consumption plans infrequently but completely once they choose to adjust, whereas investors with finite capacity adjust their plans frequently but incompletely in every period. In addition, in the 6D model, the optimal fraction of savings invested in the risky asset is assumed to be fixed at the standard Merton solution

$$\alpha = \frac{\pi}{\gamma\omega^2},$$

where π is the equity premium, γ is CRRA, and ω^2 is the variance of the equity return, whereas optimal portfolio choice under RI reflects the larger long-term consumption risk caused by slow adjustments and thus the share invested in the risky asset is less than the Merton solution. Abel, Eberly, and Panageas (2007) derived a unique solution for the optimal interval of time between consecutive observations of the value of the portfolio with observation and transaction costs, and showed that even a small observation cost can lead to a substantial (eight-month) decision interval. They assume that the investment portfolio of riskless bonds and risky stocks is managed by a portfolio manager who continuously rebalances the portfolio, which is similar to the assumption used in Gabaix and Laibson (1999). In other words, they do not examine how infrequent adjustments affect the optimal asset allocation via the channel of the long-run consumption risk. In all of these infrequent adjustments, aggregate consumption can have low contemporaneous correlation with the equity return because individual investors adjust their consumption plans infrequently and only a faction of the agents adjust their consumption in each period.

4 Two Extensions

4.0.3 Correlated Shock and Noise

In the above analysis, we assumed that the exogenous shock to the equity return and RI-induced noise are uncorrelated. We now discuss how correlated shocks and noises affect the implications of RI for long-run consumption risk and optimal asset allocation. In reality, we do observe correlated shocks and noises. For example, if the system is an airplane and winds are buffeting the plane, the random gusts of wind affect both the process (the airplane dynamics) and the measurement (the sensed wind speed) if people use an anemometer to measure wind speed as an input to the Kalman filter. In our model economy, it seems reasonable to assume that given the same level of capacity, when the economy moves into a recession (or financial crisis), both the innovation to the equity return and the noise due to finite capacity (the measurement or the perceived/sensed signal) will also be affected by the recession. In the RI problem, the correlation generalizes the assumption in Sims (2003) on the uncorrelated RI-induced noise.

Specifically, we consider the case in which the process shock (ε) and the noise (ξ) are correlated as follows:

$$\operatorname{corr}\left(\varepsilon_{t+1}, \xi_{t+1}\right) = \rho_{u\xi}, \tag{45}$$

$$\operatorname{cov}\left(\varepsilon_{t+1}, \xi_{t+1}\right) = \rho_{u\xi}\omega\omega_{\xi}, \tag{46}$$

where $\rho \in [-1, 1]$ is the correlation coefficient between ε_{t+1} and ξ_{t+1}, and $\omega_{\xi}^2 = \operatorname{var}\left[\xi_{t+1}\right]$. Substituting (45) and (46) into the pricing equation (24), we obtain

$$\pi = \lim_{S\to\infty}\left\{\sum_{j=0}^{S}\operatorname{cov}{}_t\left[\frac{\rho}{\sigma}\Delta c_{t+1+j} + (1-\rho)\left(\sum_{j=0}^{S} r_{p,t+1+j}\right), u_{t+1}\right]\right\} = (\gamma + \Gamma)\,\alpha\omega^2,$$

where

$$\Gamma = \frac{\gamma - 1}{1 - \sigma}(\varsigma - 1) + \frac{\rho\rho_{u\xi}}{\sigma}\left(1 - \frac{1}{\phi}\varsigma\right)\sqrt{\frac{\theta}{1/(1-\theta) - (1/\phi)^2}}. \tag{47}$$

measures the long-run consumption risk in the presence of the correlation between the equity return and the noise. (See Appendix 6.4.) Figure 6 illustrates how RI affects the long-run consumption risk Γ for different values of the correlation when $\beta = 0.91$, $\sigma = 0.99999$, and $\gamma = 1.01$. The figure clearly shows that the positive correlation will reduce the long-run consumption risk and thus increase the optimal share invested in the risky asset. For example, when $\theta = 0.8$ and $\rho_{u\xi} = 0.1$, $\Gamma = 20$; if $\rho_{u\xi}$ reduces to -0.1, $\Gamma = 31$.

What is a reasonable sign for this correlation? If we assume that capacity is fixed when the state of the economy changes, it seems more reasonable that $\rho_{u\xi}$ is positive because it would be more difficult to observe a more volatile economy given fixed capacity. However, if we relax the assumption that κ is fixed, some capacity from other sources will be reallocated to monitor the state of the economy to increase the economic efficiency because an increase in the underlying uncertainty leads to larger marginal welfare losses due to RI. In this case, $\rho_{u\xi}$ could be negative as the Kalman gain θ will increase with capacity κ.

4.1 Nontradable Labor Income

It is known that some of the anomalous predictions of the portfolio model can be reduced, although not eliminated, by the introduction of nontradable labor income. Following Viceira (2001) and Campbell and Viceira (2002), we assume that labor income Y_t is uninsurable and nontradable in the sense that investors cannot write claims against future labor income; thus, labor income can be viewed as a dividend on the implicit holdings of human wealth. We will only sketch the results here; formal derivations are a straightforward extension of our existing results and are omitted.

We assume that the process for labor income is

$$Y_{t+1} = Y_t \exp\left(\nu_{t+1} + g\right) \tag{48}$$

where g is a deterministic growth rate and ν_{t+1} is an iid normal random variable with mean zero and variance ω_ν^2. Log labor income therefore follows a random walk with drift; to keep the exposition simpler, we abstract from any transitory income shocks. In order to permit the risky asset to play a hedging role against labor income risk, we suppose that the two shocks are potentially correlated contemporaneously:

$$\mathrm{cov}_t\left(u_{t+1}, \nu_{t+1}\right) = \omega_{u\nu}.$$

If $\omega_{u\nu} = 0$ then labor income can be viewed as purely idiosyncratic. The flow budget constraint then becomes

$$A_{t+1} = R_{t+1}^p \left(A_t + Y_t - C_t\right) \tag{49}$$

Log-linearizing (49) around the long-run means of the log consumption-income ratio and the log wealth-income ratio, $c - y = E\left[c_t - y_t\right]$ and $a - y = E\left[a_t - y_t\right]$, and defining a new state variable, $s_t = a_t + \lambda y_t$, where $\lambda = \frac{1-\rho_a+\rho_c}{\rho_a-1}$, we adopt the same solution method in our benchmark model to solve this model with uninsurable labor income. The following proposition summarizes the results on the optimal consumption and portfolio rules under RI:

Proposition 6 *Suppose that* γ *is close to* 1 *and Assumption (2) is satisfied (see below). The optimal share invested in the risky asset is*

$$\alpha^* = \frac{1}{\widetilde{\varsigma}}\left[\frac{1}{b_1}\left(\frac{\mu - r^f + 0.5\omega^2}{\omega^2}\right) + \left(1 - \frac{1}{b_1}\right)\frac{\widetilde{\varsigma}\omega_{u\nu}}{\omega^2}\right], \tag{50}$$

where $\widetilde{\varsigma} = \frac{\rho}{\sigma}\varsigma + 1 - \rho > 1$*,* $b_1 = \frac{\rho_a - 1}{\rho_c} \in (0,1]$*,* $\rho_a = \frac{\exp(a-y)}{1+\exp(a-y)-\exp(c-y)} > 0$*, and* $\rho_c = \frac{\exp(c-y)}{1+\exp(a-y)-\exp(c-y)} > 0$*; the consumption function is*

$$c_t^* = b_0 + b_1 \widehat{s}_t; \tag{51}$$

the true state evolution equation is

$$s_{t+1} = \rho_0 + \rho_a s_t - \rho_c c_t - g + \varepsilon_{t+1} + \frac{1 - \rho_a + \rho_c}{\rho_a - 1} \nu_{t+1} + r_{t+1}^p, \tag{52}$$

where $\rho_0 = -(1 - \rho_a + \rho_c)\log(1 - \rho_a + \rho_c) - \rho_a \log(\rho_a) + \rho_c \log(\rho_c)$*; and the estimated state* $\widehat{s}_t$ *is characterized by the following Kalman filtering equation*

$$\widehat{s}_{t+1} = (1 - \theta)\widehat{s}_t + \theta(s_{t+1} + \xi_{t+1}) + \Upsilon, \tag{53}$$

where $\psi = \log(\phi) - (1 - 1/\phi)\log(1 - \phi)$*,* $\theta = 1 - \frac{1}{\exp(2\kappa)}$ *is the optimal weight on a new observation,* ξ_t *is an* iid *idiosyncratic noise shock with* $\omega_\xi^2 = \mathrm{var}[\xi_{t+1}] = \Sigma/\theta$*,* $\Sigma = \frac{\alpha^{*2}\omega^2}{\exp(2\kappa)-(1/\phi)^2}$ *is the steady state conditional variance, and* Υ *is an irrelevant constant term.*

Proof. See Online Appendix 6.5. ■

Note that to obtain these results, we require the following assumption.

Assumption 2:

$$1 - (1 - \theta)\rho_a > 0. \tag{54}$$

Comparing with the assumption used in the benchmark model, here ρ_a has replaced β^{-1}, but otherwise Equation (54) is the same as Equation (20). When $\rho = 1$ (or $\gamma = 1/\sigma$), the RU solution reduces to the expected utility solution:

$$\alpha^* = \frac{1}{\varsigma}\left[\frac{1}{b_1}\left(\frac{\mu - r^f + 0.5\omega^2}{\omega^2}\right) + \left(1 - \frac{1}{b_1}\right)\frac{\varsigma\omega_{u\nu}}{\omega^2}\right] \tag{55}$$

where $\varsigma = \frac{\theta}{1-(1-\theta)\rho_a} > 1$.

$\widetilde{\varsigma} > 1$ measures the long-run (accumulated) impacts of financial shocks on consumption. It is clear that our key result – that the presence of rational inattention combined with a preference for early resolution of uncertainty will dramatically reduce the share of risky assets and increase the required equity premium – survives the introduction of labor income risk. Expression 50 contains two components. The first part is the so-called speculative asset demand, driven by the gap between the return to equity and the riskfree rate. Note that without labor income risk, the optimal asset allocation is solely determined by the speculative demand; that is, the allocation is proportional to the expected excess return of the risky asset, and is inversely related to the variance of the equity return and to the elasticity of consumption to perceived wealth, b_1.

The second part is the hedging demand, governed by the correlation between returns and labor income. Given that $\rho_a > 1$ and $\theta \in (0,1)$, RI affects the optimal allocation in the risky asset via the following two channels:

1. Reducing both the speculative demand and the income-hedging demand by the long-run consumption risk parameter $\widetilde{\varsigma}$.

2. In addition, as shown in the second term in the bracket of (50), RI increases the income hedging demand by $\widetilde{\varsigma}$ because u_t and ν_t are correlated and consumption reacts to the shock to total wealth $\zeta_t = \alpha u_t + \lambda \nu_t$ gradually and indefinitely.

To make these points clear, we rewrite (50) as:

$$\alpha^* = \frac{1}{\widetilde{\varsigma} b_1}\left(\frac{\mu - r^f + 0.5\omega^2}{\omega^2}\right) + \left(1 - \frac{1}{b_1}\right)\frac{\omega_{u\nu}}{\omega^2} \tag{56}$$

This expression clearly shows that RI increases the *relative* importance of the income-hedging demand to the speculative demand via the long-run consumption risk $\widetilde{\varsigma}$; under RI, the ratio of the income hedging demand to the speculative demand increases by $\widetilde{\varsigma}$. As inattention increases (θ declines), the hedging aspect of the demand for risky assets increases in importance, since $\frac{\partial \widetilde{\varsigma}}{\partial \theta} < 0$. To see where this positive relationship derives from, results from Luo (2008) and Luo and Young (2009b) imply that the welfare cost of labor income uncertainty is increasing in the degree of inattention (as θ falls, the cost rises). If equity returns are positively correlated with labor income, the agent will decrease demand for the asset as an insurance vehicle; similarly, a negative correlation will increase hedging demand. The data suggest this correlation is negative, but so small as to be quantitatively unimportant.[27] In addition, the second term in (56) also shows that RI has no effect on the absolute value of the income hedging demand. The reason is simple: under RI the innovation to the equity return not only affects the amount of long-run consumption risk measured by $1/\varsigma$ but also affects the long-run correlation between the shocks to the equity return and labor income measured by $\varsigma\omega_{u\nu}$ as both shocks affect consumption growth; consequently, RI does not change the hedging demand of labor income. It is clear from Expression (55) that ςs in the term $(1/\varsigma)$ and in the term $(\varsigma\omega_{u\nu})$ are just cancelled out.

As in Section 3.5, we can examine the asset pricing implications of the twin assumptions of recursive utility and rational inattention in the presence of nontradable labor income. Given that every investor has the same degree of RI, the following pricing equation linking consumption

[27] For example, Heaton and Lucas (2000) find that individual labor income is weakly correlated with equity returns, with support for both positive and negative correlations. Aggregate wages have a correlation of -0.07 with equity returns.

growth and the equity premium holds when $\gamma \simeq 1$:

$$\pi = \alpha^* \widetilde{\varsigma} b_1 \omega^2 - \widetilde{\varsigma} (b_1 - 1) \omega_{u\nu} - 0.5\omega^2. \tag{57}$$

Under the same assumptions made above (zero net supply of bonds so that $\alpha^* = 1$), (57) becomes

$$\pi = \widetilde{\varsigma} \left[b_1 \omega^2 + (1 - b_1) \omega_{u\nu} \right] - 0.5\omega^2, \tag{58}$$

which clearly shows that the positive correlation between the equity return and labor income, $\omega_{uv} > 0$, increases the equilibrium equity premium. Specifically, the magnitude of the hedging demand, $(1 - b_1) \omega_{uv}$, is increased by ς in the presence of information-processing constraints. In sum, the interaction between RI and positive correlations between the equity return and labor income will increase the equity premium in equilibrium by

$$\varpi = \widetilde{\varsigma} \left[1 + \left(\frac{1}{b_1} - 1 \right) \frac{\rho_{uv} \omega \omega_\nu}{\omega^2} \right]. \tag{59}$$

Note that in the case without RI and $\rho_{uv} = 0$, $\pi + 0.5\omega^2 = b_1 \omega^2$.

5 Conclusion

In this paper we have studied the portfolio choice of a household with Kreps-Porteus/Epstein-Zin preferences and limited information-processing capacity (rational inattention). Rational inattention interacts with a preference for early resolution of uncertainty to generate significant decreases in the demand for risky assets; small deviations from indifference over timing and infinite channel capacity are magnified over the infinite future to produce empirically-reasonable portfolios with actual risk aversion essentially equal to 1, whether the agent has nontradable labor income or not. This result raises important questions about empirical assessment, such as how to identify risk aversion separately from channel capacity, that we will not pursue here.

We have focused on solutions in which *ex post* uncertainty is Gaussian. Recent results in the rational inattention literature (Matejka and Sims 2010 and Saint-Paul 2010) have noted that there exist *discrete* optimal solutions to the decision problem, even in the LQ-Gaussian case, that may dominate the Gaussian one; the intuition for this result is that information costs can be reduced by dividing the state space into regions and only permitting solutions to differ across these regions instead of inside them. With these solutions in mind, Batchuluun, Luo, and Young (2008) show that fully-nonlinear portfolio decisions are discrete in a simple two-period economy; these solutions have the property that agents will place positive measure on only a small number of different portfolio shares. For reasonable degrees of risk aversion and low enough channel capacity, one group of these points involves zero equity holdings, because agents who want to borrow from future income will do so using the risk-free asset and there is always a positive probability that wealth is actually such that borrowing would be optimal. A second group of points involves a small amount of risky assets (and generally this set of points has the most mass), while a third group has a significant amount of risky assets. Extending these results to study portfolios in long-horizon models has the potential to rationalize why few households hold assets that do not appear very risky (in terms of consumption or utility), why those that hold these assets hold so few of them, and why these assets pay such high rates of return. The mechanism identified here will still be present, if somewhat obscured by the numerical solution.

6 Online Appendix (Not for Publication)

6.1 Deriving The Perceived State Evolution Equation

Here, we detail the straightforward steps omitted in the main part of the paper that derive the perceived state evolution equation. The Kalman filtering equation can be written as:

$$\widehat{a}_{t+1} = (1-\theta)\left[\frac{1}{\phi}\widehat{a}_t + \left(1-\frac{1}{\phi}\right)c_t\right] + \theta a^*_{t+1}. \tag{60}$$

Combining this Kalman filtering equation with the true state evolution equation yields:

$$\widehat{a}_{t+1} = \frac{1}{\phi}\widehat{a}_t + \left(1-\frac{1}{\phi}\right)c^*_t + \psi + \eta_{t+1}, \tag{61}$$

where η_{t+1} is the innovation to the perceived state:

$$\eta_{t+1} = \theta\left[\alpha^*\left(r^e_{t+1} - r^f\right) + r^f + \frac{1}{2}\alpha^*\left(1-\alpha^*\right)\omega^2 + \xi_{t+1}\right] + \frac{\theta}{\phi}\left(a_t - \widehat{a}_t\right),$$

and $a_t - \widehat{a}_t$ is the estimation error:

$$a_t - \widehat{a}_t = \frac{(1-\theta)\left[\alpha^*\left(r^e_{t+1} - r^f\right) + r^f + \frac{1}{2}\alpha^*\left(1-\alpha^*\right)\omega^2\right]}{1-((1-\theta)/\phi)\cdot L} - \frac{\theta\xi_t}{1-((1-\theta)/\phi)\cdot L}.$$

6.2 Deriving Long-term Euler Equation within the Recursive Utility Framework

Within the recursive utility framework, when wealth is allocated efficiently across assets, the marginal investment in any asset yields the same expected increase in future utility,

$$E_t\left[\frac{U_{2,t}U_{1,t+1}}{U_{1,t}}\left(R_{e,t+1} - R_f\right)\right] = 0, \tag{62}$$

which means that

$$E_t\left[U_{1,t+1}\left(R_{e,t+1}-R_f\right)\right]=0, \tag{63}$$

where $U_{i,t}$ for any t denotes the derivative of the aggregator function with respect to its $i-th$ argument, evaluated at $(C_t, E_t[U_{t+1}])$.

Using the Euler equation for the risk free asset between $t+1$ and $t+1+S$,

$$\begin{aligned} U_{1,t+1} &= E_{t+1}\left[\left(\beta_{t+1}\cdots\beta_{t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\right] \\ &= E_{t+1}\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\right], \end{aligned} \tag{64}$$

where we denote $\beta_{t+1+j}=U_{2,t+j}$, for $j=0,\cdots,S$. Substituting (64) into (63) yields

$$\begin{aligned} & E_t\left[E_{t+1}\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R^f\right)^S U_{1,t+1+S}\right]\left(R^e_{t+1}-R^f\right)\right] \\ &= E_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R^f\right)^S U_{1,t+1+S}\left(R^e_{t+1}-R^f\right)\right]=0. \end{aligned}$$

Hence, the expected excess return can be written as

$$\begin{aligned} E_t\left[R_{e,t+1}-R_f\right] &= -\frac{\operatorname{cov}_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}, R_{e,t+1}-R_f\right]}{E_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\right]} \\ &= -\frac{\operatorname{cov}_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}, R_{e,t+1}-R_f\right]}{E_t\left[U_{1,t+1}\right]} \\ &= -\frac{\operatorname{cov}_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}, R_{e,t+1}-R_f\right]}{U_{1,t}/\left(U_{2,t}R_f\right)} \\ &= -\operatorname{cov}_t\left[\left(U_{2,t}\cdots U_{2,t+S}\right)\left(R_f\right)^{S+1}\frac{U_{1,t+1+S}}{U_{1,t}}, R_{e,t+1}-R_f\right] \\ &= -\operatorname{cov}_t\left[\left(R_f U_{2,t}\frac{U_{1,t+1}}{U_{1,t}}\right)\cdots\left(R_f U_{2,t+S}\frac{U_{1,t+1+S}}{U_{1,t+S}}\right), R_{e,t+1}-R_f\right] \\ &\simeq \operatorname{cov}_t\left[\left(\frac{\theta}{\rho}\Delta c_{t+1}+(1-\theta)\,r_{p,t+1}\right)+\cdots+\left(\frac{\theta}{\rho}\Delta c_{t+1+S}+(1-\theta)\,r_{p,t+1+S}\right), u_{t+1}\right] \\ &= \operatorname{cov}_t\left[\frac{\theta}{\rho}\left(\sum_{j=0}^{S}\Delta c_{t+1+j}\right)+(1-\theta)\left(\sum_{j=0}^{S} r_{p,t+1+S}\right), u_{t+1}\right]. \end{aligned}$$

6.3 Deriving the Stochastic Properties of Consumption Dynamics

Taking unconditional variance on both sides of (31) yields

$$\begin{aligned}
\operatorname{var}\left[\Delta c_t^*\right] &= \theta^2\left\{\frac{\alpha^{*2}\omega^2}{1-(1-\theta)^2/\phi^2}+\left[1+\frac{\theta^2/\beta^2}{1-(1-\theta)^2/\beta^2}\right]\omega_\xi^2\right\} \\
&= \theta^2\left\{\frac{1}{1-(1-\theta)^2/\phi^2}+\left[\frac{1}{\left(1-(1-\theta)/\phi^2\right)\theta}-\frac{1}{1-(1-\theta)^2/\phi^2}\right]\right\}\alpha^{*2}\omega^2 \\
&= \frac{\theta\phi^2}{\phi^2+\theta-1}\alpha^{*2}\omega^2.
\end{aligned}$$

Note that in the absence of the endogenous noise shocks, we have

$$\begin{aligned}
\operatorname{var}\left[\Delta c_t^*\right] &= \theta^2\frac{\alpha^{*2}\omega^2}{1-(1-\theta)^2/\phi^2} \\
&= \frac{\theta^2}{1-(1-\theta)/\phi^2}\alpha^{*2}\omega^2.
\end{aligned}$$

Using (31), we can compute the first-order autocovariance of consumption growth:

$$\begin{aligned}
&\operatorname{cov}\left(\Delta c_t^*,\Delta c_{t+1}^*\right)=\operatorname{cov}\left(\begin{array}{c}\theta\left\{\frac{\alpha^* u_t}{1-((1-\theta)/\phi)\cdot L}+\left[\xi_t-\frac{(\theta/\phi)\xi_{t-1}}{1-((1-\theta)/\phi)\cdot L}\right]\right\}, \\ \theta\left\{\frac{\alpha^* u_{t+1}}{1-((1-\theta)/\phi)\cdot L}+\left[\xi_{t+1}-\frac{(\theta/\phi)\xi_t}{1-((1-\theta)/\phi)\cdot L}\right]\right\}\end{array}\right) \\
&= \frac{1-\theta}{\phi}\operatorname{cov}\left(\frac{\theta\alpha^* u_t}{1-((1-\theta)/\phi)\cdot L},\frac{\theta\alpha^* u_t}{1-((1-\theta)/\phi)\cdot L}\right) \\
&+\operatorname{cov}\left(\theta\left[\xi_t-\frac{(\theta/\phi)\,\xi_{t-1}}{1-((1-\theta)/\phi)\cdot L}\right],-\frac{\theta\,(\theta/\phi)\,\xi_t}{1-((1-\theta)/\phi)\cdot L}\right) \\
&= \frac{1-\theta}{\phi}\operatorname{var}\left(\frac{\theta\alpha^* u_t}{1-((1-\theta)/\phi)\cdot L}\right)+\operatorname{cov}\left(\theta\left[\xi_t-\frac{(\theta/\phi)\,\xi_{t-1}}{1-((1-\theta)/\phi)\cdot L}\right],-\frac{\theta\,(\theta/\phi)\,\xi_t}{1-((1-\theta)/\phi)\cdot L}\right) \\
&= \frac{1-\theta}{\phi}\frac{(\theta\alpha^*)^2\omega^2}{1-(1-\theta)^2/\phi^2}-\frac{\theta^3}{\phi}\frac{\alpha^{*2}\omega^2}{\left(1/(1-\theta)-1/\phi^2\right)\theta} \\
&+\frac{\theta^4(1-\theta)}{\phi^3}\frac{1}{1-(1-\theta)^2/\phi^2}\frac{\alpha^{*2}\omega^2}{\left(1/(1-\theta)-1/\phi^2\right)\theta} \\
&= \left[\begin{array}{c}\frac{1-\theta}{\phi}\frac{\theta^2}{1-(1-\theta)^2/\phi^2}-\frac{\theta^2}{\phi}\frac{1}{1/(1-\theta)-1/\phi^2} \\ +\frac{\theta^3(1-\theta)}{\phi^3}\frac{1}{1-(1-\theta)^2/\phi^2}\frac{1}{1/(1-\theta)-1/\phi^2}\end{array}\right]\alpha^{*2}\omega^2 \\
&= 0.
\end{aligned}$$

We thus have

$$\operatorname{corr}\left(\Delta c_t^*,\Delta c_{t+1}^*\right)=\frac{\operatorname{cov}\left(\Delta c_t^*,\Delta c_{t+1}^*\right)}{\sqrt{\operatorname{var}\left(\Delta c_t^*\right)}\sqrt{\operatorname{var}\left(\Delta c_{t+1}^*\right)}}=0.$$

Note that in the absence of the endogenous noise shocks, we have

$$
\begin{aligned}
\operatorname{corr}\left(\Delta c_t^*, \Delta c_{t+1}^*\right) &= \frac{1-\theta}{\phi} \frac{\left(\theta \alpha^*\right)^2 \omega^2}{1-(1-\theta)^2/\phi^2} \left(\frac{\theta \phi^2}{\phi^2+\theta-1} \alpha^{*2} \omega^2\right)^{-1} \\
&= \frac{1-\theta}{\phi} \frac{\theta^2}{1-(1-\theta)^2/\phi^2} \left(\frac{\theta}{1-(1-\theta)^2/\phi^2}\right)^{-1} = \frac{\theta(1-\theta)}{\phi} > 0
\end{aligned}
$$

because

$$
\begin{aligned}
&\operatorname{cov}\left(\Delta c_t^*, \Delta c_{t+1}^*\right) = \operatorname{cov}\left(\frac{\theta \alpha^* u_t}{1-((1-\theta)/\phi)\cdot L}, \frac{\theta((1-\theta)/\phi)\alpha^* u_t}{1-((1-\theta)/\phi)\cdot L}\right) \\
&= \frac{1-\theta}{\phi} \operatorname{cov}\left(\frac{\theta \alpha^* u_t}{1-((1-\theta)/\phi)\cdot L}, \frac{\theta \alpha^* u_t}{1-((1-\theta)/\phi)\cdot L}\right) \\
&= \frac{1-\theta}{\phi} \frac{\left(\theta \alpha^*\right)^2 \omega^2}{1-(1-\theta)^2/\phi^2}.
\end{aligned}
$$

Finally, using (31), it is straightforward to show that

$$
\begin{aligned}
\operatorname{corr}\left(\Delta c_{t+1}^*, u_{t+1}\right) &= \frac{\operatorname{cov}\left(\Delta c_{t+1}^*, u_{t+1}\right)}{\operatorname{sd}\left(\Delta c_{t+1}^*\right) \operatorname{sd}\left(u_{t+1}\right)} \\
&= \frac{\theta \alpha^* \omega^2}{\sqrt{\frac{\theta \phi^2}{\phi^2+\theta-1} \alpha^* \omega^2}} \\
&= \sqrt{\theta\left(1-(1-\theta)/\phi^2\right)},
\end{aligned}
$$

where we use the result that

$$
\operatorname{cov}\left(\Delta c_{t+1}^*, u_{t+1}\right) = \operatorname{cov}\left(\theta \alpha^* u_{t+1}, u_{t+1}\right) = \theta \alpha^* \omega^2.
$$

Note that in the absence of the endogenous noise shocks, we have

$$
\operatorname{corr}\left(\Delta c_{t+1}^*, u_{t+1}\right) = \sqrt{\left(1-(1-\theta)/\phi^2\right)},
$$

6.4 Deriving the Long-run Risk in the Presence of the Correlation

Using (24), we have

$$
\begin{aligned}
\pi &= \lim_{S\to\infty}\left\{\sum_{j=0}^{S}\operatorname{cov}_t\left[\begin{array}{c}\frac{\rho}{\sigma}\theta\left\{\frac{\alpha u_{t+1+j}}{1-((1-\theta)/\phi)\cdot L}+\left[\xi_{t+1+j}-\frac{(\theta/\phi)\xi_{t+j}}{1-((1-\theta)/\phi)\cdot L}\right]\right\}\\ +(1-\rho)\left(\sum_{j=0}^{S} r_{p,t+1+j}\right),u_{t+1}\end{array}\right]\right\}\\
&= \frac{\rho}{\sigma}\theta\left\{\frac{1}{1-(1-\theta)/\phi}\alpha\omega^2+\left[1-\frac{\theta}{\phi}\frac{1}{1-(1-\theta)/\phi}\right]\rho\omega\omega_\xi\right\}+(1-\rho)\,\alpha\omega^2\\
&= \frac{\rho}{\sigma}\theta\left\{\frac{1}{1-(1-\theta)/\phi}+\rho_{u\xi}\left[1-\frac{\theta}{\phi}\frac{1}{1-(1-\theta)/\phi}\right]\sqrt{\frac{1}{\left[1/(1-\theta)-(1/\phi)^2\right]\theta}}\right\}\alpha\omega^2+(1-\rho)\,\alpha\omega^2\\
&= \left\{\left[\frac{\rho}{\sigma}\frac{\theta}{1-(1-\theta)/\phi}+(1-\rho)\right]+\frac{\rho\rho_{u\xi}}{\sigma}\theta\left[1-\frac{\theta}{\phi}\frac{1}{1-(1-\theta)/\phi}\right]\sqrt{\frac{1}{\left[1/(1-\theta)-(1/\phi)^2\right]\theta}}\right\}\alpha\omega^2,
\end{aligned}
$$

which will reduce to (47) in the text. Note that here we use the fact that $\omega_\xi=\sqrt{\frac{1}{\left[\exp(2\kappa)-(1/\phi)^2\right]\theta}}\alpha\omega$.

6.5 Deriving Optimal Consumption and Portfolio Rules in the Presence of Uninsurable Labor Income

Log-linearizing the flow budget constraint, $A_{t+1}=R^p_{t+1}(A_t+Y_t-C_t)$, around the long-run means of the log consumption-income ratio and the log wealth-income ratio, $c-y=E[c_t-y_t]$ and $a-y=E[a_t-y_t]$, yields the approximate budget constraint

$$a_{t+1}-y_{t+1}=\rho_0+\rho_a(a_t-y_t)+\rho_c(c_t-y_t)-\Delta y_{t+1}+r^p_{t+1} \tag{65}$$

where ρ, ρ_a, and ρ_c are constants:

$$\rho_a=\frac{\exp(a-y)}{1+\exp(a-y)-\exp(c-y)}>0 \tag{66}$$

$$\rho_c=\frac{\exp(c-y)}{1+\exp(a-y)-\exp(c-y)}>0 \tag{67}$$

$$\rho_0=-(1-\rho_a+\rho_c)\log(1-\rho_a+\rho_c)-\rho_a\log(\rho_a)+\rho_c\log(\rho_c). \tag{68}$$

Starting from the standard time-separable rational expectations model ($\gamma=\sigma$ and $\theta=1$) we obtain the decision rules

$$
\begin{aligned}
c_t &= y_t+b_0+b_1(a_t-y_t)\\
\alpha^* &= \frac{1}{b_1}\left(\frac{\mu-r_f+\frac{1}{2}\omega^2}{\gamma\omega^2}\right)+\left(1-\frac{1}{b_1}\right)\frac{\omega_{u\nu}}{\omega^2}
\end{aligned}
$$

where

$$b_1 = \frac{\rho_a - 1}{\rho_c} \in (0, 1] \tag{69}$$

$$b_0 = \frac{1}{1-\rho_a}\left[\left(\frac{1}{\gamma} - b_1\right) E\left[r^p_{t+1}\right] + \frac{1}{\gamma}\log(\beta) + \frac{\Xi}{2\gamma} - \rho_0 - (1-b_1)\, g\right];$$

b_1 is the elasticity of consumption with respect to financial wealth, making $1 - b_1$ the elasticity with respect to labor income, and Ξ is an irrelevant constant term. If labor income is tradable, $b_1 = 1$ and the model reduces to the one studied previously.

To introduce rational inattention, we define a new state variable

$$s_t = a_t + \lambda y_t, \tag{70}$$

where

$$\lambda = \frac{1 - \rho_a + \rho_c}{\rho_a - 1}.$$

(As we have noted earlier, multivariate rational inattention models are analytically intractable, so the reduction of the state space to a single variable is critical for our results). Using this new state variable, the log-linearized budget constraint (65) can be rewritten as

$$s_{t+1} = \rho_0 + \rho_a s_t - \rho_c c_t - g + \rho_\varepsilon \varepsilon_{t+1} + \lambda \nu_{t+1} + r^p_{t+1}. \tag{71}$$

The consumption function then becomes

$$c_t = b_0 + b_1 s_t. \tag{72}$$

Applying the separation principle yields

$$c_t = b_0 + b_1 \widehat{s}_t \tag{73}$$

and we obtain the law of motion for the conditional mean of permanent income

$$\widehat{s}_{t+1} = (1-\theta)\,\widehat{s}_t + \theta\,(s_{t+1} + \xi_{t+1}) + \Upsilon, \tag{74}$$

where Υ is an irrelevant constant and all other notation is the same as above.

Given the assumption that $1 - (1-\theta)\,\rho_a > 0$, applying the same long-term Euler equation we used in the benchmark model, we can solve for the optimal share invested in equity in the presence of labor income as:

$$\alpha^* = \frac{1}{\widetilde{\varsigma}}\left[\frac{1}{b_1}\left(\frac{\mu - r^f + 0.5\omega^2}{\omega^2}\right) + \left(1 - \frac{1}{b_1}\right)\frac{\widetilde{\varsigma}\omega_{u\nu}}{\omega^2}\right] \tag{75}$$

where $\widetilde{\varsigma} = \frac{\rho}{\sigma}\varsigma + 1 - \rho$, $\varsigma = \frac{\theta}{1-(1-\theta)\rho_a} > 1$, and (66). This expression is just (75) in the text. Comparing with the result in the benchmark model, here ρ_a has replaced ϕ^{-1}, but otherwise the assumption, (54), is the same as 20.

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Table 1: Effects of RU and RI on Consumption and the Equity Premium

	U.S. data	Full-information	$\theta = 25\%$	$\theta = 35\%$	$\theta = 45\%$	$\theta = 55\%$
π ($\sigma = 0.998$)	0.06	0.023	0.073	0.050	0.039	0.034
π ($\sigma = 0.996$)	0.06	0.023	0.048	0.036	0.031	0.028
μ	0.07	1	0.81	0.75	0.77	0.81
$\rho_{\Delta c(1)}$	0.22	0	0.21	0.25	0.27	0.27
corr $\left(\Delta c^*_{t+1}, u_{t+1}\right)$	0.21	1	0.31	0.46	0.58	0.68

Note: The U.S. data are based on Campbell (2003) and we set $\gamma = 1.01$.

Figure 1: The Effects of RI and RU on Long-run Consumption Risk

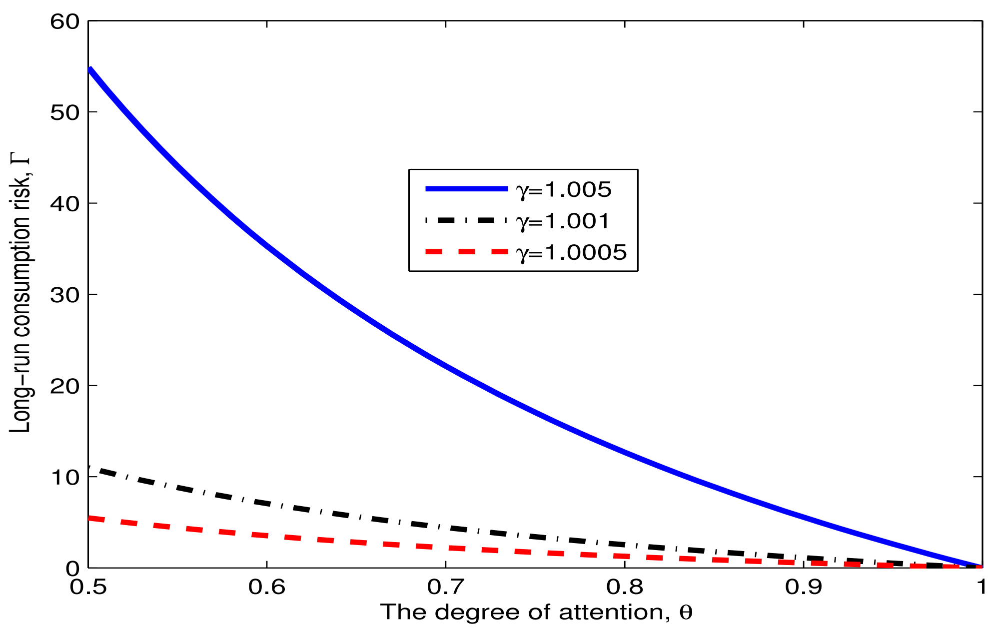

Figure 2: The Effects of RI and RU on Long-run Consumption Risk

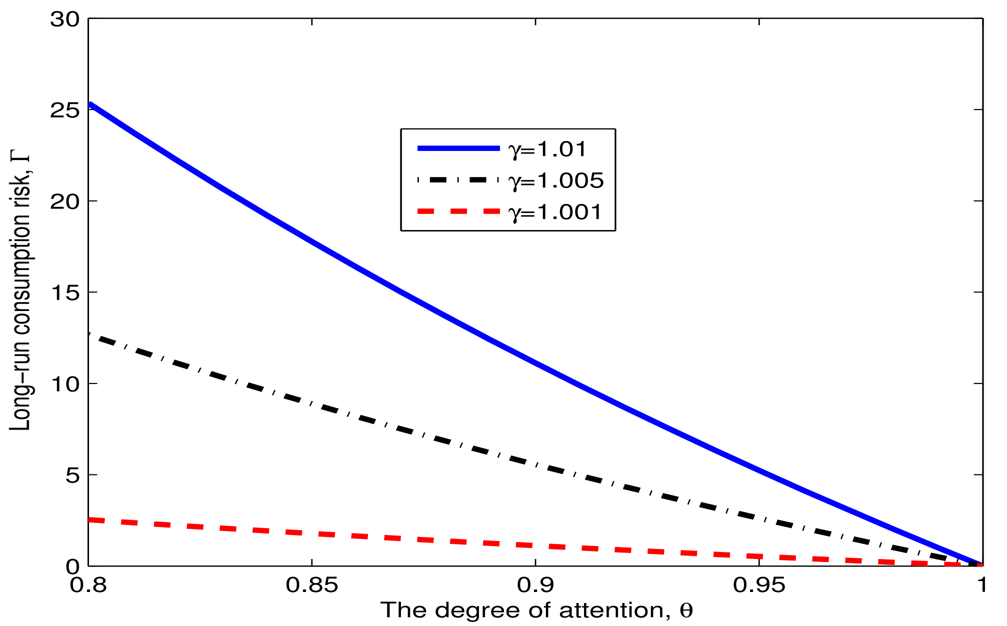

Figure 3: The Effects of the Investment Horizon on Long-run Consumption Risk

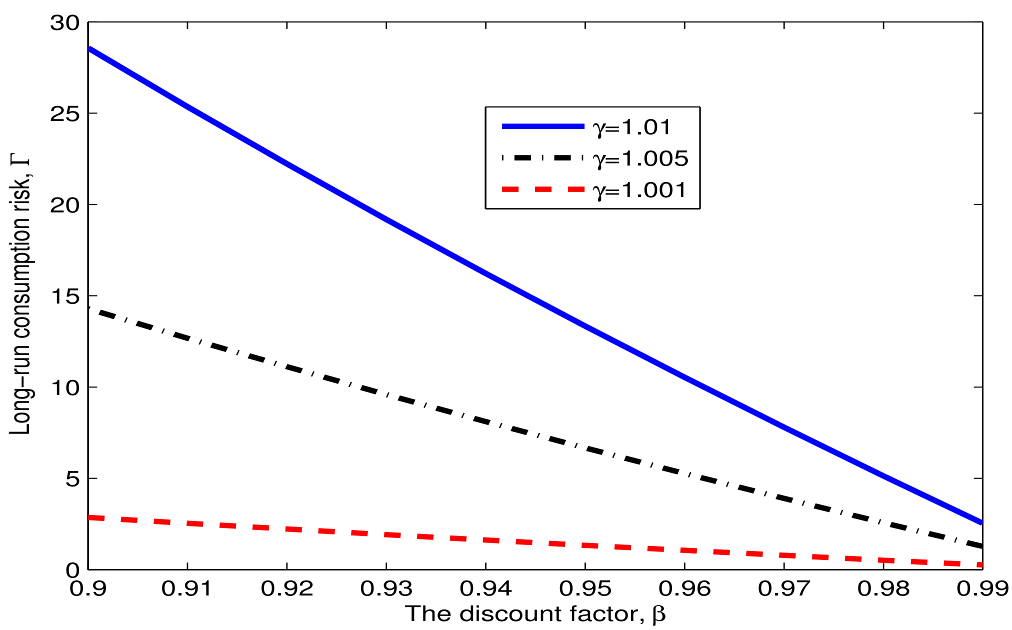

Figure 4: The Effects of RI on Consumption Volatility

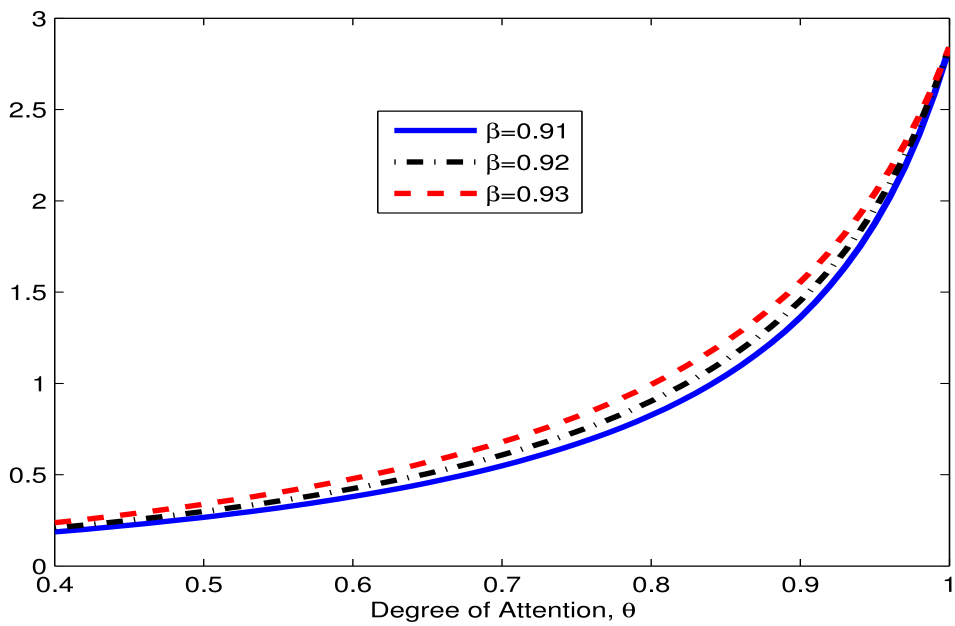

Figure 5: The Effects of RI on Consumption Correlation

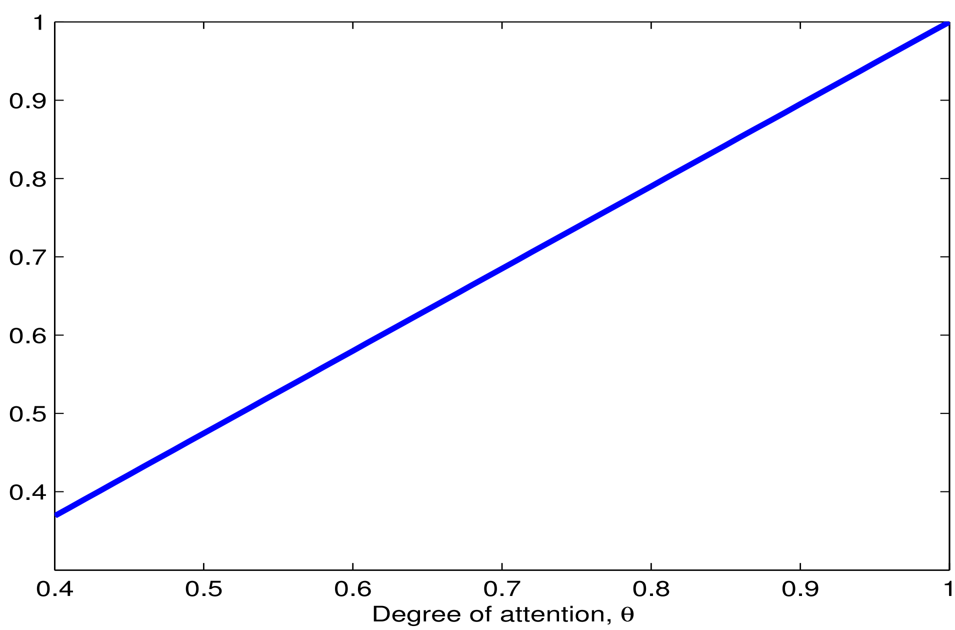

Figure 6: The Effects of RI and RU on Long-run Consumption Risk when $\rho_{u\xi} \neq 0$