Why Branded Firm may Benefit from Counterfeit Competition

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Abstract

A durable good monopolist sells its branded product over two periods. In period 2, when there is entry of a counterfeiter, the branded firm may charge a high price to signal its quality. Counterfeit competition thus enables the branded firm to commit to high future price in period 2, alleviating the classic time-inconsistency problem under durable good monopoly. This can increase the branded firm's profit by encouraging consumer purchase without delay, despite the revenue loss to the counterfeiter. Total welfare can also increase, because early purchase eliminates delay cost and consumers enjoy the good for both periods.

JEL: D82, L11, L13

Keywords: Coase Conjecture, Counterfeit, Quality Signaling

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1 Introduction

Today, counterfeit has became a fast growing multi-hundred billion dollars business. In the 2007 OECD counterfeit report, the volume of counterfeits was around 200 billion dollars in international trade, 2% of world trade.* This figure does not include domestic consumption of counterfeits or digital products distributed via internet. The U.S. government estimated that counterfeit trade increased more than 17 folds in the past decade (U.S. Customs and Border Protection 2008).

Counterfeits are generally viewed as harmful to both the authentic producers and consumers, especially when they are deceptive, such as counterfeits of pharmaceutical products, eyeglasses, luxury goods or even normal textile products manufactured by famous brands.* There have been, however, some recent empirical evidences suggesting that (deceptive) counterfeit could actually benefit the branded firm. In particular, Qian (2008) finds that the average profit for branded shoes in China is higher after the entry of counterfeit. Qian (2011) shows that the impact of counterfeits on profit depends on the quality gap between authentic good and counterfeit good; the branded firm benefits from the counterfeit when the quality gap is sufficiently large. In this paper, I show that a branded firm can indeed benefit from competition of a deceptive counterfeiter when their quality difference is large enough.

I consider a model with an authentic durable-good firm who sells in two periods. Without counterfeits, the branded durable good monopolist faces the classic time-inconsistency problem (Coase, 1972): After selling to high-value consumers in the first period at a high

*The Economic Impact of Counterfeiting and Piracy `http://www.oecd.org/industry/ind/38707619.pdf`

*This does not mean consumers can not distinguish the product at all. It is just hard for buyers to tell without any other information. For example, a consumer may not be able to tell a genuine Chanel bag from a fake one only by appearance. However, if one is priced at $3,000 and the other is sold for $50, she will know that the expensive one is more likely to be authentic ex post. On the other hand, non-deceptive counterfeits are those that consumers can easily recognize when purchasing, such as many digital products.

price, it cannot resist cutting its price in the second period. But then rational consumers will delay their purchase, forcing the monopolist to reduce its price in the first period and lower the monopolist's overall profit. Now suppose that a counterfeiter will enter the market. In order to separate its product from the counterfeit, the branded firm needs to set a high price to signal its quality in period 2. Thus the presence of the counterfeit enables the branded firm to commit to a high price in period 2, providing a solution to the time-inconsistency problem. This then motivates more consumers to purchase in period 1 instead of waiting to buy in period 2, even if the first period's price is high. When the quality gap is sufficiently large, this front loading effect will dominate the profit loss from competition in the second period. In terms of total welfare, counterfeits are likely to decrease surplus in the second period. However, first period welfare increases due to front loaded purchases. Early purchase contributes twice surplus compared to late one because consumers can use the good for two periods. Therefore, if the quality gap is not too large, total welfare will be elevated by counterfeits.

The results in this paper shed some light on the policy towards counterfeits. Both branded firms and consumers respond to counterfeits strategically. In the present paper, the authentic firm signals itself from the counterfeiter through high price when the quality gap is large enough. Therefore, consumers will not be fooled by counterfeits with extremely low quality and consumer confusion may not as severe as we think. Besides, with awareness of the entry of counterfeits, consumers are more inclined to purchase early, which benefits both the authentic firm and total welfare in a dynamic context.

Many papers have investigated strategies the durable good monopolist can use to resolve the commitment problem (see, e.g., Waldman, 2003 for an excellent survey). Solutions include leasing rather than selling the durable good (Coase, 1972; Bulow, 1982), special contract between the monopoly and consumers (Butz, 1990), offering an inferior version (Karp and Perloff, 1996; Hahn, 2006) and product-line management (Huhn and Padilla, 1996). All of them involves tactics that the monopoly is willing to adopt to alleviate the problem. The present paper suggests a novel commitment mechanism through the competition from another firm.

Several other papers have discussed the counter-intuitive result of price or profit increasing competition (e.g., Chen and Riordan, 2008; Gaibaix et al., 2005; Perloff et al., 2005; Thomadsen, 2007, 2012). In those papers, the competition changes the demand curve of the incumbent firm. When the competitor attracts some price elastic consumers, the incumbent can concentrate on price insensitive consumers with higher price. However, in my paper, the higher price is created by quality signaling. In addition, in these static models, competition generally will not increase a firm's profit even if prices go up, because the monopolist always earns higher profit than a duopoly if price is the same. Since the monopoly price is the first best for an incumbent, it cannot get higher payoff at any other price, even if that price is higher than the monopoly price. However, in a dynamic model, the monopoly profit is not the first best. Price increasing competition helps the monopoly to overcome time-inconsistency problem and eventually drives up profit.

While many papers have discussed counterfeit or piracy issues, most of them are about the non-deceptive piracy of digital products (see, e.g., Peitz and Waelbroeck, 2006 for a suvery). In terms of deceptive counterfeits, Grossman and Shapiro (1988a) discuss the problem in international trade. They show counterfeits will decrease the total welfare and the authentic firm's profit without the consideration of innovation. Qian (2012) focuses on the brand-protection strategies against counterfeits, including price, quality upgrade, etc. She uses a vertical differentiation model similar to my second period competition. The difference is that I investigate the counterfeit problem in a dynamic context. The new feature yields entirely opposite results. In her paper, the authentic firm's profit decreases with the threat of counterfeits. Total welfare also drops when the ratio of uninformed consumer is high. However, in the present paper, the brand firm's profit and total surplus might increase even if all consumers are uninformed.

Finally, the second period counterfeit competition is related to the literature of duopoly signaling game. Hertzendorf and Overgaard (2001), Fluet and Garella (2002) and Yehezkel (2008) study the similar game with advertising. These papers focus on the role of dissipative advertising on expanding the separating equilibrium regime while I try to answer how counterfeits influence profit and welfare. As Qian (2012), those papers only investigate the static game while my paper incorporates the signaling game into a durable good model.

The paper is organized as follows. Section 2 presents the model and reviews the monopoly benchmark. Section 3 investigates the effect of counterfeit competition on profit and welfare in a specific equilibrium. Section 4 checks the robustness of results. Section 5 concludes. All proofs are relegated to the Appendix.

2 The Model and Monopoly Benchmark

I adapt the two-period durable good model in Tirole (1988). A branded firm sells a durable good that can be used in two periods. The branded firm's quality Q_A is normalized to 1. In the second period, a counterfeit firm producing a lower quality clone $Q_C = C < 1$ will enter and compete with the branded firm.[⋆] Both firms have no marginal cost to produce the good. Consumers know the quality of both products from the beginning of the game.[⋆] However, they are not able to tell which good is produced by the brand firm only by appearance when they purchase.[⋆] This contrasts the standard assumption that consumers can trace the producer of the good, which is the problem of deceptive counterfeit. The online market is a typical example with this feature. The authentic firm sells through many retailers in each period. It is too costly for a consumer to track every retailer. After

[⋆]It implicitly assumes that the authentic product has some lead time advantage. Many firms would have special designs on their new product so that imitators have to spend more time to produce the similar counterfeit.

[⋆]They are aware of the quality of counterfeit product at the first period.

[⋆]The underling assumption is that the counterfeit is deceptive and all consumers are uninformed. An alternative assumption is that part of consumers are informed. As long as the proportion of uninformed consumers are large enough, my qualitative conclusion will hold.

a while, the counterfeiter builds its own website and sell the fake good. Inexperienced consumers can not tell whether a website has been selling the genuine product from the first period or just enters to sell counterfeit. Besides, it is also possible that a website that sells authentic version turns to spread counterfeits later.

There is a unit mass of heterogeneous consumer indexed by the taste parameter $\theta_l \sim U[0,1]$. Consumer's utility is defined as the linear function form:

$$U_l = \theta_l Q_i - P_i, i \in \{\text{Authentic}(A), \text{Counterfeit}(C)\}$$

Consumer decides whether to buy in each period. The discount factor of both firms and consumers are assumed to be 1.

Before analyzing the model, it is important to specify consumer's belief. Let $\mu_i(P_A, P_C)$ be the probability that consumers believe the good she purchased from firm i is the authentic good given the branded firm's price is P_A and the counterfeiter's price is P_C. Unlike the traditional monopoly signaling model, there are two signal senders here. Consumer belief is based on price and the number of firms charge that price. Consumers are aware that two firms sell the good and one of them is the counterfeiter. $\mu_A(P_A, P_C) + \mu_C(P_A, P_C) = 1$ must be satisfied in equilibrium. When $P_A = P_C$, consumers can not identify two products and $\mu_A = \mu_C = \frac{1}{2}$. In the separating equilibrium, when $P_A \neq P_C$, consumers believe that the expensive good is authentic and the cheap one is counterfeit.

Given consumer's belief, firm's profit is represented by $\Pi^k_{it}(P_A, P_C, \mu_i), t \in \{1,2\}, k \in \{\text{Pooling Equilibrium}(PE), \text{Separating Equilibrium}(SE)\}$, where the subscript i, t stands for firm type and time respectively, the superscript k denotes equilibrium type in the second period. The variables with superscript k are in equilibrium value. This notation form also applies to other variables like price, surplus etc. Note that for both belief and profit, the first position in parentheses is always the authentic firm's price and the second one is the counterfeiter's price. Assuming the authentic firm chooses separating equilibrium when profits are the same over two types of equilibriums.

The time-line of the game is as follows: The authentic firm sets the first period price P_1

at t=1. Consumers decide whether to buy or wait. The counterfeiter enters in the second period and both firms set price simultaneously. Then consumers are aware of both prices and make a purchasing decision based on their beliefs.

Before analyzing the game with counterfeit competition, let's firstly review the benchmark monopoly model without entry. Initially proposed by Coase (1972), then formalized by Bulow (1982), it is well-known that the monopoly faces a time-inconsistency problem (Coase Conjecture) if it cannot make a commitment on future price. In the first period, the monopoly sells to consumers with high valuation at a high price. Low valuation consumers will be left to the second period. However, the monopoly fails to take first period's demand into account and has an incentive to decrease the price to reap the residual demand in the second period. As a consequence, high valuation consumers will anticipate the price reduction in the future and some of them postpone purchase to the second period. The monopoly's profit under intertemporal price discrimination is lower than if it can commit. The argument can be proved as follows.[⋆]

(1) When the monopoly lacks commitment power. $P_2 = \frac{1}{2}\theta_1$

Consumers who purchase in the first period must satisfy the intertemporal incentive compatibility constraint such that their first period surplus is higher.

$$2\theta_1 - P_1 \geq \theta_1 - P_2$$

The marginal consumer θ_1^M purchases in the first period is the one makes the constraint binding.

Then the monopoly's aggregate profit will be determined by:

$$\Pi = (2\theta_1 - \theta_1 + P_2)(1 - \theta_1) + P_2(\theta_1 - P_2)$$

The marginal buyer and the monopoly's profit are $\theta_1^M = \frac{3}{5}$ and $\Pi^M = \frac{9}{20}$ respectively.

[⋆]Since there is only one firm here, the subscript represents time and the superscript stands for the equilibrium value in monopoly case.

(2) When the monopoly can commit to none intertemporal price discrimination:

$$\Pi = 2\theta(1-\theta)$$

This gives a profit $\Pi^M = \frac{1}{2} > \frac{9}{20}$ and $\theta_1 = \frac{1}{2}$

3 Equilibrium Analysis With Counterfeit Competition

In this section, I will firstly characterize all Perfect Bayesian Equilibriums (PBE) and then show that there exists a reasonable equilibrium such that the counterfeit could increase the authentic firm's profit and social welfare.

The standard backward induction is applied to analyze the counterfeit game. As in the benchmark, there is a marginal consumer θ_1, such that all consumers with taste parameter above θ_1 will purchase in the first period. The rest consumers are left to be served in the future. θ_1 can be interpreted as the market size of the second period.

3.1 Signaling Game in Second Period

In t=2, there is a signaling game played between a pair of vertically differentiated duopoly and consumers. Consumers use market prices to construct their beliefs. If both firms have the same price, counterfeits are indistinguishable ex post and pooling equilibrium is sustained. If the counterfeiter sets a lower price than the branded firm and reveals itself, there will be a separating equilibrium where consumers know for sure which goods are counterfeits.

In a pooling equilibrium, assume consumers randomly pick a product and the expected quality of the product is $\frac{1+C}{2}$. The profit function is given by the following equation.

$$\Pi_{A2} = \Pi_{C2} = \frac{1}{2}(\theta_1 - \frac{2P_2}{1+C})P_2$$

In a separating equilibrium, profit functions of both firms are the same as vertical price competition with complete information.

$$\Pi_{A2}(P_{A2}, P_{C2}, 1) = (\theta_1 - \frac{P_{A2} - P_{C2}}{1 - C})P_{A2}$$

$$\Pi_{C2}(P_{A2}, P_{C2}, 0) = (\frac{P_{A2} - P_{C2}}{1 - C} - \frac{P_{C2}}{C})P_{C2}$$

The counterfeiter's best response function is always $P_{C2} = \frac{C}{2}P_{A2}$ in the separating equilibrium.

The key question is when the separating equilibrium can be sustained. In the standard monopoly signaling game, the separation is attained if the single cross condition is satisfied. The firm with high marginal cost is willing to distort price further than the low cost firm because the profit only depends on own price and consumer belief. However, in a duopoly case, a firm's profit is also affected by the other firm's price. When one sets a high price, the other one faces a trade off between consumer belief and demand: If the counterfeiter decides to pool with the authentic firm, it is believed to produce authentic goods at 50% but sells less because of high price. The counterfeiter can reveal itself by a lower price, which may be better because the upward distorted price of the branded firm mitigates competition and leaves a large market for the counterfeiter. Two incentive compatibility constraints must be satisfied to support a separating equilibrium. The first equation assures that the counterfeiter does not deviate to the authentic price and the second one implies the branded firm do want to maintain the high price. Otherwise, pooling equilibrium is the only candidate in the second period.

$$\Pi_{C2}(P_{A2}, P_{C2}, 0) \geq \Pi_{C2}(P_{A2}, P_{A2}, \frac{1}{2}) \tag{1}$$

$$\Pi_{A2}(P_{A2}, P_{C2}, 1) \geq \Pi_{A2}(P_{C2}, P_{C2}, \frac{1}{2}) \tag{2}$$

Lemma 1. When the quality of counterfeit is low ($C \leq C_1 \approx 0.604$), a set of separating equilibriums can be sustained. $P_{A2}^{SE} \in S^{SE} = [\underline{P_2}(\theta_1, C), \overline{P_2}(\theta_1, C)]$,
$\underline{P_2}(\theta_1, C) = \frac{2(1-C^2)}{C^2-3C+4}\theta_1$, $\overline{P_2}(\theta_1, C) = \frac{(4-C)(1-C^2)}{2(2-C)(1+C)-C^2(1-C)}\theta_1$, $P_{C2}^{SE} = \frac{C}{2}P_{A2}^{SE}$. For any

quality C, there exists a set of pooling equilibriums.

$P_{A2}^{PE} = P_{C2}^{PE} = P_2^{PE} \in S^{PE} = [0, \underline{P_2}(\theta_1, C))$.

All equilibriums listed in Lemma 1 can be supported by a system of out of equilibrium beliefs, such as the most pessimistic belief. For any separating equilibrium with $\widetilde{P}_{A2} \in [\underline{P_2}, \overline{P_2}]$ and $\widetilde{P}_{C2} = \frac{C}{2} P_{A2}$, if the out of equilibrium belief is that any deviating price $P^{'} \neq \{\widetilde{P}_{A2}, \widetilde{P}_{C2}\}$ is conceived as a sign of counterfeits, then no firm would deviate and that particular separating equilibrium is stable. Similarly, the belief that $\mu(P^{'}, \widetilde{P}_2) = 0$, $\forall P^{'} \neq \widetilde{P}_2$ can support all pooling equilibriums.

This lemma is the non-advertising result in Hertzendorf and Overgaard (2001). In that paper, the use of dissipative advertising by the authentic firm serves as an extra tool to signal its quality and could expand the separating equilibrium to the entire range. However, in the context of counterfeit, it is hard for the branded firm to effectively advertise because consumers can not track the firm or tell which one is burning money.

The result is very intuitive: When the quality gap is large, the profit from pooling equilibrium is low because of the low expected quality. The authentic firm just needs to slightly distort the price upward, which will reduce the price competition and leave the counterfeiter enough profit under separating regime. For the branded firm, since price distortion is moderate, the cost of signaling is not too high to them to afford. However, if two products are close substitutes, the cost of signaling for the branded firm is so high that it would rather pool with the counterfeiter.

As in other signaling games, there is multiple-equilibrium problem in this model as well. In some low price pooling equilibriums, counterfeit competition is detrimental to the branded firm's profit. In this section, I will show that there exist an equilibrium in which both the authentic firm and the society benefit from counterfeit under certain conditions. In the next section, it is proved that all equilibriums surviving from the Competitive Intuitive Criterion refinement have similar properties.

The equilibrium selected here is the one with the highest second period profit for authentic firm, which is defined as *profit maximizing equilibrium.* Assuming that consumers are rational enough to figure out all equilibriums and believe that the authentic firm will choose the price that maximizes its second period profit.* Therefore, consumers believe the firm charging that price is the authentic firm. If both firms set that price, the good has 50% probability to be genuine. Any good with other price is conceived as fake. This is the pessimistic belief that supports profit maximizing price in t=2. Formally, consumer belief is defined as follow.

$$\mu_A(P_{A2}^*, P_2) = 1, \forall P_2 \neq P_{A2}^*; \mu_C(P_2, P_{A2}^*) = 1, \forall P_2 \neq P_{A2}^*$$
$$\mu(P_{A2}^*, P_{A2}^*) = \frac{1}{2}; \mu_i(P_2, P_2^{'}) = 0, \forall P_2, P_2^{'} \neq P_{A2}^*$$

In this section, an extra asterisk is used in superscript to denote variables in profit maximizing equilibrium. Let P_{A2}^{SE*} and P_2^{PE*} be the authentic price in the optimal separating equilibrium and pooling equilibrium respectively. $P_{A2}^* = Max[P_{A2}^{SE*}, P_2^{PE*}]$ is the price that maximizes the branded firm's second period profit, which is illustrated in the following lemma.

Lemma 2. In profit maximizing equilibrium, if the counterfeit's quality is low enough ($C \leq C_3 \approx 0.512$), the separating equilibrium is supported as the PBE of signaling game in t=2. $P_{A2}^* = P_{A2}^{SE*} = \underline{P_2}(\theta_1, C)$, $\Pi_{A2}^* = \Pi_{A2}^{SE*} = \frac{4(1-C)^2(1-C^2)}{C^2-3C+4}\theta_1^2$. If the counterfeit's quality is high ($C > C_3$), the pooling equilibrium will be selected. For $C_3 < C \leq C_2 \approx 0.702$, $P_2^* = P_2^{PE*} = \frac{1+C}{4}\theta_1$, $\Pi_2^* = \Pi_{A2}^{PE*} = \frac{1+C}{16}\theta_1^2$. For $C > C_2$, $P_2^* = P_2^{PE*} = \underline{P_2}(\theta_1, C)$, $\Pi_2^* = \Pi_{A2}^{PE*} = \frac{C(1+C)(1-C^2)}{2(C^2-3C+4)^2}\theta_1^2$.

When $C \in [0, C_3]$, the price has an inverted U-shape and higher than the monopoly price in benchmark. $\underline{P_2}(\theta_1, C)$ is the minimum price that prevents the counterfeiter from mimicking the branded firm. The counterfeiter's profit under pooling increases faster with

*The firm has no commitment ability. Since the first period sell is the sunk history, consumers can only adjust their beliefs to help the authentic firm in the second period.

C than that under separating equilibrium when C is close to 0.[⋆] Therefore, the authentic firm is forced to increase the price in order to reduce competition and increase the competitor's profit under separating equilibrium. As C gets larger, the condition will be reversed and the authentic firm has no need to incur a large distortion to support separating equilibrium. When $C \in (C_3, C_2]$, the price increases with C because of higher expected quality. When C is close to 1, the game converges to Bertrand Competition of homogeneous good, and the price goes down to 0.

3.2 The Dynamic Game

In this subsection, I will analyze the dynamic game and illustrate why the entry of counterfeiter may generate higher profit for the incumbent. Given the second period consumer surplus and the first period price, the marginal buyer in the first period will be determined. The authentic firm's decision is to choose this marginal consumer to maximize total profit. In the monopoly benchmark, the firm can not resist the temptation to cut price, which makes consumer surplus too high in period 2 and decreases revenue from rational high valuation consumers in period 1. However, with the competition from a deceptive counterfeiter, consumer surplus is likely to decrease in period 2, which alleviate the time-inconsistency problem.

Pooling Equilibrium

In the first segment of pooling equilibrium ($C_3 < C \leq C_2$), consumer surplus in period 2 decreases because the market is flooded with counterfeits. This pushes more consumers to buy in the first period since the authentic good can be guaranteed. However, the market price is lower than the benchmark, which makes late purchase more attractive.[⋆] Overall, consumer surplus falls below the benchmark case and the time-inconsistency problem is mitigated. The effect that makes consumer buy early is named as *Front Loading Effect*.

[⋆]When C is close to 0, $\frac{d\Pi_{C2}^{PE}}{dC} = \frac{1}{(1+C)^2} P_{A2}^2 \geq \frac{d\Pi_{C2}^{SE}}{dC} = \frac{1}{4(1-C)^2} P_{A2}^2$.

[⋆]$P_2^{PE*} = \frac{1+C}{4}\theta_1 \leq \frac{1}{2}\theta_1$

On the other hand, the counterfeit competition will decrease the branded firm's revenue in the second period, which I called *Competition Effect*. The change of the authentic firm's profit is determined by the magnitude of two effects.

The marginal consumer who purchases at t=1 in pooling equilibrium is determined by the binding incentive compatibility constraint:

$$2\theta_1 - P_1 = \frac{1+C}{2}\theta_1 - P_2^{PE*}$$

The authentic firm's maximization problem is:

$$\max_{\theta_1} \ \Pi_A^{PE*}(\theta_1) = (1-\theta_1)(2\theta_1 - \frac{1+C}{2}\theta_1 + P_2^{PE*}) + \frac{1}{2}(\theta_1 - \frac{2P_2^{PE*}}{1+C})P_2^{PE*}$$

The marginal buyer θ_1^{PE*} and equilibrium profit Π_A^{PE*} are:

$$\theta_1^{PE*} = \begin{cases} \frac{1+\frac{3-C}{4}}{2(1+\frac{11-5C}{16})} & C \in (C_3, C_2] \\ \frac{[\frac{3-C}{2}+\frac{2(1-C^2)}{C^2-3C+4}]}{2[\frac{3-C}{2}+\frac{(1-C^2)}{C^2-3C+4}+\frac{4(1-C^2)(1-C)}{(C^2-3C+4)^2}]} & C \in (C_2, 1) \end{cases}$$

$$\Pi_A^{PE*} = \begin{cases} \frac{(1+\frac{3-C}{4})^2}{4(1+\frac{11-5C}{16})} & C \in (C_3, C_2] \\ \frac{[\frac{3-C}{2}+\frac{2(1-C^2)}{C^2-3C+4}]^2}{4[\frac{3-C}{2}+\frac{(1-C^2)}{C^2-3C+4}+\frac{4(1-C^2)(1-C)}{(C^2-3C+4)^2}]} & C \in (C_2, 1) \end{cases}$$

As Figure 1 shows, when $C \in (C_3, C_2]$, θ_1^{PE*} increases with C for two reasons. Individual surplus in the second period increases with the quality of counterfeit and more customers tend to wait, which decreases the wedge between P_1 and θ_1. On the other hand, the branded firm balances the profit in each period to maximize total profit by properly choosing θ_1. When C increases, there is a higher second period profit. It is optimal to leave more customers in the second period (increase second period market size).

When $C \in (C_2, 1)$, θ_1^{PE*} firstly increases and then decreases in this range. When C gets close to 1, the front loading effect disappears because P_2 is close to 0. The branded firm decreases the market size in period 2 due to fierce competition. It can be inferred that the incumbent does not benefit from counterfeit competition in this range.

Separating Equilibrium

In separating equilibrium, the competition effect is similar to the pooling equilibrium but less prominent. As the high quality producer, the branded firm would earn higher profit compared to the head-to-head competition in pooling equilibrium. The mechanism of the front loading effect is slightly different. Consumers will not be fooled ex post but face a super monopoly price in the second period. The binding intertemporal incentive compatibility constraint reflects that the marginal buyer θ_1^{PE} faces two outside options in the second period—buy the authentic good or the counterfeit.

$$2\theta_1 - P_1 = max\{\theta_1 - \underline{P_2}(C, \theta_1), C\theta_1 - \frac{C}{2}\underline{P_2}(C, \theta_1)\}$$

However, the marginal consumer who is indifferent between a genuine product and a counterfeit in the second period definitely has a lower taste parameter than θ_1. Therefore, the outside option must be the authentic good in intertemporal constraint. The incumbent's profit maximization is as follows.

$$\max_{\theta_1} \ \Pi_A(\theta_1) = (1 - \theta_1)(\theta_1 + \underline{P_2}(C, \theta_1)) + \Pi_{A2}^{SE*}(\theta_1)$$

In equilibrium,

$$\theta_1^{SE*} = \frac{1 + \frac{2(1-C^2)}{C^2-3C+4}}{2[1 + \frac{2(1-C^2)(-C^2+C+2)}{(C^2-3C+4)^2}]}$$

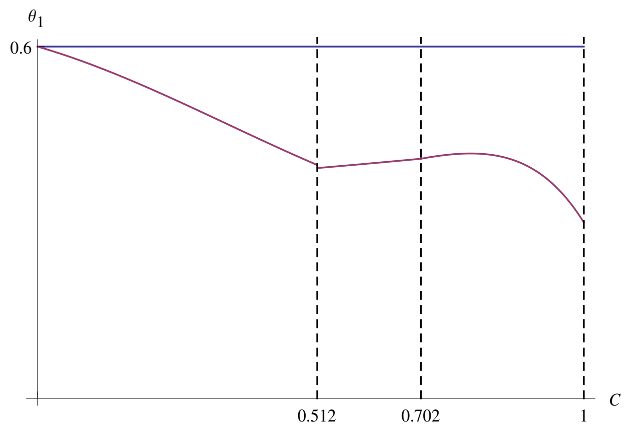
$$\Pi_A^{SE*} = \frac{[1 + \frac{2(1-C^2)}{C^2-3C+4}]^2}{4[1 + \frac{2(1-C^2)(-C^2+C+2)}{(C^2-3C+4)^2}]}$$

The left segment of lower curve in Figure 1 informs that θ_1^{SE*} monotonically decreases with C. As the quality gap getting closer, the branded firm's profit in the second period decreases. It would be better to assign less weight on the second period by decreasing θ_1^{SE*}.

Profit Comparison

Proposition 1. In the profit maximizing equilibrium, the authentic firm's profit will be higher than the monopoly benchmark if the quality of counterfeit is sufficiently low

Figure 1: Marginal Buyer at t=1

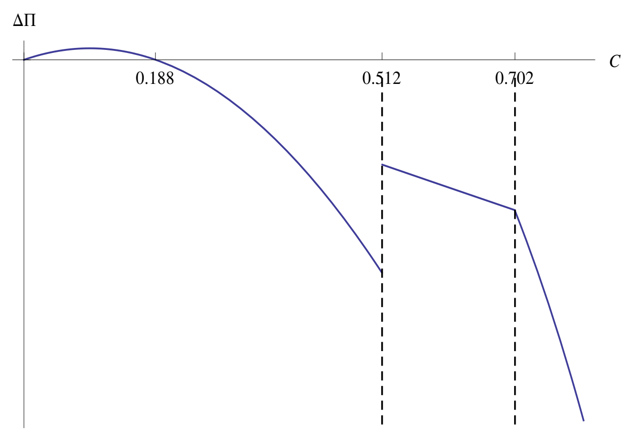


($C < C_4 \approx 0.188$). When the counterfeit's quality is above that threshold, no matter which equilibrium exists in the second period, the competition always decreases the incumbent's profit.

In the first segment of pooling equilibrium, the front loading effect gets weaker when the quality increases (θ_1^{PE*} increases with C) and the time-inconsistency problem is reinforced. However, the high quality counterfeit also weakens competition effect and raises the second period profit. In the second segment, the competition effects gets too strong and the front loading effect disappears. When pooling equilibrium emerges in the second period, the competition effect is too strong and always dominates the front loading effect. The authentic firm suffers from entry of the counterfeiter.

However, as Figure 2 shows, the branded firm's profit has an inverted-U shape and can be higher than the monopoly benchmark. When the quality of the counterfeit is 0, the result with counterfeit competition is the same as the monopoly benchmark. In the first period, since the high second period price makes consumers less likely to wait, the front loading effect will be stronger when $\underline{P_2}(\theta_1, C)$ is high. Recall that $\underline{P_2}(\theta_1, C)$ has an inverted-U shape, which implies the branded firm's profit will has the same curvature. On the other hand, the magnitude of negative competition effect monotonically increases with C. Therefore, when the quality of counterfeits is low, the combination of strong front loading effect and weak competition effect raises the branded firm's profit above the benchmark. As C increases, this condition will be reversed and the incumbent's profit falls below the monopoly case.

Figure 2: Profit Difference



3.3 Welfare and Policy Implication

In terms of welfare, the conventional wisdom is that without the consideration of future R&D incentive or monitoring cost, the non-deceptive counterfeit may increase welfare while the deceptive counterfeit is more likely to have a negative effect.* In short run, the non-deceptive counterfeit acts like a low quality competitor, which serves low-end consumers at a low price and increases total surplus. However, the deceptive counterfeit fools consumers to buy the low quality product at a relatively high price, which is the reason that the trademark law is legislated to prevent consumer confusion. This paper shows that the impact on welfare can be quite different in a dynamic context.

*Many papers studying software piracy confirm non-deceptive piracy could increase social surplus in short term, such as Johnson (1985) and Belleflamme (2002). Grossman and Shapiro (1988a) show deceptive counterfeits decrease welfare with free entry in trade.

In the monopoly benchmark, total surplus is given as follows.

$$TS^M = \int_{\theta_1^M}^{1} 2\theta d\theta + \int_{\theta_2^M}^{\theta_1^M} \theta d\theta$$

The first term represents the surplus created by first period transaction and the second one is welfare in the second period.* Clearly, $\frac{dTS^M}{d\theta_1^M} < 0$. It is always better to let a consumer buy early because she could enjoy double surplus. Given the marginal buyer in each period, $TS^M = 0.775$.

The welfare under deceptive counterfeit competition is a piecewise function.

$$TS(C) = \begin{cases} TS^{SE*}(C) = \int_{\theta_1^{SE*}}^{1} 2\theta d\theta + \int_{\bar{\theta}_2^{SE*}}^{\theta_1^{SE*}} \theta d\theta + \int_{\underline{\theta}_2^{SE*}}^{\bar{\theta}_2^{SE*}} C\theta d\theta & C \leq C_3 \\ TS^{PE*}(C) = \int_{\theta_1^{PE*}}^{1} 2\theta d\theta + \int_{\theta_2^{PE*}}^{\theta_1^{PE*}} \frac{1+C}{2}\theta d\theta & C > C_3 \end{cases}$$

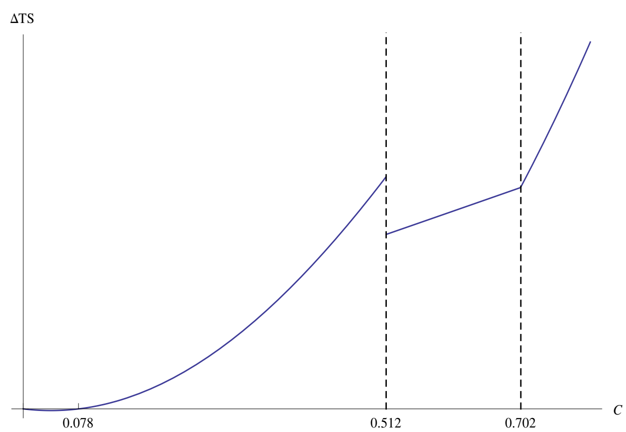
In separating equilibrium, there are two marginal consumers in the second period. $\bar{\theta}_2^{SE*}$ denotes the marginal consumer who is indifferent between the genuine good and the counterfeit. $\underline{\theta}_2^{SE*}$ stands for the one who is indifferent between buying the counterfeit and buying nothing. Surplus is discounted by C if the counterfeit is purchased. In pooling equilibrium, expected surplus is discounted by $\frac{1+C}{2}$ for all consumers because of confusion. Comparing the welfare under two cases yields the next Proposition.

Proposition 2. The deceptive counterfeit will increase total welfare iff the quality of the counterfeit is not too low ($C \geq C_5 \approx 0.078$).

Deceptive counterfeits have two effects on welfare. Firstly, in the second period, total surplus decreases because of competition with incomplete information, which is the typical critic against counterfeits. However, if the first period welfare is taken into account, the result will be quite different. As Figure 1 shows, there are always more sells in the first period once $C > 0$. The front loading effect essentially decreases first period demand elasticity while the competition effect forces the incumbent to reduce the market size in t=2 by

*Surplus is attributed to the trading period. First period buyer enjoys surplus in both periods but the purchase is made at the first period, therefore all surplus belongs to the first period.

Figure 3: Welfare Difference



decreasing first period price. Consumers who purchase in the first period provide “double” contribution on surplus since they are guaranteed with high quality for two periods, which is the reason that total welfare could be higher under bad competition.

In Figure 3, the middle segment demonstrates the welfare difference under the pooling equilibrium with $C \in (C_3, C_2]$. The downward pressure on welfare decreases with quality of the counterfeit because consumer confusion problem is alleviated. Since θ_1^{PE*} increases with C in this range, the positive effect also decreases with C. Overall, the social welfare is higher for all quality levels that sustain the pooling equilibrium in the second period. In the right segment of Figure 3, the second period price decreases with C, which implies more trade and higher welfare.

The left segment is the welfare under separating equilibrium. In Figure 1, as the counterfeit's quality improves, the positive effect increases with C roughly at the same speed ($\frac{d^2\theta_1^{SE*}}{dC^2}$ is close to 0). The second period welfare decreases because of upward distorted prices. Since the second period price has an inverted-U shape, the welfare in that period will be an U-shape curve. Combining these two effects, it is clear why total welfare also has U-shape. When the quality of counterfeit is 0, the model coincides with the benchmark. When C is small, unlike the pooling equilibrium, θ_1 is close to the benchmark value and decreases slower compared to the second period welfare. Therefore, when the quality of the counterfeit is sufficiently low, the overall welfare effect is negative.

This proposition implies that deceptive counterfeits may have a positive effect on welfare in a dynamic context, which is contrary to the traditional argument. What is more surprising is that welfare is significantly higher when counterfeits are indistinguishable ex post. The result reminds us to think deeply in the counterfeit problem. Firstly, branded firms actively adopt strategies against clones. Although counterfeits are deceptive ex ante, whether they can be recognized ex post is endogenized. If the quality of clone is low, in which case consumer confusion induced by counterfeit has a strong negative effect on welfare, the authentic firm will signal by price and rational consumer will not be fooled. If consumers can not distinguish the counterfeit ex post, it must be that the quality gap is close enough. Even if consumers are diverted to the counterfeit in that case, the welfare loss is relatively small. Secondly, consumers have rational response to the problem. In the present paper, they are aware that surplus associate with future purchase is lowered by the counterfeit competition. Thus, more people buy earlier, which is beneficial for both the branded firm and welfare. However, as I point out, when the authentic firm decides to separate itself by distorted price, the counterfeiter can also charge a higher price in the second period. This "price collusion" created by quality signaling might decrease welfare.

4 Equilibrium Refinement and Robustness

The profit maximizing equilibrium discussed above is not selected by a well-accepted refinement, which might endanger its robustness. In this section, the Intuitive Criterion (Cho and Kreps, 1987) is applied to refine equilibriums. Since there are two signal senders here, I will use a competitive version as Bontems et al. (2005) and Yehezkel (2008) . The refinement is not applicable to separating equilibriums because both firms' prices are informative. The Intuitive Criterion requires the unilateral deviation. However, since the other firm charges the equilibrium price, consumers can use that information to construct the out of equilibrium belief. Therefore, I can not simply assume a belief towards the deviating firm while the other one prices at the equilibrium path.[⋆] Previous papers use the unprejudiced belief in Bagwell and Ramey (1991) or similar mechanism to refine equilibriums. In this paper, the refinement regarding separating equilibriums is not an important issue. Our general conclusion that the competition of counterfeit may increase the branded firm's profit and social welfare holds in all separating equilibriums.

In previous discussion, both firms are assumed to have zero marginal cost. Now, let the authentic firm has a slightly higher marginal cost $\epsilon > 0$ which is arbitrarily close to 0. This is just a tie-breaker that helps us to eliminate all pooling equilibriums. By continuity of all functions in the paper, this modification will not alter any of my results except for the existence of pooling equilibrium. For simplicity of mathematics and notation, I only explicitly state this adjustment in refinement.

4.1 Refinement

Pooling Equilibrium

The basic logic of Intuitive Criterion is equilibrium dominance. It says if there exists an out of equilibrium price such that given consumer's best belief towards that deviation, one type of firm would be better off (it is willing to deviate to that price given consumers believe that it is the authentic firm) while the other type can not benefit from that deviation under the same condition, then the equilibrium should be eliminated by this criterion.

In terms of the pooling equilibrium, the Competitive Intuitive Criterion requires that there is no $P^{'}$, such that

$$\Pi_{A2}(P^{'}, P_2^{PE}, 1) \geq \Pi_{A2}(P_2^{PE}, P_2^{PE}, \frac{1}{2}) \tag{3}$$

$$\Pi_{C2}(P_2^{PE}, P^{'}, 1) < \Pi_{C2}(P_2^{PE}, P_2^{PE}, \frac{1}{2}) \tag{4}$$

[⋆]Bester and Demuth (2011), Bontems et.al (2006) and Hertzendorf and Overgaard (2001) all discuss this issue.

However, for every pooling equilibrium, there must exist a $P^{'}$ such that both equations hold. The reason is similar to monopoly signaling refinement: Once the authentic firm has a higher marginal cost ϵ, no matter how small it is, the firm will have a lower cost to signal. Since the the profit function satisfies single cross property, I can always find an upward distorted price such that the authentic firm is willing to deviate if it can convince consumers of its quality while the counterfeiter would rather pool even if deviating to that price makes people believe it produces genuine product.

Proof: Firstly, I will show $\forall P \in [0, \underline{P_2}(C, \theta_1))$, there exists a $P < P^{'} < P + (1 - C)$, such that (3) is binding.
Choosing a δ that is arbitrarily close to 0. Then $\Pi_{A2}(P + \delta, P, 1) > \Pi_{A2}(P, P, \frac{1}{2})$ and $\Pi_{A2}(P + (1 - C)\theta_1, P, 1) = 0 < \Pi_{A2}(P, P, \frac{1}{2})$. Therefore, by the continuity of profit function, there must exist a $P < P^{'} < P + (1 - C)$ that makes $\Pi_{A2}(P^{'}, P, 1) = \Pi_{A2}(P, P, \frac{1}{2})$.
Plug $P^{'}$ and Eq(3) into Eq(4),

$$\begin{aligned}
&\Pi_{C2}(P, P^{'}, 1) - \Pi_{C2}(P, P, \frac{1}{2}) \\
=&(\theta_1 - \frac{P^{'} - P}{1 - C})P^{'} - \frac{1}{2}(\theta_1 - \frac{2P}{1 + C})P \\
=&\epsilon(\theta_1 - \frac{P^{'} - P}{1 - C})(\frac{P - P^{'}}{P - \epsilon}) < 0
\end{aligned}$$

Hence, for every pooling equilibrium, there is a price $P^{'}$ that the authentic firm wants to deviate and the counterfeit firm does not given consumer's best belief. Q.E.D

Separating Equilibrium

Hertzendorf and Overgaard (2001) and Yehezkel (2008) use the Resistance to Equilibrium Defections(REDE) to select the unique and most intuitive separating equilibrium in the duopoly signaling game, which is similar to the unprejudiced equilibrium in Bagwell and Ramey (1991). Basically, REDE assumes that consumers can still make reasonable induction from the equilibrium behavior of one sender even if they see out of equilibrium behavior from the other sender. Mathematically, if consumers observe one good is sold at a price $\widetilde{P} \in [\underline{P_2}(C, \theta_1), \overline{P_2}(C, \theta_1)]$ and the other one is priced at $P \in [0, \underline{P_2}(C, \theta_1))$ but $P \neq \frac{C}{2}\widetilde{P}$, then they will believe the one with $\widetilde{P}$ is genuine and the other one is counterfeit. This gives the authentic firm an incentive to unilaterally deviate to the price that will maximize its profit within the separating equilibrium range. The counterfeiter never deviates because any deviation cannot fool consumers. Therefore, all separating equilibriums will be eliminated except the one that yields highest second period profit for the authentic firm, which is the profit maximizing equilibrium I investigate in the previous section. However, as I prove later, the refinement of separating equilibrium is not a big problem. All separating equilibriums have the desired property presented in the last section.

4.2 Robustness of Results

Firstly, It can be proved that the branded firm's profit increases with the second period price for any counterfeit's quality in every separating equilibrium because the front loading effect grows faster than the competition effect. In last section, the profit maximizing equilibrium is discussed in detail, which is the one with lowest second period price among all separating equilibriums. Since the branded firm can benefit from the entry of counterfeit under the equilibrium with lowest second period price, the result will hold under all other equilibriums. If the counterfeit's quality is below C_4, the authentic firm's profit is always higher with the presence of counterfeits, no matter which separating equilibrium emerges in the second period.

In terms of the impact on welfare, there is not such a nice monotonicity property among equilibriums as profit. However, it is verified that if C is higher than a threshold, the social welfare is higher with counterfeit competition in all equilibriums. The economic intuition is the same as the last section. All equilibriums have higher second price (more distortion) than the one selected by benevolent consumer belief. Thus, the incumbent's second period profit in other equilibriums is lower than that one. When the branded firm maximizes the profit, it tends to reduce the weight on the second period (lower θ_1). Therefore, more consumers purchase in the first period and the welfare increases.

Proposition 3. All pooling equilibriums are eliminated by the Competitive Intuitive Criterion. In every separating equilibrium, when $C \leq C_4$, the authentic firm's profit is higher with the competition of counterfeit. When $C \geq C_6 \approx 0.248$, the social welfare is higher with the presence of counterfeit.

4.3 Self-Provision of Damaged Good

In this paper, the entry of a low quality competitor can actually benefit the incumbent. The downside to the brand is that it takes away part of the revenue. An interesting question is can the branded firm overcome the competition effect by offering an inferior version itself and earn higher profit? We do observe many examples of damaged good. Armani has a premium ready-to-wear line marketed as Giorgio Armani, relatively cheaper bridges as Armani Collezioni and Emporio Armani, as well as lines distributed in shopping malls like Armani Jeans and Armani Exchange. Deneckere and McAfee (1996) analyze how damaged good can increase profit in a static model. The trade-off is that it serves low end market but may decrease the demand for premium version. Hahn (2006) discusses the role of damaged good in durable good model and shows the condition of increased profit can be relaxed because of the extra benefit that low type consumers tend to buy low quality version earlier. This part will elaborate why self-provision of an inferior good can not help to increase the monopoly's profit under my set up.

Firstly, the incumbent has no incentive to provide an inferior good in the second period. Deneckere and MacAfee (1996) points out the linear utility function fails the condition that damaged good helps to raise profit. In my model, no matter what inferior quality the branded firm chooses, the optimal decision is to sell zero damaged version in t=2. The second period price and profit are the same as monopoly benchmark. Since the price is not higher than the monopoly price, the front loading effect does not exist. Therefore, the total profit can never be higher than the benchmark. If there is any fixed cost associate with product line introduction, the profit is always lower than the monopoly case.

Secondly, damaged good introduced in the first period is not profitable as well. Although this paper has the durable good context as Hahn (2006), his argument can not be applied here because he assumes only two types of consumers but I have a continuous distribution of θ. In his paper, part of high (low) type consumers buy high (low) quality good in each period, which changes the ratio of consumer type. Since some low types have purchased damaged good in earlier period, the firm has less incentive to decrease price sharply later, which relaxes the competition between two versions and alleviates time-inconsistency problem. However, with continuous consumer type, it can be proved that if anyone buys the damaged good in the first period, then all consumers with higher θ must buy a good in that period as well. If this is violated (e.g. $[\theta_1, 1]$ buy premium goods and $[\theta_3, \theta_2]$ purchase damaged version while $[\theta_2, \theta_1]$ buy nothing in t=1), then it can be proved that no damaged good is sold. For θ_2, intertemporally, she is indifferent between buying damaged good in t=1 and premium good in t=2, otherwise the consumer who has a slightly higher taste $\theta_2 + \epsilon$ ($\epsilon \to 0$) will deviate to purchase in t=1. On the other hand, by continuity, she is indifferent between purchasing premium version and damaged one in t=2, which is the binding incentive compatibility constraint required to attain a separating menu. Therefore, θ_2 is indifferent between buying damaged good in t=1 and t=2. However, this is also the intertemporal constraint for consumer θ_3, which implies $\theta_2 = \theta_3$. Given this argument, in t=2, the firm still faces a group of cohort truncated above and the time-inconsistency problem remains the same. The mechanism that helps to solve Coase Conjecture in Hahn (2006) disappears in my model and the firm would rather just offer the original version.

5 Conclusion

This paper challenges the conventional wisdom that deceptive counterfeit is harmful for the authentic firm and total welfare. Despite business stealing effect, deceptive counterfeits mitigate the time-inconsistency problem for the incumbent. It is demonstrated that the effect of the counterfeit crucially depends on its quality. When the quality gap is sufficiently small, pooling equilibrium is sustained in the second period. The front loading effect is never strong enough to cover the loss from the competition and the authentic firm's profit always decreases with counterfeiter's entry. However, if the quality gap is sufficiently large, the low quality counterfeit only incurs a mild competition that is dominated by increased sell in the first period. The branded firm benefits from counterfeit competition in this case. Besides, the incumbent can not earn higher profit by offering a damaged good because the front loading effect disappears. In terms of welfare, contrary to traditional arguments, it is shown that in most of quality range, the deceptive counterfeit is actually beneficial to the society due to more earlier purchases. Surprisingly, if counterfeit remains indistinguishable ex post, total surplus will increase for sure. The result implies that the welfare effect may not have a straightforward one-sided conclusion. The government should take both firms and consumers rational response into account when making policies toward counterfeits.

There are several interesting directions for future research. For instance, there is no variable controlled by the government. What if the government is another active player in the game and could choose policy variables against counterfeits? How will that affect the counterfeiter's entry decision and the interplay between the authentic firm and consumers? In this paper, only one counterfeit firm enters in the second period. However, famous brands face many counterfeiters with different qualities in reality. Finally, I only investigate the short term effect of the counterfeit. How will the counterfeiter affect the authentic firm's innovation and product upgrading decision? If that is taken into account, is the total welfare still likely to increase?

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A Appendix

Proof of Lemma 1 To sustain a separating equilibrium, the incentive compatibility constraint (1) for the counterfeiter requires that:

$$(\frac{P_{A2} - \frac{C}{2}P_{A2}}{1-C} - \frac{\frac{C}{2}P_{A2}}{C})\frac{C}{2}P_{A2} \geq \frac{1}{2}(\theta_1 - \frac{2P_{A2}}{1+C})P_{A2}$$
$$P_{A2} \geq \frac{2(1-C^2)}{C^2-3C+4}\theta_1 = \underline{P_2}(\theta_1, C)$$

This equation is derived from Eq(1) by plugging the best response function of the counterfeit. Similarly, the incentive compatibility constraint for the authentic firm requires that:

$$(\theta_1 - \frac{P_{A2} - \frac{C}{2}P_{A2}}{1-C})P_{A2} \geq \frac{1}{2}(\theta_1 - \frac{C}{1+C}P_{A2})\frac{C}{2}P_{A2}$$
$$P_{A2} \leq \frac{(4-C)(1-C^2)}{2(2-C)(1+C) - C^2(1-C)}\theta_1 = \overline{P_2}(\theta_1, C)$$

Therefore, when $\overline{P_2}(\theta_1, C) \geq \underline{P_2}(\theta_1, C)$, a separating equilibrium exists. Otherwise, only pooling equilibrium can be supported.

$$\frac{(4-C)(1-C^2)}{2(2-C)(1+C)-C^2(1-C)}\theta_1 \geq \frac{2(1-C^2)}{C^2-3C+4}\theta_1$$

This implies that when $C \leq C_1 \approx 0.604$, a separating equilibrium could be supported.

For pooling equilibriums, as long as Eq(1) is violated, the counterfeiter is willing to pool with the authentic firm. Therefore, $\forall C$, if $P_{A2}^{PE} = P_{C2}^{PE} = P_2^{PE} \in S^{PE} = [0, \underline{P_2}(C, \theta_1))$, pooling equilibrium can be sustained by certain out of equilibrium beliefs. *Q.E.D.*

Proof of Lemma 2 In separating equilibriums, it can be easily shown that all authentic prices are higher than the unconstrained optimal price. Since the profit function is a concave parabola, $P_{A2}^{SE*} = \underline{P_2}(\theta_1, C)$. In any separating equilibrium, the branded firm's profit decreases with the quality of the counterfeit in the second period because of intensified competition.

In pooling equilibriums, when $C < C_2 \approx 0.702$, the unconstrained optimal price is always less than $\underline{P_2}(\theta_1, C)$. Therefore, the optimal price is the unconstrained optimal, $P_2^{PE*} = \frac{1+C}{4}\theta_1$. Fixing the market size, the authentic firm's profit increases with C within this range. That is because consumer confusion is alleviated, which enables the firm to raise the price. However, when $C > C_2$, the quality gap is small and the competition is intense. The unconstrained optimal is higher than $\underline{P_2}(\theta_1, C)$. Since the profit function is a concave parabola as well, $P_2^{PE*} = \underline{P_2}(\theta_1, C)$.

As Lemma 1 indicates, when $C \leq C_1$, both types of equilibriums exist and $\Pi_{A2}^* = Max\ \{\Pi_{A2}^{SE*}, \Pi_{A2}^{PE*}\}$. Given $C_1 < C_2$, the price of the optimal pooling equilibrium is $P_2^{PE*} = \frac{1+C}{4}\theta_1$. Since $\frac{d\Pi_{A2}^{SE*}}{dC} < 0$ and $\frac{d\Pi_{A2}^{PE*}}{dC} > 0$, there is a cut-off quality $C_3 \approx 0.512$ such that the optimal separating equilibrium is chosen if $C \leq C_3$ and the pooling equilibrium would be selected for $C_3 < C \leq C_1$. When the quality of the counterfeit is low, the profit of separating equilibrium is high because of moderate distortion while the profit of pooling equilibrium is low due to low expected quality. As the quality of the fake good increases, the pooling equilibrium becomes better for the branded firm. For $C > C_1$, the separating equilibrium can not exist and the only candidate is the pooling equilibrium. When $C_1 < C \leq C_2$, the equilibrium is characterized by the unconstrained optimal price. If $C > C_2$, the price is $\underline{P_2}(C, \theta_1)$. *Q.E.D*

Proof of Proposition 1

(1) When $C > C_3$, pooling equilibrium is sustained. There are two second period prices given different C, both of which could be written as $P_2^{PE*} = F(C)\theta_1^{PE*}$. Firstly, I will prove $\forall C \in (C_3, 1)$, $\frac{\partial \Pi_A^{PE*}}{\partial F(C)} > 0$.

$$frac\partial\Pi_A^{PE*}\partial F(C) = \frac{(\frac{3-C}{2} + F(C))[(3-C)(\frac{1}{2} - \frac{F(C)}{1+C}) + \frac{F(C)}{2}]}{4(\frac{3-C}{2} + \frac{F(C)}{2} + \frac{F(C)^2}{1+C})^2}$$

Since $0 < F(C) \leq \frac{1+C}{4}$, $\frac{\partial \Pi_A^{PE*}}{\partial F(C)} > 0$.

Given this property, it can be shown that even using the larger second period price $P_2^{PE*} = \frac{1+C}{4}\theta_1, \forall C \in (C_3, 1)$, the counterfeit competition in pooling equilibrium still can not increase the branded firm's profit.

Using $P_2^{PE*} = \frac{1+C}{4}$, $\forall C \in (C_3, 1)$.

$$\frac{d\Pi_A^{PE*}}{dC} = \frac{(1 + \frac{3-C}{4})(-2 + \frac{35C-53}{64})}{(1 + \frac{11-5C}{16})^2} < 0$$

Therefore, $\forall C \geq C_3, \Pi_A^{PE*}(C) \leq \Pi_A^{PE*}(C_3)$. Since $\Pi_A^{PE*}(C_3) < \Pi^M$, we have $\Pi_A^{PE*}(C) < \Pi^M$, $\forall C \geq C_3$.

(2) When $C \leq C_3$, separating equilibrium is supported in the second period. If C=0, the model is degenerated to the monopoly benchmark. $\Pi^M = \Pi_A^{SE*}$.

Let $\Delta\Pi(C) = \Pi_A^{SE*} - \Pi^M$, then $\frac{d\Delta\Pi(C)}{dC}|_{C=0} = 0.045 > 0$. So there must exist some C that is low enough such that the authentic firm's profit would increase under the competition.

On the other hand, the only root $C_4 \in (0, 1]$ of $\Delta\Pi(C) = 0$ is $C_4 \approx 0.188$. Henceforth, $\Delta\Pi(C) \geq 0$ if $C \leq C_4$ and vice versa. *Q.E.D.*

Proof of Proposition 2.

(1) In pooling equilibrium, similar to proof of Proposition 1, let $P_2^{PE*} = F(C)\theta_1^{PE*}$. Firstly, I will show that $\frac{\partial TS^{PE*}(C)}{\partial F(C)} < 0$ for any $C > C_2$.

$$TS^{PE*}(C) = (1-(\theta_1^{PE})^2) - \frac{1+C}{4}[1 - \frac{4}{(1+C)^2}F(C)^2]$$

$$\frac{\partial TS^{PE*}(C)}{\partial F(C)} = 2(\theta_1^{PE*})^2[\frac{\partial \theta_1^{PE*}(C)}{\partial F(C)}(\frac{1+C}{4} - \frac{F(C)^2}{1+C} - 1) - \frac{F(C)}{1+C}\theta_1^{PE*}]$$

Plugging in θ_1^{PE*} and $\frac{\partial \theta_1^{PE*}(C)}{\partial F(C)}$, it is easy to verify that $\frac{\partial TS^{PE*}(C)}{\partial F(C)} < 0$.

Given this property, it is proved that with the larger second period price $P_2^{PE*} = \frac{1+C}{4}\theta_1, \forall C \in (C_3, 1)$, the counterfeit competition with pooling equilibrium still increases the total welfare. When $P_2^{PE*} = \frac{1+C}{4}\theta_1$, $\theta_2^{PE*} = \frac{1}{2}\theta_1^{PE*}$.

$$\begin{aligned}
TS^{PE*}(C) &= \int_{\theta_1^{PE*}}^{1} 2\theta d\theta + \int_{\theta_2^{PE*}}^{\theta_1^{PE*}} \frac{1+C}{2}\theta d\theta \\
&= 1 - (\theta_1^{PE*})^2(1 - \frac{3(1+C)}{16}) \\
\Delta TS^{PE*}(C) &= TS^{PE*}(C) - TS^M(C) = \frac{5}{8}(\theta_1^M)^2 - (\theta_1^{PE*})^2(1 - \frac{3(1+C)}{16}) \\
\frac{d\Delta TS^{PE*}(C)}{dC} &= \frac{8(1+C)}{(27-5C)^3} > 0
\end{aligned}$$

Therefore, $\Delta TS^{PE*}(C) \geq \Delta TS^{PE*}(C_3)$, $\forall C > C_3$. Since, $\Delta TS^{PE*}(C_3) > 0$, deceptive counterfeit always results in a higher welfare under the pooling equilibrium.

(2) In separating equilibrium,

$$\overline{\theta}_2^{SE*} = \frac{2-C}{2(1-C)}\underline{P_2}(C,\theta_1), \underline{\theta}_2^{SE*} = \frac{1}{2}\underline{P_2}(C,\theta_1)$$

$$\begin{aligned}
TS^{SE*}(C) &= \int_{\theta_1^{SE*}}^{1} 2\theta d\theta + \int_{\frac{(2-C)(1+C)}{C^2-3C+4}\theta_1^{SE*}}^{\theta_1^{SE*}} \theta d\theta + \int_{\frac{1-C^2}{C^2-3C+4}\theta_1^{SE*}}^{\frac{(2-C)(1+C)}{C^2-3C+4}\theta_1^{SE*}} C\theta d\theta \\
&= 1 - \frac{1}{2}(\theta_1^{SE*})^2[1 + \frac{(1+C)^2(4-3C)(1-C)}{(C^2-3C+4)^2}]
\end{aligned}$$

Since $\frac{d\Delta TS^{SE*}(C)}{dC}|_{C=0} < 0$, if C is sufficiently low, $TS^{SE*}(C) < TS^{M}(C)$. Besides, there is only one root $C_5 \in (0,1]$ such that $\Delta TS^{SE*}(C) = 0$. Therefore, $\forall C \leq C_5$, $TS^{SE*}(C) \leq TS^{M}(C)$ and vice verse. Under separating equilibrium, the entry of a deceptive counterfeit increases total welfare if the quality of the counterfeit $C \geq C_5 \approx 0.078$. *Q.E.D.*

Proof of Proposition 3.

$$\underline{P_2}(C,\theta_1) = \frac{2(1-C^2)}{C^2-3C+4}\theta_1, \overline{P_2}(C,\theta_1) = \frac{(4-C)(1-C^2)}{2(2-C)(1+C)-C^2(1-C)}\theta_1$$

For notation convenience, let

$$\underline{K}(C) = \frac{2(1-C^2)}{C^2-3C+4}, \overline{K}(C) = \frac{(4-C)(1-C^2)}{2(2-C)(1+C)-C^2(1-C)}$$

Therefore, any authentic price in separating equilibrium is between $\underline{K}(C)\theta_1$ and $\overline{K}(C)\theta_1$.

(1) For incumbent's profit:

$$\Pi_A^{SE} = \frac{1}{4}\frac{[1+K(C)]^2}{[1+\frac{2-C}{2(1-C)}K(C)^2]}$$

$$\frac{\partial \Pi_A^{SE}}{\partial K(C)} = \frac{[1+K(C)][1-\frac{2-C}{2(1-C)}K(C)]}{2(1+\frac{2-C}{2(1-C)}K(C)^2)^2}$$

Since $1-\frac{2-C}{2(1-C)}K(C) \geq 1-\frac{2-C}{2(1-C)}\overline{K}(C) > 0$, $\frac{\partial \Pi_A^{SE}}{\partial K(C)} > 0$. The profit maximizing equilibrium is the one that yields lowest total profit for the incumbent. In that equilibrium, when $C \leq C_4$, the profit with counterfeit is higher. Therefor no matter which separating equilibrium is sustained in the second period, $\Delta\Pi_A^{SE}(C) \geq 0$ if $C \leq C_4$.

(2) For total welfare:

$$\Delta TS^{SE}(C, K(C)) = 0.225 - \frac{1}{8}\frac{[1+K(C)]^2[1+\frac{4-3C}{4-4C}K(C)^2]}{[1+\frac{2-C}{2-2C}K(C)^2]^2}$$

When $C=0$,

$$\Delta TS^{SE}(0, K(0)) = 0.225 - \frac{1}{8}\frac{[1+K(0)]^2}{1+K(0)^2}$$

Since $\frac{[1+K(0)]^2}{1+K(0)^2}$ increases with $K(0)$,

$$\Delta TS^{SE}(0, K(0)) \leq 0.225 - \frac{1}{8}\frac{[1+\underline{K}(0)]^2}{1+\underline{K}(0)^2} = 0$$

When $C = C_1$, $\forall K(C_1)$,

$$\begin{aligned}
\Delta TS^{SE}(C_1, K(C_1)) &= 0.225 - \frac{1}{8}\frac{[1+K(C_1)]^2[1+\frac{4-3C_1}{4-4C_1}K(C_1)^2]}{[1+\frac{2-C_1}{2-2C_1}K(C_1)^2]^2} \\
&> 0.225 - \frac{1}{8}\frac{[1+K(C_1)]^2}{[1+\frac{2-C_1}{2-2C_1}K(C_1)^2]} \\
&\geq 0.225 - \frac{1}{8}\frac{[1+\overline{K}(C_1)]^2}{[1+\frac{2-C_1}{2-2C_1}\overline{K}(C_1)^2]} > 0
\end{aligned}$$

Therefor, when $C = 0$, the welfare differences under all equilibriums are negative. However, when $C = C_1$, the welfare differences under all equilibriums are strictly positive. By continuity of the welfare difference function, there must exist a threshold C_6 such that as long as $C \geq C_6$, the welfare is higher with the entry of deceptive counterfeit for any separating equilibrium. Numerically, I find that $C_6 \approx 0.248$. *Q.E.D.*