The Kreps-Scheinkman game in mixed duopolies

Bakó, Barna and Tasnádi, Attila

Corvinus University of Budapest

15 January 2014

The Kreps-Scheinkman game in mixed duopolies

Barna Bakó[*] Attila Tasnádi[†]

January 15, 2014

Abstract

In this paper we generalize the results of Kreps and Scheinkman (1983) to mixed-duopolies. We show that quantity precommitment and Bertrand competition yield Cournot outcomes not only in the case of private firms but also when a public firm is involved.

Keywords: mixed duopoly, Cournot, Bertrand-Edgeworth.

JEL Classification Number: D43, L13.

1 Introduction

One of the most cited papers in the oligopoly related theoretical literature is that of Kreps and Scheinkman (1983). In this seminal paper, the authors claim that Cournot competition leads to an outcome which is equivalent to the equilibrium of a two-stage game, where there is simultaneous production after which price competition occurs. This is an important result given the popularity of the Cournot model, as it solves the price-setting problem represented by the mythical Walrasian auctioneer in quantity-setting games.

Since then, many papers dealt with this equivalence trying to exploit its boundaries. Firstly, Davidson and Deneckere (1986) challenged the validity of the result by replacing the efficient rationing rule used by Kreps and Scheinkman (1983) and showed that the result

[*]Department of Microeconomics, Corvinus University of Budapest and MTA-BCE „Lendület" Strategic Interactions Research Group, 1093 Budapest, Fővám tér 8., *e-mail:* `barna.bako@uni-corvinus.hu`. This research was supported by the European Union and the State of Hungary, co-financed by the European Social Fund in the framework of TÁMOP-4.2.4.A/ 2-11/1-2012-0001 'National Excellence Program'.

[†]Department of Mathematics, Corvinus University of Budapest and MTA-BCE „Lendület" Strategic Interactions Research Group, 1093 Budapest, Fővám tér 13-15., *e-mail:* `attila.tasnadi@uni-corvinus.hu`. Financial support from the Hungarian Scientific Research Fund (OTKA K-101224) is gratefully acknowledged.

fails to hold.[1] Furthermore, Reynolds and Wilson (2000) introduced demand uncertainty to the model and pointed out that equilibrium capacities are not equal to the Cournot quantities. In their model the uncertainty prevails only at the time when firms choose capacities. However, at the beginning of the second stage the demand is observed and prices are set in a deterministic way.[2] On the other hand, when uncertainty persists in the price-setting stage, de Frutos and Fabra (2011) illustrate that under mild assumptions the total welfare is equivalent to the Cournot case, yet the capacity levels are asymmetric even when firms are ex-ante identical.

Boccard and Wauthy (2000 and 2004) generalize Kreps and Scheinkman's (1983) result to multi-player markets assuming efficient rationing and identical cost functions. Moreover, under similar conditions Loertscher (2008) proves that the equivalence result holds when firms compete in the input and the output market at the same time. More recently, Wu, Zhu and Sun (2012) generalized the celebrated equivalency result by relaxing the assumptions imposed on the demand and cost functions.

In this paper we extend the Kreps and Scheinkman (1983) result to the case in which a private firm competes with a public firm, that is, to the case of a so-called mixed duopoly. The idea of mixed oligopolies as a possible form of regulation was introduced by Merrill and Schneider (1966). Its relevance stems from the possibility of increasing social welfare through the presence of a public firm in the market. Indeed, it is common to observe public and private firms competing in the same industry.[3]

As for studies of mixed oligopolies, the Cournot game was examined by Harris and Wiens (1980), Beato and Mas-Colell (1984), Cremer, Marchand and Thisse (1989) and de Fraja and Delbono (1989). Balogh and Tasnádi (2012) studied the price-setting game for given capacities. Therefore, in order to extend the Kreps and Scheinkman (1983) result for mixed duopolies, the solution of the capacity game is required. For linear demand and cost functions this solution was given by Bakó and Tasnádi (2014), but that requires the private firm to be more cost-efficient than the public firm. However, as we will see, in the case of strictly convex cost and general demand functions there is no need for such an assumption.[4]

In the remainder of the paper we first present our setup and then solve the mixed Cournot game followed by the results on the price-setting game. Finally, we determine the equilibrium capacity levels and summarize our results.

[1] For more about rationing rules see, for instance, Vives (1999) or Wolfstetter (1999).

[2] Lepore (2012) generalizes Reynolds and Wilson (2000) results for a wide range of demand uncertainties with different rationing rules.

[3] A few notable examples for public firms are: the Kiwibank, which is a state owned commercial bank in New-Zealand; Amtrak, the railway company in USA; the Indian Drugs and Pharmaceuticals Limited, which is owned by the Indian Government; the Norwegian Statoil, owned in 60% by the national government; or in the aviation industry Aeroflot, Air New-Zealand, Finnair, Qatar Airways are all owned in majority by their national government.

[4] Tomaru and Kiyono (2010) analyzes the linear and convex case separately in timing games for mixed duopolies for exactly the same reason.

2 The Model

Consider a mixed duopoly in which two firms, A and B, produce perfectly substitutable products. Firm A is a private firm and maximizes its profit, while Firm B is a public firm and aims to maximize total surplus.

The market demand function is given by D on which we impose the following assumptions:

Assumption 1 *(i) D intersects the horizontal axis at quantity a and the vertical axis at price b; (ii) D is strictly decreasing, concave and twice-continuously differentiable on $(0, a)$; (iii) D is right-continuous at 0 and left-continuous at b; and (iv) $D(p) = 0$ for all $p \geq b$.*

Note, that based on this assumption none of the firms sets its price above b. Denote by P the inverse demand function, that is $P(q) = D^{-1}(q)$ for $0 < q \leq a$, $P(0) = b$, and $P(q) = 0$ for all $q > a$.

The firms cost functions are given by C_i $(i = A, B)$ and we assume that:

Assumption 2 *(i) $C_i(0) = 0$; (ii) $C_i'(0) < b$ and (iii) C_i is strictly increasing, convex and twice-continuously differentiable on $[0, \infty)$.*

3 The mixed Cournot duopoly

The private firm is a profit-maximizer and its profit function can be given as:

$$\pi_A(q_A, q_B) = P(q_A + q_B)q_A - C_A(q_A) \tag{1}$$

while the public firm intends to maximize social welfare, hence its objective function is as follows:

$$\pi_B(q_A, q_B) = \int_0^{q_A+q_B} P(z)dz - C_A(q_A) - C_B(q_B) \tag{2}$$

In equilibrium firms produce quantities which satisfy the equation system derived from the first order conditions:

$$\begin{aligned} \tfrac{\partial \pi_A(q_A,q_B)}{\partial q_A} &= P'(q_A + q_B)q_A + P(q_A + q_B) - C_A'(q_A) = 0, \\ \tfrac{\partial \pi_B(q_A,q_B)}{\partial q_B} &= P(q_A + q_B) - C_B'(q_B) = 0. \end{aligned} \tag{3}$$

Example 1 *Let $P(q) = 1 - p$, $C_A(q_A) = \frac{1}{4}q_A^2$ and $C_B(q_B) = \frac{1}{6}q_A^2$.*

The social welfare for this example is depicted in Figure 1 by the shaded area when firms produce quantities $q_A = 1/2$ and $q_B = 1/3$.

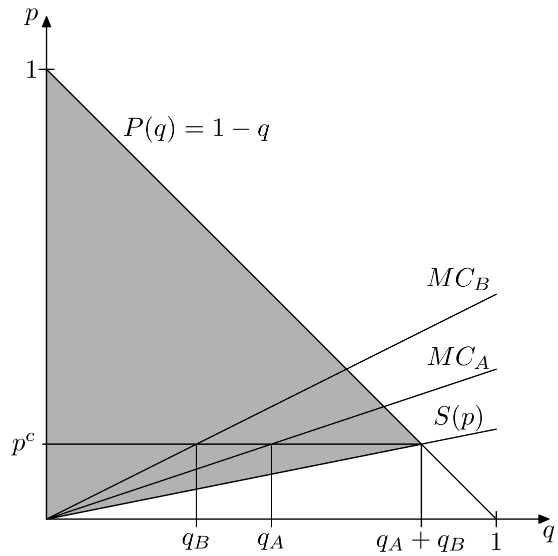

Figure 1: Social welfare in a mixed Cournot duopoly.

4 The mixed Kreps–Scheinkman game

Now, we assume that firms are involved in the following two-stage game: firstly they choose their capacity level simultaneously and non-cooperatively and secondly they compete in a Bertrand fashion. Moreover, we assume that firms can produce up to their capacity levels at zero unit cost in the subsequent production stage, however producing beyond their capacity levels is impossible (or stated otherwise, unit costs rise discontinuously to infinity).

To solve the game we use backward induction. Suppose that in the first stage firms install capacities k_A and k_B and these decisions are common knowledge. Without loss of generality we can assume that $k_A, k_B \in [0, a]$. Taking capacities as given, we analyze the price-setting game in which firms choose their prices $p_i \in [0, P(0)]$ $(i = A, B)$ to maximize their payoffs.

To determine firms' demand and profit functions, we employ the efficient rationing rule.[5] The firm which sets the lower price faces the market demand, while the firm with the higher price has a residual demand of $D_i^r(p_i) = \max\{0, D(p_i) - k_j\}$. In the case of $p_A = p_B$ the following tie-breaking rule is used for mixed duopolies: if prices are higher than a threshold $\overline{p}$ (explicitly determined later on) the demand is allocated in proportion

[5] Suppose firm i charges the lowest price (p_i). If $S_i(p_i) < D(p_i)$, where $S_i(p_i)$ denotes i's competitive supply, not all consumers who want to buy from firm i are able to do so. The efficient rationing rule suggests that the most eager consumers are the ones who are able to purchase from firm i, that is the residual demand function of firm $j \neq i$ can be obtained by shifting the market demand function to the left by $S_i(p_i)$. This rationing rule is called efficient because it maximizes consumer surplus. For more details we refer to Vives (1999) or Wolfstetter (1999).

of the firms' capacities, however if prices are not higher than $\overline{p}$ the public firm allows the private firm to serve the entire demand up to its capacity level.[6] Formally,

$$q_i = \Delta_i(p_i, p_j) = \begin{cases} \min\{k_i, D(p_i)\} & \text{if} \quad p_i < p_j, \\ \min\{k_i, D_i^r(p_i)\} & \text{if} \quad p_i > p_j, \\ \min\{k_i, \frac{k_i}{k_i+k_j} D(p_i)\} & \text{if} \quad p_i = p_j > \overline{p}, \\ \min\{k_i, D(p_i)\} & \text{if} \quad p_i = p_j \leq \overline{p} \text{ and } i = A, \\ \min\{k_i, D_i^r(p_i)\} & \text{if} \quad p_i = p_j \leq \overline{p} \text{ and } i = B. \end{cases} \tag{4}$$

Firms' objective functions can be given as:

$$\pi_A(p_A, p_B) = p_A q_A, \tag{5}$$

and

$$\pi_B(p_A, p_B) = \int_0^{\min\{k_j, \max\{0, D(p_j) - k_i\}\}} R_j(q)dq + \int_0^{\min\{k_i, a\}} P(q)dq, \tag{6}$$

where $0 \leq p_i \leq p_j \leq b$ and $R_j(q) = (D_j^r)^{-1}(q)$. Note that, since capacities are set in the first stage of the game and their costs are already sunk, firms objective functions are free of costs.

For Example 1 we illustrate firms' profits and consumers' surplus in Figure 2. The lightest-grey triangle corresponds to the surplus realized by the consumers who purchase the product at the highest price, while the light-grey area depicts the surplus realized by the other consumers. On the producers' side, the low-price firm's surplus is given by the darkest-grey rectangular and the high-price firm's surplus by the dark-grey area. Note that total welfare is determined by the higher price, except when the residual demand equals zero at the higher price.

Let us denote the market clearing price by p^c and the firm i's ($i = A, B$) unique revenue maximizing price on the firm's residual demand curve by p_i^m, hence:

$$p^c = P(k_A + k_B) \quad \text{and} \quad p_i^m = \arg \max_{p \in [0, P(0)]} p D_i^r(p).$$

Furthermore, let p_i^d be the lowest price satisfying equation

$$p_i^d \min\{k_i, D(p_i^d)\} = p_i^m D_i^r(p_i^m),$$

[6]For prices higher than $\overline{p}$ we could have used many other tie-breaking rules, e.g. the tie-breaking rule used by Kreps and Scheinkman (1983), the only requirement is that none of the firms should have the possibility to serve the market entirely. For more about the employed tie-breaking rule we refer to Balogh and Tasnádi (2012).

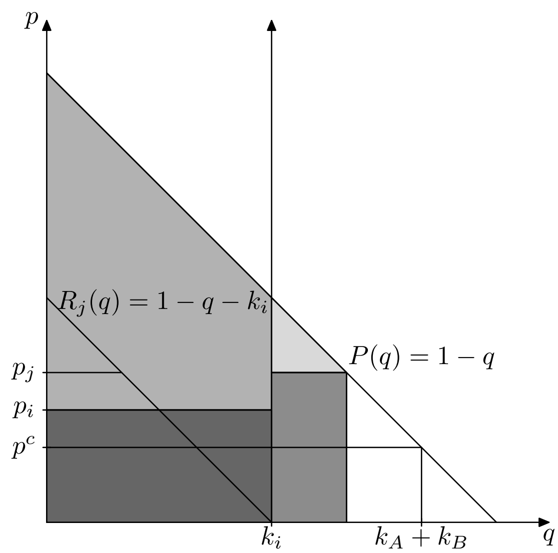

Figure 2: Total welfare in price-setting game

thus, by choosing p_i^d and selling $\min\{k_i, D(p_i^d)\}$, firm i generates the same amount of profit as it would by setting p_i^m and serving the residual demand.[7]

Based on Berge's Maximum Theorem $\pi_A^r(p_A^m) = \max_{p_A} \pi_A^r(p_A) = \max_{p_A} p_A D_A^r(p_A)$ is continuous in (k_A, k_B) and since p_A^m is unique p_A^m is continuous function of (k_A, k_B) as well. Therefore, p_i^d is continuous in (k_A, k_B), whenever p_i^d is well defined.[8]

4.1 Solving the price-setting game

In this subgame k_A and k_B are given parameters. If $p_A^m \geq p^c$, firms set equilibrium prices as follows:

$$p_A^* = p_B^* = p_A^d \tag{7}$$

or

$$p_A^* = p_A^m \quad \text{and} \quad p_B^* \leq p_A^d. \tag{8}$$

Moreover, if $k_B \leq k_A$ and $k_B \leq D(p^M)$, where p^M is the price set by a monopolist without capacity constraints, i.e. $p^M = \arg\max_{p \in [0, P(0)]} pD(p)$, the following price-profiles are also part of the equilibrium:

$$p_A^* = \max\{p^M, P(k_A)\} \quad \text{and} \quad p_B^* > \max\{p^M, P(k_A)\}. \tag{9}$$

[7]To abbreviate our expressions we omit the variables k_A and k_B of D_i^r, π_i, p^c, p_i^m and p_i^d. However, keep in mind that these expressions depend on the capacity levels chosen in the first stage of the game.

[8]p_i^d is well defined, whenever $p_i^m \geq p^c$. Note that, if $p_A^m = p^c$ then $p_A^d = p^c$. See Balogh and Tasnádi (2012).

If, however $p_A^m < p^c$, in equilibrium firms set prices as follows:

$$p_A^* = p_B^* = p^c. \tag{10}$$

Henceforward, we will refer to the first case ($p_A^m \geq p^c$) as the *strong private firm case* and to the latter ($p_A^m < p^c$) as the *weak private firm case.* At this point we can already define $\overline{p}$ introduced before Equation (4): let $\overline{p} = p_A^d$ if $p_A^m \geq p^c$ and $\overline{p} = 0$ otherwise.

In the strong private firm case the equilibrium given by (7) Pareto dominates the one given by (8). Furthermore, the not always existing (9) describes situations when the public firm is inactive. Therefore in what follows we consider (7) as the solution of the price-setting game in the strong private firm case.[9]

Hence, firms' equilibrium quantities can be given as:

$$q_A^* = \min\{k_A, D(p_A^d)\} \quad \text{and} \quad q_B^* = \min\{k_B, D_B^r(p_B^*)\}. \tag{11}$$

4.2 The capacity-choice game

Let us denote the set of capacity-profiles compatible with the weak private firm case as

$$K^c = \left\{(k_A, k_B) \in [0,a]^2 \mid p_A^m\left(k_A, k_B\right) \leq P(k_A + k_B)\right\}$$

and with the strong private firm case as

$$K^d = \left\{(k_A, k_B) \in [0,a]^2 \mid p_A^m\left(k_A, k_B\right) > P(k_A + k_B)\right\}$$

Notice that K^c is a closed set, since p_A^m and P are continuous.

To determine p_A^d, first we need to consider p_A^m, which by definition is the price maximizing $p(D(p) - k_B)$.[10] That is, p_A^m satisfies the following first order condition:

$$\frac{\partial \pi_A^r}{\partial p}\left(p_A^m\right) = p_A^m D'\left(p_A^m\right) + D\left(p_A^m\right) - k_B = 0. \tag{12}$$

Based on Assumption 1, $\frac{\partial \pi_A^r}{\partial p}$ is strictly decreasing, p_A^m is unique and, as it can be checked easily, p_A^m is independent from k_A.

The boundary curve dividing the strong and the weak private firm case is given by $p_A^m(k_A, k_B) = p^c = P(k_A + k_B)$. For any given k_B, if k_A satisfies $p_A^m(k_A, k_B) = P(k_A + k_B)$, then for every capacity $k'_A \in [0, k_A)$ we have that $p_A^m(k'_A, k_B) < P(k'_A + k_B)$, which is the case because the left-hand side is independent of k'_A and the right-hand side is decreasing in k'_A. Thus, for every k_B there exists a k_A^* such that the projection of K^c at k_B equals $[0, k_A^*]$.

[9]For more details on selecting (7) as the most plausible equilibrium we refer to Balogh and Tasnádi (2012).

[10]Bear in mind that we have to consider the case $k_B = a$ separately. In this situation p_A^m is not unique, since any price leads to zero profit. Let $p_A^m(k_A, a) = 0$, since then p_A^m will be left-continuous at a.

We show that the boundary curve, which is defined by the implicit equation $p_A^m(k_A, k_B) = p^c = P(k_A + k_B)$, is strictly decreasing in (k_A, k_B). The implicit equation defining the boundary curve can be expressed as

$$D'\left(P(k_A + k_B)\right) P(k_A + k_B) + k_A + k_B - k_B = 0$$

from which under Assumption 1 by the Implicit Function Theorem we obtain

$$\begin{aligned}
\frac{\partial k_B}{\partial k_A} &= -\frac{D''(P(k_A + k_B))P'(k_A + k_B)P(k_A + k_B) + D'(P(k_A + k_B))P'(k_A + k_B) + 1}{D''(P(k_A + k_B))P'(k_A + k_B)P(k_A + k_B) + D'(P(k_A + k_B))P'(k_A + k_B)} \\
&= -1 - \frac{1}{P'(k_A + k_B)\left(D''(P(k_A + k_B))P(k_A + k_B) + D'(P(k_A + k_B))\right)} < 0.
\end{aligned}$$

Furthermore, let us divide K^d into two subsets as follows:

$$\begin{aligned}
K_1^d &= \left\{(k_A, k_B) \in K^d \mid k_A \leq D\left(p_A^d\left(k_A, k_B\right)\right)\right\} \text{ and} \\
K_2^d &= \left\{(k_A, k_B) \in K^d \mid k_A > D\left(p_A^d\left(k_A, k_B\right)\right)\right\}.
\end{aligned}$$

We turn to determining the projection of the set K_2^d for any given k_B. The condition $D(p_A^d) < k_A$ defining K_2^d is equivalent to the condition $p_A^d > P(k_A)$. We thus define:

$$f(k_A) = p_A^d - P(k_A) = \frac{p_A^m\left(D(p_A^m) - k_B\right)}{k_A} - P(k_A) = \frac{c}{k_A} - P(k_A),$$

where $c = \pi_A^r(p_A^m)$ depends only on k_B. While the sign of f' is ambiguous, $f'' > 0$, that is f is strictly convex. Moreover, $\lim_{k_A \to 0^+} f(k_A) = \infty$ and $f(a) > 0$. Let us denote the capacity levels on the boundary of sets K^c and K^d by k_A', that is $p_A^m(k_A', k_B) = p^c = P(k_A' + k_B)$. It can be shown that $f(k_A') < 0$, thus for any given k_B there exists a k_A^* so that the projection of the set K_2^d equals $(k_A^*, a]$. Based on these results Figure 3 illustrates the spatial arrangement of K^c, K_1^d and K_2^d for Example 1.

Now, if $k_A \leq D(p_A^d)$, then

$$p_A^d k_A = p_A^m\left(D(p_A^m) - k_B\right) \iff p_A^d = \frac{p_A^m\left(D(p_A^m) - k_B\right)}{k_A}, \tag{13}$$

while for $k_A > D(p_A^d)$, p_A^d is defined by the minimum price satisfying the following condition:

$$p_A^d D(p_A^d) = p_A^m\left(D(p_A^m) - k_B\right). \tag{14}$$

Note, however, that this latter case cannot be part of the equilibria, since p_A^d given by (14) is independent of k_A, and for that reason the private firm could increase its profit by choosing a lower capacity level equal to $k_A' = k_A - \varepsilon > D(p_A^d)$. Thus, in equilibrium $k_A \leq D(p_A^d)$ holds.

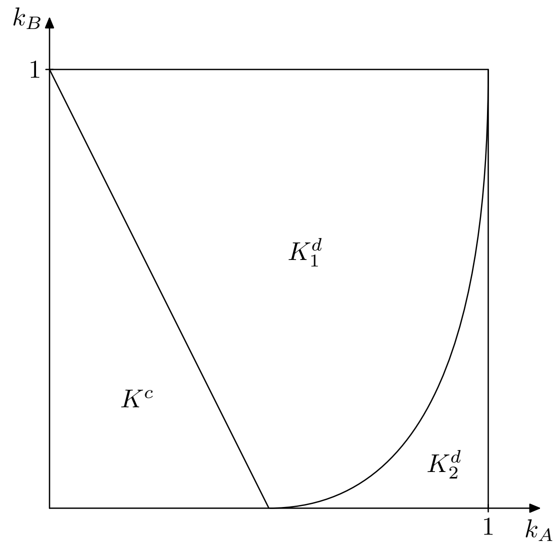

Figure 3: Set of capacities

Given the equilibrium prices, for any (k_A, k_B) capacity profile the firms' objective functions are as follows:

$$\overline{\pi}_A(k_A, k_B) = \begin{cases} p_A^d k_A - C_A(k_A) & \text{if } (k_A, k_B) \in K^d, \\ p^c k_A - C_A(k_A) & \text{if } (k_A, k_B) \in K^c \end{cases} \tag{15}$$

and

$$\overline{\pi}_B(k_A, k_B) = \begin{cases} \int_0^{D(p_A^d)} P(q)dq - C_A(k_A) - C_B(k_B) & \text{if } (k_A, k_B) \in K^d, \\ \int_0^{\min\{k_A+k_B,a\}} P(q)dq - C_A(k_A) - C_B(k_B) & \text{if } (k_A, k_B) \in K^c. \end{cases} \tag{16}$$

For simplicity we neglect the arguments k_A and k_B of functions p_A^d and p^c, moreover, we did not substitute the already determined expressions for these functions in the objective functions.

Since solutions from K^c and K_1^d dominate the capacity levels from K_2^d we focus our attention only on K^c and K_1^d. However, by determining $\frac{\partial}{\partial k_A}\overline{\pi}_A(k_A, k_B)$ on the interior of K_1^d we can exclude capacities belonging to K_1^d as well. To see this, consider the private firm's profit function on the above mentioned interval:

$$\overline{\pi}_A(k_A, k_B) = p_A^d k_A - C_A(k_A) = p_A^m \left(D(p_A^m) - k_B\right) - C_A(k_A),$$

thus

$$\frac{\partial}{\partial k_A}\overline{\pi}_A(k_A, k_B) = -C'(k_A) < 0.$$

Hence, π_A is decreasing in k_A on K_1^d for any given k_B, which implies that the equilibrium solution is necessary in K^c.

Notice that the objective functions given by (15) and (16) are identical to (1) and (2) determined for the mixed Cournot duopoly case. Yet, we have to show that the equilibrium Cournot outcomes are in set K^c as well. To do so, let us express the second period residual profit function defining p_A^m in terms of quantities and maximize:

$$\pi_A^r(q_A) = P(q_A + k_B)q_A$$

with respect to q_A. The solution is denoted as q_A^m. For this problem the sufficient first order condition yields

$$P'(q_A^m + k_B)q_A^m + P(q_A^m + k_B) = 0. \tag{17}$$

Observe that $P(q_A^m + k_B)$ coincides with p_A^m, since we have solved the same profit maximization problem in two different ways. If we compare (17) with the first equation of (3) we can see that for any $k_B \in [0, a)$ we have $p_A^m = p^c$. Therefore, for capacities given by $P(q_A^m + k_B) = p^c$ we have $\frac{\partial \pi_A}{\partial k_A}(q_A^m, k_B) < 0$, and thus taking Assumptions 1 and 2 into account, we can see that the first order conditions given by (3) are satisfied within K^c.

We summarize our results as follows:

Theorem 1 *Given Assumptions 1 and 2, quantity precommitment and Bertrand competition yield Cournot outcomes not only in duopolies with private firms (see Kreps and Scheinkman (1983)) but also in mixed duopolies.*

References

Bakó, B. and Tasnádi, A. (2014): A Kreps-Scheinkman állítás érvényessége lineáris keresletű vegyes duopóliumok esetén? (The Kreps and Scheinkman result remains valid for mixed duopolies with linear demand, in Hungarian), *MPRA Paper* No. 52746.

Balogh, T.L. and Tasnádi, A. (2012): Does timing of decisions in a mixed duopoly matter?, *Journal of Economics*, Vol. 106, No. 3, 233–249.

Beato, P. and Mas-Colell, A. (1984): The marginal cost pricing as a regulation mechanism in mixed markets, in Marchand, M., Pestieau, P. and Tulkens, H. eds., *The Performance of Public Enterprises*, North-Holland, Amsterdam, 81–100.

Boccard, N. and Wauthy, X. (2000): Bertrand competition and Cournot outcomes: further results, *Economics Letters*, Vol. 68, No. 3, 279–285.

Boccard, N. and Wauthy, X. (2004): Bertrand competition and Cournot outcomes: a correction, *Economics Letters*, Vol. 84, No. 2, 163–166.

Cremer, H., Marchand, M. and Thisse, J.-F. (1989): The Public Firm as an Instrument for Regulating an Oligopolistic Market, *Oxford Economic Papers*, Vol. 41, No. 2, 283–301.

Davidson, C. and Deneckere, R. (1986): Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model, *Rand Journal of Economics*, Vol. 17, No. 3, 404–415.

de Fraja, G. and Delbono, F. (1989): Alternative Strategies of a Public Enterprise in Oligopoly, *Oxford Economic Papers*, Vol. 41, No. 2, 302–311.

de Frutos, M.-A. and Fabra, N. (2011): The role of demand uncertainty, *International Journal of Industrial Organization*, Vol. 29, No. 4, 399–411.

George, K. and La Manna, M.M.A. (1996): Mixed Duopoly, Inefficiency, and Public Ownership, *Review of Industrial Organization*, Vol. 11, No. 6, 853–860.

Harris, R.G. and Wiens, E.G. (1980): Government enterprise: an instrument for the internal regulation of industry, *Canadian Journal of Economics*, Vol. 13, No. 1, 125–132.

Kreps, D.M. and Scheinkman, J.A. (1983): Quantity Precommitment and Bertrand Competition Yiels Cournot Outcomes, *The Bell Journal of Economics*, Vol. 14, No. 2, 326–337.

Lepore, J.J. (2012): Cournot outcomes under Bertrand-Edgeworth competition with demand uncertainty, *Journal of Mathematical Economics*, Vol. 48, No. 3, 177–186.

Loertscher, S. (2008): Market Making Oligopolies, *Journal of Industrial Economics*, Vol. 56, No. 2, 263–289.

Merrill, W.C. and Schneider, N. (1966): Government Firms in Oligopoly Industries: A Short-run Analysis, *Quarterly Journal of Economics*, Vol. 80, No. 3, 400–412.

Reynolds, S.S. and Wilson, B.J. (2000): Bertrand-Edgeworth Competition, Demand Uncertainty, and Asymmetric Outcomes, *Journal of Economic Theory*, Vol. 92, No. 1, 122–141.

Tomaru, Y. and Kiyono, K. (2010): Endogenous Timing in Mixed Duopoly with Increasing Marginal Costs, *Journal of Institutional and Theoretical Economics*, Vol. 166, No. 4, 591–613.

Vives, X. (1999): *Oligopoly Pricing: Old Ideas and New Tools*, MIT Press, Cambridge MA.

Wolfstetter, E. (1999): *Topics in Microeconomics*, Cambridge University Press, Cambridge UK.

Wu, Xin-wang, Zhu, Quan-tao and Sun, Laixiang (2012): On equivalence between Cournot competition and the Kreps–Scheinkman game, *International Journal of Industrial Organization*, Vol. 30, No. 1, 116–125.