Uncertain Longevity and Investment in Education

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Abstract

It has been argued that increased life expectancy raises the rate of return on education, causing a rise in the investment in education followed by an increase in lifetime labor supply. Empirical evidence of these relations is rather weak. Building on a lifecycle model with uncertain longevity, this paper shows that increased life expectancy does not suffice to warrant the above hypotheses. We provide assumptions about the change in survival probabilities, specifically about the age dependence of hazard rates, which determine individuals' behavioral response w.r.t. education, work and age of retirement. Comparison is made between the case when individuals have access to a competitive annuity market and the case of no insurance.

JEL Classification: D11, D91, E21, G23

Key Words: Longevity, Survival Functions, Education, Work, Age of Retirement, Annuities.

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1 Introduction[1]

The causal effect of increased life expectancy on the rise of investment in human capital has been extensively studied. The evidence is mixed: Acemoglu and Johnson (2006) and Lorentzen, McMillan and Wacziarg (2008), using cross-country regressions, find no systematic relation. Bils and Klenow (2000) and Manuelli and Seshadri (2005) find a positive relation, although of different magnitudes.

The theoretical reason why such a relation is expected was first formulated by Ben-Porath (1967), followed by a number of studies in the development literature (e.g. Cervellati and Sunde (2005) and Hall and Jones (1999)). The argument is that "falling mortality... raises the rate of return on investments in a child's human capital and thus can induce households to make quality-quantity trade-off" (Galor and Weil (1999)). Hazan (2009) observes that the higher returns to education have to be realized via a longer working life. He therefore hypothesizes that higher life expectancy leads to larger investment in education accompanied by an increase in lifetime labor supply. However, his time-series study of American and European men (1840-1970) finds no relation between life expectancy and lifetime labor.

The motivation for this paper is to provide general conditions on the direction of behavioral response, in particular investment in education and retirement age, to increased longevity. On the basis of an individual lifetime model with uncertain longevity, it is demonstrated that the above relations cannot be based solely on rises in life expectancy. Rather, they require specific conditions on the changes in survival probabilities. The salient feature of this paper is a representation of uncertain longevity by an age-dependent survival function. When survival probabilities rise at all ages, behavioral response depends crucially on the change in the *Hazard-Rate*. An increase in life expectancy may lead to different decisions about investment in education and retirement age, depending on the change in the Hazard-Rate at different ages. We demonstrate that this is the missing feature in previous studies of individuals' response to higher longevity.

Section 2 introduces survival functions, their associated life expectancy and the hazard-rate. It characterizes the relation between a shift of a survival function and the change in the hazard-rate.

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[1] I want to thank my colleague Moshe Hazan for suggesting the question analyzed here, a natural extension of Sheshinski (2008).

In Section 3 we build a standard lifetime model with uncertain longevity, where the individual chooses the lifetime consumption path, the length of time devoted to education, followed by a period of work and then the age of retirement. Perfectly competitive insurance markets are assumed to be available. Three propositions provide sufficient conditions to enable in some cases a determination of the effects of a rise in longevity on consumption, the level of education and retirement.

Section 4 conducts the same analysis under the assumption that no annuity markets are available. While a limited number of private annuity products are available in advanced economies, their market is characterized by adverse selection and moral hazard. It is worth, therefore, to study individuals' response in the absence of this market. The results in this case are similar, and even 'sharper', than in previous sections.

The general conclusion that emerges from this paper is that while a rise in life expectancy is inadequate to explain the increased investment in education, certain (testable) conditions formulated in the paper may confirm conventional wisdom.

2 Survival Functions and Longevity Changes

Age is a continuous variable, z, whose range is from 0 to maximum lifetime, T. Formally, it is possible to allow $T = \infty$. Age 0 should be interpreted as the age at which individuals make decisions about consumption, an initial period of education followed by a working period and then retirement. Uncertainty about longevity, that is the age of death, is represented by a *survival distribution function*, $F(z, \alpha)$, which is the probability of survival to age z. The exogenous parameter α represents factors which affect longevity, such as health and family circumstances. The focus is on how individuals respond to changes in these factors.

The function $F(z, \alpha)$ satisfies $F(0, \alpha) = 1$, $F(T, \alpha) = 0$ and $F(z, \alpha)$ strictly decreases in z, for any α[2]. We shall assume that $F(z, \alpha)$ is differentiable in z and α and hence the probability of death at age z, $f(z, \alpha)$, which is the density of function of $1 - F(z, \alpha)$, exists for all z, $f(z, \alpha) = -\dfrac{\partial F(z, \alpha)}{\partial z} > 0$, $0 < z < T$.

[2] Thus, when T is finite it generally depends on α.

Life expectancy, denoted $\bar{z}$, is given by

$$\bar{z}(\alpha) = \int_0^T z f(z,\alpha) dz$$

or, integrating by parts,

$$= \int_0^T F(z,\alpha) dz \tag{1}$$

A commonly used survival function is

$$F(z,\alpha,T) = \frac{e^{-\alpha z} - e^{-\alpha T}}{1 - e^{-\alpha T}} \qquad 0 \leq z \leq T \tag{2}$$

is a function of two parameters, $\alpha > 0$ and $T > 0$. When $T = \infty$ this becomes the well known exponential function $F(z,\alpha) = e^{-\alpha z}$ for which $\bar{z} = \frac{1}{\alpha}$.

As in (2), we take an increase in α to decrease survival probabilities at all ages: $\frac{\partial F(z,\alpha)}{\partial \alpha} < 0$, for all $0 < z < T$. Clearly, a decrease in α increases life expectancy, $\frac{\partial \bar{z}}{\partial \alpha} > 0$.

It will be seen that individuals' response to a change in α depends on the magnitude of the changes in survival probabilities in different ages. Notice, for example, that in (2), changes in α and in T have very different effects on $F(z,\alpha)$ (Figure 1).

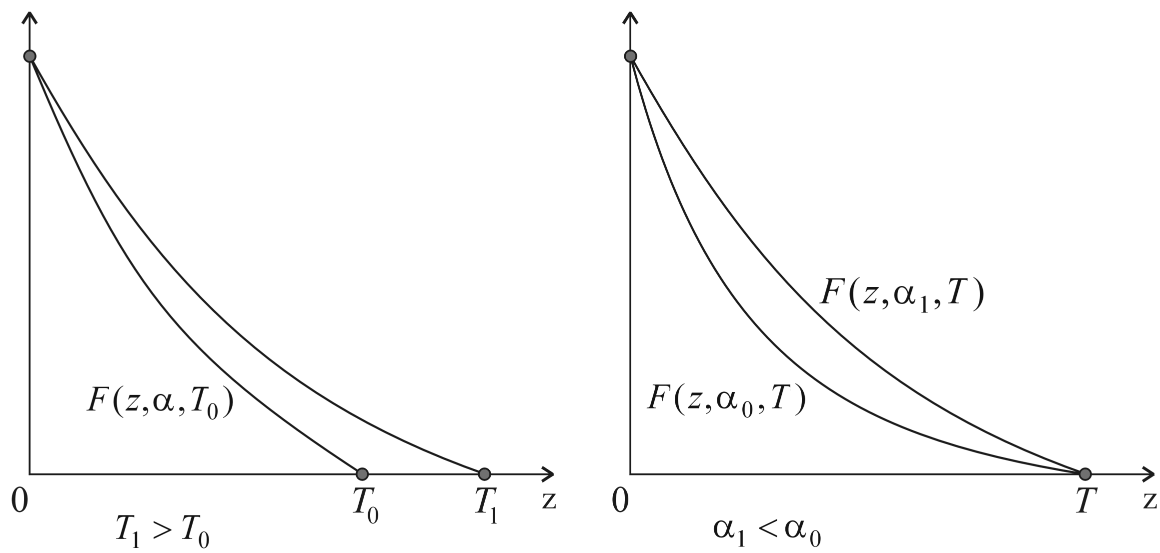

Figure 1

A change in α affects mainly medium ages while a change in T affects largely older ages.

Each survival function has an associated *Hazard-Rate*, the conditional probability of dying at age z: $f(z,\alpha)/F(z,\alpha)$. The effect of a change in α on the hazard rate is:

$$\frac{\partial}{\partial\alpha}\left(\frac{f(z,\alpha)}{F(z,\alpha)}\right) = -\frac{\partial\mu(z,\alpha)}{\partial z}$$

where

$$\mu(z,\alpha) = \frac{1}{F(z,\alpha)}\frac{\partial F(z,\alpha)}{\partial\alpha} \tag{3}$$

The function $\mu(z,\alpha)$ (< 0) is the relative decrease of survival probability at age z due to a small increase in α. The shape of this function plays an important role in our analysis. In particular, individual behavior will be shown to depend on whether $\mu(z,\alpha)$ increases or decreases with z.

Figure 2 displays the case when $\mu(z,\alpha)$ decreases in z, which means that the relative increase in survival probabilities due to a decrease in α rises with age.

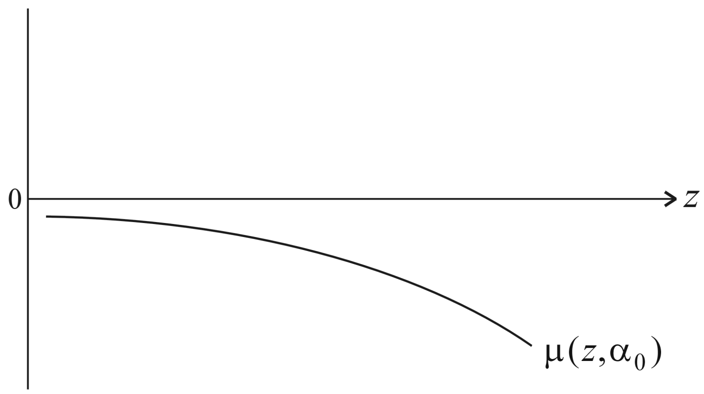

Figure 2

Notably, the historical pattern of $\mu(z,\alpha)$ has been uneven. Cutler, Deaton and Lleras-Muney (2006) and others distinguish three phases in modern history: in the early 20th century, infants' survival probabilities improved dramatically, followed in mid century by major improvements of survival probabilities in middle-ages (50-70) due to the cardio-vascular revolution. During recent decades, improvements of life prospects due to new medical technologies focused mainly on the old and the very old[3]. These historical phases suggest that models in which longevity is represented only by life expectancy are missing the important relation between the rise in survival probabilities at different ages.

3 Lifecycle Model with Longevity Risk

Denote consumption at age z by $c(z)$. Utility of consumption, $u(c)$, is assumed to be positive, independent of age, strictly increasing and concave: $u'(c) > 0$, $u''(c) < 0$. Assume further that when working the individual provides 1 unit of labor. Disutility of work, $a(z)$, is independent of consumption and increasing with age, $a'(z) > 0$. Individuals spend ages 0 to e in education. The focus is on the productivity enhancing value of education disregarding its possible direct consumption value.

With no subjective discount rate (time preference) and no bequest motive, expected lifetime utility, V, is

$$V = \int_0^T u(c(z))F(z,\alpha)dz - \int_e^R a(z)F(z)dz \tag{4}$$

When working, an individual with a level of education e receives a wage of $w(z,e)$ at age z. Education is productive and has diminishing marginal productivity: $\dfrac{\partial w(z,e)}{\partial e} = w_1(z,e) > 0$, $\dfrac{\partial^2 w(z,e)}{\partial e^2} = w_{11}(z,e) < 0$.

With a zero rate of return on non-annuitized assets and perfectly competitive insurance markets that allow annuitization of savings (including contingent loans[4]), the individuals' budget constraint is[5]

$$\int_0^T c(z)F(z,\alpha)dz - \int_e^R w(z,e)F(z,\alpha)dz = 0 \tag{5}$$

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[3] The incorporation of direct utility from education would essentially not affect the results below.

[4] In Section 4 we discuss the case when such annuitization is not available.

[5] We disregard tuition fees for e. Adding such fees would not affect the results below.

Maximization of (4) s.t. (5) w.r.t. $c(z)$ yields an optimum constant consumption flow, $c(z) = c$, $0 \leq z \leq T$. The level of c depends, via the budget constraint, (5), on e and R:

$$c = \frac{\int_e^R w(z,e)F(z,e)dz}{\bar{z}(\alpha)} \tag{6}$$

Consumption equals expected wages divided by life expectancy.

The first order condition for an optimum retirement age, R, is

$$u'(c)w(R,e) - a(R) = 0 \tag{7}$$

At the optimum, the marginal benefit of a small postponement in retirement is equal to marginal labor disutility.

The condition for the optimum level of education, e, is

$$u'(c)\frac{1}{F(e,\alpha)}\int_e^R w_1(z,e)F(z,\alpha)dz - u'(c)w(e,e) + a(e) = 0 \tag{8}$$

where $w_1(z,e) = \dfrac{\partial w(z,e)}{\partial z}$. At the optimum, the marginal benefit of an additional investment in education, equal to the marginal utility of the conditional expected increase in lifetime wages, is equated to the marginal cost, which is the marginal utility times the wage at e less the disutility of work.

Conditions (6)-(8) jointly determine optimum consumption, retirement age and the level of education, denoted (c^*, R^*, e^*).

To facilitate the subsequent analysis, we shall make two simplifying (inessential) assumptions. First, let wages be independent of age, $w(z,e) = w(e)$ $(w'(e) > 0$, $w''(e) < 0)$. Second, assume that there is no labor disutility at the beginning of the working phase: $a(e) = 0$[6]. With these assumptions, (6)-(8) are rewritten:

$$c = \frac{w(e)\int_e^R F(z,e)dz}{\bar{z}(\alpha)} \tag{9}$$

$$\varphi(R,e;\alpha) \equiv u'(c)w(e) - a(R) = 0 \tag{10}$$

[6]It seems reasonable that labor disutility at age e does not play a significant role, compared to income effects, in the marginal benefit-cost between education and work.

$$\psi(R,e;\alpha) \equiv \frac{w'(e)}{F_1(e,\alpha)} \int_e^R F(z,\alpha)dz - w(e) = 0 \tag{11}$$

where in (10), c is substituted from (9). Given α, the functions φ and ψ are curves in the (R,e) plane. In the Appendix it is shown that they are both upward sloping and, by the second-order conditions, φ is steeper than ψ at the unique intersection point (R^*, e^*) (Figure 3).

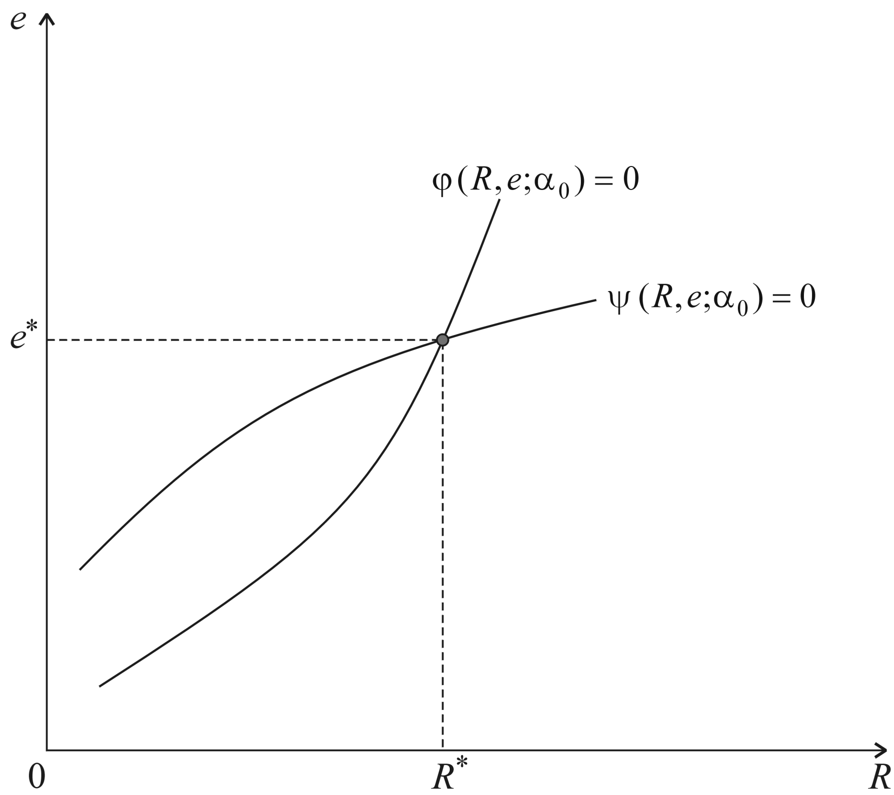

Figure 3

Generally, R^* and e^* depend on α. The following proposition provides a sufficient condition which enables determination of the direction of this dependence:

Proposition 1 *When $\mu(z,\alpha)$ strictly decreases in z, then $\frac{dR^*}{d\alpha} < 0$ and $\frac{de^*}{d\alpha} < 0$.*

Proof. Appendix ■

The result in Proposition 1 is depicted in Figure 4. As shown in the Appendix, a decrease in α shifts the curve $\varphi = 0$ downward and to the right. The curve $\psi = 0$

shifts upward and to the left. The new solution, (R^{**}, e^{**}), has $R^{**} > R^{*}$ and $e^{**} > e^{*}$.

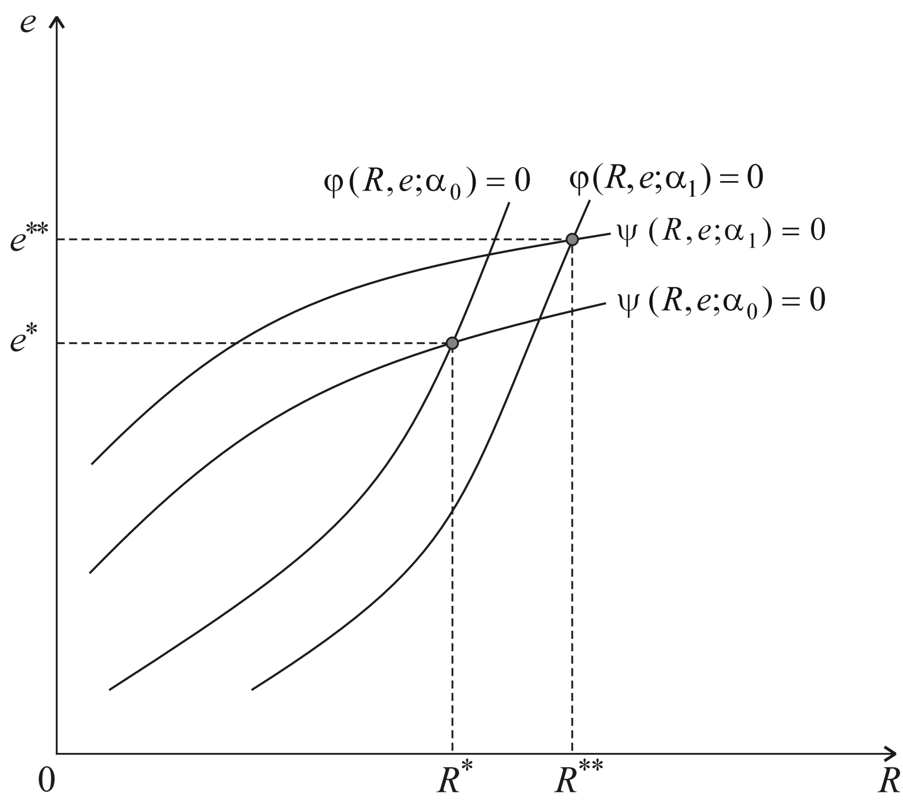

Figure 4: $\alpha_1 < \alpha_0$

Proposition 1 seems to confirm the conventional wisdom (see Hazan (2009)) that, abstracting from other effects, increased life expectancy causes, a rise in investment in human capital (i.e. education). However, it is shown below that when the condition in Proposition 1 does not hold then the sign of $\dfrac{de^*}{d\alpha}$ and $\dfrac{dR^*}{d\alpha}$ is indeterminate. The implication is that the conventional wisdom has to be qualified: an increase in life expectancy, $\bar{z}$, may or may not lead to a rise in e, the direction depending on additional conditions.

It is always possible to predict the direction of the change in e^* when retirement age is held constant. Specifically, the opposite assumption to the one in Proposition 1 is shown to lead to the opposite result w.r.t. e^*:

Proposition 2 *Holding retirement age constant, when $\mu(z, \alpha)$ increases (decreases) in z then* $\dfrac{de^*}{d\alpha} > (<)0$.

Proof. Appendix. ■

Proposition 2 takes retirement age as given and therefore is more limited than Proposition 1. The explanation for this restriction is the following. As proved in the Appendix, when $\mu(z,\alpha)$ increases in z then a decrease in α shifts the curve ψ downwards and to the right. It follows that holding R constant (in particular at R^*), optimum e decreases. The opposite result obtains when $\mu(z,\alpha)$ decreases in z, as assumed in Proposition 1. When $\mu(z,\alpha)$ increases in z, the direction of the shift in φ is indeterminate. If φ shifts upward and to the left then, in contrast to the result in Proposition 1, R^* and e^* *both decrease.* If, however, φ shifts downward and to the right then R^* and e^* may increase or decrease. The only impossible configuration is a decrease in R^* coupled with an increase in e^*. This is the formal reason why Proposition2 is restricted to a constant R. The economic explanation is straightforward. When improvements in survival probabilities are concentrated at younger ages, conditional expected length of work *decreases* as longevity rises (in (11), $\frac{1}{F(e,\alpha)}\int_e^R F(z,\alpha)dz$ decreases as α decreases). Hence, the marginal benefit from education decreases, leading to a reduction in the investment in education.

When retirement is also endogenous, one has to consider the interaction of increased longevity with optimum retirement. Given R and e, a decrease in α increases life expectancy, $\bar{z}$, *and* expected lifetime work. From the budget constraint, (5), consumption increases if the former rises more than the latter and *vice-versa.* Two effects are at work: on the one hand, expected lifetime is based on a longer time span than expected lifetime work. On the other hand, by assumption, improvements in survival probabilities are concentrated at the younger working ages. As seen from (10), the change in consumption affects the marginal benefit of postponing retirement. Consequently, since the direction of the change in consumption is indeterminate so is the age of retirement. This ambiguity feeds, in turn, on the conditional expected lifetime work.

The general conclusion that emerges from the above discussion is that behavioral response to a rise in longevity, in particular investment in education and age of retirement, depends on the age-related changes in survival probabilities, specifically, changes in *hazard rates* at different ages.

Another conventional wisdom is that *"...as individuals live longer, they invest more in human capital, if and only if, their lifetime labor supply increases"* (Hazan, 2009). The logic of this conclusion is clear: starting at an initially optimum level of education, additional investment in education is justified only when future earnings rise by extending then over a longer period.

As stated, this conclusion may be incorrect. The reason is that it is not the length of the working life that is correlated in the level of education, but rather the *conditional expected lifetime work*. Consider an example of equations (10)-(11). Let $F(z, \alpha) = e^{-\alpha z}$, $0 \leq z \leq \infty$. For this function, $\mu(z, \alpha)$ decreases with z. Let $a(R)$ be highly inelastic around R^* and let $w(e) = e^{\beta}$, β a constant, $0 < \beta < 1$. The solution to (10)-(11) can be seen to have αe^* constant while R^* is approximately constant, independent of α. Thus, a reduction in α raises e^* proportionately while R^* is approximately unchanged. Clearly then, the length of the working period, $R^* - e^*$, decreases as longevity increases.

We now write formally the correct statement about the relation between investment in education and expected lifetime labor supply:

Proposition 3 *Optimum education, e^*, is positively correlated with conditional expected lifetime work.*

Proof. From eq.(11), since $w''(e) < 0$, the sign of the change in e^* is the same as the sign of the change in conditional lifetime work: $\dfrac{1}{F(e^*, \alpha)} \int\limits_{e^*}^{R^*} F(z, \alpha) dz$. ■

4 No Annuities

Perfect annuitization of savings, assumed in previous sections, is not always available. Let us conduct the analysis in the absence of longevity insurance markets. It is assumed that borrowing and lending is possible, hence we disregard liquidity constraints[7]. The individual is constrained by a lifetime budget,

$$\int\limits_{0}^{T} c(z) dz - w(e)(R - e) = 0 \tag{12}$$

[7] The assumption that the interest rate on non annuitized assets is zero is not realistic since the death of borrowers may leave lenders with unpaid loan balances. However, incorporation of a positive interest rate in (12) will not change essentially the following analysis. Note also that there are now *unintended bequests*, and hence the analysis below cannot be carried-over to the economy as a whole without explicitly addressing this issue.

which replaces (5). The F.O.C. w.r.t. $c(z)$ is now

$$u'(c(z))F(z,\alpha) - \lambda = 0, \qquad 0 \leq z \leq T \tag{13}$$

for $\lambda > 0$ constant: $\lambda = u'(c(0))$. Differentiating (13) w.r.t. z,

$$\frac{\dot{\hat{c}}(z)}{\hat{c}(z)} = -\frac{1}{\sigma}\frac{f(z,\alpha)}{F(z,\alpha)} < 0, \qquad 0 \leq z \leq T \tag{14}$$

where $\sigma = -\dfrac{u''(c)c}{u'(c)} > 0$ is the coefficient of relative risk aversion. In general, σ depends on c.

In the absence of insurance, risk aversion leads the individual to decrease consumption with age. The rate of decrease is inversely proportional to σ and proportional to the hazard-rate, $\dfrac{f(z)}{F(z)}$.

To simplify the subsequent analysis, we specialize to $u(c) = \ln c$. With $\sigma = 1$, (14) and (12) solve for the optimum $c(z)$[8], denoted $\hat{c}(z)$,

$$\hat{c}(z) = \frac{w(R-e)}{\bar{z}(\alpha)}F(z,\alpha) \tag{15}$$

The F.O.C. w.r.t. R and e, become, respectively

$$\varphi(R,e;\alpha) = \frac{\bar{z}(\alpha)}{(R-e)F(R,\alpha)} - a(R) = 0 \tag{16}$$

and (with $a(e) = 0$)

$$\psi(R,e) = w'(e)(R-e) - w(e) = 0 \tag{17}$$

As before, these conditions equate marginal benefits and costs of changes in R and e. Denote the solution to (16)-(17) by $(\hat{R},\hat{e})$. Importantly, ψ is now independent of α. This enables a sharper result than in Propositions 1 and 2:

Proposition 4 *In the absence of annuities, when $\mu(z,\alpha)$ decreases (increases) in z, a decrease in α increases (decreases) $\hat{R}$ and $\hat{e}$.*

Proof. *Appendix.* ■

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[8] Using $\dfrac{f(z,\alpha)}{F(z,\alpha)} = -\dfrac{d\ln F(z,\alpha)}{dz}$.

As shown in the Appendix the curves φ and ψ are upward sloping with φ steeper than ψ at the unique intersection point, $(\hat{R}, \hat{e})$. A decrease in α shifts φ downward and to the right, hence the new solution has $\widehat{\widehat{R}} > \widehat{R}$ and $\widehat{\widehat{e}} > \widehat{e}$ (Figure 5a). When $\mu(z, \alpha)$ increases in z this obtain an opposite result: $\widehat{\widehat{R}} < \widehat{R}$ and $\widehat{\widehat{e}} < \widehat{e}$ (Figure 4b).

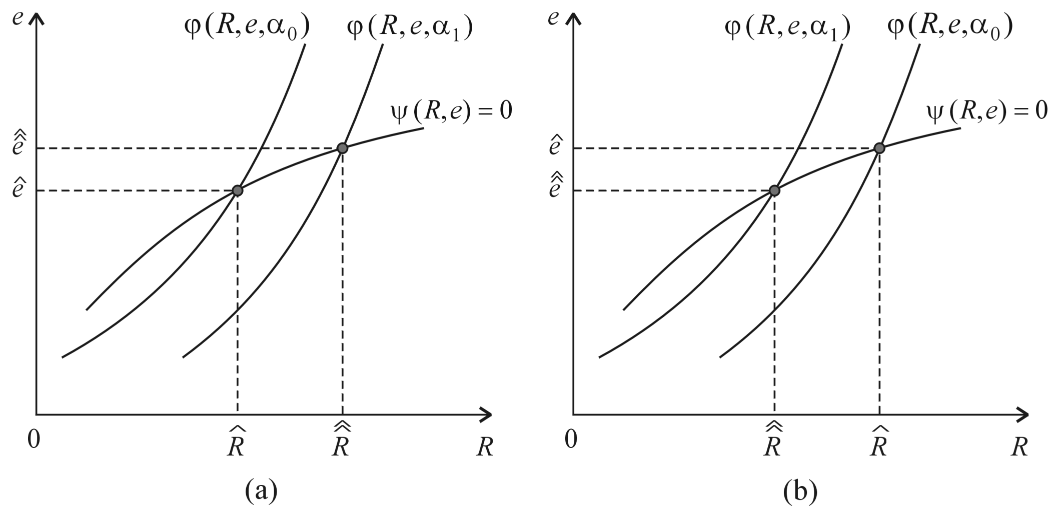

Figure 5: $\alpha_1 < \alpha_0$

In the absence of contingent savings, the optimum level of education is determined by equating (17), the rise in the wage rate caused by a small increase in e times the length of the working phase, with the marginal costs, namely the wage rate. While annuity prices depend on survival prospects, these do not affect directly the budget constraint (12). A change in α affects the optimum e only through its effect on the age of retirement. The optimum consumption path depends on the hazard-rate which changes with α.

We conclude that in the absence of annuitization, conventional wisdom that rise in longevity causes an increase in education may be wrong but, as see in (17), an increase in the investment in education always leads to a longer work phase, $\hat{R} - \hat{e}$.

Appendix

Two equations determine the optimum (R^*, e^*) :

$$\varphi(R, e; \alpha) = u'(c)w(e) - a(R) = 0 \tag{A.1}$$

and

$$\psi(R, e; \alpha) = \frac{1}{F(e, \alpha)} \int_e^R F(z, \alpha)dz - \frac{w(e)}{w'(e)} = 0 \tag{A.2}$$

where

$$c = \frac{w(e) \int_e^R F(z, e)dz}{\bar{z}(\alpha)} \tag{A.3}$$

and $\bar{z}$ is expected lifetime $\bar{z}(\alpha) = \int_0^T F(z, e)dz$.

The functions φ and ψ are the first-order conditions for maximization of V, (4), w.r.t. R and e, respectively. Using (A.1) and (A.2), second-order conditions are (subscripts denote partial derivatives):

$$\varphi_R = -u'(c)w(e) \left(\sigma \frac{F(R, \alpha)}{\int_e^R F(z, \alpha)dz} + \frac{a'(R)}{a(R)} \right) < 0 \tag{A.4}$$

where $\sigma = -\frac{u''(c)c}{u'(c)} > 0$ is the coefficient of relative risk aversion;

$$\psi_e = \frac{w(e)}{w'(e)} \left(\frac{w''(e)}{w'(e)} - \frac{F_1(e, \alpha)}{F(e, \alpha)} \right) - 2 < 0 \tag{A.5}$$

Since $F_1(e, \alpha) < 0$, a sufficient condition for ψ_e to be negative is that the term in brackets is negative: $\frac{w''(e)}{w'(e)} - \frac{F_1(e, \alpha)}{F(e, \alpha)} < 0$.

The cross partial derivatives are:

$$\varphi_e = u'(c) \left((1 - \sigma)w'(e) + \sigma w(e) \frac{F(e, \alpha)}{\int_e^R F(z, \alpha)dz} \right)$$

A sufficient condition for $\varphi_e > 0$ is $\sigma < 1$ for all c.

When (A.2) holds (at (R^*, e^*)), this expression simplifies to

$$\varphi_e = u'(c)w'(e) > 0 \tag{A.6}$$

The cross partial of ψ is

$$\psi_R = \frac{F(R,\alpha)}{F(e,\alpha)} > 0 \tag{A.7}$$

The remaining condition is $\Delta = \varphi_R \psi_e - \varphi_e \psi_R > 0$. From (A.4)-(A.7),

$$\begin{aligned}\Delta \;=\; & u'(c)w(e)\left\{\left(\sigma\frac{F(R,\alpha)}{\int_e^R F(z,\alpha)dz} + \frac{a'(R)}{d(R)}\right)\left(2 + \frac{w(e)}{w'(e)}\left(\frac{F_1(e,\alpha)}{F(e,\alpha)} - \frac{w''(e)}{w'(e)}\right)\right) - \right.\\ & \left. - \frac{w'(e)}{w(e)}\frac{F(R,\alpha)}{F(e,\alpha)}\right\}\end{aligned} \tag{A.8}$$

Using (A.2) it can be seen that sufficient conditions for $\Delta > 0$ are $2\sigma - 1 > 0$ and the condition on (A.5), $\dfrac{w''(e)}{w'(e)} - \dfrac{F_1(e,\alpha)}{F(e,\alpha)} < 0$.

Conditions (A.4)-(A.7) and $\Delta > 0$ imply that the curves φ and ψ are upward sloping $\left(\left.\dfrac{de}{dR}\right|_{\varphi=0} = -\dfrac{\varphi_R}{\varphi_e} > 0,\ \left.\dfrac{de}{dR}\right|_{\psi=0} = -\dfrac{\psi_R}{\psi_e} > 0\right)$ and that $\varphi = 0$ is steeper than $\psi = 0$ at (R^*, e^*). Hence, (R^*, e^*) is unique.

How do the curves φ and ψ shift as α changes? Differentiating φ partially w.r.t. α,

$$\varphi_\alpha = \left(\frac{u''(c)w(e)^2}{\bar{z}}\int_e^R F(z,\alpha)dz\right)\eta(R,e;\alpha) \tag{A.9}$$

where $\eta(R,e;\alpha) \equiv \int_e^R \frac{\partial F(z,\alpha)}{\partial\alpha}dz \Big/ \int_e^R F(z,\alpha)dz - \int_0^T \frac{\partial F(z,\alpha)}{\partial\alpha}dz \Big/ \int_0^T F(z,\alpha)dz.$

Since for any $e > 0$, $\int\limits_e^R \frac{\partial F(z,\alpha)}{\partial \alpha} dz \Big/ \int\limits_e^R F(z,\alpha) dz > \int\limits_0^T \frac{\partial F(z,\alpha)}{\partial \alpha} dz \Big/ \int\limits_0^T F(z,\alpha) dz$, it follows that $\eta(T, e; \alpha) > 0$. Calculating the change in η w.r.t. R:

$$\frac{\partial \eta(R, e; \alpha)}{\partial R} = \frac{F(R,\alpha)}{\int\limits_e^R F(z,\alpha)dz} \int\limits_e^R \left[\frac{1}{F(R,\alpha)} \frac{\partial F(R,\alpha)}{\partial \alpha} - \frac{1}{F(z,\alpha)} \frac{\partial F(z,\alpha)}{\partial \alpha} \right] \frac{F(z,\alpha)}{\int\limits_e^R F(z,\alpha)dz} dz \tag{A.10}$$

When $\mu(z,\alpha) = -\frac{1}{F(z,\alpha)} \frac{\partial F(z,\alpha)}{\partial \alpha}$ decreases in z, the term in square brackets in (A.10) is negative for all z. Hence, $\frac{\partial \eta}{\partial R} < 0$ for all R. Since $\eta(T, e; \alpha) > 0$, it follows that $\eta(R, e; \alpha) > 0$ for all $0 \le R \le T$.

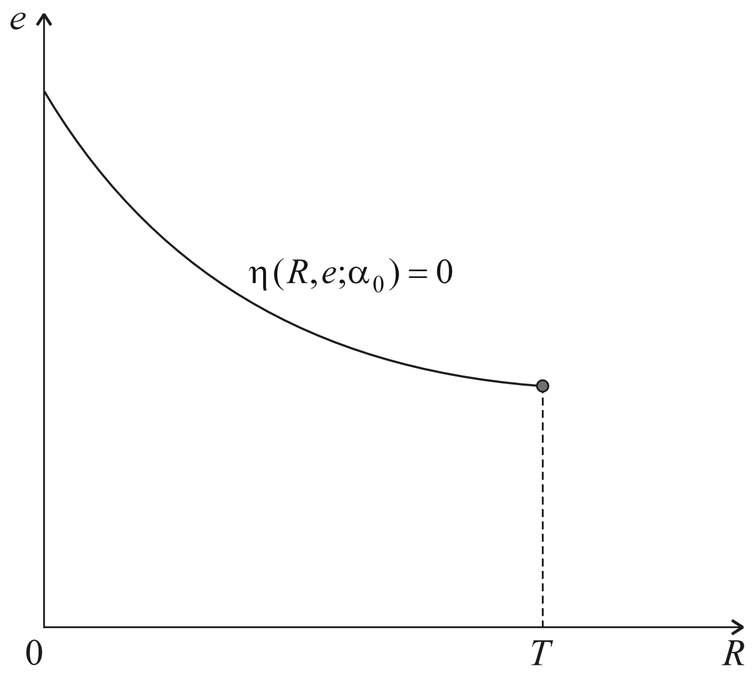

Figure A.1

By (A.9) it now follows that $\varphi_\alpha < 0$. This explains why in Figure 3, φ shifts upwards and to the left as α decreases.

Note that under the alternative assumption, namely, that $\mu(z,\alpha)$ increases in z, $\frac{\partial \eta}{\partial R} > 0$. Hence, while $\eta(T, e; \alpha) > 0$, it is impossible to infer whether η is positive for all R and, consequently, it is impossible to determine the direction of the change in φ.

Calculating the direction of the shift in ψ due to a change in α yields, after some manipulations:

$$\psi_\alpha = \frac{F(e,\alpha)\int\limits_e^R F(z,\alpha)dz}{F(e,\alpha)} \int\limits_e^R \left[\frac{1}{F(z,\alpha)}\frac{\partial F(z,\alpha)}{\partial\alpha} - \frac{1}{F(e,\alpha)}\frac{\partial F(e,\alpha)}{\partial\alpha}\right] \frac{F(z,\alpha)}{\int\limits_e^R F(z,\alpha)dz} dz \tag{A.11}$$

When $\mu(z,\alpha)$ decreases in z, the term in square brackets in (A.11) is negative. It follows that ψ shifts downward and to the right. When $\mu(z,\alpha)$ increases in z, ψ shifts in the opposite direction. This proves Propositions 1 and 2.

Proposition 2 considers the case that $\mu(z,\alpha)$ increases with z, but the proposition is restricted to a given R. The reason why the joint effect of a change in α on (R^*, e^*) is indeterminate in this case is because the direction of the shift in φ is in determinate.

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