Quarterly Bayesian DSGE Model of Pakistan Economy with Informality

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Abstract

In this paper we use the Bayesian methodology to estimate the structural and shocks' parameters of the DSGE model in Ahmad et al. (2012). This model includes formal and informal firms both at intermediate and final goods production levels. Households derive utility from leisure, real money balances and consumption. Each household is treated as a unit of labor which is a composite of formal (skilled) and informal (unskilled) labor. The formal (skilled) labor is further divided into types "r" and households have monopoly over each type "r" labor which depends upon degree of education. We go a step further by converting the existing annually calibrated model to quarterly frequency. As a result our impulse response functions have more relevant and realistic policy implications. From the results we do find the shock absorbing role of the informal sector, however, with short term existence. The model estimation diagnostics also confirm robustness and reasonability of the estimation results.

Key words: *Bayesian Estimation., DSGE Model, Shock Process.*
JEL Classification: *E17*

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1. Introduction

In Ahmad et al. (2012) Pakistan's economy has been modeled while focusing on informality in the labor and production markets. The model has been calibrated as per standard calibration norms and impulse response functions have been generated in response to three shocks, i.e., the technology shock, the government spending shock and the monetary shock. It is important to note that (in that paper) the calibration of model parameters and shock parameters was handled with care so that the micro-evident behavior of certain economic agents involved in the model can be captured as appropriately as possible. Nevertheless, in many cases, the data issue remained since we were constrained by the availability of higher frequency of data, i.e., preferably quarterly time series data. Instead, we used the data at annual frequency and cautiously mentioned that the model outcomes cannot have policy implications, however, they can mimic the medium to long run behavior of an economy which is an important outcome in itself since it can confirm the performance of the DSGE model as satisfactory which, in itself, is a reasonable landmark for a data scarce developing country with no significant theoretical economic modeling references.

When it comes to modern approaches in DSGE modeling, Bayesian estimation of DSGE models' parameters is one of the latest as well as most widely practiced developments after standard calibration. Almost all recent literature which deals with estimation of DSGE models terms it as a big leap forward in making DSGE models more intuitive by estimating those parameters scientifically/ statistically which were first being calibrated based on guess work. However, calibrated models are still important when the calibration exercise presents the true reflection of the micro level economic behavior in an economy. This is the reason that for developed countries we still see many papers even with extensive models having lots of parameters being calibrated in the standard method. This is only because of availability of extensive good high frequency data which is easily accessible to researchers. But when it comes to countries like Pakistan, as also mentioned above, instead of fixing parameter values, as we do in standard calibration, there are few methods that incorporate distributions for estimation of these parameters, e.g. maximum likelihood method and Bayesian estimation method. However, the success of Bayesian estimation is due to some of the advantages this method has over its competitors. These advantages are as follow:

a. Bayesian estimation fits the complete DSGE model as compared to GMM estimation which is based on a particular equilibrium relationship such as the relevant Euler equation.

b. Estimation in the Bayesian method is generated by whole of the DSGE system which is a clear improvement over implied DSGE and VAR based impulse response functions.

c. The use of priors in Bayesian estimation work as weights in the estimation process. Consequently the posterior distribution is much stable and avoids peaking at strange points where the likelihood peaks. This, thus, advantages the Bayesian estimates to be free from the issue of absurd parameter estimates.

d. Use of priors helps identify the parameters. Since different values of parameters can lead to similar outcomes, thus, their identification remains an issue in calibrated models.

e. Bayesian estimation explicitly addresses model misspecification by including shocks, which can be interpreted as observation errors, in the structural equations.

f. The posterior distributions of competing models can easily be used to determine which model fits the data best.

Recent and important literature on estimating DESGE models using Bayesian estimation techniques are Schorfheide (2000) which compares the fit of two competing DSGE models of consumption, Lubik and Schorfheide (2003) which deals with central banks' responses to changes in exchange rate, Smets and Wouters (2003) which applies the technique to the Eurozone, Auroba et al. (2004) which deals with the econometric properties of Bayesian estimates, Lubik and Schorfheide (2005) which deals with matters of misspecification and identification of Bayesian estimation, and Rabanal and Rubio-Ramirez (2005) which compares the fit based on posterior distributions of New Keynesian models with nominal rigidities.

Since we have mentioned the usefulness and relevance of Bayesian estimation for DSGE models, we can gauge their efficacy for policy implications, as a result we selected this method for the way forward in modeling Pakistan's Economy meaningfully. Keeping the data frequency constraint in mind, we have made efforts to attain as much data as possible in quarterly frequency. As a result we have been able to use not only the Bayesian estimation technique but also on quarterly data, hence, generating impulse response functions having policy implications. In the next section we briefly mention the model, which has already been laid out in much detail in Ahmad et al. (2012), and then continue with the sections explaining Bayesian method of estimation, data used, prior and posterior distributions, model diagnostics and impulse response functions.

2. Model

The model comes directly from Ahmad et al. (2012). That is why we just briefly outline the model and leave the equations of the final model for the appendix at the end of the paper. The model is such that the economy consists of households, firms, government, and monetary authority. There are two types of firms; formal and informal. Both formal and informal firms are further classified as intermediate goods producing firms; and final goods producing firms. Households derive utility from leisure, real money balances and consumption. Each household is treated as a unit of labor which is a composite of formal (skilled) and informal (unskilled) labor. The formal (skilled) labor is further divided into types "r". Households have monopoly over each type "r" labor which depends upon degree of education.

The final goods are produced using intermediate goods. Formal intermediate firms produce differentiated goods employing hired labor and capital. These goods are then sold to formal final producers in a monopolistically competitive market. Informal intermediaries sell their output to informal final producers in a perfectly competitive market.

In both formal and informal sectors, final goods are produced by packaging intermediate goods; albeit under different technologies. Final output of both sectors is sold in a perfectly competitive environment. The combination of all these prices, both for formal and informal goods, then form the formal and informal aggregate price as well as the overall aggregate price.

Government finances its consumption partly through taxes on formal sector and partly through printing money. Monetary authority follows Taylor type rule.

There are three shocks. First is the technology shock (technology only resides with the formal sector production) which affects the productivity of the formal sector, however, there are spillovers in the informal sector as well. Second is the fiscal shock and third is the interest rate shock that operates through the Taylor rule.

3. Estimation Methodology

Our approach, in this study, relies on the "Bayesian Maximum Likelihood" (B-MLE) methodology for parameter estimates of the model and therefore it is imperative to focus on the procedures associated with it. Conceptually, in Bayesian methodology, things are radically different from the standard estimation of parameters. In classical wisdom, priors can be considered as additional data. Bayesian methodology actually updates likelihood (data), with priors (subjective believe) via simple probability rule known as Baye's Rule given below:

$$P(\Psi \setminus \mathrm{Y}^{\mathrm{T}}, \Lambda) = \frac{L(\mathrm{Y}^{\mathrm{T}} \setminus \Psi, \Lambda)\mathrm{P}(\Psi \setminus \Lambda)}{L(\mathrm{Y}^{\mathrm{T}} \setminus \Lambda)} \tag{1}$$

Here Y^{T} and $L(\mathrm{Y}^{\mathrm{T}} \setminus \Lambda)$ represent vector of variables and marginal distribution of observables (variables used) selected for our DSGE model respectively. The random vector[1] Ψ consists of model parameters illustrating relationship among variables of linearized system of equations. The likelihood $\mathrm{L}(\mathrm{Y}^{\mathrm{T}} \setminus \Psi, \Lambda)$ corresponds to the joint density of Y^{T} in the sample with T observations, conditional on the structure (Λ) and parameter vector Ψ of our DSGE model.

As marginal distribution $L(\mathrm{Y}^{\mathrm{T}} \setminus \Lambda)$ is independent of vector Ψ , so equation (1) can safely be written as:

$$P(\Psi \setminus \mathrm{Y}^{\mathrm{T}}, \Lambda) \propto L(\mathrm{Y}^{\mathrm{T}} \setminus \Psi, \Lambda)\mathrm{P}(\Psi \setminus \Lambda) \tag{2}$$

From equation (2) it is clear that in order to evaluate posterior distribution $\mathrm{P}(\Psi \setminus \mathrm{Y}^{\mathrm{T}}, \Lambda)$ one needs to have a likelihood function $L(\mathrm{Y}^{\mathrm{T}} \setminus \Psi, \Lambda)$ and a prior distribution$\mathrm{P}(\Psi \setminus \Lambda)$. We discuss them one by one:

3.1. Estimation of Likelihood Function $\boldsymbol{L(\mathrm{Y}^{\mathrm{T}} \setminus \Psi, \Lambda)}$

As mentioned above, likelihood function $\mathrm{L}(\mathrm{Y}^{\mathrm{T}} \setminus \Psi, \Lambda)$ corresponds to the joint density of data variables Y^{T} in a sample with T observations, conditional on the structure (Λ) and parameters Ψ of our DSGE model. So, in order to derive the likelihood function we need to establish a relationship between data and the model. We can do this by assuming that the observables can be explained partly by the model's variables and partly by some factors that the model is unable to measure, which we term as measurement errors. Mathematically we can write it as:

[1] Unlike classical, Bayesian econometrics considers parameter as random variables.

$$\mathrm{Y}^{\mathrm{T}} = \mathrm{A}\widehat{\mathrm{Y}^{\mathrm{T}}} + \mathrm{B}\varepsilon^{\mathrm{T}} \qquad (3)$$

In equation (3) matrix A defines the role of model's endogenous variables $\widehat{\mathrm{Y}^{\mathrm{T}}}$ in explaining the data Y^{T}. The other vector ε^{T} represents the measurement errors and matrix B explains the relationship between measurement errors and data. For simplicity we assume that ε^{T} follows Gaussian White Noise process. We know that DSGE model can be summarized as:

$$E_t(f(\widehat{\mathrm{Y}^{\mathrm{T}+1}}, \widehat{\mathrm{Y}^{\mathrm{T}}}, \widehat{\mathrm{Y}^{\mathrm{T}-1}}, \mu^{\mathrm{T}}) = 0$$

Here model innovations (μ^{T}) are also assumed to follow Gaussian White Noise process.

The solution of this system can be written as:

$$\widehat{\mathrm{Y}^{\mathrm{T}}} = \mathrm{C}\widehat{\mathrm{Y}^{\mathrm{T}-1}} + \mathrm{D}\mu^{\mathrm{T}} \qquad (4)$$

The system (3) and (4) constitutes a linear State Space model where equation (3) is a measurement equation and equation (4) is a transition equation. This system can be evaluated by Kalman filter, a powerful technique which not only gives us optimal estimates of $\widehat{\mathrm{Y}^{\mathrm{T}}}$ but also provides the likelihood function ($L(\mathrm{Y}^{\mathrm{T}} \setminus \Psi, \Lambda)$).

3.2. Prior Distributions

After estimating the likelihood function the next step is the specification of prior distributions which is also the starting point for Bayesian component of the estimation process.

The first step in specification of prior distributions is selection of the most adequate functional forms for their distributions. This can be done on the basis of different criteria, and most common practices are the following:

1) ***Gamma or inverse Gamma distributions*** are used for parameters which are bounded to be non negative;

2) ***Beta distributions*** are for the parameters that are restricted between the 0-1 range (for example probabilities or frictions);

3) ***Normal distributions*** are used when more informative priors are necessary or when the parameters are not bounded; and

4) ***Uniform distributions*** are used for non-informative priors.

Next step is to choose the defining values for each prior distribution which mostly consist of location (mean, mode, etc.) and dispersion (variance or probability intervals) parameters. Literature specifies the defining values of priors' densities on the bases of past studies or occurrences or simply on subjective views of the researcher without using the data set utilized in analysis. *The spirit behind the prior elicitation is to use other sources of information that do not directly enter the likelihood function*.

To locate means of prior distributions of those parameters for which researchers have strong *a-priori* convictions, usually the *related* existing empirical literature is used and weighted averages are used as location and dispersion parameters. If we suppose that all empirical studies are equally likely to be relevant then their weights would be equal[2]. In many cases, however, we may have reason to believe that some features may innately be more or less likely to be relevant. We give weights according to their *relevance* (Meta features). Such priors are known as Meta priors. Relevance is defined on the bases of economic or subjective views.

The application of subjective view for few parameters can be seen in Harrison and Oomen (2010). They borrow values of model parameters from the related empirical studies of UK, USA and Euro area for their weighting scheme. This study adopts a mechanical approach and reflects a "meta prior" that has two components. First it attaches more weight to the parameter estimates from studies of the US economy because features of UK economy are more or less similar to the US economy, secondly it attaches higher weight to the parameter estimates from studies that use Bayesian Maximum likelihood than on studies that match the model's impulse responses to those from an estimated VAR.

For the calculation of variance of the shocks to TFP, government spending, and monetary policy rule Harrison and Oomen (2010) uses a weighting scheme based on economic theory by estimating the contribution of these shocks to output and nominal interest rate variability in previous studies of US and Euro data. Del Negro and Schorfheide (2008) discuss these issues in more detail.

In the case for Pakistan, and specifically for the model in Ahmad et al. (2012), there is not much relevant empirical literature for many parameters which can be used as priors. Therefore, we calibrate (generally) the parameters over different bands of data[3] and use simple average of estimates to locate the means of prior distributions[4]. The standard errors are set so that the domain covers estimated range of parameter values. Details are in the sections to follow.

Generally, in order to set mean of the prior distribution under uncertainty and data scarcity, the strategy is mostly to set prior's location parameter on the basis of information from (a) countries enjoying similar economic conditions as the one being modeled or (b) to use a reasonable mean with large value of the dispersion parameter so that the distribution can cover a considerable range of parameter values, reflecting the lack of knowledge.

After estimation of likelihood and specifying the prior distribution we are able to evaluate the posterior kernel:

$$P(\Psi \setminus \mathrm{Y}^{\mathrm{T}}, \Lambda) \propto L(\mathrm{Y}^{\mathrm{T}} \setminus \Psi, \Lambda)\mathrm{P}(\Psi \setminus \Lambda) = \kappa(\Psi \setminus \mathrm{Y}^{\mathrm{T}}, \Lambda) \qquad (5)$$

Here $\kappa(\Psi \setminus \mathrm{Y}^{\mathrm{T}}, \Lambda)$ is the posterior kernel which is proportional to the posterior by the factor$1/\mathrm{L}(\mathrm{Y}^{\mathrm{T}}, \Lambda)$. Taking the log on both sides:

[2] See for example Kaelbling (2003).
[3] The corresponding bands are, 1981-1995, 1981-2000, 1991-2005, and 1991-2011. Each Parameter estimation procedure is thoroughly discussed in Ahmad et al. (2012).
[4] We take simple averages for simplicity.

$$\text{Ln}\ \kappa(\Psi \setminus \text{Y}^{\text{T}}, \Lambda) = LnL(\text{Y}^{\text{T}} \setminus \Psi, \Lambda) + \text{lnP}(\Psi \setminus \Lambda)$$

Or, since the priors are independently distributed, the above equation can be written as:

$$\text{Ln}\ \kappa(\Psi \setminus \text{Y}^{\text{T}}, \Lambda) = LnL(\text{Y}^{\text{T}} \setminus \Psi,, \Lambda) + \sum_{\theta=1}^{\text{r}} \text{lnP}(\Psi_{\theta} \setminus \Lambda) \qquad (6)$$

Here $'\text{r}'$ represents the number of priors in the model. This equation allows us to estimate posterior kernel, however it is nonlinear and has complicated functional form, therefore, the analysis has to be performed with numerical methods. To estimate the posterior kernel an optimization routine known as "csmiwell" developed by Christopher Sims is used. This optimization routine gives us point estimates of parameter vector Ψ, however, our objective is to calculate means, standard errors and confidence interval or simply we want to find the posterior kernel distribution of parameters. To simulate the posterior distribution Monte-Carlo Markov-Chain (MCMC) sampling method with Metropolis-Hastings (MH) algorithm is used. The basic idea of the MCMC algorithm is to generate a Markov-Chain that represents a sequence of possible parameter estimates, in a way that the whole domain of the parameter space is explored, and then use the frequencies associated with each estimate to build a histogram that mimics the posterior distribution. Functioning of the MCMC algorithm has been outlined in Appendix B.

4. Data

We estimated our model from Ahmad et al. (2012), presented briefly in section 2, at quarterly frequencies from the period 1980Q1 to 2010Q4 on three macroeconomic variables:

1) Real per Capita GDP (at 1999-2000 prices);

2) Real per Capita Private Investment[5]; and

3) Year on year (YoY) CPI inflation (1999-2000 as base).

Quarterly GDP and Private Investment series are obtained from State Bank of Pakistan's Research Department that is conducting a detailed study to convert national accounts data into quarterly frequency. CPI inflation is taken from Haver DLXVG3. We apply X-12 ARIMA filter to eliminate seasonality from these series. Since DSGE model specifies log deviations from the steady state of all variables, we use HP filtered series of Real per Capita GDP and Private Investment and demean CPI inflation. All the series are in log terms.

5. Choice of Priors

As usual practice in literature[6] , we split the structural parameters into two groups. The first group (table A in appendix C) contains parameters that play a role in determining the steady state of the model with little or no influence over its dynamic properties.

[5] We transform annual population into quarterly by using compounded growth methodology and use it to translate variables into quarterly per capita.

While, second group (table B and table C in appendix C) contains parameters that predominantly influence the dynamic behavior of the model with little or no effect on its steady state. We choose priors only for parameters in these two tables B and C while values of parameters in table A are considered as strict priors.

Using the standard criteria followed in this line of literature, we use ***beta distributions*** to describe our priors about the persistence parameters of the shock processes and ***inverse gamma distributions*** with two degrees of freedom for the standard deviations. Elasticities and weights (assigned to variables in Taylor rule) are assumed to have ***normal distributions*** whereas preference parameter of money is assumed to have a ***beta distribution***.

To locate means of the distributions for priors for which we have corresponding data we use the average of corresponding estimated values of parameter over different bands of (quarterly) data[7]. The standard errors are set so that the domain covers an estimated range of parameter values. We specifically discuss the parameters in all the three tables in appendix C below.

5.1. Discussion on Calibration of Steady State Deterministic Parameters (Table A)

i) Discount factor (β)

Ahmed et al. (2012) calibrated the values of Discount factor, β, to be 0.9882 quarterly.

ii) Capital depreciation rate (δ)

Ahmad et al. (2012) estimated 15% annual depreciation rate for Pakistan, so for a quarter it is safe to use 3.75%.

iii) We suppose that the *share of capital in total production (α), share of formal consumption in total consumption (ω), share of formal labor in total labor (η) and formal wage markup over informal wage (rw)* are frequency invariant. In estimation of model we use their estimated annual values reported in "Pakistan economy DSGE model with informality (2012)" which are 0.5, 0.55 and 0.29, 0.25 respectively.

5.2. Discussion on Priors of Shocks' Parameters (Table B)

Monetary shock's persistence parameter is estimated and set at 0.59. We set rather strict standard error, i.e. 0.1, to have a clear separation between persistent and non-persistent parameters. To estimate the technology persistence parameter we need aggregate output (GDP). As we have utilized this data in our likelihood function therefore we cannot use it in prior estimation process. Also we have not found any other relevant quarterly study. Therefore, we borrow this parameter from Levine and Gabriel (2011)[8] and set it equal to 0.75 with dispersion parameter equal to 0.1. Similarly, we do not have quarterly government consumption data,

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[6] Many previous studies have fixed a subset of model parameters in a similar way, for instance see Smets and Wouters (2003), Richard Harrison (2010), and Levine and Gabriel (2011).

[7] The corresponding bands are, 1981-1995, 1981-2000,1991-2005,1991-2011

[8] Levine (2011) used a model with very similar structures to our own model, besides the features of India and Pakistan economies are more or less similar.

therefore, we also borrow it from Levine and Gabriel (2011) and set it equal to 0.75 with its standard error as 0.1.

5.3. Discussion on Priors of Dynamic Behaving Parameters (Table C)

i) For estimation of Preferences parameter on money we applied GMM methodology on money demand Euler equation of the model. We set this parameter equal to 0.056. This value is close to 0.01 which is used by McCandless (2008) for US economy.

ii) Weight assigned to inflation and output gap in Taylor rule are borrowed from Malik and Ahmed (2007) and set at 0.58 and 0.42 respectively.

iii) Elasticity of substitution b/w formal and informal labor $\boldsymbol{\upsilon}$ has mean 1.89, which we compiled from the average of annual estimates of the parameter ranges from 1997-98 to 2008-09 using LFS data and we assume safely, being it a long-run characteristic, that it remains same over quarterly frequency.

iv) We borrow the value of inverse of elasticity of labor supply ***(ϕ)*** at 1.5 from Fagan and Messina (2009). This value was calibrated for the US, Germany, Portugal, Belgium, and Finland. We set its SD at 0.5 so that it covers the range of these studies.

v) The mean of parameter, μ, the elasticity of substitution between formal and informal consumption is set at its steady state value 0.7 with larger standard deviation to ensure that it covers reasonable range for the parameter values.

6. Bayesian Estimation of the Model

We use Dynare to estimate the Bayesian model. To keep thing simple we replicated two parallel chains of MH algorithms for 20000 times each. For analysis we use last 12500 values of each chain. The average acceptance rate per chain was 0.310 and 0.313. The posteriors and their distributions, thus, obtained.

Table 1 reports type of prior distributions, their defining parameters, i.e., mean and variance along with two sets of results concerning the parameter estimates. The first set contains entries under the column “Estimated Maximum Posterior”. The entries in these columns report the parameters mode and standard deviation and are obtained by directly maximizing the log of the posterior distribution with respect to the parameters, and an approximate standard error based on the corresponding Hessian.

The second set contains the 50th, 5th and 95th percentile of the posterior distribution computed with the MH sampling algorithm based on 100000 draws. The estimates of posterior distributions in Table 1 are usually compared with corresponding prior distributions. Figure 1 summarizes this information visually by plotting the prior distribution and the posterior distribution. Solid gray lines denote the prior distributions, solid black lines denote the posterior distributions and green dashed lines represent the posterior modes.

Table 1: Parameter Estimates using Bayesian Methodology

Prior Distribution				Estimated Maximum Post.		Posterior Distribution MH		
Parameters	Type	Mean	St.error	Mode	St.error	Mean	5%	95%
Tech. shock persistence ρ_A	Beta	0.75	0.1	0.89	0.0017	0.90	0.85	0.94
Fiscal shock persistence ρ_G	Beta	0.75	0.1	0.95	0.001	0.96	0.94	0.98
Monetary shock persistence ρ_R	Beta	0.59	0.1	0.96	0.0069	0.955	0.93	0.9759
Pref. parameter on money χ	Beta	0.05	0.02	0.037	0.002	0.0398	0.018	0.0571
Inverse of elasticity of labor supply (ϕ)	Normal	1.5	0.5	1.82	0.07	1. 59	1.006	2.16
Weight assigned to inflation in Taylor rule ψ^{π}	Normal	0.58	0.04	0.5815	0.0044	0. 57	0.53	0.62
Weight assigned to output gap in Taylor rule ψ^{Y}	Normal	0.42	0.04	0.4344	0.0051	0.4208	0.34	0.49
Elasticity of substitution b/w formal and informal consumption μ	Normal	0.7	0.1	1	0.0004	0.99	0.998	1
Elasticity of substitution b/w formal and informal labor υ	Normal	1.89	0.46	2.03	0.048	1.92	1.72	2.6
SD of Tech. shock σ_A	Inv gamma	0.1	2.0	0.0187	0.0017	0.0193	0.016	0.022
SD of Fiscal shock σ_G	Inv gamma	0.1	2.0	0.0131	0.001	0.013	0.011	0.015
SD of Monetary shock σ_R	Inv gamma	0.1	2.0	0.44	0.1007	0.58	0.29	0.84

For better estimates posterior distributions should be close to normal or at least not display a shape that is clearly non-normal which is evident here only for some of the graphs in Figure 1 in appendix D.. However, there is not any serious deviation from normality in most of the cases. Also for almost all cases the mode, represented by the green dotted line, calculated from the numerical optimization of the posterior kernel is not too much far away from the posterior distribution.

Some of the results are noteworthy. Starting with persistence parameters of technology, fiscal and interest rate shocks, it is possible to see that our prior conviction that the shocks are highly persistence is confirmed by data. However this does not seem to be the case for the standard deviations of these shocks. Data emphasizes for tight priors. The priors and posteriors for weights assign to inflation and output gap in Taylor rule are almost the same, which means this parameter may not be truly identified. The elasticity of substitution between formal and informal consumption is unit elastic which is close to our prior conviction. Turning to the parameters of Taylor rule our model confirms the results of Malik and Ahmed (2007).

6.1. Diagnostics

Figures 2 and 3 (see appendix D) represent MCMC univariate and multivariate diagnostics. Overall visual inspection indicates that the optimization procedure was able to obtain a robust maximum for the posterior kernel. Red and blue lines in Figures 2 and 3, although, have some variations but they do converge except for phi and psi2. This was also evident in prior and posterior distributions in Figure 1.

In Figure 4 (see appendix D) two features are noteworthy. First, it can be easily observed that the maximum in the quadratic log-likelihood remains around the true parameter values while the linear log-likelihood attains its peak around the pseudo-true parameter values. For some parameters, like Phi1 and Phi2, the quadratic log-likelihood function is more concave than the linear one which implies that the nonlinear approach is able to extract more information from the data. Smoothened estimated shocks shown in Figure 5 (see appendix D) are constructed via Kalman smoother from the values of unobserved shocks over the sample, incorporating all the information contained in the data. The assumption of the model is that they have zero mean. Figure 5 indicates that all except output 'y' are centered on zero. Variable output 'y' is systematically getting away from zero which shows some problem. It might be a mismatch between the meaning of the variable in model and data. In model, variable 'y' represents output of non-agricultural manufacturing; however, data represents aggregate output which includes agriculture sector as well. Using the appropriate data may improve the results but it is unavailable.

7. Bayesian IRFs

As in Ahmad et al. (2012), here as well, we have the similar three shocks introduced to the model. The main difference is the frequency of data. Here we are now dealing with quarterly data and as a result the impulse response functions are over quarters as well rather than over years. This is a step forward as it allows one to interpret the impulse response functions with policy implications in reasonably the short run. Figures 6, 7 and 8 (see appendix D) show the impact of technology, fiscal spending and interest rate shocks respectively on variables of interest.

The standard positive technology shock (see figure 6) results in an overall increase in output. Since technology is only embedded in the formal sector we see that the formal sector output rises whereas that of the informal sector falls below the steady state. Similarly, the overall consumption falls initially, because initially inflation rises resulting in an inflationary tax on informal output which then reduces its consumption as well, but then recovers within 2-3 quarters and stays close to its steady state. Inflation also falls by the 2nd quarter resulting in more demand for formally produced goods which results in two substitutions. One is that the households substitute the informal goods for formal goods and secondly the technology shock results in making households substitute more labor hours from the informal sector to the formal sector where the real wages have gone up due to higher labor demand as well as due to lower inflation up till 5 quarters. The real wage of the informal sector, despite being affected by the initial inflation tax and consequently lower demand, also rises because labor supply for informal sector gets reduced in terms of hours provided for work at household level as well as due to negative inflation after the 2nd quarter. Investment rises in response to a technology shock which is due to the need of capital in formal sector and it then results in higher output in the formal sector.

The positive fiscal or government spending shock (see figure 7) impacts inflation and the formal output, which is what the government consumes. Since the formal output rises so does the investment. However, personal consumption from both formal and informal sectors falls. One reason for this is due to the crowding in effect of private investment substituting for consumption and the other reason is inflation resulting in crowding out of consumption due to the inflation tax. Since government is consuming more, the working hours in the formal sector rise as opposed to the informal sector where hours fall due to fall in its output as well. Real wages in aggregation as well in both formal and informal sectors fall. It is to note that the initial jump, which last for about 2 quarters, in inflation reduces the real wages of both the sectors. This implication is justifiable looking at the shape of the impulse response functions of wages. Initially they fall but rise steeply within the first couple of quarters and then become quite flat.

The positive interest rate shock (see figure 8) impacts inflation and private investment negatively. However, the impact on inflation is abrupt and becomes negligible in 2 quarters. As a result real output and rises abruptly, due to a sustained rise in informal output, but then falls below its steady state within 2 quarters as the fall in formal output overtakes the rise in informal output. The abrupt rise and then fall of formal consumption is also supported by lower real prices due to the initial fall in inflation which subsides quickly. Here the shock absorbing capacity of the informal sector comes in to play as households substitute consumption from formal to informal goods but this substitution effect starts diminishing right after the first couple of quarters. The overall labor also show the same picture, since, the aggregate wages as well as formal and informal sector wages fall. Since output and consumption of the formal sector falls so does its demand for labor resulting in lower real wages. On the other hand, due to its shock absorbing nature, the informal sector does employ more from the households' labor hours but as supply exceeds demand, the real wages of the informal sector also fall.

8. Conclusion

We have upgraded, both theoretically and empirically, our model in Ahmad et al. (2012) with two extensions. We have not only been able to incorporate quarterly analysis of impulse responses by converting model and shock related parameters to quarterly frequency but also we have done so through much more clarity and objectivity due to the use of Bayesian estimation technique for DSGE models. We have applied certain diagnostics in order to check the performance of our Bayesian estimations and have found them to be reasonably satisfactory. The resulting impulse response functions have been interpreted with some short run policy implications. We have noticed general reduction in magnitudes of impulse response functions as compared to those in Ahmad et al. (2012) which can be attributed both to the use of quarterly frequency and Bayesian estimation. We have also found the short run shock absorbing role of the informal sector in response to the interest rate shock.

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Appendix

A. Complete Model

Financial assets optimization equation

$$\frac{1}{c_t} = \beta(1+R_t)E_t\frac{1}{\pi_{t+1}c_{t+1}}$$

Physical assets optimization equation

$$E_t\left[\frac{\pi_{t+1}}{(1+R_t)}\left\{(1-\delta)+r^k_{t+1}\right\}\right] = 1$$

Hours worked optimization equation

$$h_t = \left(\frac{w_t}{c_t}\right)^{\frac{1}{\phi}}$$

Money holding optimization equation

$$\frac{\chi}{M_t/P_t} = \frac{1}{c_t} - \beta E_t\frac{1}{\pi_{t+1}c_{t+1}}$$

Capital accumulation constraint

$$k_{t+1} = (1-\delta)k_t + i_t$$

Composite of formal and informal labor equation

$$h_t = \left[\eta^{-\vartheta}(h^F_t)^{1+\vartheta} + (1-\eta)^{-\vartheta}(h^I_t)^{1+\vartheta}\right]^{\frac{1}{1+\vartheta}}$$

Supply of formal labor

$$h^F_t = \eta\left(\frac{w^F_t}{w_t}\right)^{\frac{1}{\vartheta}}h_t$$

Supply of informal labor

$$h^I_t = (1-\eta)\left(\frac{w^I_t}{w_t}\right)^{\frac{1}{\vartheta}}h_t$$

Composite wage rate

$$w_t = \left[\eta (w_t^F)^{\frac{\vartheta}{1+\vartheta}} + (1-\eta)(w_t^I)^{\frac{\vartheta}{1+\vartheta}} \right]^{\frac{1+\vartheta}{\vartheta}}$$

Informal wage rate

$$w_t^I = \gamma$$

Formal wage rate

$$w_t^F = (w_t^I)^{(1+r^w)}$$

Formal price level

$$P_t^F = \frac{1}{(1-\tau)} \left(\frac{\epsilon}{\epsilon - 1} \right) MC_t^F$$

Informal price level

$$p_t^I = \frac{W_t^I}{\gamma}$$

General Price equation

$$P_t^{1-\mu} = \omega (P_t^F)^{1-\mu} + (1-\omega)(P_t^I)^{1-\mu}$$

Formal consumption

$$c_t^F = \omega \left(\frac{P_t^F}{P_t} \right)^{-\mu} c_t$$

Informal consumption

$$c_t^I = \omega \left(\frac{P_t^I}{P_t} \right)^{-\mu} c_t$$

Sectoral relative price

$$\frac{P_t^I}{P_t^F} = \varsigma_t$$

Formal price relative to general price

$$\frac{P_t^F}{P_t} = \frac{1}{\left[\omega + (1-\omega)\varsigma_t^{1-\mu}\right]^{\frac{1}{1-\mu}}}$$

Informal price relative to general price

$$\frac{P_t^I}{P_t} = \frac{1}{\left[\omega\varsigma_t^{\mu-1} + (1-\omega)\right]^{\frac{1}{1-\mu}}}$$

Gross general inflation rate

$$\pi_t = \frac{P_t}{P_{t-1}}$$

Gross sectoral inflation rates

$$\pi_t^F = \frac{P_t^F}{P_{t-1}^F} \text{ and } \pi_t^I = \frac{P_t^I}{P_{t-1}^I}$$

Relative inflation

$$\frac{\varsigma_t}{\varsigma_{t-1}} = \frac{\pi_t^I}{\pi_t^F}$$

Formal inflation relative to general inflation

$$\frac{\pi_t^F}{\pi_t} = \left[\frac{\omega + (1-\omega)\varsigma_{t-1}^{1-\mu}}{\omega + (1-\omega)\varsigma_t^{1-\mu}}\right]^{\frac{1}{1-\mu}}$$

Informal aggregate price

$$P_t^I = \left[\int (p_{(I)}^I)^{1-\upsilon} d_I\right]^{\frac{1}{1-\upsilon}}$$

Formal production function

$$y_t^F = a_t k_t{}^{\alpha} h_t^{F\,1-\alpha}$$

Capital-labor ratio

$$\frac{r_t^k}{w_t^F} = \frac{\alpha}{1-\alpha}\frac{h_t^F}{k_t}$$

Marginal cost

$$mc_t^F = \frac{1}{a_t}(\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)}(w_t^F)^{1-\alpha}\left(r_t^k\right)^{\alpha}$$

Informal production function

$$y_t^I = \gamma h_t^I$$

Demand for informal labor equation

$$\gamma p_t^I = W_t^I$$

Informal marginal cost

$$MC_t^I = p_t^I = \frac{W_t^I}{\gamma}$$

Taylor type rule

$$R_t^c = r\pi^*\left(\frac{\pi_t}{\pi_t^*}\right)^{\psi_1}\left(\frac{y_t}{y_t^*}\right)^{\psi_2}$$

Fiscal budget constraint

$$G_t + TR_t + \frac{B_{t-1}}{P_t} = \tau Y^F + \frac{B_t}{P_t} + \frac{M_t - M_{t-1}}{P_t}$$

Resource Constraints

$$y_t^F = c_t^F + i_t + g_t$$
$$y_t^I = c_t^I$$

$$y_t = y_t^I + y_t^F = c_t^F + c_t^I + i_t + g_t$$

B. Functioning of the MCMC Algorithm

1. Draw a proposal Ψ^* from a candidate density. As a rule random walk (RW) process is used to migrate from previous parameters to new one. So Jumping distribution for model parameters can be suppose as:

$$J\left(\Psi^* \upharpoonright \Psi^{t-1}\right) = \mathrm{N}(\Psi^{t-1}, c\Sigma_{\Psi^m})$$

Where Ψ^{t-1} is the distributions' jumping mean, whose initial value is set to the previously estimated posterior mode, Σ_{Ψ^m} is the variance of the distribution, computed as the inverse of the previously estimated Hessian matrix, and c is a scale factor.

2. Draw a proposal Ψ^1 from a candidate density $\mathrm{N}(\Psi^0, c\Sigma_{\Psi^m})$ and compute the posterior kernel.

3. Compute h, which is the ratio of the posterior kernel evaluated at the new proposal estimate over the posterior kernel evaluated at the previous proposal estimate:

$$h = \frac{\kappa(\Psi^* \setminus \mathrm{Y}^{\mathrm{T}})}{\kappa\left(\Psi^{t-1} \setminus \mathrm{Y}^{\mathrm{T}}\right)}$$

4. Accept or reject Ψ^* according to acceptance probability:

$$ap = min(\, r, 1)$$

5. If Ψ^* accepted update the mean of the distribution otherwise keep the previous one.

6. Loop on steps 2 to 5.

7. Having done enough iteration, use the accepted draws to build a histogram[9].

[9] It has been proved in the literature that this distribution corresponds to posterior distribution, for more detail see understanding Metropolis- Hastings Algorithm by S.Chib, E. Greenberg (2005).

C. Model Parameters and Data Quality for their Estimation

	Sr.	Parameter Description	Data Quality (Quarterly)			Data Quality (Annual)		
			Poor	**Satisfactory**	**Good**	**Poor**	**Satisfactory**	**Good**
Table A: Steady State Deterministic Parameters	1	Discount factor (β)			√			√
	2	Capital depreciation rate (δ)		√			√	
	3	***Share of capital in total production (α)***	√			√		
	4	***Share of formal consumption in total consumption (ω)***	√			√		
	5	***Share of formal labor in total labor (η)***			√			√
	6	***Formal wage markup over informal wage (rw)***			√			√
Table B: Shock Related Parameters	7	Tech shock persistence ρ_A and SD σ_A	√				√	
	8	Fiscal shock persistence ρ_G and SD σ_G	√			√		
	9	M shock persistence ρ_R and SD σ_R		√				√
Table C: Dynamic Parameters	10	***Preferences parameter on money χ***	√			√		
	11	***Elasticity of substitution b/w formal and informal consumption μ***	√			√		
	12	***Elasticity of substitution b/w formal and informal labor υ***		√			√	
	13	Weight assigned to inflation in Taylor rule ψ^{π}		√				√
	14	Weight assigned to output gap in Taylor rule ψ^{y}		√				√
	15	Preferences parameter on money χ	√			√		
	16	***Inverse of elasticity of labor supply (ϕ)***		√		√		

Note

The above (sub) tables A, B and C describe data quality available to calibrate, approximate or estimate deep parameters of Pakistan DSGE model on annual and quarterly frequencies.

Variables that are displayed in ***bold italic*** are considered as Frequency invariant variables. Estimated or calibrated values of these parameters do not depend, in short term, on frequency and therefore will remain same for both annual and quarterly frequencies. Their values can be used for quarterly Bayesian DSGE model as they are.

D. Figures on Diagnostics and Impulse Responses

Figure 1: Prior and Posterior Distribution Plots

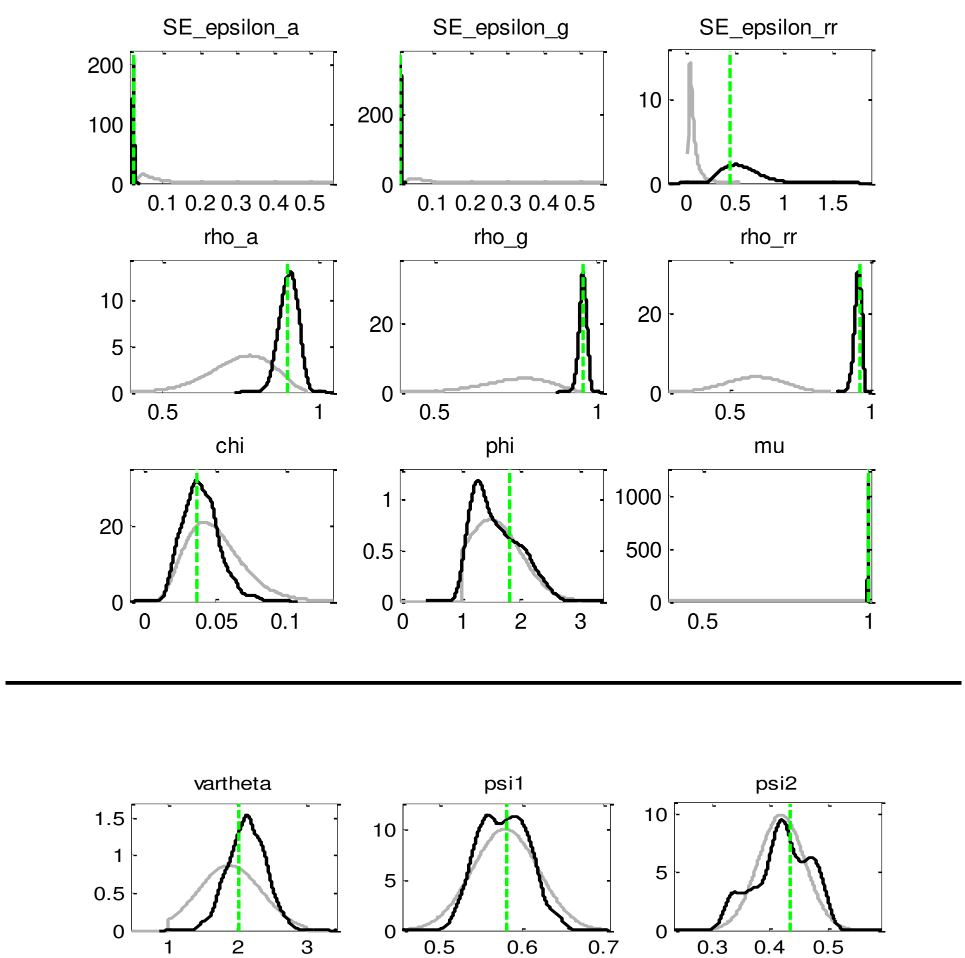

Figure 2: MCMC Univariate Diagnosis

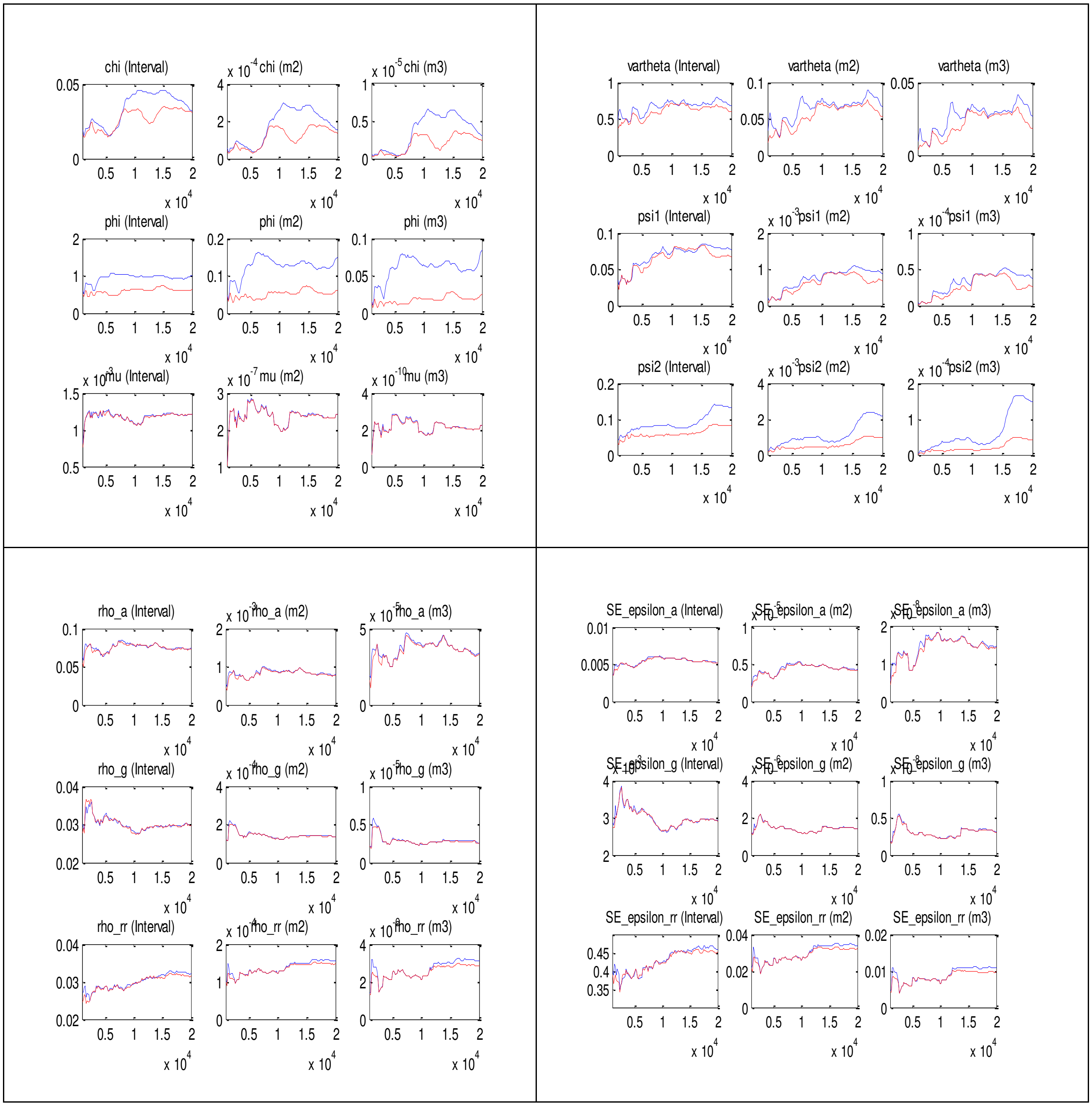

Figure 3: MCMC Multivariate Diagnosis

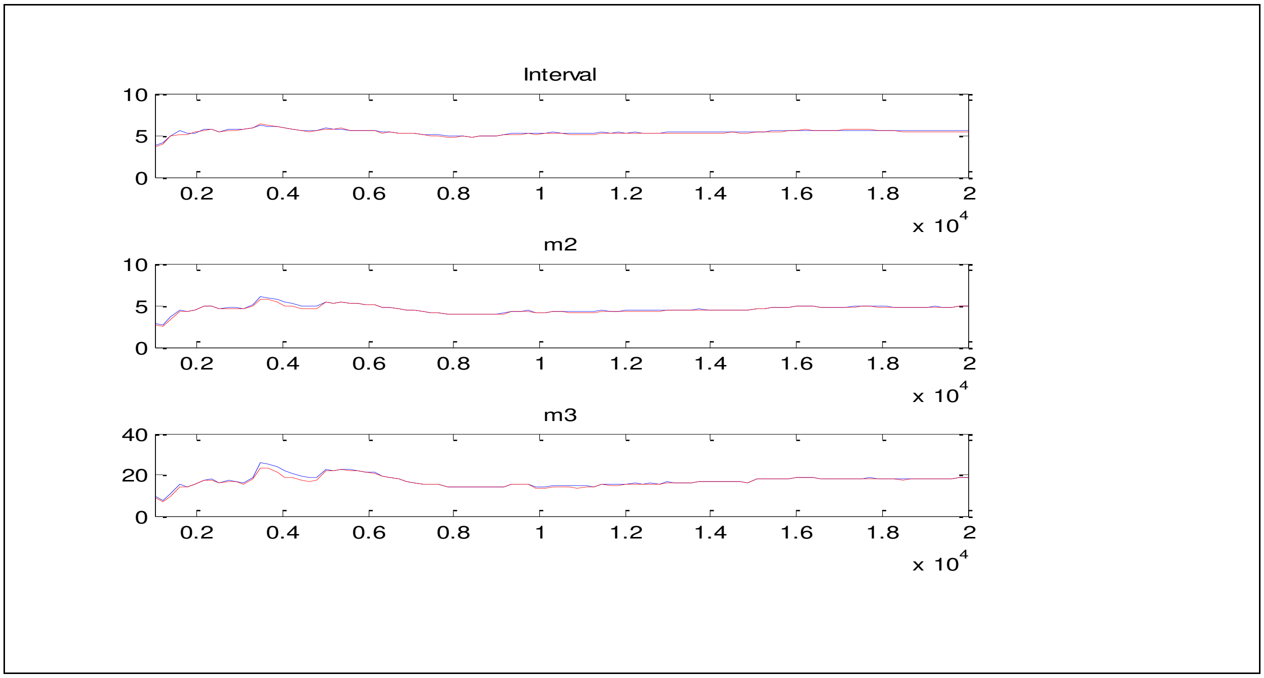

Figure 4: Check Plots

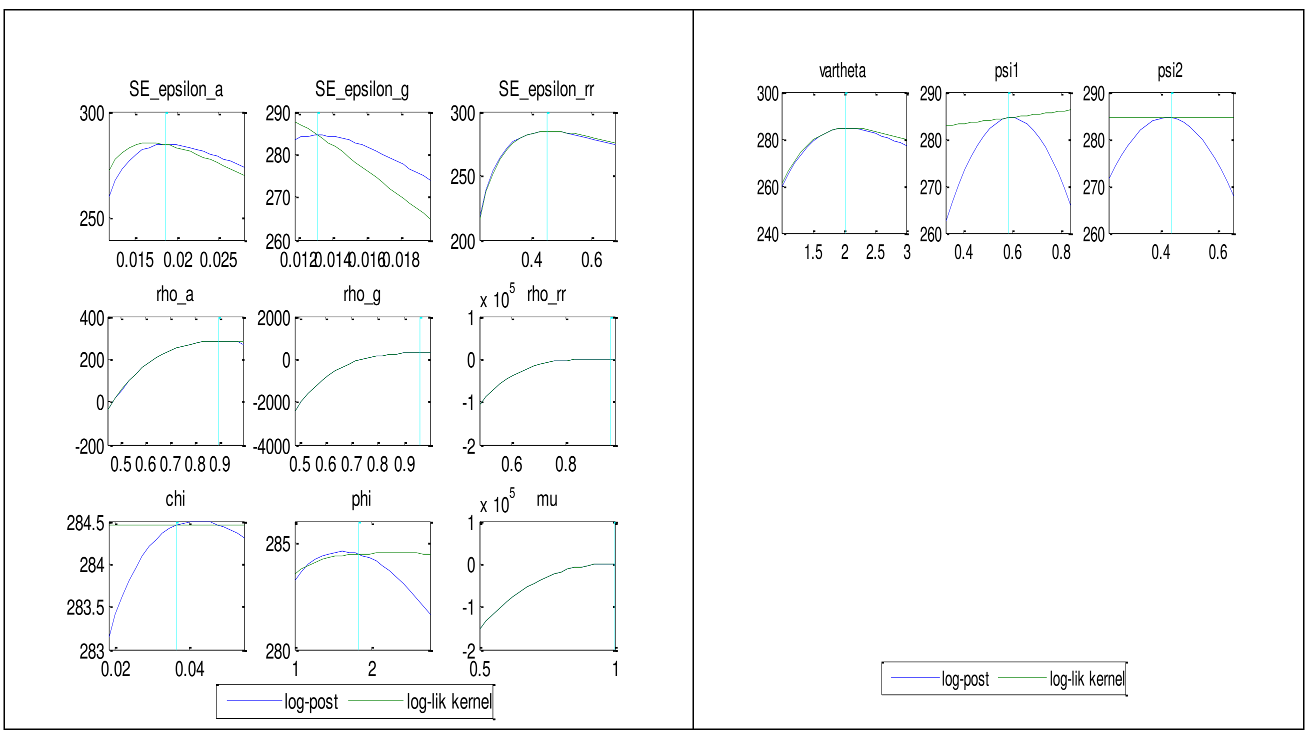

Figure 5: Historical and Smooth Variables/ Smooth Shocks

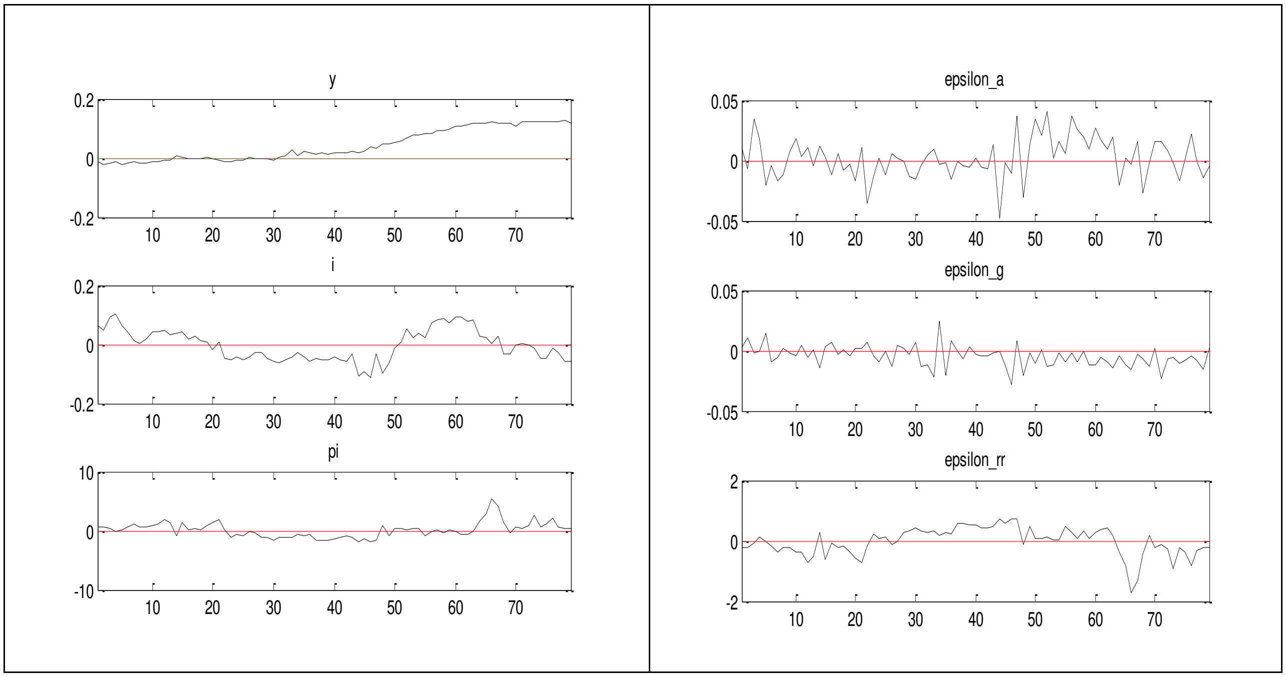

Figure 6: IRFs of the Technology Shock

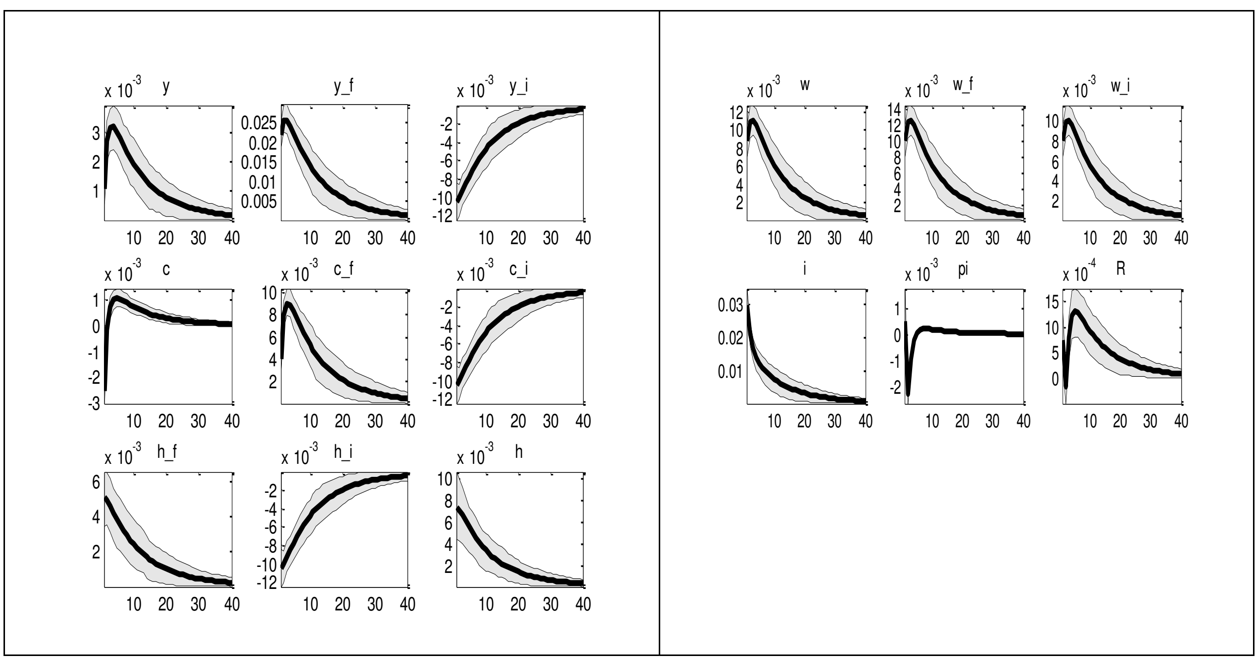

Figure 7: IRFs of the Fiscal Shock

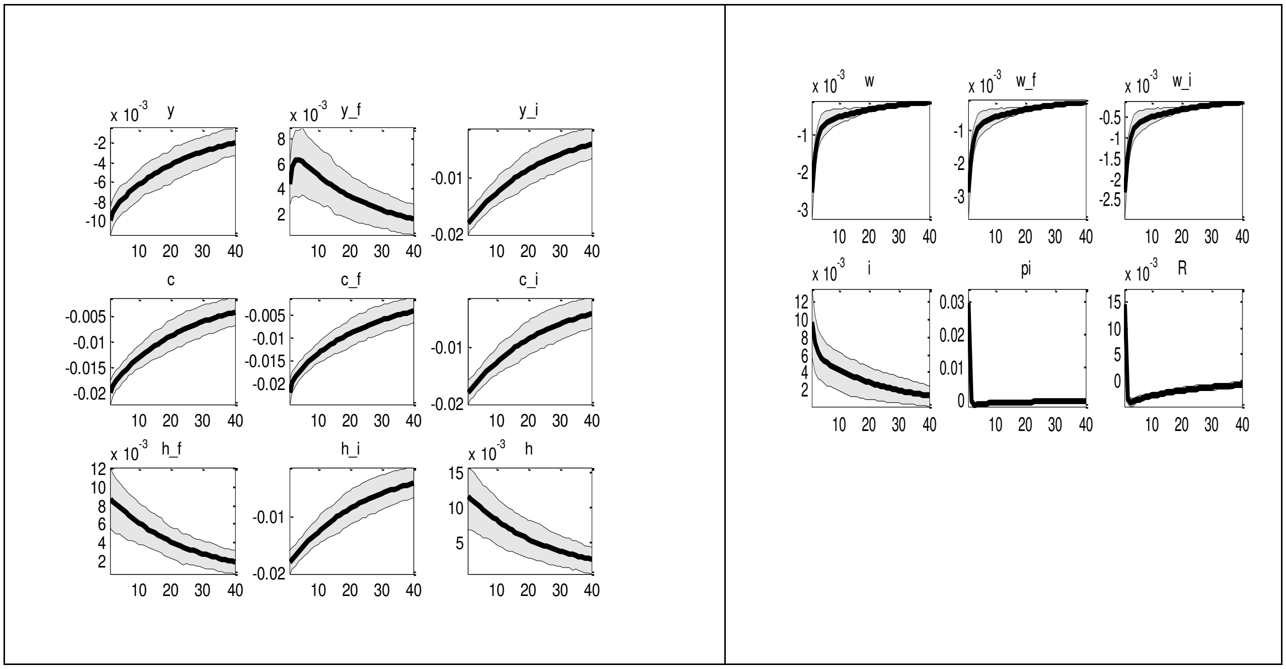

Figure 8: IRFs of the Interest Rate Shock