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Inflation, R&D and Economic Growth in an Open Economy

Angus C. Chu Guido Cozzi Ching-Chong Lai Chih-Hsing Liao

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Abstract

This study explores the long-run effects of monetary policy on economic growth and social welfare in a two-country Schumpeterian growth model with cash-in-advance constraints on consumption and R&D investment. We find that an increase in the domestic nominal interest rate decreases domestic R&D investment and the growth rate of domestic technology. Given that economic growth in a country depends on both domestic and foreign technologies, an increase in the foreign nominal interest rate also decreases economic growth in the domestic economy. When each government conducts its monetary policy unilaterally to maximize the welfare of only domestic households, the Nash-equilibrium nominal interest rates are generally higher than the optimal nominal interest rates chosen by cooperative governments who maximize the welfare of both domestic and foreign households. This difference is caused by a cross-country spillover effect of monetary policy arising from trade in intermediate goods. Under the CIA constraint on consumption (R&D investment), a larger market power of firms decreases (increases) the wedge between the Nash-equilibrium and optimal nominal interest rates. We also calibrate the two-country model to data in the Euro Area and the UK to quantify the welfare effects of decreasing the nominal interest rates from the Nash equilibrium to the optimal level.

JEL classification: O30, O40, E41, F43

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1 Introduction

In this study, we explore the long-run effects of monetary policy on economic growth and social welfare in an open economy. Specifically, we develop a two-country version of the Schumpeterian growth model and introduce money demand into the model via a cash-in-advance (CIA) constraint on R&D investment in each country.¹ Empirical evidence supports the view that R&D investment is severely affected by liquidity requirements. For example, Hall (1992), Himmelberg and Petersen (1994), Opler *et al.* (1999) and Brown and Petersen (2009) find a positive and significant relationship between R&D and cash flows in US firms. According to Bates *et al.* (2009), the average cash-to-assets ratio in US firms increased substantially from 1980 to 2006, and this increase is partly due to their increased expenditures on R&D. Recent studies by Brown and Petersen (2011) and Brown *et al.* (2012) explain this phenomenon by providing evidence that firms smooth R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves. Furthermore, Berentsen *et al.* (2012) argue that information frictions and limited collateral value of intangible R&D capital prevent firms from financing R&D investment through debt or equity forcing them to fund R&D projects with cash reserves. We capture these cash requirements on R&D using a CIA constraint. Given this CIA constraint on R&D, the nominal interest rate that determines the opportunity cost of cash holdings affects R&D investment, economic growth and social welfare. In an open economy, monetary policy may also have spillover effects across countries through international trade.

The results from our growth-theoretic analysis confirm the above intuition and can be summarized as follows. An increase in the domestic nominal interest rate decreases domestic R&D investment and the growth rate of domestic technology. Given that economic growth in a country depends on the growth rate of domestic technology as well as the growth rate of foreign technology, an increase in the foreign nominal interest rate also decreases economic growth in the domestic economy. When each government conducts its monetary policy unilaterally to maximize the welfare of only domestic households, the Nash-equilibrium nominal interest rates are generally different from the optimal nominal interest rates chosen by cooperative governments who maximize the aggregate welfare of domestic and foreign households. Specifically, we find that under the special case of inelastic labor supply, the Nash-equilibrium nominal interest rates coincide with the optimal nominal interest rates. However, under the more general case of elastic labor supply, the Nash-equilibrium nominal interest rates are generally higher than the optimal nominal interest rates because there exists a cross-country spillover effect of monetary policy. The intuition of this result can be explained as follows. When the government in a country reduces the nominal interest rate, the welfare gain from higher economic growth is shared by the other country through trade in intermediate goods, whereas the welfare cost of increasing labor supply falls entirely on domestic households. As a result, the governments do not lower the nominal interest rates sufficiently in the Nash equilibrium.

The Nash-equilibrium nominal interest rates depend on the market power of firms. Under the CIA constraint on consumption, a larger markup reduces the wedge between the Nash-

¹See also Chu and Cozzi (2013), who introduce a CIA constraint on R&D investment into a closed-economy version of the Schumpeterian growth model and analyze the effects of monetary policy.

equilibrium and optimal nominal interest rates. This finding is consistent with the interesting insight of Arseneau (2007), who shows that the market power of firms has a dampening effect on the inflationary bias from monetary policy competition analyzed in an influential study by Cooley and Quadrini (2003). However, under the CIA constraint on R&D investment, we have the opposite result that a larger markup amplifies the inflationary bias from monetary policy competition. These different implications highlight the importance of the differences between the two CIA constraints. The main difference between the CIA constraint on consumption and the CIA constraint on R&D is that under the latter, an increase in the nominal interest rate leads to a reallocation of labor from R&D to production. As a result, higher nominal interest rates would be chosen by governments in the Nash equilibrium to depress R&D as the negative R&D externality in the form of a business-stealing effect determined by the markup becomes stronger. In contrast, under the CIA constraint on consumption, this reallocation effect is absent because an increase in the nominal interest rate decreases both R&D and production labors. Given that the economy features an undersupply of labor due to the distortion of monopolistic competition, governments would reduce nominal interest rates in the Nash equilibrium to increase labor supply as this monopolistic distortion determined by the markup becomes stronger.

We also calibrate the two-country model to aggregate data in the Euro Area and the United Kingdom (UK) to quantify the welfare effects of decreasing the nominal interest rates from the Nash equilibrium to the optimal level. We find that the Friedman rule is optimal (i.e., zero nominal interest rates maximize welfare). In this case, decreasing the nominal interest rates in both economies from the Nash equilibrium to zero would lead to nonnegligible welfare gains that are equivalent to a permanent increase in consumption of 0.292% in the UK and 0.443% in the Euro Area. However, a unilateral deviation to decrease the nominal interest rate from the Nash equilibrium would hurt the domestic economy and only benefit the foreign economy. For example, we find that a unilateral decrease in the nominal interest rate in the Euro Area would reduce its welfare by 0.028% but increase the UK's welfare by 0.347%.

This study relates to the literature of inflation and economic growth; see Stockman (1981) and Abel (1985) for seminal studies of the CIA constraint on capital investment in the Neoclassical growth model. Instead of analyzing the effects of monetary policy in the Neoclassical growth model, we consider an R&D-based growth model in which economic growth is driven by innovation and endogenous technological progress. The seminal study in this literature of inflation and innovation-driven growth is Marquis and Reffett (1994), who explore the effects of a CIA constraint on consumption in a Romer variety-expanding model.² In contrast, we consider a Schumpeterian quality-ladder model and analyze the effects of monetary policy via a CIA constraint on R&D investment as in Chu and Cozzi (2013).³ Huang *et al.* (2013) also analyze the effects of monetary policy via CIA constraints on R&D investment in a Schumpeterian model with endogenous market structure. The present study differs from the closed-economy analyses in Chu and Cozzi (2013) and Huang *et al.* (2013) by considering a two-country setting with trade in intermediate goods across

²Chu, Lai and Liao (2013) provide an analysis of the CIA constraint on consumption in a hybrid growth model in which economic growth in the long run is driven by both variety expansion and capital accumulation.

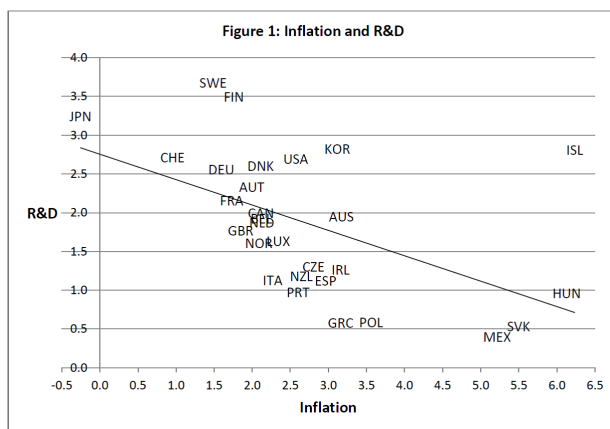
³See Chu and Lai (2013) for an analysis of the money-in-utility approach to model money demand in the quality-ladder growth model.

countries. This open-economy model allows us to explore the interesting implications of policy coordination across countries and cross-country spillover effects of monetary policy, which are absent in a closed economy. To our knowledge, this is the first study that analyzes the effects of monetary policy in a growth-theoretic framework that features R&D-driven innovation in an open economy.

The rest of this study is organized as follows. Section 2 documents stylized facts. Section 3 presents the model. Section 4 explores the effects on economic growth. Section 5 analyzes the effects on social welfare. Section 6 provides a quantitative analysis. Section 7 concludes.

2 Stylized facts

In this section, we document the empirical relationship between inflation and R&D across countries. According to the Fisher identify, the nominal interest rate is equal to the sum of the real interest rate and the inflation rate. Fisher (1930) hypothesizes a one-for-one relation between the inflation rate and the nominal interest rate, and this is known as the Fisher effect in the literature. Although an exact one-for-one relation is unlikely to hold, subsequent studies have provided empirical evidence for a positive long-run relationship between inflation and the nominal interest rate; see for example Mishkin (1992) who consider US data and Booth and Ciner (2001) who consider European data. Our model also predicts a positive steady-state relationship between inflation and the nominal interest rate. Therefore, our theoretical result of a negative effect of the nominal interest rate on domestic R&D investment implies a negative relationship between inflation and R&D, which is consistent with the following stylized facts. Figure 1⁴ plots the relationship between inflation of the consumer price index and R&D as a percentage of GDP across OECD countries.⁵ To mitigate the effects of business-cycle fluctuations, we take a 10-year average of each variable from 2000 to 2009.⁶ We consider all countries that are members of the OECD in this period.⁷



⁴Data source: World Development Indicators.

⁵Acemoglu and Zilibotti (2001) find that 90% of R&D in the world is performed in OECD countries.

⁶We have also considered other years and found that the negative relationship is robust.

⁷In Figure 1, we have removed one outlier, Turkey, that had an average inflation rate of 23.5%. However, including this country would not affect the negative relationship given that the average R&D share of GDP in Turkey was also very low at 0.60%.

3 An open-economy monetary Schumpeterian model

In this section, we develop an open-economy version of the monetary Schumpeterian growth model. The underlying quality-ladder model is based on the seminal work of Grossman and Helpman (1991).⁸ In summary, we modify the Grossman-Helpman model by introducing money demand via CIA constraints on consumption and R&D investment as in Chu and Cozzi (2013) and extending the closed-economy model into a two-country setting with trade in intermediate goods. The home country is denoted with a superscript h , whereas the foreign country is denoted with a superscript f . Both countries invest in R&D, but we allow for asymmetry across the two countries in a number of structural parameters. Following a common treatment in this type of two-country models, we assume balanced trade and labor immobility across countries.⁹ Given that the quality-ladder model has been well-studied, we will describe the familiar components briefly but discuss the new features in details. Furthermore, to conserve space, we will only present the equations for the home country h , but the readers are advised to keep in mind that for each equation that we present, there is an analogous equation for the foreign country f .

3.1 Households

In each country, there is a representative household. The lifetime utility function of the household in country h is given by

$$U^h = \int_0^{\infty} e^{-\rho t} [\ln C_t^h + \theta^h \ln(1 - L_t^h)] dt, \quad (1)$$

where C_t^h denotes consumption goods in country h at time t . L_t^h denotes the supply of labor in country h . The parameters $\rho > 0$ and $\theta^h \geq 0$ determine respectively subjective discounting and leisure preference. We allow for asymmetry in θ^h across the two countries, so that the countries may have different sizes of the labor force.

The asset-accumulation equation is given by

$$\dot{V}_t^h + \dot{M}_t^h = R_t^h V_t^h + W_t^h L_t^h + T_t^h - P_t^h C_t^h + R_t^h B_t^h. \quad (2)$$

V_t^h is the nominal value of financial assets (in the form of equity shares in monopolistic firms in country h) owned by the household.¹⁰ R_t^h is the nominal interest rate in country h . W_t^h is the nominal wage rate. T_t^h is the nominal value of a lump-sum transfer (or tax if $T_t^h < 0$) from the government to the household. P_t^h is the price of consumption goods in country h . M_t^h

⁸See also Aghion and Howitt (1992) and Segerstrom *et al.* (1990) for other seminal studies of the quality-ladder model.

⁹In the case of the UK and the Euro Area, workers are legally mobile across countries, but there are nonetheless cultural and language barriers as well as other costs associated with moving to a foreign country. Given that we are essentially considering the R&D-intensive rich countries, like Germany and France, in the Euro Area, it is not clear that the income differences between the UK and these countries are large enough to induce the majority of workers to move across countries.

¹⁰Here we assume home bias in asset holding (i.e., domestic monopolistic firms are owned by domestic households) in order to allow the rates of return on assets to differ across countries.

is the nominal value of domestic currency held by the household partly to facilitate money lending to R&D entrepreneurs subject to the following constraint: $B_t^h + \xi^h P_t^h C_t^h \leq M_t^h$, where B_t^h is the nominal value of domestic currency borrowed by R&D entrepreneurs to finance their R&D investment, and $\xi^h \geq 0$ parameterizes the strength of the CIA constraint on consumption. As the household accumulates more money M_t^h , its money lending B_t^h to R&D entrepreneurs also increases, and the rate of return on B_t^h is the nominal interest rate R_t^h .¹¹ The household also accumulates money to facilitate the payment of consumption goods that are purchased domestically.¹²

The household maximizes (1) subject to (2) and $B_t^h + \xi^h P_t^h C_t^h \leq M_t^h$, which becomes a binding constraint. From standard dynamic optimization, the optimality condition for consumption in country h is

$$P_t^h C_t^h = \frac{1}{\eta_t^h (1 + \xi^h R_t^h)}, \quad (3)$$

where η_t^h is the Hamiltonian co-state variable on (2). The optimality condition for labor supply is

$$L_t^h = 1 - \frac{\theta^h P_t^h C_t^h (1 + \xi^h R_t^h)}{W_t^h}. \quad (4)$$

Finally, the intertemporal optimality condition is

$$-\frac{\dot{\eta}_t^h}{\eta_t^h} = R_t^h - \rho. \quad (5)$$

In the case of a constant nominal interest rate R^h , (3) and (5) simplify to the familiar Euler equation $\dot{C}_t^h / C_t^h = r_t^h - \rho$, where $r_t^h = R^h - \dot{P}_t^h / P_t^h$ is the real interest rate in country h .

3.2 Consumption goods

Consumption goods in country h are produced by a unit continuum of competitive firms that aggregate two types of final goods using a Cobb-Douglas aggregator¹³ given by

$$C_t^h = \frac{(Y_t^{h,h})^{1-\alpha} (Y_t^{h,f})^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha}, \quad (6)$$

where $Y_t^{h,f}$ denotes country h 's final goods that are produced with intermediate goods imported from country f , and $Y_t^{h,h}$ denotes country h 's final goods that are produced with domestic intermediate goods. The parameter $\alpha \in [0, 1]$ determines the importance of foreign goods in domestic consumption. From profit maximization, the conditional demand functions for $Y_t^{h,h}$ and $Y_t^{h,f}$ are respectively

$$Y_t^{h,h} = (1 - \alpha) P_t^h C_t^h / P_t^{h,h}, \quad (7)$$

¹¹It can be shown as a no-arbitrage condition that the rate of return on B_t^h must be equal to R_t^h .

¹²In reality, consumers do purchase consumption goods in other countries using foreign currencies; however, this represents a relatively small share of consumption expenditures of the average household. Therefore, in this model, we do not consider the use of foreign currencies for the purchase of consumption goods in other countries.

¹³We consider a Cobb-Douglas aggregator instead of a more general CES aggregator in order to allow $Y_t^{h,h}$ and $Y_t^{h,f}$ to grow at different rates on the balanced growth path.

$$Y_t^{h,f} = \alpha P_t^h C_t^h / P_t^{h,f}, \quad (8)$$

where $P_t^{h,h}$ is the price of $Y_t^{h,h}$, and $P_t^{h,f}$ is the price of $Y_t^{h,f}$. The familiar price index of consumption goods in country h is $P_t^h = (P_t^{h,h})^{1-\alpha} (P_t^{h,f})^\alpha$.

3.3 Final goods

Final goods $Y_t^{h,h}$ and $Y_t^{h,f}$ are also produced by a unit continuum of competitive firms. Competitive firms in country h produce $Y_t^{h,h}$ by aggregating a unit continuum of domestic intermediate goods $X_t^{h,h}(i)$ for $i \in [0, 1]$. The standard Cobb-Douglas aggregator is given by

$$Y_t^{h,h} = \exp \left(\int_0^1 \ln X_t^{h,h}(i) di \right). \quad (9)$$

Similarly, competitive firms in country h produce $Y_t^{h,f}$ by aggregating a unit continuum of foreign intermediate goods $X_t^{h,f}(j)$ for $j \in [0, 1]$. The Cobb-Douglas aggregator is given by

$$Y_t^{h,f} = \exp \left(\int_0^1 \ln X_t^{h,f}(j) dj \right). \quad (10)$$

From profit maximization, the conditional demand functions for $X_t^{h,h}(i)$ and $X_t^{h,f}(j)$ are respectively

$$X_t^{h,h}(i) = P_t^{h,h} Y_t^{h,h} / P_t^{h,h}(i), \quad (11)$$

$$X_t^{h,f}(j) = P_t^{h,f} Y_t^{h,f} / P_t^{h,f}(j), \quad (12)$$

where $P_t^{h,h}(i)$ is the price of $X_t^{h,h}(i)$, and $P_t^{h,f}(j)$ is the price of $X_t^{h,f}(j)$. Finally, the standard price index of final goods $Y_t^{h,h}$ is $P_t^{h,h} = \exp \left(\int_0^1 \ln P_t^{h,h}(i) di \right)$, and the standard price index of final goods $Y_t^{h,f}$ is $P_t^{h,f} = \exp \left(\int_0^1 \ln P_t^{h,f}(j) dj \right)$.¹⁴ All these prices are denominated in the domestic currency.

3.4 Intermediate goods

In country h , there is a unit continuum of differentiated intermediate goods indexed by $i \in [0, 1]$. In each industry $i \in [0, 1]$, there is an industry leader who dominates the market temporarily until the arrival of the next innovation.¹⁵ The industry leader employs workers in country h to produce $X_t^{h,h}(i)$ for sales in country h and $X_t^{f,h}(i)$ for sales in country f .¹⁶ The industry leader's production of $X_t^{h,h}(i)$ and $X_t^{f,h}(i)$ uses the same technology except for the presence of an iceberg transportation cost $c \in (0, 1)$ for $X_t^{f,h}(i)$. Specifically, the production functions are given by

$$X_t^{h,h}(i) = (z^h)^{q_t^h(i)} L_{x,t}^{h,h}(i), \quad (13)$$

¹⁴Derivations available upon request.

¹⁵This is known as the Arrow replacement effect in the literature; see Cozzi (2007) for a discussion.

¹⁶In order to keep the analysis tractable, we do not consider production offshoring in this study; see Chu, Cozzi and Furukawa (2013) for a North-South analysis of monetary policy with production offshoring.

$$X_t^{f,h}(i) = (1 - c)(z^h)^{q_t^h(i)} L_{x,t}^{f,h}(i), \quad (14)$$

where $\{L_{x,t}^{h,h}(i), L_{x,t}^{f,h}(i)\}$ denote production labor for $\{X_t^{h,h}(i), X_t^{f,h}(i)\}$. $z^h > 1$ is the step size of innovation in country h , and we allow this parameter to differ across countries. $q_t^h(i)$ is the number of quality improvements that have occurred in industry i as of time t .¹⁷ The total number of production workers employed in industry i of country h is $L_{x,t}^h(i) = L_{x,t}^{h,h}(i) + L_{x,t}^{f,h}(i)$.

Given $(z^h)^{q_t^h(i)}$ in industry i , the leader's marginal cost functions for $X_t^{h,h}(i)$ and $X_t^{f,h}(i)$ are respectively

$$MC_t^{h,h}(i) = \frac{W_t^h}{(z^h)^{q_t^h(i)}}, \quad (15)$$

$$MC_t^{f,h}(i) = \frac{W_t^h}{(1 - c)(z^h)^{q_t^h(i)}}. \quad (16)$$

Standard Bertrand price competition leads to markup pricing. The markup ratio is assumed to equal the step size z^h of innovation in the original Grossman-Helpman model. Here we allow for variable patent breadth similar to Li (2001) and Iwaisako and Futagami (2013) by assuming that the markup $\mu^h > 1$ is a policy instrument determined by the patent authority.¹⁸ For simplicity, we focus on the case in which $\mu^h = \mu^f = \mu$, and this assumption can be partly justified by the harmonization of patent protection across countries as a result of the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS) effective since 1996.¹⁹ Furthermore, given that patent policy is not designed by the monetary authority in reality,²⁰ we treat μ as exogenous when deriving optimal monetary policy.

Given the markup ratio μ , the price of $X_t^{h,h}(i)$ is

$$P_t^{h,h}(i) = \mu MC_t^{h,h}(i) = \mu \frac{W_t^h}{(z^h)^{q_t^h(i)}}. \quad (17)$$

Similarly, the price of $X_t^{f,h}(i)$ denominated in country h 's currency is

$$\mathcal{E}_t P_t^{f,h}(i) = \mu MC_t^{f,h}(i) = \mu \frac{W_t^h}{(1 - c)(z^h)^{q_t^h(i)}}, \quad (18)$$

where $P_t^{f,h}(i)$ is the price of $X_t^{f,h}(i)$ denominated in country f 's currency, and \mathcal{E}_t denotes the nominal exchange rate. Given (17), the amount of monopolistic profit from selling $X_t^{h,h}(i)$

¹⁷It is useful to note that we here adopt a cost-reducing view of quality improvement as in Peretto (1998).

¹⁸To model patent breadth, we first make a standard assumption in the literature, see for example Howitt (1999) and Segerstrom (2000), that once the incumbent leaves the market, she cannot threaten to reenter the market due to a reentry cost. As a result of the incumbent stopping production, the entrant is able to charge the unconstrained monopolistic markup, which is infinity due to the Cobb-Douglas specification in (9) and (10), under the case of complete patent breadth. However, with incomplete patent breadth, potential imitation limits the markup. Specifically, the presence of monopolistic profits attracts imitation; therefore, stronger patent protection allows monopolistic producers to charge a higher markup without the threat of imitation. This formulation of patent breadth captures Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".

¹⁹See Grossman and Lai (2004) for an analysis of the harmonization of patent protection under TRIPS.

²⁰See Chu (2008) for a discussion of the political process in determining patent policy in the US.

in country h is

$$\omega_t^{h,h}(i) = \left(\frac{\mu-1}{\mu}\right) P_t^{h,h}(i) X_t^{h,h}(i) = \left(\frac{\mu-1}{\mu}\right) P_t^{h,h} Y_t^{h,h}, \quad (19)$$

where the second equality follows from (11). Similarly, the amount of monopolistic profit (denominated in country f 's currency) from selling $X_t^{f,h}(i)$ in country f is

$$\omega_t^{f,h}(i) = \left(\frac{\mu-1}{\mu}\right) P_t^{f,h}(i) X_t^{f,h}(i) = \left(\frac{\mu-1}{\mu}\right) P_t^{f,h} Y_t^{f,h}, \quad (20)$$

where the second equality follows from country f 's analogous condition of (12). Therefore, the total amount of monopolistic profits (denominated in country h 's currency) earned by the leader in industry i is

$$\omega_t^h(i) = \omega_t^{h,h}(i) + \mathcal{E}_t \omega_t^{f,h}(i) = \frac{\mu-1}{\mu} \left(P_t^{h,h} Y_t^{h,h} + \mathcal{E}_t P_t^{f,h} Y_t^{f,h} \right). \quad (21)$$

Finally, wage income paid to industry i 's workers in country h is

$$W_t^h L_{x,t}^h(i) = W_t^h L_{x,t}^{h,h}(i) + W_t^h L_{x,t}^{f,h}(i) = \frac{1}{\mu} \left(P_t^{h,h} Y_t^{h,h} + \mathcal{E}_t P_t^{f,h} Y_t^{f,h} \right). \quad (22)$$

3.5 R&D

Denote $V_t^h(i)$ as the nominal value of the monopolistic firm in industry $i \in [0, 1]$ of country h . Because $\omega_t^h(i) = \omega_t^{h,h}(i)$ for $i \in [0, 1]$ from (21), $V_t^h(i) = V_t^h$ in a symmetric equilibrium that features an equal arrival rate of innovation across industries within a country.²¹ In this case, the familiar no-arbitrage condition for V_t^h is

$$R_t^h = \frac{\omega_t^h + \dot{V}_t^h - \lambda_t^h V_t^h}{V_t^h}. \quad (23)$$

This condition equates the nominal interest rate R_t^h in country h to the rate of return per unit of domestic asset. The asset return is the sum of (a) monopolistic profit ω_t^h , (b) potential capital gain \dot{V}_t^h , and (c) expected capital loss $\lambda_t^h V_t^h$ due to creative destruction, where λ_t^h is the arrival rate of the next innovation in country h .

There is a unit continuum of R&D entrepreneurs indexed by $k \in [0, 1]$ in each country, and they hire R&D labor for innovation. In country h , entrepreneur k 's wage payment to R&D labor is $W_t^h L_{r,t}^h(k)$. However, to facilitate this wage payment, the entrepreneur needs to borrow $B_t^h(k) = W_t^h L_{r,t}^h(k)$ units of domestic currency from the domestic household.²²

²¹We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the Schumpeterian growth model.

²²Given that this is wage payment to workers in the domestic economy, the wage payment is naturally paid in domestic currency.

Following Chu and Cozzi (2013), we impose a CIA constraint on R&D investment, and the cost of borrowing is $R_t^h B_t^h(k)$. Therefore, the total cost of R&D is $(1 + R_t^h)W_t^h L_{r,t}^h(k)$. Free entry implies zero expected profit such that

$$V_t^h \lambda_t^h(k) = (1 + R_t^h)W_t^h L_{r,t}^h(k), \quad (24)$$

where the *firm-level* arrival rate of innovation is $\lambda_t^h(k) = \bar{\varphi}_t^h L_{r,t}^h(k)$. We follow Jones and Williams (2000) to model a duplication externality in R&D that is commonly discussed in the literature by assuming that $\bar{\varphi}_t^h = \varphi^h (L_{r,t}^h)^{-\delta^h}$, where $L_{r,t}^h$ is aggregate R&D labor, $\delta^h \in [0, 1)$ is the duplication externality parameter, and $\varphi^h > 0$ is the R&D productivity parameter. We allow $\{\varphi^h, \delta^h\}$ to differ across countries. The *aggregate* arrival rate of innovation in country h is

$$\lambda_t^h = \int_0^1 \lambda_t^h(k) dk = \varphi^h (L_{r,t}^h)^{1-\delta^h}. \quad (25)$$

For tractability, we will focus on the special case of $\delta^h = 0$ in the rest of the analytical derivations;²³ however, we will consider the general case of $\delta^h \in (0, 1)$ in the quantitative analysis.

3.6 Monetary authority

The growth rate of the nominal money supply in country h is \dot{M}_t^h/M_t^h . By definition, the real money balance in country h is $m_t^h = M_t^h/P_t^h$, where P_t^h is the price of consumption goods in country h . Therefore, the growth rate of m_t^h is $\dot{m}_t^h/m_t^h = \dot{M}_t^h/M_t^h - \pi_t^h$, where $\pi_t^h \equiv \dot{P}_t^h/P_t^h$ is the inflation rate of the price of consumption goods in country h . The policy instrument that we consider is the nominal interest rate R_t^h , which is exogenously chosen by the monetary authority in country h . Given R_t^h , the inflation rate in country h is endogenously determined according to the Fisher identity $\pi_t^h = R_t^h - r_t^h$ where r_t^h is the real interest rate in country h and is determined by the Euler equation $r_t^h = \dot{C}_t^h/C_t^h + \rho$. Then, the growth rate of the nominal money supply M_t^h in country h is endogenously determined according to $\dot{M}_t^h/M_t^h = \dot{m}_t^h/m_t^h + \pi_t^h$.²⁴ Finally, the monetary authority in country h returns the seigniorage revenue as a lump-sum transfer $T_t^h = \dot{M}_t^h$ to the domestic household.

3.7 Decentralized equilibrium

The equilibrium is a time path of allocations $\{L_t^h, L_t^f, C_t^h, C_t^f, Y_t^{h,h}, Y_t^{h,f}, Y_t^{f,f}, Y_t^{f,h}, X_t^{h,h}(i), X_t^{h,f}(j), X_t^{f,f}(j), X_t^{f,h}(i), L_{x,t}^h(i), L_{x,t}^f(j), L_{r,t}^h(k), L_{r,t}^f(k)\}_{t=0}^\infty$, a time path of prices $\{W_t^h, W_t^f, P_t^h,$

²³Under $\delta^h \in (0, 1)$, there does not exist a closed-form solution for the equilibrium labor allocations.

²⁴In the long run, the Fisher identity simplifies to $R_t^h = r_t^h + \pi_t^h = \rho + \dot{M}_t^h/M_t^h$ implying that the monetary authority could target the nominal interest rate via money growth (and inflation). It is also useful to note that in the model, it is the *growth rate* of the money supply that affects the real economy in the long run, and a one-time change in the *level* of money supply has no long-run effect on the real economy. This is the well-known distinction between the neutrality and superneutrality of money. Empirical evidence generally favors neutrality and rejects superneutrality, consistent with our model; see Fisher and Seater (1993) for a discussion on the neutrality and superneutrality of money.

$P_t^f, P_t^{h,h}, P_t^{h,f}, P_t^{f,f}, P_t^{f,h}, P_t^{h,h}(i), P_t^{h,f}(j), P_t^{f,f}(j), P_t^{f,h}(i), V_t^h, V_t^f, \mathcal{E}_t\}_{t=0}^\infty$ and a time path of policies $\{R_t^h, R_t^f, T_t^h, T_t^f\}_{t=0}^\infty$. Also, at each instance of time,

- the representative household in country h maximizes lifetime utility taking $\{R_t^h, W_t^h, P_t^h, T_t^h\}$ as given;
- the representative household in country f maximizes lifetime utility taking $\{R_t^f, W_t^f, P_t^f, T_t^f\}$ as given;
- competitive consumption-good firms in country h produce $\{C_t^h\}$ to maximize profit taking $\{P_t^h, P_t^{h,h}, P_t^{h,f}\}$ as given;
- competitive consumption-good firms in country f produce $\{C_t^f\}$ to maximize profit taking $\{P_t^f, P_t^{f,f}, P_t^{f,h}\}$ as given;
- competitive final-good firms in country h produce $\{Y_t^{h,h}, Y_t^{h,f}\}$ to maximize profit taking $\{P_t^{h,h}, P_t^{h,f}, P_t^{h,h}(i), P_t^{h,f}(j)\}$ as given;
- competitive final-good firms in country f produce $\{Y_t^{f,f}, Y_t^{f,h}\}$ to maximize profit taking $\{P_t^{f,f}, P_t^{f,h}, P_t^{f,f}(j), P_t^{f,h}(i)\}$ as given;
- monopolistic intermediate-good firm $i \in [0, 1]$ in country h produces $\{X_t^{h,h}(i), X_t^{f,h}(i)\}$ and chooses $\{P_t^{h,h}(i), P_t^{f,h}(i)\}$ to maximize profit taking $\{W_t^h\}$ as given;
- monopolistic intermediate-good firm $j \in [0, 1]$ in country f produces $\{X_t^{f,f}(j), X_t^{h,f}(j)\}$ and chooses $\{P_t^{f,f}(j), P_t^{h,f}(j)\}$ to maximize profit taking $\{W_t^f\}$ as given;
- competitive R&D entrepreneurs $k \in [0, 1]$ in country h employ $\{L_{r,t}^h(k)\}$ to maximize expected profit taking $\{R_t^h, W_t^h, V_t^h\}$ as given;
- competitive R&D entrepreneurs $k \in [0, 1]$ in country f employ $\{L_{r,t}^f(k)\}$ to maximize expected profit taking $\{R_t^f, W_t^f, V_t^f\}$ as given;
- the market-clearing condition for labor holds in both countries such that $L_{x,t}^h + L_{r,t}^h = L_t^h$ and $L_{x,t}^f + L_{r,t}^f = L_t^f$; and
- the value of trade in intermediate goods is balanced such that $\int_0^1 P_t^{h,f}(j) X_t^{h,f}(j) dj = \mathcal{E}_t \int_0^1 P_t^{f,h}(i) X_t^{f,h}(i) di$.

3.8 Aggregate economy

Substituting (13) into (9) yields the aggregate production function for final goods $Y_t^{h,h}$ given by

$$Y_t^{h,h} = Z_t^h L_{x,t}^{h,h}, \quad (26)$$

where aggregate technology Z_t^h in country h is defined as

$$Z_t^h \equiv \exp \left(\int_0^1 q_t^h(i) di \ln z^h \right) = \exp \left(\int_0^t \lambda_v^h dv \ln z^h \right). \quad (27)$$

The second equality of (27) applies the law of large numbers. Differentiating the log of (27) with respect to t yields the growth rate of aggregate technology in country h given by

$$\frac{\dot{Z}_t^h}{Z_t^h} = \lambda_t^h \ln z^h = (\varphi^h \ln z^h) L_{r,t}^h. \quad (28)$$

Similarly, substituting country f 's analogous condition of (14) into (10) yields the aggregate production function for final goods $Y_t^{h,f}$ given by

$$Y_t^{h,f} = (1 - c) Z_t^f L_{x,t}^{h,f}, \quad (29)$$

where aggregate technology Z_t^f in country f is defined as

$$Z_t^f \equiv \exp \left(\int_0^1 q_t^f(j) dj \ln z^f \right) = \exp \left(\int_0^t \lambda_v^f dv \ln z^f \right). \quad (30)$$

Differentiating the log of (30) with respect to t yields the growth rate of aggregate technology in country f given by

$$\frac{\dot{Z}_t^f}{Z_t^f} = \lambda_t^f \ln z^f = (\varphi^f \ln z^f) L_{r,t}^f. \quad (31)$$

As for the dynamics of the model, Proposition 1 shows that the economy jumps to a unique and saddle-point stable balanced growth path (BGP). On the BGP, the allocations of labor in both countries are stationary. Therefore, taking the log of (6) and differentiating it with respect to t yields the balanced growth rate of consumption goods C_t^h in country h given by

$$\frac{\dot{C}_t^h}{C_t^h} = (1 - \alpha) \frac{\dot{Y}_t^{h,h}}{Y_t^{h,h}} + \alpha \frac{\dot{Y}_t^{h,f}}{Y_t^{h,f}} = (1 - \alpha) \frac{\dot{Z}_t^h}{Z_t^h} + \alpha \frac{\dot{Z}_t^f}{Z_t^f} = (1 - \alpha)(\varphi^h \ln z^h) L_r^h + \alpha(\varphi^f \ln z^f) L_r^f. \quad (32)$$

Proposition 1 *Given constant nominal interest rates $\{R^h, R^f\}$ in the two countries, the economy immediately jumps to a unique and saddle-point stable balanced growth path along which each variable grows at a constant (possibly zero) rate.*

Proof. See Appendix A. ■

3.9 Equilibrium labor allocations

In this subsection, we sketch out the derivations of the equilibrium labor allocations in country h and relegate the detailed proof to Appendix A. From (24), the free-entry condition in the R&D sector becomes $\varphi^h V_t^h = (1 + R^h)W_t^h$, where the value of an innovation is $V_t^h = \omega_t^h / (\rho + \lambda^h) = \omega_t^h / (\rho + \varphi^h L_r^h)$ on the BGP as implied by (23), (25) and the Euler equation $r_t^h = \dot{C}_t^h / C_t^h + \rho$. Then, substituting ω_t^h from (21) and W_t^h from (22) into the R&D free-entry condition, we derive the following equilibrium relationship between L_r^h and L_x^h :

$$L_r^h = \frac{\mu - 1}{1 + R^h} L_x^h - \frac{\rho}{\varphi^h}. \quad (33)$$

The second condition is the household's labor supply function in (4), which can be reexpressed as

$$L^h = 1 - \mu\theta^h(1 + \xi^h R^h)L_x^h. \quad (34)$$

The third condition for solving the equilibrium labor allocations is the resource constraint on labor given by

$$L_x^h + L_r^h = L^h, \quad (35)$$

where $L_x^h = L_x^{h,h} + L_x^{f,h}$. Finally, we combine the conditional demand functions in (7) and (8) with (22) and the balanced-trade condition that can be reexpressed as $P_t^{h,f} Y_t^{h,f} = \mathcal{E}_t P_t^{f,h} Y_t^{f,h}$ to solve for the following equilibrium relationship between $L_x^{h,h}$ and $L_x^{f,h}$:²⁵

$$L_x^{f,h} = \frac{\alpha}{1 - \alpha} L_x^{h,h}. \quad (36)$$

Solving (33)-(36), we derive the equilibrium labor allocations in country h .

Proposition 2 *The equilibrium labor allocations in country h are given by*

$$L_r^h = \frac{\mu - 1}{\mu + R^h + \mu\theta^h(1 + \xi^h R^h)(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right) - \frac{\rho}{\varphi^h}, \quad (37)$$

$$L_x^{h,h} = (1 - \alpha)L_x^h = \frac{(1 - \alpha)(1 + R^h)}{\mu + R^h + \mu\theta^h(1 + \xi^h R^h)(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right), \quad (38)$$

$$L_x^{f,h} = \alpha L_x^h = \frac{\alpha(1 + R^h)}{\mu + R^h + \mu\theta^h(1 + \xi^h R^h)(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right), \quad (39)$$

$$L^h = 1 - \frac{\mu\theta^h(1 + \xi^h R^h)(1 + R^h)}{\mu + R^h + \mu\theta^h(1 + \xi^h R^h)(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right). \quad (40)$$

Proof. See Appendix A. ■

²⁵To be more precise, we need to use (7), (8), (11)-(14), (17), (18) and (22).

4 Effects of monetary policy on economic growth

In this section, we explore the effects of the domestic and foreign nominal interest rates on economic growth. Equation (37) shows that R&D labor L_r^h is decreasing in the domestic nominal interest rate R^h through the CIA constraints on R&D investment and consumption. Given the CIA constraint on R&D investment, an increase in the domestic nominal interest rate R^h increases the financing cost of R&D, and hence, it has a direct negative effect on R&D labor L_r^h . This financing-cost channel via the CIA constraint on R&D investment is present regardless of whether labor supply is elastic (i.e., $\theta^h > 0$) or inelastic (i.e., $\theta^h = 0$). Under the CIA constraint on consumption, the negative effect of the domestic nominal interest rate R^h on R&D operates through elastic labor supply. Equation (40) shows that labor supply L^h is decreasing in the nominal interest rate R^h partly due to the CIA constraint on consumption (i.e., $\xi^h > 0$). In other words, an increase in the domestic nominal interest rate raises the cost of consumption and causes the household to increase leisure. The resulting decrease in labor supply L^h in turn reduces R&D labor L_r^h . This labor-supply channel via the CIA constraint on consumption is present only if labor supply is elastic (i.e., $\theta^h > 0$). Given that R&D labor L_r^h is decreasing in the domestic nominal interest rate, the growth rate of technology in country h is also decreasing in the domestic nominal interest rate R^h ; in other words,

$$\frac{\partial \dot{Z}_t^h / Z_t^h}{\partial R^h} = (\varphi^h \ln z^h) \frac{\partial L_r^h}{\partial R^h} < 0. \quad (41)$$

Equation (37) also shows that R&D labor is independent of the foreign nominal interest rate R^f . Therefore, the growth rate of technology in country h must be also independent of R^f ; in other words,

$$\frac{\partial \dot{Z}_t^h / Z_t^h}{\partial R^f} = (\varphi^h \ln z^h) \frac{\partial L_r^h}{\partial R^f} = 0. \quad (42)$$

Nevertheless, economic growth in country h (i.e., the growth rate of consumption goods) is decreasing in the foreign nominal interest rate R^f . To see this result, we first write down country f 's analogous expression of (37) for the equilibrium allocation of R&D labor L_r^f given by

$$L_r^f = \frac{\mu - 1}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)} \left(1 + \frac{\rho}{\varphi^f} \right) - \frac{\rho}{\varphi^f}, \quad (43)$$

which is decreasing in the foreign nominal interest rate R^f . Therefore, the growth rate of consumption goods in country h is decreasing in the foreign nominal interest rate R^f through the growth rate of foreign technology; in other words,

$$\frac{\partial g^h}{\partial R^f} = (1 - \alpha)(\varphi^h \ln z^h) \frac{\partial L_r^h}{\partial R^f} + \alpha(\varphi^f \ln z^f) \frac{\partial L_r^f}{\partial R^f} = \alpha(\varphi^f \ln z^f) \frac{\partial L_r^f}{\partial R^f} < 0, \quad (44)$$

where $g^h \equiv \dot{C}_t^h / C_t^h$. The growth rate of consumption goods in country h is also decreasing in the domestic nominal interest rate R^h through the growth rate of domestic technology; in other words,

$$\frac{\partial g^h}{\partial R^h} = (1 - \alpha)(\varphi^h \ln z^h) \frac{\partial L_r^h}{\partial R^h} + \alpha(\varphi^f \ln z^f) \frac{\partial L_r^f}{\partial R^h} = (1 - \alpha)(\varphi^h \ln z^h) \frac{\partial L_r^h}{\partial R^h} < 0. \quad (45)$$

As foreign goods become more important in domestic consumption (i.e., a larger α), the negative growth effect of the foreign nominal interest rate becomes stronger, whereas the negative growth effect of the domestic nominal interest rate becomes weaker. Proposition 3 summarizes the above results.

Proposition 3 *The growth rate of domestic technology is decreasing in the domestic nominal interest rate but independent of the foreign nominal interest rate. The growth rate of foreign technology is decreasing in the foreign nominal interest rate but independent of the domestic nominal interest rate. The growth rate of domestic consumption is decreasing in both the domestic and foreign nominal interest rates. The relative magnitude of these negative growth effects of the domestic and foreign nominal interest rates depends on α (i.e., the importance of foreign goods in domestic consumption).*

Proof. Proven in text. ■

Using the Fisher identity $\pi^h = R^h - r^h$ and the Euler equation $r^h = g^h + \rho$, we can write down an expression for the equilibrium inflation rate given by $\pi^h = R^h - g^h(R^h) - \rho$, where g^h is decreasing in the nominal interest rate R^h . Therefore, differentiating π^h with respect to R^h yields

$$\frac{\partial \pi^h}{\partial R^h} = 1 - \underbrace{\frac{\partial g^h}{\partial R^h}}_{> 0} > 0.$$

It is important for us to emphasize that this comparative statics refers to a *long-run* relationship between inflation and the nominal interest rate.²⁶ Together with Proposition 3, we have the following empirical implications. First, an increase in the nominal interest rate requires an increase in the inflation rate and causes a decrease in R&D investment. This finding is consistent with the stylized facts documented in Section 2 and the empirical evidence in Chu and Lai (2013). Second, an increase in the nominal interest rate requires an increase in the inflation rate and causes a decrease in the growth rate of technology. This finding is consistent with the empirical results in Bruno and Easterly (1998), who provide evidence for a negative relationship between inflation and the growth rate of total factor productivity. Finally, an increase in the nominal interest rate requires an increase in the inflation rate and causes a decrease in the growth rate of output. This negative relationship between inflation and economic growth is supported by the empirical results in Vaona (2012).

²⁶As mentioned before, Mishkin (1992) and Booth and Ciner (2001) provide empirical evidence for a positive long-run relationship between inflation and the nominal interest rate. Given that we consider a long-run equilibrium, we have purposefully ignored important business cycle issues - in which inflation π_t can diverge from its target level, denoted as π_t^* , and aggregate output y_t can diverge from its full-employment trend level, denoted as y_t^* . Nevertheless, it could be useful to compare the properties of our monetary framework with that of the seminal Taylor rule: $R_t = r^* + \pi_t + a_\pi (\pi_t - \pi^*) + a_y (y_t - y_t^*)$, in which r^* is the long-run equilibrium real interest rate and a_π and a_y are the (positive) monetary policy parameters. On the one hand, the Taylor rule generally implies a *negative* relationship between the target inflation rate and the nominal interest rate when inflation does not immediately adjust to the target rate. On the other hand, our model implies a *positive* long-run relationship as a result of the Fisher identity: $R = r^* + \pi^*$. These contrasting implications are clearly due to the different time horizons of the two approaches; however, it is well-known that in the long-run the Taylor rule coincides with the Fisher identity.

5 Effects of monetary policy on social welfare

In this section, we analyze the effects of the domestic and foreign nominal interest rates on social welfare. Given the balanced-growth behavior of the economy, the lifetime utility of the representative household in country h simplifies to

$$U^h = \frac{1}{\rho} \left[\ln C_0^h + \frac{g^h}{\rho} + \theta^h \ln(1 - L^h) \right]. \quad (46)$$

Substituting (6), (26), (29) and (32) into (46) and then dropping the exogenous terms (including Z_0^h and Z_0^f) yield

$$\rho U^h = (1 - \alpha) \ln L_x^{h,h} + \alpha \ln L_x^{h,f} + \frac{(1 - \alpha)(\varphi^h \ln z^h)}{\rho} L_r^h + \frac{\alpha(\varphi^f \ln z^f)}{\rho} L_r^f + \theta^h \ln(1 - L^h). \quad (47)$$

The analogous condition of (39) in country f implies that $L_x^{h,f}$ is given by

$$L_x^{h,f} = \frac{\alpha(1 + R^f)}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)} \left(1 + \frac{\rho}{\varphi^f} \right), \quad (48)$$

which depends on the foreign nominal interest rate R^f . In summary, the set of variables $\{L_x^{h,h}, L_r^h, L^h\}$ in (47) depends on the domestic nominal interest rate R^h , whereas the set of variables $\{L_x^{h,f}, L_r^f\}$ depends on the foreign nominal interest rate R^f .

In the following subsections, we will derive (a) the nominal interest rate that is unilaterally chosen by each government to maximize domestic welfare and (b) the nominal interest rates that are chosen by cooperative governments who maximize the aggregate welfare of the two countries. Given that the results differ under the following three scenarios,²⁷ we analyze them separately. In Section 5.1, we consider the case of inelastic labor supply. In Section 5.2, we consider elastic labor supply with only the CIA constraint on R&D investment. In Section 5.3, we consider elastic labor supply with only the CIA constraint on consumption.

5.1 Inelastic labor supply

In this subsection, we consider the case of inelastic labor supply (i.e., $\theta^h = \theta^f = 0$). In this case, (37), (38), (43) and (48) simplify to

$$L_r^h = \frac{\mu - 1}{\mu + R^h} \left(1 + \frac{\rho}{\varphi^h} \right) - \frac{\rho}{\varphi^h}, \quad (49)$$

$$L_x^{h,h} = \frac{(1 - \alpha)(1 + R^h)}{\mu + R^h} \left(1 + \frac{\rho}{\varphi^h} \right), \quad (50)$$

$$L_r^f = \frac{\mu - 1}{\mu + R^f} \left(1 + \frac{\rho}{\varphi^f} \right) - \frac{\rho}{\varphi^f}, \quad (51)$$

²⁷Unfortunately, there does not exist a closed-form solution for the welfare-maximizing nominal interest rate in the most general case of elastic labor supply with CIA constraints on both R&D and consumption. We will analyze this scenario in the quantitative analysis.

$$L_x^{h,f} = \frac{\alpha(1 + R^f)}{\mu + R^f} \left(1 + \frac{\rho}{\varphi^f} \right), \quad (52)$$

and $L^h = 1$. Due to inelastic labor supply, the effect of the nominal interest rate operates solely through the CIA constraint on R&D investment. Substituting (49)-(52) into (47) and then differentiating U^h with respect to R^h , we obtain the following domestic nominal interest rate that is unilaterally chosen by the government in country h to maximize the domestic household's welfare:

$$R_{ne}^h = \max \left\{ \frac{\mu - \Omega^h}{\Omega^h - 1}, 0 \right\}, \quad (53)$$

where $\Omega^h \equiv (1 + \varphi^h/\rho) \ln z^h > 1$.²⁸ Equation (53) also takes into account the zero lower bound on the nominal interest rate, which would be binding if $\Omega^h > \mu$. The analogous foreign nominal interest rate that is unilaterally chosen by country f 's government to maximize the welfare of the household in country f is given by

$$R_{ne}^f = \max \left\{ \frac{\mu - \Omega^f}{\Omega^f - 1}, 0 \right\}, \quad (54)$$

where $\Omega^f \equiv (1 + \varphi^f/\rho) \ln z^f > 1$.²⁹ The comparative statics are quite intuitive. Recall that domestic R&D investment is decreasing in the domestic nominal interest rate. Therefore, a larger innovation step size z^h (z^f) would decrease the nominal interest rate R_{ne}^h (R_{ne}^f) because R&D has a larger social benefit in this case. Similarly, a higher R&D productivity φ^h (φ^f) would also decrease R_{ne}^h (R_{ne}^f) for the same reason. In contrast, a higher discount rate ρ would increase R_{ne}^h and R_{ne}^f because R&D that leads to economic growth has a smaller social benefit when households discount future consumption more heavily.

We refer to the pair $\{R_{ne}^h, R_{ne}^f\}$ as the Nash-equilibrium nominal interest rates because each government pursues its own objective taking the other government's action as given. An interesting observation is that R_{ne}^f is also the foreign nominal interest rate that would be preferred by the government in country h . To see this result, we differentiate U^h with respect to R^f and find that the optimal foreign nominal interest rate for country h is also R_{ne}^f . Finally, we consider cooperative governments who choose $\{R^h, R^f\}$ to maximize aggregate welfare defined as $U^h + U^f$, and we refer to these nominal interest rates as the optimal nominal interest rates denoted as $\{R_*^h, R_*^f\}$. We find that $\{R_*^h, R_*^f\} = \{R_{ne}^h, R_{ne}^f\}$; in other words, the unilateral action of each government gives rise to an internationally optimal outcome; however, in the next subsection, we will show that this special result is due to the restriction of inelastic labor supply. We summarize the above results in Proposition 4.

Proposition 4 *Under inelastic labor supply, the Nash-equilibrium nominal interest rate unilaterally chosen by each government coincides with the optimal nominal interest rate chosen by cooperative governments who maximize aggregate welfare of the two countries. The optimal nominal interest rate in each country is decreasing in the domestic step size of innovation and domestic R&D productivity but increasing in the discount rate.*

²⁸In Appendix B, we derive the first-best allocation of R&D labor in country h and show that the parameter restriction on this optimal R&D labor to be positive also implies $\Omega^h > 1$.

²⁹We show in Appendix B that this parameter restriction holds for a similar reason as $\Omega^h > 1$.

Proof. See Appendix A. ■

5.2 Elastic labor supply with CIA on R&D only

In this subsection, we consider the case of elastic labor supply with the CIA constraint on R&D. However, we remove the CIA constraint on consumption by setting $\xi^h = \xi^f = 0$. In this case, (37), (38), (40), (43) and (48) simplify to

$$L_r^h = \frac{\mu - 1}{\mu + R^h + \mu\theta^h(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right) - \frac{\rho}{\varphi^h}, \quad (55)$$

$$L_x^{h,h} = \frac{(1 - \alpha)(1 + R^h)}{\mu + R^h + \mu\theta^h(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right), \quad (56)$$

$$L^h = 1 - \frac{\mu\theta^h(1 + R^h)}{\mu + R^h + \mu\theta^h(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h}\right), \quad (57)$$

$$L_r^f = \frac{\mu - 1}{\mu + R^f + \mu\theta^f(1 + R^f)} \left(1 + \frac{\rho}{\varphi^f}\right) - \frac{\rho}{\varphi^f}, \quad (58)$$

$$L_x^{h,f} = \frac{\alpha(1 + R^f)}{\mu + R^f + \mu\theta^f(1 + R^f)} \left(1 + \frac{\rho}{\varphi^f}\right). \quad (59)$$

Substituting these conditions into (47) and then differentiating U^h with respect to R^h , we obtain the following domestic nominal interest rate that is unilaterally chosen by the government in country h to maximize the domestic household's welfare:

$$R_{ne}^h = \max \left\{ \frac{\mu - \Phi^h}{\Phi^h - 1}, 0 \right\}, \quad (60)$$

where³⁰

$$\Phi^h \equiv \frac{1 - \alpha}{1 - \alpha + \theta^h} \left(1 + \frac{\varphi^h}{\rho}\right) (1 + \mu\theta^h) \ln z^h - \mu\theta^h > 1.$$

The analogous foreign nominal interest rate that is unilaterally chosen by country f 's government to maximize the welfare of the household in country f is given by

$$R_{ne}^f = \max \left\{ \frac{\mu - \Phi^f}{\Phi^f - 1}, 0 \right\}, \quad (61)$$

where³¹

$$\Phi^f \equiv \frac{1 - \alpha}{1 - \alpha + \theta^f} \left(1 + \frac{\varphi^f}{\rho}\right) (1 + \mu\theta^f) \ln z^f - \mu\theta^f > 1.$$

³⁰In Appendix B, we derive the condition under which this parameter restriction holds.

³¹In Appendix B, we derive the condition under which this parameter restriction holds.

We also consider cooperative governments who choose $\{R^h, R^f\}$ to maximize aggregate welfare $U^h + U^f$, and these optimal nominal interest rates are given by

$$R_*^h = \max \left\{ \frac{\mu - \Psi^h}{\Psi^h - 1}, 0 \right\}, \quad (62)$$

$$R_*^f = \max \left\{ \frac{\mu - \Psi^f}{\Psi^f - 1}, 0 \right\}, \quad (63)$$

where³²

$$\Psi^h \equiv \frac{1}{1 + \theta^h} \left(1 + \frac{\varphi^h}{\rho} \right) (1 + \mu\theta^h) \ln z^h - \mu\theta^h > 1,$$

$$\Psi^f \equiv \frac{1}{1 + \theta^f} \left(1 + \frac{\varphi^f}{\rho} \right) (1 + \mu\theta^f) \ln z^f - \mu\theta^f > 1.$$

We see that $\Psi^h > \Phi^h$ and $\Psi^f > \Phi^f$ because $\alpha > 0$; therefore, $R_*^h \leq R_{ne}^h$ and $R_*^f \leq R_{ne}^f$. In other words, the unilateral action of each government generally leads to excessively high nominal interest rates in the Nash equilibrium due to a cross-country spillover effect of monetary policy under elastic labor supply, and the degree of this cross-country spillover effect is measured by α (i.e., the importance of foreign goods in domestic consumption). Intuitively, when a country lowers its nominal interest rate, the welfare gain from higher economic growth is shared by the other country, whereas the welfare cost of increasing labor supply (L_h in (57) is decreasing in R^h) falls entirely on the domestic household. As a result, the government does not lower the domestic nominal interest rate sufficiently in the Nash equilibrium. In contrast, cooperative governments would internalize the welfare gain from a higher growth rate in the other country. We summarize these results in Proposition 5.

Proposition 5 *Under elastic labor supply with only a CIA constraint on R&D, the Nash-equilibrium nominal interest rate unilaterally chosen by each government is higher than the optimal nominal interest rate chosen by cooperative governments who maximize aggregate welfare of the two countries. The Nash-equilibrium nominal interest rates are increasing in α , whereas the optimal nominal interest rates are independent of α .*

Proof. See Appendix A. ■

Substituting Φ^h into R_{ne}^h in (60), we find that R_{ne}^h is increasing in the markup μ . Similarly, substituting Ψ^h into R_*^h in (62), we find that R_*^h is also increasing in the markup μ . Intuitively, a larger markup strengthens the negative R&D externality, commonly known as the business-stealing effect, in the Schumpeterian growth model. To see this externality effect, (55) shows that L_r^h is increasing in μ , whereas (56) and (59) show that $L_x^{h,h}$ and $L_x^{h,f}$ are decreasing in μ . Given that μ determines the private benefit of R&D but not its social benefit, a larger μ causes an increase in L_r^h relative to its socially optimal level capturing the

³²In Appendix B, we derive the first-best allocations of R&D labor in countries h and f and show that the parameter restrictions on these optimal R&D labors to be positive also imply $\Psi^h > 1$ and $\Psi^f > 1$.

effect of a negative R&D externality. As a result of this stronger negative R&D externality, the government would raise the nominal interest rate to decrease the equilibrium allocation of R&D labor. Taking the difference between R_{ne}^h and R_*^h and differentiating it with respect to μ , we find that

$$\frac{\partial(R_{ne}^h - R_*^h)}{\partial\mu} = \frac{1 + \theta^h}{(1 + \mu\theta^h)^2} \left[\frac{1}{\frac{1-\alpha}{1-\alpha+\theta^h} \left(1 + \frac{\varphi^h}{\rho}\right) \ln z^h - 1} - \frac{1}{\frac{1}{1+\theta^h} \left(1 + \frac{\varphi^h}{\rho}\right) \ln z^h - 1} \right] > 0.$$

In other words, the wedge between the Nash-equilibrium and optimal nominal interest rates is monotonically increasing in the market power of firms. This result differs from the interesting result in Arseneau (2007), who shows that a larger market power of firms tends to reduce the wedge between the Nash-equilibrium and optimal nominal interest rates. The different implications between the two studies are due to the different CIA constraints. We have analyzed a CIA constraint on R&D investment, whereas Arseneau (2007) analyzes a CIA constraint on consumption. In the next subsection, we show that our model also delivers the insight of Arseneau (2007) under a CIA constraint on consumption, and we will explain the difference between the two CIA constraints then.

5.3 Elastic labor supply with CIA on consumption only

In this subsection, we consider the case of elastic labor supply with the CIA constraint on consumption. However, we remove the CIA constraint on R&D. In this case, (33) becomes

$$L_r^h = (\mu - 1)L_x^h - \frac{\rho}{\varphi^h}. \quad (64)$$

Solving (34), (35), (36) and (64) yields the equilibrium labor allocations in country h , and we can follow the same procedure to solve for the equilibrium allocations in country f . In this case, the equilibrium allocations of $\{L_r^h, L_x^{h,h}, L^h, L_r^f, L_x^{h,f}\}$ are given by

$$L_r^h = \frac{(\mu - 1)/\mu}{1 + \theta^h(1 + \xi^h R^h)} \left(1 + \frac{\rho}{\varphi^h}\right) - \frac{\rho}{\varphi^h}, \quad (65)$$

$$L_x^{h,h} = \frac{(1 - \alpha)/\mu}{1 + \theta^h(1 + \xi^h R^h)} \left(1 + \frac{\rho}{\varphi^h}\right), \quad (66)$$

$$L^h = 1 - \frac{\theta^h(1 + \xi^h R^h)}{1 + \theta^h(1 + \xi^h R^h)} \left(1 + \frac{\rho}{\varphi^h}\right), \quad (67)$$

$$L_r^f = \frac{(\mu - 1)/\mu}{1 + \theta^f(1 + \xi^f R^f)} \left(1 + \frac{\rho}{\varphi^f}\right) - \frac{\rho}{\varphi^f}, \quad (68)$$

$$L_x^{h,f} = \frac{\alpha/\mu}{1 + \theta^f(1 + \xi^f R^f)} \left(1 + \frac{\rho}{\varphi^f}\right). \quad (69)$$

Substituting (65)-(69) into U^h in (47) and the analogous conditions in country f into U^f , we find that the cooperative governments' optimization problem yields

$$\frac{\partial(U^h + U^f)}{\partial R^h} < 0$$

for all values of $R^h \geq 0$. Therefore, the optimal nominal interest rates are $\{R_*^h, R_*^f\} = \{0, 0\}$ implying that the Friedman rule³³ is always optimal under the CIA constraint on consumption.

As for the Nash-equilibrium nominal interest rate, we differentiate U^h with respect to R^h and find that

$$\frac{\partial U^h}{\partial R^h} = -\frac{\theta^h \xi^h (1 - \alpha) (1 + \rho/\varphi^h)}{\mu [1 + \theta^h (1 + \xi^h R^h)]^2} \left[\frac{(1 - \alpha) (1 + \xi^h R^h) - 1}{L_x^{h,h} (1 + \xi^h R^h)} + \frac{(\mu - 1) \varphi^h}{\rho} \ln z^h \right], \quad (70)$$

where $L_x^{h,h}$ is given by (66). Suppose $\mu \rightarrow 1$. In this case, $\partial U^h / \partial R^h > 0$ when $R^h = 0$. In other words, the government has incentives to choose a strictly positive nominal interest rate. This effect captures the important result of an inflationary bias due to monetary policy competition in Cooley and Quadrini (2003). Equation (70) also shows that a larger markup μ would reduce this inflationary bias capturing the dampening effect of monopolistic distortion raised by Arseneau (2007). If the markup μ becomes sufficiently large, then $\partial U^h / \partial R^h < 0$ for all values of $R^h \geq 0$. In this case, the Nash-equilibrium nominal interest rate would be also zero. We summarize these results in Proposition 6.

Proposition 6 *Under elastic labor supply with only a CIA constraint on consumption, the optimal nominal interest rates are always zero. If the markup $\mu \rightarrow 1$, then the Nash-equilibrium nominal interest rate in country h is $R_{ne}^h = \alpha / [\xi^h (1 - \alpha)] > 0$, which is increasing in α . As the markup μ increases, the Nash-equilibrium nominal interest rates decrease. If the markup μ is sufficiently large, then the Nash-equilibrium nominal interest rates would also be zero.*

Proof. See Appendix A. ■

Proposition 6 together with the analysis in the previous subsection show that under the two CIA constraints, the market power of firms has very different implications on the inflationary bias from monetary policy competition. The difference between the CIA constraint on consumption and the CIA constraint on R&D is that under the latter, an increase in the nominal interest rate leads to a reallocation of labor from R&D to production. As a result, a positive nominal interest rate would be chosen by the government if the business-stealing effect measured by the markup μ is strong. In contrast, under the CIA constraint on consumption, this reallocation effect is absent because an increase in the nominal interest rate decreases both R&D and production labors. Given that the economy features an undersupply of labor due to the distortion of monopolistic competition, the government would avoid setting too high a nominal interest rate that worsens this monopolistic distortion.

³³In his seminal study, Friedman (1969) argues that the optimal nominal interest rate is zero.

6 Quantitative analysis

In this section, we provide a numerical illustration on the growth and welfare effects of monetary policy across countries. We consider the general case with elastic labor supply and both CIA constraints on R&D and consumption. To make this quantitative analysis more realistic, we allow for the duplication externality (i.e., $\{\delta^h, \delta^f\} > 0$) discussed in Section 3.5.³⁴ We calibrate the model using aggregate data from 1999 to 2007³⁵ in the Euro Area and the UK,³⁶ which are two relatively open economies. To fix notation, we consider the UK as the home country h and the Euro Area as the foreign country f .

The two-country model features the following set of parameters $\{\alpha, \rho, \mu, z^h, z^f, \varphi^h, \varphi^f, \theta^h, \theta^f, \xi^h, \xi^f, R^h, R^f, \delta^h, \delta^f\}$. Given the calibrated parameter values, we then perform a counterfactual policy experiment on the effects of decreasing the nominal interest rates on economic growth and social welfare in the two economies. The average value of imports from the Euro Area to the UK as a percentage of GDP in the UK is 14.9%,³⁷ and we use this empirical moment to calibrate the parameter α . As for the discount rate ρ , we consider a standard value of $\rho = 0.05$; see for example Acemoglu and Akgigit (2012). As for the aggregate markup in the economy, we consider $\mu = 1.03$ in accordance with the aggregate profit share of 3% estimated by Basu (1996). The average growth rates of total factor productivity in the UK and the Euro Area are respectively 0.9% and 0.8%,³⁸ and we use these values to calibrate the step-size parameters $\{z^h, z^f\}$. The average value of R&D expenditures as a percentage of GDP in the UK and the Euro Area are respectively 1.8% and 1.9%,³⁹ and we use these values to calibrate the R&D productivity parameters $\{\varphi^h, \varphi^f\}$. As for the leisure parameters $\{\theta^h, \theta^f\}$, we calibrate them by setting the supply of labor $\{L^h, L^f\}$ to a standard value of 0.33. The ratios of M1 to GDP in the UK and the Euro Area are respectively 0.541 and 0.361,⁴⁰ and we use these values to calibrate the consumption-CIA parameters $\{\xi^h, \xi^f\}$. The average inflation rates in the UK and the Euro Area are respectively 2.1% and 2.8%,⁴¹ and we use these values to calibrate $\{R^h, R^f\}$. Given these calibrated values of $\{R^h, R^f\}$, we choose $\{\delta^h, \delta^f\}$ in order to set $\{R_{ne}^h, R_{ne}^f\} = \{R^h, R^f\}$. We report the parameter values in Table 1.

α	μ	ρ	z^h	z^f	φ^h	φ^f	θ^h	θ^f	ξ^h	ξ^f	R^h	R^f	δ^h	δ^f
0.149	1.03	0.05	1.12	1.09	0.27	0.33	1.92	1.95	0.52	0.34	0.080	0.086	0.77	0.76

The policy experiments are as follows. First, we lower the nominal interest rates in both economies from the Nash equilibrium to their globally optimal level and examine the effects

³⁴We present the equilibrium conditions in Appendix C.

³⁵We do not include data from 2008 onwards due to the international financial and debt crises.

³⁶The analysis here focuses on the European economies. However, the two-country model can also be calibrated to other economies.

³⁷Data source: The OECD Database.

³⁸Data source: The Conference Board Total Economy Database.

³⁹Data source: Eurostat (European Commission).

⁴⁰Data source: Bank of England Statistics Database and ECB Statistical Data Warehouse.

⁴¹Data source: The OECD Database.

on economic growth $\{g^h, g^f\}$ and social welfare $\{U^h, U^f\}$. Second, we consider a unilateral deviation from the Nash equilibrium to the optimal nominal interest rate that maximizes aggregate welfare of the two economies and examine the asymmetric implications on the two economies. Under the current set of calibrated parameter values, the optimal nominal interest rates in both economies are zero; in other words, the Friedman rule is socially optimal.

We first consider the case in which the two governments are cooperative and agree to decrease the nominal interest rates from the Nash equilibrium to the globally optimal level of zero. In this case, the welfare gains are nonnegligible and equivalent to a permanent increase in consumption of 0.292% in the UK and 0.443% in the Euro Area. These results together with the growth effects are reported in Table 2.⁴² However, a unilateral deviation to decrease the nominal interest rate from the Nash equilibrium would hurt the domestic economy and only benefit the foreign economy, and the cross-country spillover effects are quantitatively significant. For example, we find that a unilateral decrease in the nominal interest rate R^f in the Euro Area would improve welfare in the UK by 0.347% but reduce its own welfare by 0.028% despite a larger increase in the growth rate g^f in the Euro Area than the growth rate g^h in the UK. Intuitively, a decrease in R^f raises labor supply L^f via the CIA constraints. The resulting expansion in production in the Euro Area increases consumption in both countries, in which the increase in consumption in the UK is via the term $L_x^{h,f}$. It is useful to note that the welfare cost of decreasing leisure is borne by the Euro Area but by not the UK. As a result, the UK experiences a welfare gain whereas the Euro Area experiences a welfare loss. The opposite is true when the UK unilaterally decreases R^h .

	Δg^h	Δg^f	ΔU^h	ΔU^f
Cooperative policy ($R^h = R^f = 0$)	0.024%	0.022%	0.292%	0.443%
Unilateral policy ($R^f = 0$)	0.003%	0.019%	0.347%	-0.028%
Unilateral policy ($R^h = 0$)	0.020%	0.004%	-0.054%	0.471%

7 Conclusion

In this study, we have analyzed the growth and welfare effects of monetary policy in an open-economy version of the Schumpeterian growth model with CIA constraints on consumption and R&D investment. We find that economic growth and social welfare in the domestic economy are affected by both domestic and foreign monetary policies. Furthermore, the cross-country welfare effects of monetary policy are quantitatively significant. These spillover effects give rise to positive nominal interest rates in the Nash equilibrium and could prevent noncooperative governments from implementing optimal policies in the long run.

Finally, our analysis is based on a first-generation R&D-based growth model that features scale effects, under which a growing population gives rise to a rising growth rate.⁴³ We could

⁴²Changes in the growth rate are expressed as changes in percentage points. Welfare gains are expressed as the usual equivalent variation in consumption.

⁴³See Jones (1999) and Laincz and Peretto (2006) for a discussion of scale effects in R&D growth models.

allow for population growth and remove scale effects by introducing a dilution effect as in Chu and Cozzi (2013). Our results are robust to this modification. Nevertheless, it could be fruitful to explore the effects of monetary policy in other vintages of the Schumpeterian growth model, such as the semi-endogenous growth version of the Schumpeterian model in Segerstrom (1998) and the second-generation Schumpeterian growth model in Peretto (1998) and Howitt (1999). We leave these interesting extensions to future research.

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Appendix A

Proof of Proposition 1. Taking the log of the free-entry condition $\varphi^h V_t^h = (1 + R^h)W_t^h$ in the R&D sector and then differentiating it with respect to t yield

$$\frac{\dot{V}_t^h}{V_t^h} = \frac{\dot{W}_t^h}{W_t^h}. \quad (\text{A1})$$

Substituting (23) and then (5) into (A1) yields

$$\rho - \frac{\dot{\eta}_t^h}{\eta_t^h} + \lambda_t^h - \frac{\omega_t^h}{V_t^h} = \frac{\dot{W}_t^h}{W_t^h}, \quad (\text{A2})$$

where $\lambda_t^h = \varphi^h L_{r,t}^h$ from (25). Taking the log of (3) and differentiating with respect to t yield

$$-\frac{\dot{\eta}_t^h}{\eta_t^h} = \frac{\dot{P}_t^h}{P_t^h} + \frac{\dot{C}_t^h}{C_t^h}. \quad (\text{A3})$$

Substituting the balanced-trade condition $P_t^{h,f} Y_t^{h,f} = \mathcal{E}_t P_t^{f,h} Y_t^{f,h}$ into (21) and (22) yields

$$\omega_t^h = \frac{\mu - 1}{\mu} \left(P_t^{h,h} Y_t^{h,h} + P_t^{h,f} Y_t^{h,f} \right) = \frac{\mu - 1}{\mu} P_t^h C_t^h, \quad (\text{A4})$$

$$W_t^h L_{x,t}^h = \frac{1}{\mu} \left(P_t^{h,h} Y_t^{h,h} + P_t^{h,f} Y_t^{h,f} \right) = \frac{1}{\mu} P_t^h C_t^h, \quad (\text{A5})$$

where the second equality of (A4) and (A5) applies (7) and (8). Taking the log of (A5) and differentiating it with respect to t yield

$$\frac{\dot{W}_t^h}{W_t^h} = \frac{\dot{P}_t^h}{P_t^h} + \frac{\dot{C}_t^h}{C_t^h} - \frac{\dot{L}_{x,t}^h}{L_{x,t}^h}. \quad (\text{A6})$$

Substituting (A3) and (A6) into (A2) and then rearranging terms yield

$$\frac{\dot{L}_{x,t}^h}{L_{x,t}^h} = \frac{\omega_t^h}{V_t^h} - \varphi^h L_{r,t}^h - \rho. \quad (\text{A7})$$

Substituting $\varphi^h V_t^h = (1 + R^h)W_t^h$, (A4) and (A5) into (A7) yields

$$\frac{1}{\varphi^h} \frac{\dot{L}_{x,t}^h}{L_{x,t}^h} = \frac{\mu - 1}{1 + R^h} L_{x,t}^h - L_{r,t}^h - \frac{\rho}{\varphi^h}, \quad (\text{A8})$$

where $L_{r,t}^h = L_t^h - L_{x,t}^h$ from the resource constraint. Substituting (A5) into (4) yields

$$L_t^h = 1 - \mu \theta^h (1 + \xi^h R^h) L_{x,t}^h. \quad (\text{A9})$$

Substituting (A9) into the resource constraint on labor yields

$$L_{r,t}^h = 1 - \mu \theta^h (1 + \xi^h R^h) L_{x,t}^h - L_{x,t}^h. \quad (\text{A10})$$

Substituting (A10) into (A8) and then rearranging terms yield

$$\dot{L}_{x,t}^h = \varphi^h L_{x,t}^h \left\{ \left[\frac{\mu + R^h}{1 + R^h} + \mu\theta^h(1 + \xi^h R^h) \right] L_{x,t}^h - \left(1 + \frac{\rho}{\varphi^h} \right) \right\}, \quad (\text{A11})$$

which is a one-dimensional differential equation in $L_{x,t}^h$. Drawing $\dot{L}_{x,t}^h$ as a function of $L_{x,t}^h$ on a phase diagram, one can easily show that the dynamics of $L_{x,t}^h$ is characterized by saddle-point stability such that $L_{x,t}^h$ must jump to its interior steady-state value, which in turn implies that L_t^h and $L_{r,t}^h$ also jump to their steady-state values according to (A9) and (A10). An analogous proof would show that labor allocations in country f also jump to their steady-state values. ■

Proof of Proposition 2. Setting $\dot{L}_{x,t}^h = 0$ in (A11) yields the steady-state value of $L_{x,t}^h$ given by

$$L_x^h = \frac{1 + R^h}{\mu + R^h + \mu\theta^h(1 + \xi^h R^h)(1 + R^h)} \left(1 + \frac{\rho}{\varphi^h} \right). \quad (\text{A12})$$

Substituting (A12) into (33) and rearranging terms yield the steady-state value of $L_{r,t}^h$ in (37). Similarly, substituting (A12) into (34) yields the steady-state value of L_t^h in (40). Finally, we combine $L_x^{h,h} + L_x^{f,h} = L_x^h$ and (36) to derive the steady-state values of $L_x^{h,h}$ and $L_x^{f,h}$ in (38) and (39). ■

Proof of Proposition 4. The analogous expression of (47) for U^f is given by

$$\rho U^f = (1 - \alpha) \ln L_x^{f,f} + \alpha \ln L_x^{f,h} + \frac{(1 - \alpha)(\varphi^f \ln z^f)}{\rho} L_r^f + \frac{\alpha(\varphi^h \ln z^h)}{\rho} L_r^h + \theta^f \ln(1 - L^f). \quad (\text{A13})$$

The analogous expressions of (37)-(40) in country f are

$$L_r^f = \frac{\mu - 1}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)} \left(1 + \frac{\rho}{\varphi^f} \right) - \frac{\rho}{\varphi^f}, \quad (\text{A14})$$

$$L_x^{f,f} = (1 - \alpha) L_x^f = \frac{(1 - \alpha)(1 + R^f)}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)} \left(1 + \frac{\rho}{\varphi^f} \right), \quad (\text{A15})$$

$$L_x^{h,f} = \alpha L_x^f = \frac{\alpha(1 + R^f)}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)} \left(1 + \frac{\rho}{\varphi^f} \right), \quad (\text{A16})$$

$$L^f = 1 - \frac{\mu\theta^f(1 + \xi^f R^f)(1 + R^f)}{\mu + R^f + \mu\theta^f(1 + \xi^f R^f)(1 + R^f)} \left(1 + \frac{\rho}{\varphi^f} \right). \quad (\text{A17})$$

Under inelastic labor supply, we set $\theta^h = \theta^f = 0$ in (37)-(40) and (A14)-(A17). Then, we substitute the resulting expressions into $U^h + U^f$ from (47) and (A13) and differentiate it with respect to $\{R^h, R^f\}$ to obtain the optimal nominal interest rates given by

$$R_*^h = \max \left\{ \frac{\mu - \Omega^h}{\Omega^h - 1}, 0 \right\}, \quad (\text{A18})$$

$$R_*^f = \max \left\{ \frac{\mu - \Omega^f}{\Omega^f - 1}, 0 \right\}, \quad (\text{A19})$$

where $\Omega^h \equiv (1 + \varphi^h/\rho) \ln z^h > 1$ and $\Omega^f \equiv (1 + \varphi^f/\rho) \ln z^f > 1$. Therefore, $\{R_*^h, R_*^f\} = \{R_{ne}^h, R_{ne}^f\}$ in (53) and (54). As for the comparative statics, R_*^h is decreasing in Ω^h , which in turn is increasing in φ^h and z^h but decreasing in ρ . Similarly, R_*^f is decreasing in Ω^f , which in turn is increasing in φ^f and z^f but decreasing in ρ . ■

Proof of Proposition 5. In the absence of the CIA constraint on consumption, we set $\xi^h = \xi^f = 0$ in (37)-(40) and (A14)-(A17). The government in country h chooses R^h to maximize the welfare of the representative household in country h . We substitute (37), (38), (40), (A14) and (A16) under $\xi^h = \xi^f = 0$ into U^h in (47) and then differentiate it with respect to R^h to obtain the Nash-equilibrium nominal interest rate R_{ne}^h in country h given by (60). Similarly, the government in country f chooses R^f to maximize the welfare of the representative household in country f . We substitute (A14), (A15), (A17), (37) and (39) under $\xi^h = \xi^f = 0$ into U^f in (A13) and then differentiate it with respect to R^f to obtain the Nash-equilibrium nominal interest rate R_{ne}^f in country f given by (61). As for the comparative statics, R_{ne}^h in (60) is decreasing in Φ^h , which in turn is decreasing in α ; therefore, R_{ne}^h is increasing in α . Similarly, R_{ne}^f in (61) is decreasing in Φ^f , which in turn is also decreasing in α ; therefore, R_{ne}^f is also increasing in α . The cooperative governments choose $\{R^h, R^f\}$ to maximize the welfare of both domestic and foreign households. We set $\xi^h = \xi^f = 0$ in (37)-(40) and (A14)-(A17) and substitute the resulting expressions into $U^h + U^f$ from (47) and (A13). Then, we differentiate $U^h + U^f$ with respect to $\{R^h, R^f\}$ to obtain the optimal nominal interest rates given by (62) and (63), in which Ψ^h and Ψ^f are both independent of α . ■

Proof of Proposition 6. In the absence of the CIA constraint on R&D, the equilibrium labor allocations $\{L_r^h, L_x^{h,h}, L^h, L_r^f, L_x^{h,f}\}$ are given by (65)-(69). For the Nash-equilibrium nominal interest rate R_{ne}^h in country h , we substitute (65)-(69) into U^h in (47) and then differentiate it with respect to R^h . We find that

$$\frac{\partial U^h}{\partial R^h} = - \frac{\theta^h \xi^h (1 - \alpha) (1 + \rho/\varphi^h)}{\mu [1 + \theta^h (1 + \xi^h R^h)]^2} \left[\frac{(1 - \alpha) (1 + \xi^h R^h) - 1}{L_x^{h,h} (1 + \xi^h R^h)} + \frac{(\mu - 1) \varphi^h}{\rho} \ln z^h \right]. \quad (\text{A20})$$

Equation (A20) shows that as $\mu \rightarrow 1$, the Nash-equilibrium nominal interest rate is $R_{ne}^h = \alpha/[\xi^h (1 - \alpha)] > 0$, which is increasing in α . Furthermore, if μ is sufficiently large, then $\partial U^h/\partial R^h < 0$ for all values of R^h ; in this case, $R_{ne}^h = 0$. The analogous expressions of (65)-(69) for $\{L_r^f, L_x^{f,f}, L^f, L_r^h, L_x^{f,h}\}$ are

$$L_r^f = \frac{(\mu - 1)/\mu}{1 + \theta^f (1 + \xi^f R^f)} \left(1 + \frac{\rho}{\varphi^f} \right) - \frac{\rho}{\varphi^f}, \quad (\text{A21})$$

$$L_x^{f,f} = \frac{(1 - \alpha)/\mu}{1 + \theta^f (1 + \xi^f R^f)} \left(1 + \frac{\rho}{\varphi^f} \right), \quad (\text{A22})$$

$$L^f = 1 - \frac{\theta^f(1 + \xi^f R^f)}{1 + \theta^f(1 + \xi^f R^f)} \left(1 + \frac{\rho}{\varphi^f}\right), \quad (\text{A23})$$

$$L_r^h = \frac{(\mu - 1)/\mu}{1 + \theta^h(1 + \xi^h R^h)} \left(1 + \frac{\rho}{\varphi^h}\right) - \frac{\rho}{\varphi^h}, \quad (\text{A24})$$

$$L_x^{f,h} = \frac{\alpha/\mu}{1 + \theta^h(1 + \xi^h R^h)} \left(1 + \frac{\rho}{\varphi^h}\right). \quad (\text{A25})$$

For the Nash-equilibrium nominal interest rate R_{ne}^f in country f , we substitute (A21)-(A25) into U^f in (A13) and differentiate it with respect to R^f . We find that

$$\frac{\partial U^f}{\partial R^f} = -\frac{\theta^f \xi^f (1 - \alpha) (1 + \rho/\varphi^f)}{\mu [1 + \theta^f (1 + \xi^f R^f)]^2} \left[\frac{(1 - \alpha) (1 + \xi^f R^f) - 1}{L_x^{f,f} (1 + \xi^f R^f)} + \frac{(\mu - 1) \varphi^f \ln z^f}{\rho} \right]. \quad (\text{A26})$$

Equation (A26) shows that as $\mu \rightarrow 1$, the Nash-equilibrium nominal interest rate is $R_{ne}^f = \alpha/[\xi^f (1 - \alpha)] > 0$, which is increasing in α . Furthermore, if μ is sufficiently large, then $\partial U^f/\partial R^f < 0$ for all values of R^f ; in this case, $R_{ne}^f = 0$. As for the optimal nominal interest rates $\{R_*^h, R_*^f\}$, we add U^h from (47) and U^f from (A13) and substitute (65)-(69) and (A21)-(A25) into $U^h + U^f$. Then, we differentiate $U^h + U^f$ with respect to $\{R^h, R^f\}$ and find that

$$\frac{\partial(U^h + U^f)}{\partial R^h} = -\theta^h \xi^h \left[\frac{\xi^h R^h}{[1 + \theta^h (1 + \xi^h R^h)] (1 + \xi^h R^h)} + \frac{(\mu - 1) (1 + \varphi^h/\rho) \ln z^h}{\mu [1 + \theta^h (1 + \xi^h R^h)]^2} \right] < 0, \quad (\text{A27})$$

$$\frac{\partial(U^h + U^f)}{\partial R^f} = -\theta^f \xi^f \left[\frac{\xi^f R^f}{[1 + \theta^f (1 + \xi^f R^f)] (1 + \xi^f R^f)} + \frac{(\mu - 1) (1 + \varphi^f/\rho) \ln z^f}{\mu [1 + \theta^f (1 + \xi^f R^f)]^2} \right] < 0. \quad (\text{A28})$$

Therefore, given the zero lower bound on the nominal interest rates, it must be the case that $\{R_*^h, R_*^f\} = \{0, 0\}$. ■

Appendix B

In this appendix, we show the conditions under which $\{\Omega^h, \Omega^f, \Psi^h, \Psi^f, \Phi^h, \Phi^f\} > 1$. First, we derive the first-best allocations of labor. Combining (47) and (A13) yields

$$\begin{aligned} \rho(U^h + U^f) &= (1 - \alpha)(\ln L_x^{h,h} + \ln L_x^{f,f}) + \alpha(\ln L_x^{h,f} + \ln L_x^{f,h}) + \frac{\varphi^h \ln z^h}{\rho} L_r^h + \frac{\varphi^f \ln z^f}{\rho} L_r^f \\ &\quad + \theta^h \ln(1 - L^h) + \theta^f \ln(1 - L^f). \end{aligned} \quad (\text{B1})$$

The social planner maximizes (B1) subject to the following resource constraints:

$$L_x^{h,h} + L_x^{f,h} + L_r^h = L^h, \quad (\text{B2})$$

$$L_x^{f,f} + L_x^{h,f} + L_r^f = L^f. \quad (\text{B3})$$

The first-best optimal allocations of R&D labor and labor supply in country h are

$$L_r^h = 1 - \frac{\rho(1 + \theta^h)}{\varphi^h \ln z^h} > 0, \quad (\text{B4})$$

The parameter restriction on $L_r^h > 0$ implies $\Psi^h > 1$. To see this result, from (63),

$$\Psi^h > 1 \Leftrightarrow \left(1 + \frac{\varphi^h}{\rho}\right) \frac{\ln z^h}{1 + \theta^h} > 1. \quad (\text{B5})$$

From (B4), we see that

$$L_r^h > 0 \Leftrightarrow \left(\frac{\varphi^h}{\rho}\right) \frac{\ln z^h}{1 + \theta^h} > 1. \quad (\text{B6})$$

Therefore, if $L_r^h > 0$, then $\Psi^h > 1$. An analogous procedure in country f would show that if $L_r^f > 0$, then $\Psi^f > 1$. In the case of inelastic labor supply, we have

$$\Omega^h > 1 \Leftrightarrow \left(1 + \frac{\varphi^h}{\rho}\right) \ln z^h > 1, \quad (\text{B7})$$

$$L_r^h|_{\theta^h=0} > 0 \Leftrightarrow \left(\frac{\varphi^h}{\rho}\right) \ln z^h > 1. \quad (\text{B8})$$

Therefore, if $L_r^h|_{\theta^h=0} > 0$, then $\Omega^h > 1$. An analogous procedure for country f would show that if $L_r^f|_{\theta^f=0} > 0$, then $\Omega^f > 1$.

In the rest of this appendix, we consider the following alternative maximization problem: a social planner representing only country h unilaterally maximizes

$$\rho U^h = (1 - \alpha) \ln L_x^{h,h} + \alpha \ln L_x^{h,f} + \frac{(1 - \alpha)(\varphi^h \ln z^h)}{\rho} L_r^h + \frac{\alpha(\varphi^f \ln z^f)}{\rho} L_r^f + \theta^h \ln(1 - L^h) \quad (\text{B9})$$

subject to (B2) and the equilibrium condition in (36), which ensures that $L_x^{f,h}$ and $L_x^{h,f}$ are positive. In this case, the planner's allocation of R&D labor in country h is

$$L_r^h = 1 - \frac{\rho(1 - \alpha + \theta^h)}{(1 - \alpha)(\varphi^h \ln z^h)} > 0. \quad (\text{B10})$$

The parameter restriction on $L_r^h > 0$ implies $\Phi^h > 1$. To see this result, from (60),

$$\Phi^h > 1 \Leftrightarrow \frac{1 - \alpha}{1 - \alpha + \theta^h} \left(1 + \frac{\varphi^h}{\rho} \right) \ln z^h > 1. \quad (\text{B11})$$

From (B10), we see that

$$L_r^h > 0 \Leftrightarrow \frac{1 - \alpha}{1 - \alpha + \theta^h} \left(\frac{\varphi^h}{\rho} \right) \ln z^h > 1. \quad (\text{B12})$$

Therefore, if $L_r^h > 0$, then $\Phi^h > 1$. An analogous procedure in country f would show that if $L_r^f > 0$, then $\Phi^f > 1$.

Appendix C

In this appendix, we provide the equilibrium conditions for the general version of the model with duplication externality in R&D, where $\delta^h \in (0, 1)$ is the duplication externality parameter. Under $\delta^h \in (0, 1)$, (33) becomes

$$L_r^h = \frac{\mu - 1}{1 + R^h} L_x^h - \frac{\rho}{\varphi^h} (L_r^h)^{\delta^h}. \quad (\text{C1})$$

Equations (C1), (34) and (35) solve $\{L_r^h, L^h, L_x^h\}$. Then, (36) and $L_x^{h,h} + L_x^{f,h} = L_x^h$ solve $\{L_x^{h,h}, L_x^{f,h}\}$. There is a set of analogous equations in country f . Using (C1), (34) and (35), one can show that R&D labor L_r^h satisfies the following condition:

$$L_r^h + \frac{\rho}{\varphi^h} (L_r^h)^{\delta^h} = \frac{\mu - 1}{1 + R^h} \frac{1 - L_r^h}{1 + \mu\theta^h(1 + \xi^h R^h)}, \quad (\text{C2})$$

which implicitly determines the unique equilibrium value of L_r^h . A simple graphical analysis shows that $\partial L_r^h / \partial R^h < 0$. Furthermore, we have $\partial L_r^h / \partial R^f = 0$. By analogous inference, one can show that $\partial L_r^f / \partial R^f < 0$ and $\partial L_r^f / \partial R^h = 0$.

The steady-state equilibrium growth rate of consumption goods C_t^h in country h is

$$g^h \equiv \frac{\dot{C}_t^h}{C_t^h} = (1 - \alpha) \frac{\dot{Z}_t^h}{Z_t^h} + \alpha \frac{\dot{Z}_t^f}{Z_t^f} = (1 - \alpha)(\varphi^h \ln z^h)(L_r^h)^{1-\delta^h} + \alpha(\varphi^f \ln z^f)(L_r^f)^{1-\delta^f}. \quad (\text{C3})$$

Differentiating (C3) with respect to R^h yields

$$\frac{\partial g^h}{\partial R^h} = (1 - \alpha)(1 - \delta^h)(\varphi^h \ln z^h)(L_{r,t}^h)^{-\delta^h} \frac{\partial L_r^h}{\partial R^h} < 0, \quad (\text{C4})$$

which shows that the growth rate of consumption goods in country h is decreasing in the domestic nominal interest rate R^h . Differentiating (C3) with respect to R^f yields

$$\frac{\partial g^h}{\partial R^f} = \alpha(1 - \delta^f)(\varphi^f \ln z^f)(L_{r,t}^f)^{-\delta^f} \frac{\partial L_r^f}{\partial R^f} < 0, \quad (\text{C5})$$

which shows that g^h is also decreasing in the foreign nominal interest rate R^f . One can also obtain analogous results for the growth rate of consumption goods in country f .

Social welfare in country h in (47) can be reexpressed as

$$\rho U^h = (1 - \alpha) \ln L_x^{h,h} + \alpha \ln L_x^{h,f} + \frac{g^h}{\rho} + \theta^h \ln(1 - L^h), \quad (\text{C6})$$

where $\partial L_x^{h,f} / \partial R^h = 0$. Using (C6), one can show that the Nash-equilibrium nominal interest rate in country h is implicitly determined by the following condition:

$$\rho \frac{\partial U^h}{\partial R^h} = \frac{1 - \alpha}{L_x^{h,h}} \frac{\partial L_x^{h,h}}{\partial R^h} + \frac{1}{\rho} \frac{\partial g^h}{\partial R^h} - \frac{\theta^h}{1 - L^h} \frac{\partial L^h}{\partial R^h} = 0. \quad (\text{C7})$$

There is also an analogous condition that implicitly determines the Nash-equilibrium nominal interest rate in country f .