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Formation of SEZ, Agricultural Productivity and Urban Unemployment

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Abstract: Formation of SEZ using agricultural land to promote industrialization has recently been one of the most controversial policy issues in many developing economies including India. This paper critically theoretically evaluates the consequences of this policy in terms of a three-sector Harris-Todaro type general equilibrium model reasonable for a developing economy. It finds that agriculture and SEZ can grow simultaneously provided the government spends more than a critical amount on irrigation projects and other infrastructural development designed for improving the efficiency of land. Agricultural wage and aggregate employment in the economy may also improve.

JEL classifications: R13, R14, H54

Keywords: Special economic zone; fiscal concessions; agricultural productivity; rural wage; urban unemployment.

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1. Introduction

Special Economic Zones (SEZs) are specifically defined duty free enclaves and are considered to be foreign territory for the purposes of trade operations and duties and tariffs. Formation of SEZs is an important constituent of the new industrial and export policies of India during the liberalized regime. The overwhelming success of China achieved through formation of several SEZs\(^1\) has attracted the attention of the policymakers in the developing countries\(^2\) like India. The Union government policy on SEZ in India, came into effect in April 2000. It is the latest thinking so far on India’s export policy and may even represent the future of industrial development strategy. SEZs need to be seen in the context of attempts by the Government of India, to launch second-generation reforms and also a continuation of earlier initiatives to boost exports. Export Processing Zone, Export Oriented Units, Free Trade Zones etc. SEZs allow the government to experiment with radical (in the Indian context) economic reform in a sufficiently large geographical area (minimum size 5,000 hectares) but on a localized basis, without the difficulty of introducing such reforms at the national level. This may be viewed as an ambitious government plan to revitalize industry in its quest for 10 per cent annual economic growth.

It is argued that well-implemented and designed SEZ can bring about many desired benefits for a host-country: increases in employment, FDI attraction, general economic growth, foreign exchange earnings, international exposure, and the transfer of new technologies and skills. In just a year, the government of India approved nearly 400 SEZs, and received

\(^1\) See Li et al. (2005) for a detailed description of the role of SEZs in China’s overall economic development.

\(^2\) According to World Bank estimates, as of 2007 there are more than 3,000 projects taking place in SEZs in 120 countries worldwide.
proposals from state governments for 304 more. But the plans of the government came to a grinding halt after thousands of farmers across the country protested against the government's compulsory land acquisition at some sites, claiming they had not been adequately compensated. There are controversies regarding the role played by the state governments in snatching of arable land or buying land at much discounted rates from farmers or poor people and then selling it to corporate houses at exorbitant prices. The protesters insisted that the policy would not be able to create the number of jobs as suggested by the government. Besides, it is also argued that this procedure of industrialization would affect agriculture seriously given the land size of the economy. Such a dilemma has been observed in many predominantly agricultural countries that intend to industrialize using agricultural land.³

This has led to serious policy debate among economists and policymakers as to whether the SEZ policy is at all beneficial for the country. The major concerns are as follows: (i) Can industry (SEZs) and agriculture grow simultaneously without hurting one another (ii) Would this policy affect the unemployment situation adversely? (iii) How would the economic condition of the workers in the rural sector be affected due to this policy? As the experiment with this policy has just set off, it would be premature to find empirical answers to these queries. However, it is possible to predict the consequences of the SEZ policy by building up theoretical models based on the salient characteristics of a developing economy like India.

The present paper purports to provide answers to the above questions in terms of a three-sector Harris-Todaro (HT hereafter) type general equilibrium model with an SEZ located in the rural sector. The analysis of the paper finds that agriculture and SEZ can grow simultaneously under certain conditions. However, in the absence of any government spending on irrigation projects and other infrastructural development designed for improving the efficiency of land, formation of the SEZ affects agriculture adversely. It is also found that if agriculture receives government assistance greater than a critical level both agricultural

wage and aggregate employment in the economy may improve. Thus, a certain balance has to be maintained between agriculture and industry so as to reap the full benefits of the SEZ policy.

2. The model and assumptions

Let us consider a small open economy in which there are three sectors. One sector is an agricultural sector and produces good 1, \( X_1 \). The second sector is an industry in the SEZ located in the rural sector that produces an industrial good, \( X_2 \). The third sector is the urban manufacturing sector and produces good 3, \( X_3 \). It is supposed that sector 1 uses labor and land, sector 2 uses labor, land and capital, and sector 3 uses labor and capital.

The notations and the equations of the GE model are as follows;

\[ W = \text{rural sector wage}; \]
\[ R = \text{return to land in physical unit (say, per acre)}; \]
\[ h = \text{productivity of land}; \]
\[ R = \text{return to land in efficiency unit}; \]
\[ hR = \text{return to land in physical unit}; \]
\[ r = \text{return to capital}; \]
\[ W^\ast = \text{unionized urban wage of labour}; \]
\[ S = \text{total subsidy or govt. expenditure}; \]
\[ \alpha S = \text{subsidy given to SEZ with } \alpha; \]
\[ (1-\alpha)S = \text{Govt. expenditure for improving productivity of land}; \]
\[ s = \text{rate of price subsidy given to encourage formation of SEZ}; \]
\[ \bar{N} = \text{given endowment of land in physical unit}; \]
\[ h(.)\bar{N} = \text{land endowment in efficiency unit}. \]

Under perfect competition, we have

\[ Wa_{11} + Ra_{N1} = 1 \quad (1) \]
\[ Wa_{22} + h(.)Ra_{N2} + ra_{X2} = P_2(1+s) \quad (2) \]
\[ W^\ast a_{33} + ra_{X3} = P_3 \quad (3) \]
where $a_{ij}$ is the amount of the $i$th factor used in the $j$th industry to produce one unit of the output and $p_j$ is the price of the $j$th good ($j = 1,2,3$). Note that $s$ is the rate of price subsidy given to encourage formation of the SEZ.

$$\beta S = sP_2X_2, \text{ i.e. } (\beta S / P_2X_2) = s.$$ 

We will assume that subsidy is given to both the agricultural sector and the SEZ. A part of the subsidy is gone to agriculture for improving the efficiency of land. This assumption is justified on the ground that the government in a developing economy spends a substantial amount on major irrigation projects, construction of roads and for building up social infrastructure that raises the productivity of land. Simultaneously, in line with its new industrial policy sizeable amount of fiscal concessions are given on several occasions for encouraging formation of the SEZ. If $S$ be the aggregate government expenditure or subsidy, the fraction $\beta$ of it is given to the SEZ while the remaining $(1-\beta)$ fraction is spent for improvement of agriculture. It is supposed that land efficiency, $h$, is improved by the government spending on irrigation projects etc. Thus, we write

$$h = h((1-\beta)S); h = \bar{h} \text{ for } (1-\beta)S = 0 \text{ i.e. } \beta = 1; \text{ and, } h' > 0 \text{ for } (1-\beta)S > 0.$$ 

When the government spends nothing for agricultural development i.e. when $\beta = 1$, $h = \bar{h}$ (given exogenously). However, if the government’s expenditure on agriculture is positive i.e. if $\beta < 1, h' > 0$; and, $h > \bar{h}$.

We assume that all goods are tradable and then their prices are internationally given. It is also assumed for the sake of analytical simplicity that $a_{N2}$ is technologically given. Note that $hR$ is the return to land in physical unit.

In this model, it is assumed that the wage rate in urban (manufacturing) sector 3 ($W^*$) is set at a relatively high level, and it is rigid due to some political and/or institutional considerations, while the wage rate in rural (agricultural and SEZ) sectors ($W$) is flexible. In this situation, the rural workers have two alternatives of staying in rural areas in order to obtain a job at a low wage rate or migrating to urban area in order to seek a high wage income at the risk of unemployment. Thus, the labor allocation mechanism between the sectors is shown as follows:

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4 In fact one of the incentives for formation of SEZ in the rural areas is the ready availability of labour at low wage rates.
where $a_{L3}X_3$ and $L_U$ are the employed and unemployed labor in the urban sector, respectively. In the labor market equilibrium, therefore, the wage rate in the rural sectors ($W$) equals the expected wage income in sector 3, which equals the manufacturing wage rate ($W^*$) times the probability of finding a job in the urban manufacturing sector ($a_{L3}X_3/(a_{L3}X_3 + L_U)$).

Exogenously given endowments impose the following resource constraints:

$$a_{K2}X_2 + a_{K1}X_1 = K \quad (5)$$
$$a_{N1}X_1 + ha_{N2}X_2 = h\bar{N} \quad (6)$$
$$a_{L1}X_1 + a_{L2}X_2 + a_{L3}X_3 + L_U = L \quad (7)$$

where $K$ and $L$ are the endowments of labor and capital, respectively, while $\bar{N}$ is given endowment of land in physical unit and $h\bar{N}$ is land endowment in efficiency unit. We obtain from (4) and (7)

$$(W^*/W)a_{L3}X_3 + a_{L2}X_2 + a_{L3}X_3 = L. \quad (8)$$

This completes the specification of our model with the fixed endowment of factors and the internationally determined prices. We have six unknown variables, $W$, $R$, $r$, $X_1$, $X_2$, and $X_3$, which are solved by six equations (1)–(3), (5), (6) and (8) for given parameters, $W^*$, $P_2$, $P_3$, $L$, $K$, $\bar{N}$, $\beta$ and $S$.

3. Subsidy and production

Now let us examine the effects of subsidy on the change in outputs. Differentiating equations (1) – (3), (5), (6) and (8), we obtain
\[
\begin{bmatrix}
\theta_{L1} & \theta_{N1} & 0 & 0 & 0 \\
\theta_{L2} & \theta_{N2} & 0 & \bar{s} & 0 \\
B & 0 & 0 & \lambda_{K2} & \lambda_{K3} \\
C & D & \lambda_{N1} & \lambda_{N2} & 0 \\
F & G & \lambda_{L1} & \lambda_{L2} & \bar{\lambda}_{L3}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\dot{\bar{R}} \\
\dot{\bar{X}}_1 \\
\dot{\bar{X}}_2 \\
\dot{\bar{X}}_3
\end{bmatrix}
= \begin{bmatrix}
0 \\
A \\
E \\
0 \\
0
\end{bmatrix}
\]  
(9)

where \( \bar{s} = s/(1+s) \), \( \bar{\lambda}_{L3} = (W^*/W)\lambda_{L3} \), \( A = \bar{s} - \theta_{N2}\bar{e}_k \), \( B = \lambda_{K2}S_{KL}^2 > 0 \), \( C = \lambda_{N1}S_{NL}^1 > 0 \), \( D = \lambda_{N1}S_{KN}^1 < 0 \), \( E = \lambda_{N1}\bar{e}_k \geq 0 \), \( F = (\lambda_{L1}S_{LL}^1 + \lambda_{L2}S_{LL}^2 - \bar{\lambda}_{L3}) < 0 \), \( G = \lambda_{L1}S_{LN}^1 > 0 \).

The value of the determinants of the coefficient matrix of the system (9) is

\[
\Delta = |\theta|\left[-\lambda_{K3}\lambda_{L1}\lambda_{L2}\lambda_{L3} + \bar{s}\lambda_{L3}\lambda_{N1}(\theta_{L1}D - \theta_{N1}C) \\
+ \lambda_{N1}\lambda_{L3}(\theta_{N1}F - \theta_{L1}G) + \bar{\lambda}_{L3}\theta_{N1}B]\right] 
\]  
(10)

where

\[
|\theta| = \theta_{L1}\theta_{N2} - \theta_{L2}\theta_{N1} \quad \text{and} \quad |\lambda| = \lambda_{L1}\lambda_{N2} - \lambda_{L2}\lambda_{N1}
\]

It can be seen that \( \Delta < 0 \) if sector 1 is labour-intensive than sector 2 with respect to land\(^5\), i.e. \( |\theta| > 0 \) and \( |\lambda| > 0 \).

[A] The change in output 2

We are interested in the changes in outputs of three sectors. First, consider the effect of the subsidy on the output of the SEZ.

We obtain from (9)

\[
(\dot{\bar{X}}_2/\dot{\bar{S}})/[-\lambda_{L1}\lambda_{K3}E|\theta| + A(\lambda_{L1}\lambda_{K3}(\theta_{L1}D - \theta_{N1}C) \\
+ \lambda_{N1}\lambda_{K3}(\theta_{N1}F - \theta_{L1}G) - \bar{\lambda}_{N1}\bar{\lambda}_{L3}\theta_{N1}B)]/\Delta 
\]  
(11)

\(^5\) It may be noted that the SEZ use both labour and land in lesser quantities than agriculture. Despite this, the SEZ can use anyone of those two factors more intensively with respect to the other than the other sector.
This leads to the following proposition.

**Proposition 1:** The subsidy increases the output of the SEZ if $A \geq 0$.

The subsidy increases land in efficiency term. This increases the output of sector 2 through the *Rybczynski effect* under the assumption that sector 2 is land-intensive with respect to labour. The first term of (12) captures this effect. Note that this effect vanishes if $\beta = 0$ (i.e. $E=0$) and no subsidy is allocated to improve land efficiency.

The terms in the curly brackets capture the usual resource-reallocation effect (*Rybczynski-type effect*) following the *Stolper-Samuelson effect* which arises due to the subsidy given to sector 2 (SEZ). That is, the subsidy increases the producer’s price, and then it expands the production of sector 2. The larger the share of subsidy given to SEZ (i.e. the larger the value of $A$), the larger would be the increase in the output.

[B] The change in output 1

Now, let us turn to examine the effect on output 1. Solving (9) for $\hat{X}_1$ with respect to $\hat{S}$, we obtain

$$
\frac{\hat{X}_1}{\hat{S}} = \frac{-\Lambda \{E \theta + A(\theta_{N1}C - \theta_{L1}D)\} + (\lambda_{N2}A - \bar{s}E) \times \{\theta_{N1}(\bar{L}_3, -\lambda_{K3}F) + \theta_{L1}\lambda_{K3}G\} / \Delta}{\Lambda = \lambda_{K2}\lambda_{L3} - \lambda_{K3}\lambda_{L2}}.
$$

where $\Lambda = \lambda_{K2}\lambda_{L3} - \lambda_{K3}\lambda_{L2}$. Thus, the following proposition is immediate.

**Proposition 2:** The subsidy increases the output of sector 1 if (i) $\bar{s} \geq \lambda_{N2}A / E$, (ii) $A \geq 0$ and (iii) sector 2 is capital-intensive vis-à-vis sector 3 with respect to labour, i.e. $\Lambda > 0$.

As discussed previously, the subsidy given to agriculture raises the efficiency of land and hence the land endowment of the economy in efficiency unit. This increases the output of sector 2 through the *Rybczynski effect* if this sector is land-intensive than sector 1. This in turn demands capital and labour from sector 3. Consequently sector 3 contracts leading to reverse migration of labour from the urban to the rural sector. A large amount of labour is
released from sector 3 because sector 3 is labour-intensive than sector 2 with respect to capital. Sector 2 cannot absorb all the workers who have migrated back to the rural sector. This causes the output of sector 1 to expand since sector 1 is labour-intensive relative to sector 2. However, this is possible if sector 1 gets a sizeable amount of the subsidy and the land efficiency is sufficiently increased which is guaranteed by the condition: \( \bar{\sigma} \geq \lambda_{N_2} \frac{A}{E} \).

In the extreme case where the agricultural sector does not get any subsidy, \( E = 0 \); and, from (12) it follows that the subsidy policy designed to encourage formation of the SEZ leads to an unambiguous contraction of the agricultural sector.

The subsidy given to SEZ also expands the production of sector 2. This effect enforces the movement of resources, and it raises the output of sector 1 under the conditions as stated in the proposition.

[C] The change in output 3

Finally, we consider the change in output in sector 3. From (9) we obtain the following expression.

\[
\frac{\dot{X}_3}{\dot{S}} = \left[ \lambda_{l_1} \lambda_{k_2} E [\theta] + \lambda_{l_1} \theta_{n_1} B (\bar{\sigma} E - \lambda_{N_2} A) + A (\lambda_{n_1} \lambda_{l_2} \theta_{n_1} B \\
+ \lambda_{k_2} \theta_{n_1} (\lambda_{l_1} C - \lambda_{N_1} F) - \lambda_{k_2} \theta_{1_1} A (\lambda_{l_1} D - \lambda_{n_1} G) ) \right] / \Delta .
\]

(13)

The result is summarized as follows.

**Proposition 3**: The subsidy decreases the output of sector 3 if (i) \( \bar{\sigma} \geq \lambda_{N_2} \frac{A}{E} \) and (ii) \( A \geq 0 \).

Note that the output of sector 1 increases if (i) \( \bar{\sigma} \geq \lambda_{N_2} \frac{A}{E} \), (ii) \( A \geq 0 \) and (iii) sector 2 is capital intensive than sector 3 with respect to labour (i.e. \( \Lambda > 0 \)), while that of sector 2 increases if \( A \geq 0 \). Thus, both outputs increase under the condition stated in Proposition 3. This implies that the expansion of sector 2 absorbs capital from sector 3. Labour is also reallocated from sector 3 to both sectors 1 and 2. This induces the contraction of sector 3.

Thus the conditions (i) \( \bar{\sigma} \geq \lambda_{N_2} \frac{A}{E} \) and (ii) \( A \geq 0 \) play very crucial role in determining the effects of the subsidy policy on the production structure. Let us examine the
relation between the two conditions. Considering the definitions of $\bar{r}, A$ and $E$, and arranging terms we obtain that

$$\frac{\lambda_{N2}M}{(\lambda_{N2}H - \lambda_{N1}\bar{e}_h)S} \geq \beta \geq \frac{M}{HS}$$

(13)

where $M = \theta_{N2}e_h P_2 X_2$ and $H = (1 - \theta_{N2}e_h) > 0$. Note that $\beta$ is the fraction of the subsidy allocated to SEZ. Thus, two conditions are summarized in the condition that the ratio is between the two values as shown in (13). This implies that the value of $\beta$ constitutes an important part of the sufficient conditions for outputs of sectors 1 and 2 to increase while that of sector 3 to decrease.

4. Subsidy, wage rate and unemployment

The effects of the subsidy on the wage rate and unemployment are also extremely important in a developing economy like India where chronic unemployment and poor economic conditions of the informal sector workers are two of the major disconcerting problems. We will investigate how these two are affected by the subsidy policy and examine the relation between the production structure and the labour market.

[A] The change in the wage rate

From (9) we obtain the effect of the subsidy on the wage rate as

$$(\hat{W} / \hat{S}) = \theta_{N1}[\lambda_{N1}A\bar{A} + \lambda_{N1}A_{K3}(\lambda_{N2}A - \bar{r}E)]/\Delta,$$

(14)

where $\bar{A} = \lambda_{K2}\bar{r}_{L3} - \lambda_{K3}\bar{r}_{L2}$. The result is summarized in terms of the following proposition.

**Proposition 4:** The subsidy increases the wage rate if (i) $\bar{r} \geq \lambda_{N2}A / E$; (ii) $A \geq 0$; and, if (iii) $\bar{r}_{L3} \cong 0$.

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6 See the appendix for the detailed derivation.
The subsidy leads to expansion of sectors 1 and 2 under the conditions (i) and (ii) as stated above. The displaced workers from the urban sector flock to agriculture while the latter raises its demand for labour as it expands which is permissible by the increase in the effective land endowment of the economy. So two opposite effects work on the rural sector competitive wage, $W$. The possibility for $W$ to increase rises with a decrease in the number of workers released by sector 3. If the allocative share of labour in sector 3 is sufficiently small, i.e. $\lambda_{k,3} \approx 0$ and the other two sufficient conditions hold, the competitive wage rate rises owing to the subsidy policy. From (14) it is also evident that $W$ rises even if sector 2 is labour-intensive relative to sector 3. However, in this case the possibility for the agricultural sector to contract is also higher.

[B] The change in unemployment

Finally, let us examine the effect of the subsidy on unemployment. Differentiating (4) and using (13) and (14), we obtain

\[
\frac{\dot{L}_U}{\dot{S}} = \lambda_{L,U}[\lambda_{k,2}E\lambda_{k,2} - \lambda_{k,2}\theta_{l,1}A(\lambda_{l,1}D - \lambda_{N,1}G) + \lambda_{l,1}\theta_{N,1}(BE - \lambda_{N,2}A)(B + \lambda_{L,3}\lambda_{k,3} / \lambda_{L,U}) + \lambda_{k,2}\theta_{N,1}A(\lambda_{l,1}C - \lambda_{k,1}F) + \lambda_{N,1}\theta_{N,1}A(\lambda_{l,2}B - (\lambda_{L,3} / \lambda_{L,U})\lambda)] / \Delta
\]

where $\lambda_{L,U} = L_U / L$. From (15) the following proposition readily follows.

\textbf{Proposition 5:} The subsidy decreases unemployment if (i) $\bar{\kappa} \geq \lambda_{N,2}A / E$, (ii) $A \geq 0$ and (iii) $\lambda_{L,3} \approx 0$.

In the HT framework the consequence of any policy on urban unemployment crucially hinges on the relative strengths of the centrifugal and centripetal forces. Conditions (i) and (ii) as stated in proposition 5 are sufficient for the expansion of sectors 1 and 2 and contraction of sector 3 following the subsidy policy. Labour and capital move from sector 3

\footnote{That sector 2 is labour-intensive vis-à-vis sector 3 with respect to capital in the value sense (i.e. $\Lambda < 0$) implies that sector 2 is labour intensive than sector 3 also in the physical sense, (i.e. $\Lambda < 0$) where $\Lambda = (\lambda_{k,2}\lambda_{L,3} - \lambda_{k,3}\lambda_{k,2}) < 0$; and, $\Lambda = (\lambda_{k,2}\lambda_{L,3} - \lambda_{k,3}\lambda_{k,2}) < 0$.}
to the two rural sectors. As sector 3 contracts both in terms of output and employment the expected urban wage decreases for each worker thereby leading to a reverse migration from the urban to the rural sectors. Therefore the strength of the centrifugal force that draws rural workers into the urban sector weakens. On the other hand, the availability of more workers in the rural sectors exerts a downward pressure on the rural wage, $W$. On the contrary, there will also be a positive effect on $W$ as both the rural sectors expand. The second effect outweighs the first negative effect either if the allocative share of labour in sector 3 is significantly low (i.e. $\lambda_{z3} \approx 0$) or if sector 3 is capital-intensive. Thus the strength of the centripetal force that keeps the workers in the rural sector increases as $W$ rises. Thus we find that if the sufficient condition (iii) is satisfied both the centrifugal and centripetal forces work in the same direction and the additional workers in the rural workers are supplied from the pool of unemployed workers. Consequently, the level of urban unemployment falls.

5. Concluding remarks

Provision of agricultural land and fiscal concessions for setting up of SEZ have been considered to be the new industrial policy in the labour-abundant and predominantly agricultural economies like India. The advocates of the SEZ policy believe that this will usher in an industrial revolution and lead to an overall economic development of the country. Although the process was initiated with much enthusiasm it came to a standstill due to protests by the opposition parties, farmers and certain academicians including economists. The opponents believe that the costs of maintaining SEZ privileges are not offset by benefits to the rest of the economy. It is easily understandable that given the land size of the economy, the formation of SEZs using agricultural land is bound to hurt agriculture and the people dependent on it. An attempt has been made in this paper to theoretically challenge this common wisdom using a three-sector Harris-Todaro type model. The paper finds that it is possible for both the agricultural sector and the SEZ to grow simultaneously if the subsidy policy of the government is designed in an appropriate way. A part of the subsidy must be spent on irrigation projects and other infrastructural development that raise the productivity of land and hence the effective land endowment of the economy. The fraction of the
aggregate subsidy allocated to the SEZ must lie between two critical values which in turn depend on the institutional, technological and trade-related factors of the economy. The unemployment problem and the economic conditions of the common people may also improve in the process. The final outcomes, of course, rely much on the political will of the government.

Appendix

Here we derive (13) in the main text. By definitions of \( \bar{s} \), \( A \) and \( E \), \( \bar{s} \geq \lambda_{N_2} A / E \) can be written as

\[
\frac{s}{1 + s} \geq \lambda_{N_2} \left( \frac{s}{1 + s} - \theta_{N_2} \epsilon_h \right) / \lambda_{N_1} \epsilon_h \quad (A1)
\]

or

\[
\left( \frac{s}{1 + s} \left( \frac{\lambda_{N_2} - \lambda_{N_1} \epsilon_h}{\lambda_{N_1} \epsilon_h} \right) \right) \leq \frac{\lambda_{N_2}}{\lambda_{N_1}} \theta_{N_2} \quad (A2)
\]

Considering \( s = \beta S / P_2 X_2 \), we obtain the LHS of equation (13) that

\[
\left[ \frac{\lambda_{N_2} \theta_{N_2} \epsilon_h P_2 X_2}{(\lambda_{N_2} - \lambda_{N_1} \epsilon_h - \lambda_{N_2} \theta_{N_2} \epsilon_h) S} \right] \geq \beta \quad (A3)
\]

From (A3) it follows that

\[
(\lambda_{N_2} - \lambda_{N_1} \epsilon_h) > 0; \text{ and } (1 - \theta_{N_2} \epsilon_h) > 0. \quad (A4)
\]

(A3) is rewritten as follows.

\[
\frac{\lambda_{N_2} M}{(\lambda_{N_2} H - \lambda_{N_1} \epsilon_h) S} \geq \beta \quad (A5)
\]

where \( M = \theta_{N_2} \epsilon_h P_2 X_2 \) and \( H = (1 - \theta_{N_2} \epsilon_h) > 0 \).
Now it can be seen that $A \geq 0$ implies

$$s \geq \theta_{N^2} e_h (1 - \theta_{N^2} e_h) . \quad (A6)$$

Considering $s = \beta S / P_2 X_2$ again, (A6) may be written as

$$\frac{\beta S}{P_2 X_2} \geq \frac{\theta_{N^2} e_h}{(1 - \theta_{N^2} e_h)} \quad (A7)$$

or equivalently,

$$\beta S \geq \frac{\theta_{N^2} e_h P_2 X_2}{(1 - \theta_{N^2} e_h)} = \frac{M}{H} . \quad (A8)$$

Thus, from (A5) and (A8), we obtain equation (13) in the text.

References:


