A Multi-sectorial Assessment of the Static Harrod Foreign Trade Multiplier

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Abstract

With this inquiry we seek to develop a multi-sectorial version of the static Harrod foreign trade multiplier, by showing that indeed it can be derived from an extended version of the Pasinettian model of structural change to international trade. This new version highlights the connections between balance of payment and the level of employment and production. It is also shown that departing from this disaggregated version of the Harrod foreign multiplier we can arrive at the aggregated version thus proving the consistency of our analysis. By following this approach we go a step further in establishing the connections between the Structural Economic Dynamic and Balance-of-Payments Constrained Growth approaches.

Keywords: structural economic dynamics; foreign trade multiplier; balance-of-payments constrained growth.

JEL classification: O19, F12

1. Introduction

This article deals with the relationship between income determination and balance of payments equilibrium in a structural economic dynamic setting. In particular, the article delivers a multi-sectorial version of the Harrod foreign trade multiplier [Harrod (1933)] by showing that it can be derived from an extended version of the Pasinettian model (1993) that takes into account foreign trade [Araujo and Teixeira (2004)]. The disaggregated Harrod foreign trade multiplier is shown to keep the original flavor of the aggregated version since it predicts that the output of each sector is strongly affected by its export ability, which highlights the validity of the original Harrod's insight not only at an aggregated level.

Besides, in order to prove the consistency of our approach we also show that departing from the multi-sectoral Harrod foreign trade multiplier we can obtain the aggregated version, with emphasis on the role played by the structure on determining the output performance. With this approach, we intend to emphasize the view that in the presence of a favorable economic structure a country may enjoy a higher level of output, which may be reached through relaxing the balance of payments constraint.

The SED framework is adopted as the starting point for our analysis. Initially this model was conceived for studying the interactions between growth and structural change in a closed economy [see Pasinetti (1981, 1993)]. However, more recently it was formally extended to take into account international flows of goods [see Araujo and Teixeira (2003, 2004)], and a balance of payments constrained growth rate was derived in this set up under the rubric of a multi-sectoral Thirlwall's law [see Araujo and Lima (2007)]. Such extensions have proven that the insights of the Pasinettian analysis remain valid for the case of an open economy: the interaction between tastes and technical change is responsible for variations in the structure of the economy, which by its turn affect the overall growth performance.

This view is also implicit in the Balance-of-Payments Constrained Growth (BoP) approach to the extent that variations in the composition of exports and imports lead to changes in the structural of the economy and determine the output growth consistent with balance of payments equilibrium [See Thirlwall (2013)]. The BoP approach asserts that assuming that real exchange rates are constant and that trade must be balanced in the long run, there is a very close correspondence between the growth rate of output and the ratio of the growth of exports to the income elasticity of demand for imports. Indeed, this result is the prediction of a dynamic version of the Harrod trade multiplier (1933).

It can also be argued that the particular dynamics of technical change and patterns of demand is taken into account in the BoP approach since observed differences in the income elasticities of demand for exports and imports reflect the non-price characteristics of goods and, therefore, the structure of production [Thirlwall (1997, p. 383)]. But in fact, by departing from the aggregated Keynesian model, the literature on both the static and dynamic Harrod foreign trade multiplier is advanced in terms of an aggregated economy, in which it is not possible to fully consider particular patterns of demand and productivity for different goods.

Harrod (1933) considers an open economy with neither savings and investment nor government spending and taxation. In this set-up income, Y, is generated by the production of consumption goods, C, and exports, X, namely: $Y = C + X$. It is assumed that all income is spent on consumption goods and imports (M), such that $Y = C + M$. The real terms of trade are constant and balanced trade is assumed: $X = M$. If we assume a linear import function such as $M = mY$, where m is the marginal propensity to import, then we have after some algebraic manipulation:

$$Y = \frac{1}{m} X \tag{1}$$

Expression (1) is known as the static Harrod foreign trade multiplier[1]. According to it the main constraint to income determination is the level of export demand in relation to the propensity to import. McCombie and Thirlwall (1994, p. 237) claim that "Harrod put forward the idea that the pace and rhythm of industrial growth in open economies was to be explained by the principle of the foreign trade multiplier which at the same time provided a mechanism for keeping the balance-of-payments in equilibrium." Any change in X brings the balance trade back into equilibrium through changes in income and not in relative prices. According to this view the Harrod foreign trade multiplier is an alternative to the Keynesian determination of income through the investment multiplier.

The subsequent development of Harrod's analysis was to study the growth implications of his model but as pointed out by Thirlwall (2013, p. 83), Harrod himself never managed to accomplish this task. It has been carried out by a number of authors who departed from a revival of the idea and significance of the Harrod original insight by Kaldor (1975). [see e.g. Thilwall (1979), McCombie (1985) and Setterfield (2010)]. Probably the main outcome of this strand is built in terms of a dynamic version of the Harrod foreign trade multiplier that became known in the literature as Thirlwall's Law [McCombie and Thirlwall (2004)]. Professor A. Thirlwall (1979) has turned the Harrod multiplier into a theory of balance of payments constrained growth, in which the growth process is demand led rather than supply constrained. According to him, assuming that real exchange rates are constant

[1] The dynamic Harrod foreign trade multiplier is connected to the Hicks supermultiplier. While the former considers just the straight impact of the growth rate of exports on the growth rate of output the latter also takes into account the feedbacks that a higher growth rate of exports has on other components of autonomous expenditures. According to McCombie (1985, p. 63) "(…) an increase in exports will allow other autonomous expenditures to be increased until income has risen by enough to induce an increase in imports equivalent to the initial increase in exports".

and that trade must be balanced in the long run, there is a very close correspondence between the growth rate of output and the ratio of the growth of exports to the income elasticity of demand for imports. Indeed, this result may be obtained from expression (1):

$$\frac{\Delta Y}{Y} = \frac{1}{m}\frac{\Delta X}{X} \tag{2}$$

According to this expression the growth rate of ouput, namely $\frac{\Delta Y}{Y}$, is related to the growth rate of exports, that is $\frac{\Delta X}{X}$, by the inverse of the propensity to import, represented by *m*. Thus in a balanced trade framework with the real terms of trade constant, countries are constrained to grow at this rate, which in its continuous time version became widely known in the literature as the Thirlwall law[2]. According to this view the balance of payments position of a country is the main constraint on its growth rate, since it imposes a limit on demand to which supply can (usually) adapt. As it turns out, observed differences in growth performance between countries are associated with the particular elasticities of demand for exports and imports.

In this context, structural change registers as one of the sources for changes in the elasticity of income of exports and imports. Arguably, a country whose structure is concentrated on sectors that produce raw materials, for instance, will have a lower income elasticity of demand for exports than a country specialized in the production of sophisticated goods. From this perspective we may conclude that the policy implications from the SED and

[2] According to McCombie (1985, p. 71) the conciliation between Thirlwall's law and the dynamic foreign trade multiplier is not so straight since the former is based on a multiplicative import function while the latter is built in terms of a linear import function.

the BoP approaches are similar: underdeveloped countries should pursue structural changes in order to produce and export goods with higher income elasticity of demand.

Previous attempts to establish connections between these two strands have been proven fruitful. Results such as the multi-sectoral version of Thirlwall's law [Araujo and Lima (2007)] and the disaggregated version of the cumulative model [Araujo (2013) and Araujo and Trigg (2013)] have shown that demand, captured mainly by income elasticities, plays a central role in determining the growth rates even in the long run. These developments have shown that a disaggregated assessments of well establish results of that literature may give rise to new insights.

In order to carry out the present analysis we have adopted a procedure analogous to the one advanced by Trigg and Lee (2005) and extended by Araujo and Trigg (2013) to consider international trade. The former authors explore the relation between the Keynesian multiplier and Pasinetti's model of pure production in a closed economy, by showing that indeed it is possible to derive a simple multiplier relationship from multisectoral foundations in a closed version of the Pasinetti model, meaning that a scalar multiplier can legitimately be applied to a multisector economy. By departing from this result, Araujo and Trigg (2013) have derived an initial formulation of the disaggregated Harrod foreign trade multiplier.

Here we go a step further by showing that the equilibrium Pasinettian solution for the system of physical quantities may be obtained as a particular case of the solution given by multi-sectoral Harrod foreign trade multiplier derived here when the condition of trade balance is satisfied. Finally, in order to prove the consistency of our approach we show that departing from this disaggregated version of the Harrod foreign trade multiplier we can obtain the aggregated version.

This article is structured as follows: in the next section we highlight the relevance of the Harrod foreign trade multiplier not only by deriving it but also trying to emphasize its relevance. Section 3 performs the derivation of a multisectoral version of this multiplier and sector 4 concludes.

2. Systems of physical and monetary quantities in an extended version of the Pasinettian Model to International Trade

Let us consider an extended version of the pure labour Pasinettian model to foreign trade as advanced by Araujo and Teixeira (2004). Demand and productivity vary over time at a particular rate in each sector of the two countries – the advanced one is denoted by *A* and the underdeveloped one by *U*. Assume also that both countries produce *n* – 1 consumption goods in each vertically integrated sector, but with different patterns of production and consumption. In order to establish the basic notation, it is useful to choose one of the countries, let us say *U*, to express physical and monetary flows. The system of physical quantities may be expressed as:

$$\begin{bmatrix} \mathbf{I} & -(\mathbf{c}+\xi\mathbf{c}^{e}) \\ -\mathbf{a} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ X_n \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix} \qquad (3)$$

where **I** is an (*n*–1)x(*n*–1) identity matrix, **0** is an (*n*–1) null vector, $\mathbf{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_{n-1} \end{bmatrix}$ is the (*n*–1) column vector of physical quantities, $\mathbf{c} = \begin{bmatrix} a_{1n} \\ \vdots \\ a_{n-1,n} \end{bmatrix}$ is the (*n*–1) column vector of consumption coefficients, $\mathbf{c}^{\mathrm{e}} = \begin{bmatrix} a_{1\hat{n}} \\ \vdots \\ a_{n-1,\hat{n}} \end{bmatrix}$ refers to the (n–1) column vector of foreign demand coefficients, and $\mathbf{a} = \begin{bmatrix} a_{n1} & \cdots & a_{n-1,n} \end{bmatrix}$ is the (n–1) row vector of labour coefficients. X_n denotes the quantity of labour in all internal production activities. The family sector in country A is denoted by $\hat{n}$ and the population sizes in both countries are related by the coefficient of proportionality ξ. According to Pasinetti (1993), system (3) is a homogenous and linear system and, hence a necessary condition to ensure non-trivial solutions of the system for physical quantities is:

$$\det\begin{bmatrix} \mathbf{I} & -(\mathbf{c}+\xi\mathbf{c}^{\mathrm{e}}) \\ -\mathbf{a} & 1 \end{bmatrix} = 0 \tag{4}$$

Condition (4) may be equivalently written as [see Araujo and Teixeira (2004)]:

$$\mathbf{a}(\mathbf{c}+\xi\mathbf{c}^{\mathrm{e}}) = 1 \tag{4'}$$

If condition (4)' is fulfilled then there exists solution for the system of physical quantities in terms of an exogenous variable, namely $\overline{X}_n$. In this case, the solution of the system for physical quantities may be expressed as:

$$\begin{bmatrix} \mathbf{X} \\ X_n \end{bmatrix} = \begin{bmatrix} (\mathbf{c}+\xi\mathbf{c}^{\mathrm{e}})\overline{X}_n \\ \overline{X}_n \end{bmatrix} \tag{5}$$

From the first n – 1 lines of (5), we conclude that in equilibrium the physical quantity of each tradable commodity to be produced in country U, that is X_i , $i = 1,...,n-1$, will be determined by the sum of the internal and foreign demand, namely $a_{in}\overline{X}_n$ and $\xi a_{in}\overline{X}_n$ respectively. The last line of (5) shows that the labour force is fully employed. It is important to emphasize that solution (5) holds only if condition (4)' is fulfilled. If (4)' does not hold, then the non-trivial solution of physical quantities cannot be given by expression (5). The economy depicted by system (3) may also be represented by a system of monetary quantities, where total wages are spent on domestic consumption goods (represented by domestic coefficients, $\mathbf{c}$) and imports of foreign goods (represented by import coefficients, $\mathbf{c^m}$. The monetary system may be written as:

$$[\mathbf{p} \quad w]\begin{bmatrix} \mathbf{I} & -(\mathbf{c}+\mathbf{c}^{\mathrm{m}}) \\ -\mathbf{a} & 1 \end{bmatrix} = [\mathbf{0} \quad 0] \tag{6}$$

where $\mathbf{p} = [p_1 \quad \cdots \quad p_{n-1}]$ is the (*n*–1) row vector of prices, $\mathbf{c}^{\mathrm{m}} = \begin{bmatrix} a_{\hat{1}n} \\ \vdots \\ a_{\hat{n}-1,n} \end{bmatrix}$ is the (*n*–1) column vector of consumption import coefficients, and w is the uniform wage. Like system (3), system (6) is also a homogenous and linear system and, hence a necessary condition to ensure non-trivial solutions for prices should be observed, that is:

$$\det\begin{bmatrix} \mathbf{I} & -(\mathbf{c}+\mathbf{c^m}) \\ -\mathbf{a} & 1 \end{bmatrix} = 0 \tag{7}$$

Condition (7) may be equivalently written as [see Araujo and Teixeira (2004)]:

$$\mathbf{a}(\mathbf{c}+\mathbf{c}^{\mathrm{m}}) = 1 \tag{7'}$$

If condition (7)' is fulfilled then there exists a solution for the system of monetary quantities in terms of an exogenous variable, namely $\overline{w}$. In this case, the solution of the system for monetary quantities may be expressed as:

$$[\mathbf{p} \quad w] = [\mathbf{a}w \quad \overline{w}] \tag{8}$$

From the first *n* – 1 lines of (8), we conclude that in equilibrium the price of each tradable commodity is given by amount of labour employed in its production, that is $p_i = a_{in}\overline{w}$, $i = 1,...,n-1$. If expressions (5) and (8) hold simultaneously it is possible to show after some algebraic manipulation that they express a new condition, which can be viewed as embodying a notion of equilibrium in the trade balance. If $\mathbf{a}(\mathbf{c}+\xi\mathbf{c}^{\mathrm{e}})=1$ and $\mathbf{a}(\mathbf{c}+\mathbf{c}^{\mathrm{m}})=1$ then by equalizing the left hand side of both expressions we obtain:

$$\mathbf{a}(\xi\mathbf{c}^{\mathrm{e}}-\mathbf{c}^{\mathrm{m}})=0 \tag{9}$$

The fulfilment of conditions (4)' and (7)' implies the equilibrium in the trade balance but the reverse is not true. Note for instance that if $\mathbf{a}(\mathbf{c}+\xi\mathbf{c}^{\mathrm{e}})=0.9$ and $\mathbf{a}(\mathbf{c}+\mathbf{c}^{\mathrm{m}})=0.9$ the trade balance condition will also be fulfilled by equalizing the right hand side of both expressions but this situation corresponds to unemployment and under expenditure of national income. That is, the equilibrium in trade balance does not imply neither full employment of the labour force nor full expenditure of national income. This possibility has been somewhat emphasized by the BoP constrained growth approach. According to this view the main constraint on the performance of a country is related to the balance of payments that must be balanced in the long run. In this set up a poor export performance may lead to low levels of employment and national output thus showing that the external constraint may be more relevant that shortages in savings and investment mainly for developing economies. In this context the Harrod foreign trade multiplier plays a decisive role since it changes the focus of determination of national income from investment to exports.

From the first line of expression (8), we know that $\mathbf{p}=\mathbf{a}w$. Hence by assuming a wage unit, namely $w=1$, money prices equal to labour coefficients, and the equilibrium in the trade balance may be rewritten as:

$$\mathbf{p}(\xi\mathbf{c}^{\mathrm{e}}-\mathbf{c}^{\mathrm{m}})=0 \tag{9'}$$

In the next section it is derived a disaggregated version of the Harrod foreign trade multiplier from the system of physical quantities. The system of monetary quantities will be employed to show the consistency of this disaggregated version since departing from it we arrive at the aggregated version of the static Harrod foreign trade multiplier.

3. The Derivation of the Multi-sectoral static Harrod Foreign Trade Multiplier

The idea of developing a multi-sectoral version of the Keynesian multiplier dates back to Goodwin (1949) and Miyazawa (1960) who accomplished to develop a disaggregated version of the income multiplier in Leontief's framework from the relatively simple Keynesian structure. Both authors emphasized that although there are important differences between the Keynes and Leontief approaches, a bridge between them, namely a disaggregated version of the multiplier, is an important development for both views. In order to derive a multi-sectoral version of the Harrod foreing trade multiplier, let us adopt a procedure similar to the one advanced by Trigg and Lee (2005) and extended by Araujo and Trigg (2013). Dealing with the original Pasinettian model, Trigg and Lee (2005) had to assume that investment in the current period becomes new capital inputs in the next period and that the rate of depreciation is 100% (that is, all capital is circulating capital) in order to derive the Keynesian multiplier. By considering an economy extended to foreign trade we do not need this hypothesis. Let us rewrite the system of physical quantities in (3) as:

$$\begin{bmatrix} \mathbf{I} & -\mathbf{c} \\ -\mathbf{a} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ X_n \end{bmatrix} = \begin{bmatrix} \mathbf{E} \\ 0 \end{bmatrix} \qquad (3)'$$

Note that the difference between expression (3) and (3)' is that in the latter we isolate the vector of sectoral exports $\mathbf{E} = \xi \overline{X}_n \mathbf{c}^{\mathrm{e}}$ on the right hand side. We may rewrite system (3)' as:

$$\begin{cases} \mathbf{X} - \mathbf{c}X_n = \mathbf{E} \\ -\mathbf{a}\mathbf{X} + X_n = 0 \end{cases} \qquad (10)$$

From the last line of system (10), it follows that:

$$X_n = \mathbf{aX} \tag{11}$$

Note that now the employment level, namely X_n, is not exogenous as in (5) since we are solving the system by considering the possibility of unemployment. That was not admissible for the solution (5) since there, the existence of full employment is a necessary condition for the existence of non-trivial solutions. By pre-multiplying throughout the first line of (11) by **a** and by considering that $X_n = \mathbf{aX}$, one obtains: $\mathbf{aX} = \mathbf{acaX} + \mathbf{aE}$. By isolating $\mathbf{aX}$, we obtain the employment multiplier relationship:

$$\mathbf{aX} = \frac{1}{1-\mathbf{ac}}\mathbf{aE} \tag{12}$$

where $1/1-\mathbf{ac}$ is a scalar employment multiplier [Trigg and Lee (2005)]. This is an employment multiplier relationship between the employment level $\mathbf{aX}$ and the total labour embodied in exports $\mathbf{aE}$, where the scalar employment multiplier is $1/1-\mathbf{ac}$. Since $\mathbf{E} = \xi\overline{X}_n\mathbf{c}^{\mathrm{e}}$ expression (12) may be rewritten as:

$$\mathbf{aX} = \frac{\xi\mathbf{ac}^{\mathrm{e}}}{1-\mathbf{ac}}\overline{X}_n \tag{12'}$$

From expression (7)’, $1-\mathbf{ac} = \mathbf{ac}^{\mathrm{m}}$. It is worth remembering that implicit in this expression is the notion of full expenditure of national income. By substituting this result into expression (12)’ we can rewrite it as:

$$\mathbf{aX} = \frac{\xi\mathbf{ac}^{\mathrm{e}}}{\mathbf{ac}^{\mathrm{m}}}\overline{X}_n \tag{12''}$$

This result shows that if the balance of payment equilibrium condition conveyed by expression (9) is fulfilled, namely $\xi \mathbf{ac}^{\mathrm{e}} = \mathbf{ac}^{\mathrm{m}}$ then the employment level is equal to the full employment level, namely $\mathbf{aX} = \overline{X}_n$.

A further scrutiny of this result allows us to conclude that the full employment of the labour force will be reached when both the condition of full expenditure of national income and the balance of payments equilibrium are simultaneously satisfied. Another way of showing this result is to note that if $\xi \mathbf{ac}^{\mathrm{e}} = \mathbf{ac}^{\mathrm{m}}$ and $1 - \mathbf{ac} = \mathbf{ac}^{\mathrm{m}}$ then $1 - \mathbf{ac} = \xi \mathbf{ac}^{\mathrm{e}}$, which is the full employment condition given by expression (7)'. The rationale for this result may be grasped considering two main possibilities. Assume first that the condition of full expenditure is satisfied, namely $1 - \mathbf{ac} = \mathbf{ac}^{\mathrm{m}}$, but there is a trade imbalance in the sense that imports are higher than exports, that is $\xi \mathbf{ac}^{\mathrm{e}} < \mathbf{ac}^{\mathrm{m}}$. In this case, $1 - \mathbf{ac} > \xi \mathbf{ac}^{\mathrm{e}}$ which implies that $\mathbf{a}(\mathbf{c} + \xi \mathbf{c}^{\mathrm{e}}) < 1$, meaning unemployment. In this case, although the national income if fully expended the content of labour in the exports is lower than the content of labour in the imports, which gives rise to unemployment.

The other possibility is connected to the case in which the trade is balanced but the national income is not fully expended. Then $\xi \mathbf{ac}^{\mathrm{e}} = \mathbf{ac}^{\mathrm{m}}$ but $\mathbf{a}(\mathbf{c} + \mathbf{c}^{\mathrm{m}}) < 1$. It is easy to show that this case also leads to: $\mathbf{a}(\mathbf{c} + \xi \mathbf{c}^{\mathrm{e}}) < 1$, also meaning unemployment. Then it is proven that the full employment of the labour force depends on the conjunction of two other conditions, namely full expenditure of national income and balance of payments equilibrium.

This result shows that if the effective demand condition given by expression (5) is fulfilled then the employment level is equal to the full employment level, namely $\mathbf{aX} = \overline{X}_n$. While expression (12)' generates different levels of employment, only one of them will be the full employment level that corresponds to the Pasinettian solution. Through further decomposition [see Trigg (2006, Appendix 2)], (12) can be substituted into the first line of (10) to yield:

$$\mathbf{X} = \left(\mathbf{I} + \frac{\mathbf{ca}}{1 - \mathbf{ac}} \right) \mathbf{E} \tag{13}$$

From expression (7)' $\mathbf{ac}^{\mathrm{m}} = 1 - \mathbf{ac}$. Hence:

$$\mathbf{X} = \left(\mathbf{I} + \frac{\mathbf{ca}}{\mathbf{ac}^{\mathrm{m}}} \right) \mathbf{E} \tag{14}$$

This is a multiplier relationship between the vector of gross outputs, **X**, and the vector representing foreign demand $\mathbf{E}$, where $\left(\mathbf{I} + \frac{\mathbf{ca}}{\mathbf{ac}^{\mathrm{m}}} \right)$ is the output multiplier matrix. This result is a multi-sectoral version of the Harrod foreign trade multiplier whereby the output of each sector is related to the export performance of that sector. One of the main differences between this multi-sectoral multiplier for an open economy and the one derived by Trigg and Lee is that the latter is a scalar, and the former is a matrix.

The derivation of the multi-sectoral Harrod foreign trade multiplier allows us to better understand the connection between the balance of payments and the level of employment and production. Expression (12)' and (14) shows that a country may experience balance of payment equilibrium with levels of employment and production lower than those related to full employment and equilibrium. In order to show this let us rewrite expression (14) by considering that $\mathbf{E} = \xi \overline{X}_n \mathbf{c}^{\mathrm{e}}$. After some algebraic manipulation it yields:

$$\mathbf{X} = \left(\xi \mathbf{c}^{\mathrm{e}} + \mathbf{c} \frac{\xi \mathbf{ac}^{\mathrm{e}}}{\mathbf{ac}^{\mathrm{m}}} \right) \overline{X}_n \tag{14'}$$

Expression (14)' plays a central role in our analysis. It shows that if $\mathbf{ac}^{\mathrm{m}} = \xi \mathbf{ac}^{\mathrm{e}}$ then the solution given by (14)' sums up to the solution given by the first line of (5). In this vein, the equilibrium Pasinettian solution given by the first lines of expression (5) is a particular case of the solution given by multi-sectoral Harrod foreign trade multiplier (14)' when there is equilibrium in the trade balance $\mathbf{ac}^{\mathrm{m}} = \xi\mathbf{ac}^{\mathrm{e}}$.

Hence the solution put forward by Araujo and Teixeira (2004) for an open version of the Pasinetti model is in fact a particular case of the solution obtained here. That result is of key importance. Note that if $\xi\mathbf{ac}^{\mathrm{e}} > \mathbf{ac}^{\mathrm{m}}$ meaning that the $\frac{\xi\mathbf{ac}^{\mathrm{e}}}{\mathbf{ac}^{\mathrm{m}}} > 1$, a situation in which the country is running trade surpluses we should expect that the Harrodian solution given by (14)' is higher than the Pasinettian solution given by the first line of (5). Otherwise, if the country is running trade deficits, that is $\xi\mathbf{ac}^{\mathrm{e}} < \mathbf{ac}^{\mathrm{m}}$, this implies that $\frac{\xi\mathbf{ac}^{\mathrm{e}}}{\mathbf{ac}^{\mathrm{m}}} < 1$, and the Pasinettian solution is higher than the Harrodian solution. In sum, we should expect that the sectoral output given by the Harrod foreign trade multiplier deviates from the equilibrium Pasienettian output in the presence of trade deficits and surpluses.

But one of the emphasis of the BoP constrained growth theory is that in the long run trade should be balanced, namely $\xi\mathbf{ac}^{\mathrm{e}} = \mathbf{ac}^{\mathrm{m}}$, since a country cannot run permanent deficits. While the case $\xi\mathbf{ac}^{\mathrm{e}} < \mathbf{ac}^{\mathrm{m}}$ is unsustainable from the viewpoint of country *U* in the long run, the reverse $\xi\mathbf{ac}^{\mathrm{e}} > \mathbf{ac}^{\mathrm{m}}$ is unsustainable from the viewpoint of country *A*. Hence we may conclude that the Harrodian solution tends to the Pasinettian solution in the long run.

4. From the Multi-sectoral to an Aggregated version of the static Harrod Foreign Trade Multiplier

In order to prove the consistency of our approach let us show that it is possible to obtain the aggregated version of the static foreign trade multiplier from the analysis developed in the previous section. Now under a pure labour theory of value, as assumed by Pasinetti, let us say that there is a wage unit, $w = 1$ such that money prices are equal to labour coefficients. From the first line of system (8) we conclude that: $\mathbf{p} = \mathbf{a}$. By substituting this result into expression (12), a scalar output multiplier relationship can be specified as follows:

$$\mathbf{pX} = \frac{1}{1 - \mathbf{pc}} \mathbf{pE} \tag{15}$$

Note that $\mathbf{pX}$ amounts for total output, namely $Y = \mathbf{pX}$, and $\mathbf{pE}$ stands for total exports, that is $E = \mathbf{pE}$. Hence, expression (15) takes the form:

$$Y = \frac{1}{1 - \mathbf{pc}} E \tag{16}$$

Expression (16) is analogous to the aggregated Harrod foreign trade multiplier since it relates the output to total exports. But in order to prove that it is really this multiplier it is necessary to show that the denominator embodies a notion of income elasticity of imports. By also substituting $\mathbf{p} = \mathbf{a}$ into expression (4)' one obtains: $\mathbf{p}(\mathbf{c} + \xi \mathbf{c}^{\mathrm{e}}) = 1$, which yields: $\xi \mathbf{pc}^{\mathrm{e}} = 1 - \mathbf{pc}$. By substituting this result into expression (16) one obtains:

$$Y = \frac{1}{\xi \mathbf{pc}^{\mathrm{e}}} E \tag{17}$$

A key assumption to derive the static Harrod foreign trade multiplier is that of trade balance. By also substituting $\mathbf{p} = \mathbf{a}$ into expression (9) one obtains the trade balance equation in terms of prices, meaning that in a pure labor economy there is equivalence between the trade balance equilibrium in terms of prices and in terms of labour: $\mathbf{p}(\xi\mathbf{c}^{\mathrm{e}} - \mathbf{c}^{\mathrm{m}}) = 0$, which yields: $\xi\mathbf{pc}^{\mathrm{e}} = \mathbf{pc}^{\mathrm{m}}$. By substituting this result into expression (17) one obtains:

$$Y = \frac{1}{\mathbf{pc}^{\mathrm{m}}} E \tag{18}$$

Expression (18) conveys the taste of the static Harrod foreign trade multiplier since the denominator in the right hand side includes the income elasticities of demand. But by considering a disaggregated version of the linear import function, given by $M = mY$, it is possible to show that expression (18) may be made even closer to the Harrod foreign trade multiplier. Let us also assume a disaggregated linear import function, given by: $x_{\hat{i}n} = m_i X_i$, where $x_{\hat{i}n}$ stands for the amount of imported good i, and, from expression (5) $X_i = (a_{in} + \xi\, a_{i\hat{n}})X_n$. By dividing $x_{\hat{i}n}$ by X_n we obtain the per capita import coefficient for the i-th sector: $a_{\hat{i}n} = m_i(a_{in} + \xi a_{i\hat{n}})$. By considering that $\mathbf{p} = \mathbf{a}$, which yields $p_i = a_{ni}$, $\mathbf{pc}^{\mathrm{m}} = \sum_{i=1}^{n-1} p_i a_{\hat{i}n} = \sum_{i=1}^{n-1} p_i m_i (a_{in} + \xi\, a_{i\hat{n}}) = \sum_{i=1}^{n-1} a_{ni} m_i (a_{in} + \xi\, a_{i\hat{n}})$. By substituting this result into expression (18) we obtain:

$$Y = \frac{1}{\sum_{i=1}^{n-1} m_i (a_{in} + \xi\, a_{i\hat{n}}) a_{ni}} E \tag{18'}$$

Note that $(a_{in} + \xi\, a_{i\hat{n}})a_{ni}$ measures the share of the i-th sector in the national income. Then by denoting $sh_i = (a_{in} + \xi\, a_{i\hat{n}})a_{ni}$ we can rewrite expression (18)’ as:

$$Y = \frac{1}{\sum_{i=1}^{n-1} m_i sh_i} E \tag{19}$$

This expression is closer to the Harrod foreign trade multiplier since we can consider that: $m = \sum_{i=1}^{n-1} m_i (a_{in} + \xi \, a_{i\hat{n}}) a_{ni}$. Then, expression (19) may be rewritten as:

$$Y = \frac{1}{m} E \qquad (19)'$$

which is the static version of the Harrod foreign trade multiplier. Expression (19) shows that analogous to the Multi-Sectoral Thirlwall's law, changes in the composition of demand or in the structure of production also matter for income determination. In this vein, a country GDP may be higher if it shifts resources away from sectors with a high income elasticity of demand for imports.

Hence, according to the approach presented here one of the main barriers to favourable structural changes is also given by the balance of payments constraint. A country with access to foreign markets may induce changes in the structure of production that will allow the reallocation of resources from the low to high productivity sectors, thus giving rise to a propitious economic structure that will lead to higher output.

5. Concluding Remarks

The SED and the BoP-constrained growth approaches share the view that demand plays an important role in the growth process but with different emphasis. While the SED framework focuses on the structural changes accruing from the existence of particular growth rates of demand and technical change for each sector, the BoP literature considers that elasticities of demand for exports and imports are responsible for explaining particular growth experiences.

On one hand, the BoP constrained growth approach emphasizes the notion of equilibrium in the long run by considering that a country cannot experience permanently growth rates higher than those consistent with the long run balance of payment equilibrium. On the other hand, the derivation of the concept of balance of payment in the SED approach has shown that this equilibrium is subject to the particular dynamics of technical change and patterns of demand.

A common feature of both approaches is that the notion of equilibrium plays a central role but with different emphasis. While in the BoP approach, the equilibrium in the balance of payment is a required condition of sustainability in the long run, the SED approach shows that the most probable Macroeconomic consequence of the growth process is disequilibria which is translated in terms of structural unemployment. But it is undeniable that even in the SED approach the equilibrium in the balance of payment should be observed in the long run. The straightest consequence of this fact is that the evolving patterns of technical change and preferences cannot be exogenous but will be subject to the external constraint as pointed out by the BoP approach. Fortunately, the SED approach embodies a strong normative taste and the conditions for full employment of the labour force and equilibrium in the balance of payment are stated clearly although it is easy to prove that the former will not be generally satisfied.

Here we provide more foundations to the connections between the SED and BoP constrained approach by showing that a disaggregated version of the static Harrod foreign trade multiplier may be derived from an open version of the Pasinettian model. Besides, we show that the equilibrium Pasinettian solution for the system of physical quantities may be obtained as a particular case of the solution given by multi-sectoral Harrod foreign trade multiplier derived here when the full employment condition is satisfied. Finally, in order to prove the consistency of our approach we show that departing from this disaggregated version of the Harrod foreign trade multiplier we can obtain the aggregated version. With the approach developed here the outcomes from a cross-fertilization between them go beyond the disaggregated version of the Thirlwall's law.

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