Consumption efficiency hypothesis and the HOS model: Some counterintuitive results

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Abstract: We show how accommodation of the consumption efficiency hypothesis can explain the existence of involuntary unemployment in the two-by-two Heckscher-Ohlin-Samuelson (HOS) model. Although the workers consume both the commodities their nutritional efficiency depends on the consumption of one commodity only. An increase in the relative price of the capital-intensive (labour-intensive) good raises (lowers) the effective employment in the economy. The effects of commodity price changes on the output levels of the two sectors might be perverse. These results are different from the standard HOS results.

JEL classification: J41; O15

Keywords: Consumption efficiency hypothesis; General equilibrium; Heckscher-Ohlin-Samuelson model; Effective employment; Output composition.

Consumption efficiency hypothesis and the HOS model: Some counterintuitive results

1. Introduction

The 'consumption efficiency hypothesis' of Leibenstein (1957, 1958)[1] is the earliest version of the efficiency wage theory and has been widely used in the development models for explaining the existence of involuntary unemployment in a poor less developed country (hereafter, LDC). The basic tenet of the hypothesis is that the nutritional efficiency of a worker is positively related to his consumption level at least up to a certain point. Higher consumption means higher calorie intake, an increase in body mass, a reduction in morbidity as well as greater ability to work.[2]. There is now considerable evidence[3] that in a poor LDC with low levels of consumption of the workers there is a significant positive relation between workers' consumption and productivity. Therefore, an increase in the consumption level raises the nutritional efficiency i.e. productivity of the worker. Now if there is a stable relationship between the consumption level of the worker and his wage income then the worker's productivity is positively linked to the wage that he receives. If this is so, then it is in its interest the firm will not offer its profit-maximizing wage but the efficiency wage because now the wage through the nutritional efficiency function enters into the production function of the firm. The firm minimizes its unit labour cost and it is a standard result of this literature that the efficiency wage is set where the elasticity of the nutritional efficiency function of the worker is equal to unity. Hence the efficiency wage is constant. Even if there is an excess supply of labour at the efficiency wage, the firms will not lower the wage rate. Hence the labour market does not clear and the problem of involuntary unemployment crops up.

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[1] See also Stiglitz (1976), Mazumdar (1959), Mirrlees (1975), Bliss and Stern (1978) and Dasgupta and Ray (1986).

[2] One may go through Ray (1993) for the process of efficiency build-up.

[3] See Bose (1996) in this context.

However, this theory assumes a one commodity world. A pertinent question is how the basic results of the consumption efficiency literature are affected if instead of one the workers consume two commodities while the nutritional efficiency depends only on the consumption of one of the two commodities. If the two commodities are food and cloth, then the worker's nutritional efficiency depends only on his consumption of food. Another interesting theoretical exercise would be to embed the consumption efficiency theory in the simple two-by-two Heckscher-Ohlin-Samuelson (HOS) framework and examine whether the basic trade results like Stolper-Samuelson and Rybczynski theorems still hold despite this incorporation. The present paper purports to deal with these issues. The analysis explains the existence of involuntary unemployment in the two-sector general equilibrium setup. It finds that commodity price changes might produce perverse effects on output composition. In the case of fixed-coefficient technologies, output changes are unambiguously perverse. These results are different from those obtained in the HOS model.

2. The model

Let there be two commodities: X and Y. There are L number of homogeneous workers in the economy. The worker consumes both commodities but his productivity depends only on the consumption of commodity, X (say, food). The utility function is of the CES type and is given by

$$V = A\left[\delta x^{-\rho} + (1-\delta) y^{-\rho}\right]^{\frac{-1}{\rho}} \text{ with } A > 0; 0 < \delta < 1; \text{ -1}<\rho \neq 0 \qquad (1)$$

where V, x and y denote the utility level and the consumption levels of X and Y, respectively. A, δ and ρ are parameters. δ is the share of commodity X in the consumer's budget while ρ is the substitution parameter.

The budget constraint of the worker (consumer) is

$$P_x x + P_y y = (1-u)W \qquad (2)$$

where P_x, P_y and W are the two prices and the wage rate. Finally, u denotes the rate of unemployment of labour in the economy. So the right-hand side of (2) is the expected wage income of the worker.

Equation (1) is maximized with respect to x and y and subject to (2). Maximization exercise leads to the following demand function for commodity X.[4]

$$x = \frac{(1-u)W}{P_x(1+K)} \tag{3}$$

where $K = [(\frac{1-\delta}{\delta})^{\frac{1}{1+\rho}}(\frac{P_y}{P_x})^{\frac{\rho}{1+\rho}}] > 0$

The demand for X by each worker in the general form is written as

$$x = x(P_x, P_y, W, u) \tag{3.1}$$

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For quite straightforward reasons the demand for X is a negative function of the two prices and the unemployment rate while it is a positive function of the wage rate.

The nutritional efficiency of each worker, h, is assumed to be a positive function of his consumption level of commodity X and is given by

$$h = h[x(P_x, P_y, W, u)]; h^{/} > 0; h^{//} < 0 \tag{4}$$

So h increases at a decreasing rate with an increase in x.

The unit cost of labour, ω, is given by

$$\omega = \frac{W}{h} \tag{5}$$

Apart from labour, capital is used in production. Assuming capital to be perfectly mobile intersectorally and its uniform return be r economy-wide, each firm minimizes its unit labour cost given by (5). The first-order condition of minimization is

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[4] Here the two commodities are gross compliments. In other words, an increase in the price of either good lowers the quantities demanded of both the commodities and vice versa.

$$h(.) = Wh^{/}(.)x_3 \tag{6}$$

where $x_3 = (\partial x / \partial W) > 0$; with $x_{33} = 0$; and, $x_{31}, x_{32}, x_{34} < 0$.[5]

It may be checked that the second-order condition is automatically satisfied as $h^{//} < 0$.

Equation (6) may be interpreted as follows. The elasticity of the nutritional efficiency function (4) is given by

$$\varepsilon_h = ((dh / dx)(x / h)) \tag{7}$$

Using (3) and (7), equation (6) can be rewritten as follows.

$$\varepsilon_h = 1. \tag{6.1}$$

So in equilibrium the efficiency wage is set where the elasticity of the nutritional efficiency function is equal to unity. This is a standard result of the efficiency wage literature. But unlike the one commodity framework, the condition here does not imply the constancy of the wage. It rather implies constancy of the consumption of commodity X on which the nutritional efficiency of a worker depends. This establishes the following proposition.

Proposition 1: In a two-commodity world where the nutritional efficiency of the worker depends positively on the consumption of one commodity only, its demand remains constant despite changes in the parameters of the system. The nutritional efficiency of the worker is also constant.

3. General equilibrium and the consumption efficiency hypothesis

Let us introduce the nutritional efficiency function into the conventional Heckscher-Ohlin-Samuelson framework. We consider a small open poor economy with two commodities, X and Y. There are two inputs of production, labour (L) and capital (K) and their endowments are given exogenously. Commodity prices, P_x and P_y, are given by the small open economy assumption. Production functions exhibit constant returns to scale with positive but diminishing marginal productivities to each input. Markets for both

[5] See Appendix I for the derivations.

commodities and capital are perfectively competitive while firms in both sectors set wages in the labour market according to equation (6). The total number of workers, L, is fixed in the economy although the effective labour force in efficiency unit is $h(.)L$. Normalizing labour in physical unit to unity the economy's effective labour force is $h(.)$.

Given the assumption of perfectly competitive commodity markets the zero-profit conditions for the two sectors are as follows.

$$\frac{W}{h}a_{Lx} + ra_{Kx} = P_x \tag{8}$$

$$\frac{W}{h}a_{Ly} + ra_{Ky} = P_y \tag{9}$$

where a_{ji} is the amount of the j th input required to produce one unit of output of the i th sector for $j = L, K$ and $i = X, Y$ and (W/h) and r are the wage rate per efficiency unit of labour and the return to capital, respectively. Sector X is more labour-intensive than sector Y in both value and physical sense i.e. $|\theta| = (\theta_{Lx}\theta_{Ky} - \theta_{Kx}\theta_{Ly}) > 0$; and,, $|\lambda| = (\lambda_{Lx}\lambda_{Ky} - \lambda_{Kx}\lambda_{Ly}) > 0$ where θ_{ji} and λ_{ji} are the distributive and allocative shares of the j th input in the i th sector, respectively.

Capital is fully utilized in the two sectors. The full-employment condition for capital is given by

$$a_{Kx}X + a_{Ky}Y = K \tag{10}$$

where X and Y are the levels of output in the two sectors.

There is unemployment of labour in the economy and the rate of unemployment is u. The labour endowment equation is, therefore, given by

$$a_{Lx}X + a_{Ly}Y = h(1-u) \tag{11}$$

The effective employment of labour in the economy, E, is

$$E = h(1-u) \tag{12}$$

The general equilibrium system consists of equations (4), (6) and (8) – (12). The endogenous variables are: W, h, r, u, X, Y and E. If we look at the structure we find that W, h, r and u are determined from (4), (6), (8) and (9). Once h and u are determined the effective employment of labour, E is also obtained from (12). So factor prices, nutritional efficiency of worker, the unemployment rate and the effective employment of labour in the economy depend on commodity prices but not on factor endowments. Finally, X and Y are obtained from (10) and (11).

4. Comparative statics

We are now going to analyze the consequences of commodity price changes on the endogenous variables. Totally differentiating equations (4), (6), (8) and (9) the following results can be obtained.

$$(dh / dP_x), (dh / dP_y) = 0 \tag{13}$$

$$\hat{W} > (<) \hat{P}_x > (<) \hat{P}_y > (<) \hat{r} \text{ if } \hat{P}_x > (<) \hat{P}_y \tag{14}$$

where"^" implies proportional change.

The result given by (13) has already been stated in proposition 1. As the consumption of commodity X is independent of commodity prices (and other parameters) the efficiency of the worker does not change following changes in commodity prices.

From (14) we find that the Stolper-Samuelson theorem and the magnification effects of Jones (1965) are valid even in this structure.

Totally differentiating (4), (6), (8) and (9) and then (12) the following results can be obtained.[6]

$$(\hat{u} / \hat{P}_x) > 0; (\hat{u} / \hat{P}_y) < 0; (\hat{E} / \hat{P}_x) < 0; \text{and}, (\hat{E} / \hat{P}_y) > 0. \tag{15}$$

These results are stated in terms of the following proposition.

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[6] See Appendix II for detailed derivations.

Proposition 2: An increase in the price of commodity X raises the unemployment rate and lowers the effective employment in the economy. On the contrary, an increase in the price of commodity Y lowers the unemployment rate and raises the economy-wide effective employment.

Proposition 2 can be intuitively explained as follows. An increase in P_x (or in P_y) lowers the demand for X (i.e. x). The wage rate per worker, W, rises (falls) as P_x (P_y) rises as sector X is more labour-intensive vis-à-vis sector Y which in turn raises (lowers) the demand for X. There are two negative effects on x when P_y rises. On the contrary, there are two opposite effects on x when P_x rises. But from (14) we find that $\hat{W} > \hat{P}_x > 0$ (the magnification effect). So the positive effect of an increase in W on the demand for X dominates over the negative effect of an increase in P_x. However, from proposition 1 we find that the net effect of any parameter changes on the demand for commodity X must be zero. Therefore, so as to neutralize the negative effect of an increase P_y on x, the unemployment rate, u, must fall which works through an increase in the expected wage income of the worker. In contrast the unemployment rate has to increase for counterbalancing the net positive effect of an increase in P_x.

As the nutritional efficiency, h, of each worker remains the unchanged despite changes in the values of the parameters (see proposition 1), the effects of changes in commodity prices on the economy's effective employment level depend solely on how price changes affect the unemployment rate, u. As u rises (falls) following an increase in P_x (P_y), the economy-wide effective employment, E, falls (rises) as P_x (P_y) rises.

Our next task would be to analyze the consequences of commodity price changes on the composition of output in the economy. Totally differentiating (8) – (11) the following proposition and the corollary can be proved.[7]

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[7] Mathematical proofs are provided in Appendix III.

Proposition 3: An increase in P_x may lead to a contraction of sector X and an expansion of sector Y. On the contrary, sector X may expand and sector Y may contract following an increase in P_y.

Corollary 1: If technologies of production are of fixed-coefficient in nature an increase in P_x unambiguously leads to a contraction of sector X and an expansion of sector Y while an increase in P_y expands sector X and contracts sector Y unequivocally.

Proposition 3 is quite interesting as it presents results that are contrary to the standard results of the HOS model. In the HOS model a Stolper-Samuelson effect is followed by a Rybczynski effect if technologies of production are of the variable coefficient type. An increase in P_x, ceteris paribus, raises the wage rate and lowers the return to capital if sector X is labour-intensive. This is the Stolper-Samuelson effect. As the wage-rental ratio goes up, producers in both the sectors will substitute labour by capital. So the labour-output ratios fall and the capital-output ratios increase in the two sectors. Given the product-mix, there will an excess supply (a shortage) of labour (capital). Consequently, sector X expands and sector Y contracts following a Rybczynski type effect as sector X is labour-intensive. Similarly, if P_y rises, sector Y expands while sector X contracts. These are the standard results of the HOS model.[8] But in the present case where the nutritional efficiency of the worker depends positively on the consumption of commodity X, there will be a second Rybczynski effect apart from the usual one. If P_x rises, the effective employment in the economy falls that leads to a contraction of sector X and an expansion of sector Y. So there are two Rybczynski effects which work in the opposite directions to each other. If the second effect is stronger than the first effect, the net result of an increase in P_x will be an expansion (a contraction) of the capital-intensive (labour-intensive) sector Y (sector X). On the contrary, an increase in P_y raises the economy-wide effective employment thereby producing a second Rybczynski effect which may outweigh the initial Rybczynski effect that leads to an

[8] See Jones (1965) in this context.

expansion of the capital-intensive sector. Proposition 3 presents the condition under which the perverse results are obtained. In the case of fixed coefficient technologies, the initial Rybczynski effect will be absent. Only the second Rybczynski effect will be there that leads to unambiguously perverse changes in output composition in response to commodity price changes. In the HOS model with fixed-coefficient technologies, output levels are insensitive to commodity price changes.

5. Concluding remarks

This paper has introduced the consumption efficiency hypothesis in an otherwise Heckscher-Ohlin-Samuelson model where the nutritional efficiency of each worker depends positively on the consumption of one of the two commodities only. It has been found that commodity prices cannot affect the productivity of the workers although the wage is affected. An increase in the price of the labour-intensive (capital-intensive) good lowers (raises) the effective employment of labour in the economy. Commodity price changes may produce perverse effects on the output levels. In fixed-coefficient technology case these effects are unambiguously perverse. All these results are different from the standard HOS results.

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Appendix I:

The problem of the consumer is as follows:

$$\underset{x,y}{Max}\ V = A\left[\delta x^{-\rho} + (1-\delta)y^{-\rho}\right]^{\frac{-1}{\rho}}; \quad (A>0;\ 0<\delta<1;\ -1<\rho \neq 0\ ) \qquad (1)$$

subject to $P_x x + P_y y = (1-u)W$ (2)

The first-order condition is

$$(\frac{y}{x})^{1+\rho} = (\frac{P_x}{P_y})(\frac{1-\delta}{\delta}) \qquad (A.1)$$

Solving (A.2) and (A.3) the optimum demand for good X is obtained as follows.

$$x = \frac{(1-u)W}{P_x(1+K)} \qquad (3)$$

where: $K=[(\frac{1-\delta}{\delta})^{\frac{1}{1+\rho}}(\frac{P_y}{P_x})^{\frac{\rho}{1+\rho}}] > 0$ (A.2)

In general form the demand for X by each worker is written as

$$x = x(P_x, P_y, W, u) \tag{3.1}$$

Differentiating (3) the following results are obtained.

$$\left.\begin{aligned}
& x_1 = (\frac{\partial x}{\partial P_x}) = -(1-u)W\frac{[1+\frac{K}{1+\rho}]}{P_x^2(1+K)^2} < 0\,;\, x_2 = (\frac{\partial x}{\partial P_y}) = \frac{-xK}{P_y(1+K)}(\frac{\rho}{1+\rho}) < 0; \\
& x_3 = (\frac{\partial x}{\partial W}) = \frac{(1-u)}{P_x(1+K)} > 0;\; x_4 = (\frac{\partial x}{\partial u}) = \frac{-W}{P_x(1+K)} < 0; \\
& x_{31} = (\frac{\partial^2 x}{\partial W \partial P_x}) = -(1-u)\frac{[1+\frac{K}{1+\rho}]}{P_x^2(1+K)^2} < 0;\; x_{33} = (\frac{\partial^2 x}{\partial W^2}) = 0; \\
& x_{32} = (\frac{\partial^2 x}{\partial W \partial P_y}) = \frac{-x}{WP_y}(\frac{K}{1+K})(\frac{\rho}{1+\rho}) < 0;\; x_{34} = (\frac{\partial^2 x}{\partial W \partial u}) = -\frac{1}{P_x(1+K)} < 0; \\
& (x_2 - Wx_{32}) = 0\,;\;\; (x_1 - Wx_{31}) = 0\,;\, (x_4 - Wx_{34}) = 0
\end{aligned}\right\} \tag{A.3}$$

Appendix II:

Totally differentiating equations (8), (9), (4) and (6) and writing in a matrix notation one gets the following.

$$\begin{bmatrix} -\theta_{Lx} & \theta_{Lx} & \theta_{Kx} & 0 \\ -\theta_{Ly} & \theta_{Ly} & \theta_{Ky} & 0 \\ Wx_3 & -Wx_3 & 0 & -x_4 u \\ 0 & W^2 h^{//} x_3^2 & 0 & uWh^{//} x_3 x_4 \end{bmatrix} \begin{bmatrix} \hat{h} \\ \hat{W} \\ \hat{r} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} \hat{P}_x \\ \hat{P}_y \\ (x_1 P_x)\hat{P}_x + (x_2 P_y)\hat{P}_y \\ (-h^{//} P_x W x_3 x_1)\hat{P}_x + (-h^{//} P_y W x_3 x_2)\hat{P}_y \end{bmatrix} \tag{A.4}$$

Solving (A.4) and simplifying one gets the following expressions.

$$\hat{h} = 0 \tag{A.5}$$

$$\hat{u} = (\frac{1}{u|\theta| x_4})\;[(Wx_3\theta_{Kx} - |\theta| x_2 P_y)\hat{P}_y - (Wx_3\theta_{Ky} + |\theta| x_1 P_x)\hat{P}_x] \tag{A.6}$$

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$$\hat{W} = \frac{1}{|\theta|}(\theta_{Ky}\hat{P}_x - \theta_{Kx}\hat{P}_y) \quad \text{(A.7)}$$

$$\hat{r} = \frac{1}{|\theta|}(\theta_{Lx}\hat{P}_y - \theta_{Ly}\hat{P}_x) \quad \text{(A.8)}$$

where: $|\theta| = (\theta_{Lx}\theta_{Ky} - \theta_{Kx}\theta_{Ly}) > 0$. (A.9)

(as sector X is more labour-intensive relative to sector Y)

From (A.5) – (A.8) the following results trivially follow.

$$\left.\begin{array}{l} \text{(i) } (dh/dP_x),(dh/dP_y) = 0; \\ \text{(ii) } (du/dP_x) > 0;(du/dP_y) < 0; \\ \text{(iii) } (dW/dP_x) > 0;(dW/dP_y) < 0; \text{ and,} \\ \text{(iv) } (dr/dP_x) < 0;(dr/dP_y) > 0. \end{array}\right\} \quad \text{(A.10)}$$

Now we recall equation (12).

$$E = h(1-u) \quad (12)$$

Differentiating (12), using (A.5) and (A.6) and simplifying the following expression is obtained.

$$\hat{E} = \{\frac{-1}{(1-u)|\theta|x_4}\}\ [(Wx_3\theta_{Kx} - |\theta|x_2P_y)\hat{P}_y - (Wx_3\theta_{Ky} + |\theta|x_1P_x)\hat{P}_x] \quad \text{(A.11)}$$

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Assuming $\rho > 0$ and using (A.3) and (A.9) it can be easily shown that

$$(Wx_3\theta_{Ky} + |\theta|x_1P_x) > 0 \quad \text{(A.12)}$$

From (A.11) and (A.12) the following results are obtained.

(v) $(dE/dP_x) < 0;(dE/dP_y) > 0$ (A.13)

Appendix III:

Totally differentiating (10) and (11) and (A.5), (A.7) and (A.8) we get the following two expressions.

$$\lambda_{Kx}\hat{X}+\lambda_{Ky}\hat{Y}=[(\lambda_{Kx}S_{KL}^{1}+\lambda_{Ky}S_{KL}^{2})\frac{(\hat{P}_y-\hat{P}_x)}{|\theta|}] \tag{A.14}$$

$$\lambda_{Lx}\hat{X}+\lambda_{Ly}\hat{Y}=\hat{E}-[(\lambda_{Lx}S_{LK}^{1}+\lambda_{Ly}S_{LK}^{2})\frac{(\hat{P}_y-\hat{P}_x)}{|\theta|}] \tag{A.15}$$

where: $S_{ji}^{k}=$ the degree of substitution between factors j and i in the k th sector, $j,i=L,K$; and, $k=1,2$ For example, $S_{LK}^{1}\equiv(r/a_{L1})(\partial a_{L1}/\partial r)$, $S_{LL}^{1}\equiv(\omega/a_{L1})(\partial a_{L1}/\partial\omega)$ etc. $S_{ji}^{k}>0$ for $j\neq i$; and, $S_{jj}^{k}<0$;

Solving (A.14) and (A.15) and using (A.11) the following expressions are obtained.

$$\begin{aligned}\hat{X}=&\frac{1}{|\lambda|}[\frac{(\lambda_{Ly}A+\lambda_{Ky}B)}{|\theta|}+(\frac{\lambda_{Ky}}{(1-u)|\theta|x_4})(Wx_3\theta_{Ky}+|\theta|x_1P_x)]\hat{P}_x\\&-\frac{1}{|\lambda|}[\frac{(\lambda_{Ly}A+\lambda_{Ky}B)}{|\theta|}+(\frac{\lambda_{Ky}}{(1-u)|\theta|x_4})(Wx_3\theta_{Kx}-|\theta|x_2P_y)]\hat{P}_y\end{aligned} \tag{A.16}$$

and,

$$\begin{aligned}\hat{Y}=&\frac{1}{|\lambda|}[\frac{(\lambda_{Lx}A+\lambda_{Kx}B)}{|\theta|}+\{\frac{\lambda_{Kx}}{(1-u)|\theta|x_4}\}(Wx_3\theta_{Kx}-|\theta|x_2P_y)]\hat{P}_y\\&-\frac{1}{|\lambda|}[\frac{(\lambda_{Lx}A+\lambda_{Kx}B)}{|\theta|}+\{\frac{\lambda_{Kx}}{(1-u)|\theta|x_4}\}(Wx_3\theta_{Ky}+|\theta|x_1P_x)]\hat{P}_x\end{aligned} \tag{A.17}$$

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where:

$$A=(\lambda_{Kx}S_{KL}^{1}+\lambda_{Ky}S_{KL}^{2})>0;$$

$$B=(\lambda_{Lx}S_{LK}^{1}+\lambda_{Ly}S_{LK}^{2})>0;\text{ and,} \tag{A.18}$$

$$|\lambda|=(\lambda_{Lx}\lambda_{Ky}-\lambda_{Kx}\lambda_{Ly})>0.$$

(Note that sector X is more labour-intensive relative to sector Y)

$$\left.\begin{aligned}&\text{Let } C=[\frac{(Wx_3\theta_{Ky}+|\theta|x_1P_x)}{(1-u)x_4}]<0 \text{ (assuming that } \rho>0 \text{ and using (A.3)); and,}\\ &D=[\frac{(Wx_3\theta_{Kx}-|\theta|x_2P_y)}{(1-u)x_4}]<0 \text{ (using (A.3))}\end{aligned}\right\} \tag{A.19}$$

Using (A.18) and (A.19) from (A.16) and (A.17) it is easy to derive the following results.

$$\left.\begin{aligned}&(\frac{\hat{X}}{\hat{P}_x})<0 \text{ iff } -(C\lambda_{Ky})>(\lambda_{Ly}A+\lambda_{Ky}B);\\ &(\frac{\hat{X}}{\hat{P}_y})>0 \text{ iff } -(D\lambda_{Ky})>(\lambda_{Ly}A+\lambda_{Ky}B);\\ &(\frac{\hat{Y}}{\hat{P}_x})>0 \text{ iff } -(C\lambda_{Kx})>(\lambda_{Lx}A+\lambda_{Kx}B);\\ &(\frac{\hat{Y}}{\hat{P}_y})<0 \text{ iff } -(D\lambda_{Kx})>(\lambda_{Lx}A+\lambda_{Kx}B).\end{aligned}\right\} \tag{A.20}$$

If technologies of production are of the fixed-coefficient type, S_{ji}^k s are equal to zero. In that case $A, B = 0$. Then from (A.20) it follows that $(\frac{\hat{X}}{\hat{P}_x})<0$; $(\frac{\hat{X}}{\hat{P}_y})>0$; $(\frac{\hat{Y}}{\hat{P}_x})>0$; and, $(\frac{\hat{Y}}{\hat{P}_y})<0$ without any restrictions.