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Abstract

In this paper Pasinetti's model of structural economic dynamics (1981) is extended to consider international economic relations. Conditions for full employment, full expenditure of income and equilibrium of the trade balance are established for an open economy that requires capital goods to produce final commodities. Analytical results concerning the benefits from free trade and international learning are formally studied. In addition, static and dynamic aspects of the 'principle of comparative cost advantage' are analysed considering the determinants of the specialisation level.

Keywords*:* structural change, international economic relations, capital goods.

JEL classification: E12, F02, O33

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1. Introduction

The analysis presented here amounts to a formal extension of Pasinetti's model (1981, 1993) to international economic relations. When dealing with free trade and international learning, Pasinetti considers a hypothetical case of two countries, advanced and underdeveloped, producing the same set of commodities with different methods of production. One of the features of his approach to international economic relations is that it is not carried out with the same formal rigour of his treatment of a closed economy.

In this paper the version of his model that requires capital goods to produce final commodities is formally extended to consider international flows of commodities. Our aim is to establish conditions for full employment, full expenditure of national income and equilibrium of the trade balance, along with solutions of systems for physical and monetary quantities for an open economy.

We intend to investigate if the propositions stated by Pasinetti for an open pure labour economy can be formally extended to a dynamic system with capital goods. The motivation for this study is the following: if a pure labour theory resists to the addition of capital goods then the results concerning international relations obtained by Pasinetti (1981, 1993), Araujo & Teixeira (2001, 2001b) and Teixeira & Sarquis (2001) hold true for the more general case that requires capital goods to produce consumption goods. In particular, we intend to investigate if benefits from technological diffusion among countries are more significant than gains from free trade.

This paper is structured as follows: in section 2, Pasinetti's model is extended to consider international flows of commodities; section 3 focuses on the process of learning as the first source of international gains and section 4 summarises the results.

2. International Flows of Consumption and Capital Goods

Suppose that there exist only two countries: *A* (for advanced) and *U* (for underdeveloped). Both economies are assumed to produce *n*-1 consumption goods: one in each vertically integrated sector[1] but with different patterns of production and consumption. Corresponding to each consumption goods sector there is a specific capital goods sector[2].

In order to establish the basic notation, it is useful to choose one of the countries, let us say *U*, to express physical and monetary flows. Consider that X_i denotes the domestic physical quantity produced of consumption good *i* and X_n represents the quantity of labour in all internal production activities; per capita demand of consumption goods is represented by a set of consumption coefficients: both a_{in} and $a_{i\hat{n}}$ stand for the demand coefficients of final commodity *i*. The former refers to domestic and the latter to foreign demand. In the same vein, $a_{ki,n}$ and $a_{ki,\hat{n}}$ stand for the investment coefficients of capital goods *ki*. The production coefficients of consumption and capital goods are respectively a_{ni} and a_{nki}. The family sector in country *A* is denoted by $\hat{n}$ and the size of population in both countries is related by the coefficient of proportionality ξ. The physical system may be written as follows:

$$\begin{cases} X_i - (a_{in} + \xi\ a_{i\hat{n}})X_n = 0 \\ X_{ki} - (a_{ki,n} + \xi\ a_{ki,\hat{n}})X_n = 0 \\ X_n - \sum_{i=1}^{n-1} a_{ni}X_i - \sum_{i=1}^{n-1} a_{nki}X_{ki} = 0 \end{cases} \qquad (1)$$

[1] Halevi (1996, p. 194) argues that "the theory of growth based on vertical integration revolutionizes the very concept of choice of technique and by focusing on the per capita demand, it overcomes the limitations of Feldman's strategy of growth". See also Araujo & Teixeira (2001a).

[2] For the sake of convenience only, we assume that capital goods do not depreciate.

A sufficient condition to ensure non-trivial solutions[3] of the system for physical quantities in country *U* is:

$$\sum_{i=1}^{n-1}(a_{in}+\xi\ a_{i\hat{n}})a_{ni}+\sum_{i=1}^{n-1}(a_{ki,n}+\xi\ a_{ki,\hat{n}})a_{nki}=1 \tag{2}$$

This is also a condition for full employment of the labour force. The solution of the system for physical quantities may be expressed as:

$$\begin{cases} X_i=(a_{in}+\xi\ a_{i\hat{n}})X_n \\ X_{ki}=(a_{ki,n}+\xi\ a_{ki,\hat{n}})X_n \end{cases} \tag{3}$$

Considering that $a_{\hat{i}n}$ is the foreign demand coefficient for commodity *i* produced in country *A*, $a_{\hat{k}i,n}$ is the foreign investment coefficient for the capital goods *ki* produced in country *A*, p_i is the price of commodity *i* country *U* (*i* = 1,2,...,*n*-1), and *w* is the wage rate (uniform), the monetary system may be written as:

$$\begin{cases} p_i-a_{ni}w-\pi_i p_{ki}=0 \\ p_{ki}-a_{nki}w=0 \\ w+\sum_{i=1}^{n-1}a_{in}p_{ki}\pi_i-\sum_{i=1}^{n-1}(a_{in}+a_{\hat{i}n})p_i-\sum_{i=1}^{n-1}(a_{ki,n}+a_{\hat{k}i,n})p_{ki}=0 \end{cases} \tag{4}$$

A sufficient condition to ensure non-trivial solutions of the system for prices in country *U* is:

[3] As pointed out by Pasinetti (1981, p. 33), fulfilment of (1) is a sufficient condition for the system for physical quantities to have non-trivial solutions. However, non-fulfilment does not imply any meaningful solution. The particular form of the coefficient matrix (all its entries are zeros, except those in the last row, those in the last column, and along with the main diagonal) means that the solution of the system can be derived directly, without substitution, from the first 2*n*–1 equations. Therefore, relative quantities are determined independently of condition (2).

$$\sum_{i=1}^{n-1}(a_{in}+a_{\hat{i}n})a_{ni}+\sum_{i=1}^{n-1}(a_{ki,n}+a_{\hat{k}i,n})a_{nki}=1 \tag{5}$$

This is also a condition for full expenditure of national income[4]. The set of solution for prices may be expressed as:

$$\begin{cases} p_i=(a_{ni}+\pi_i a_{nki})w \\ p_{ki}=a_{nki}w \end{cases} \tag{6}$$

In general, if the rates of profit, π_i (i=1,...,n-1), are positive and the capital intensity is different from one production process to another, relative prices of consumption goods will depend both on labour inputs and on the rate of profit. In this case a pure labour theory of value is no longer valid since the price of commodity i depends not only on quantities of direct and indirect labour but also on the rate of profit. In order to develop a theory in terms of pure labour, Ricardo (1817) and Marx (1887) assumed a uniform organic composition of capital in a static framework.

In a dynamic system with capital goods a theory of dated labour[5] can be kept taking an alternative route. In this case two conditions must be verified, namely, the proportionality of the sectoral rates of profit to the rates of growth, and constant sectoral capital-output ratios. Under these two assumptions, a dynamic labour theory of value also holds strictly[6] and Pasinetti's propositions about international trade remain valid. In other words, those propositions are not confined to the restricted case of a pure labour economy, but have some wider validity.

[4] The same observation made in relation to condition (2) applies here, i.e. non-fulfilment of condition (5) does not imply the non-existence of meaningful solutions for prices.

[5] To understand the relationship between vertical integration and reduction of prices to a sum of weighted quantities of labour see Pasinetti (1973). He assesses the process of vertical integration in the theory of value and income distribution.

[6] See Pasinetti (1981a, p. 51)

The solution of the monetary system shows that the wage rate can be factored out from all price components. The price of each consumption good is expressed as the sum of two unweighted physical quantities of labour: labour required directly in sector i, a_{ni}, plus labour required to produce one unit of productive capacity for sector i, a_{nki}. The profit component computes amounts of labour indirectly required for the equilibrium production of consumption good i[7]. The proportionality between the rate of profit to the sectoral rate of growth emerges as a natural requirement to endow the economic system with the necessary productive capacity to fulfil the expansion of demand. Therefore, a growing economy does imply a natural rate of profit[8], which is given by the following expression (see Pasinetti, 1981, p. 131):

$$\pi_i^* = g + r_i \tag{7}$$

where π_i^* represents the natural rate of profit for sector i, g is the growth rate of population and r_i is the rate of change of demand, particular for sector i.

This view of the natural rate of profit contradicts the concept of equilibrium since classical long run is characterised by a uniform rate of profit π for all sectors. This point is beyond the aim of this paper. However, Pasinetti (1990, p. 244) shows that a complete generalisation of the pure labour theory is possible by considering a uniform rate of profit. This author reported that "such a labour theory of value would be general in the sense that it would realise, for an advanced society, the fundamental characteristic

[7] See Pasinetti (1981, p. 132).

[8] The concept of 'natural rate of profit', introduced by Adam Smith, was reinterpreted by Pasinetti (1981, 1988). Whereas Adam Smith (1776) argues that – due to the competition amongst capitalists – the ordinary rate of profit is – in the long run – uniform across sectors, Pasinetti (1981, p. 130) postulates that "there are as many natural rates of profit as there are rates of expansion of demand (and production) of the various consumption goods."

of the pure labour theory of value originally proposed by Adam Smith with reference to a primitive society: namely a set of values that realise a universal equality of 'labour commanded' and 'labour embodied'. The analytical step that allows the achievement of this result is of course a redefinition of the concept of 'labour embodied' which must be intended as the quantity of labour required directly, indirectly and hyper-indirectly to obtain the corresponding commodity as a consumption good." [9]

The hypothesis of constant sectoral capital-output ratios requires that the capital intensity of the production processes remain constant over time in each sector. The empirical relevance of the constancy of these ratios turns out to be a test on the kind of technical progress that the economies experience. If technical progress affects both the consumer sector and the corresponding capital goods sector at the same rate, then technical progress for that sector is 'Harrod neutral' and the above ratios would remain indeed constant.

With these two assumptions, which seem quite reasonable in a growing economic system, a 'dynamic' pure labour theory of value is obtained. Under these conditions, the results concerning the benefits of international learning are extendable to the case of production with capital goods, in spite of heterogeneity of capital intensity from sector to sector.

Now it is worth to compare expressions (2) and (5). Pasinetti (1981) shows that the conditions emerging from both systems for a closed economy are identical. This can be shown in the present analysis by considering a closed economy. In this case,

[9] Harris (1982) emphasises that "(...) it is now shown that it is logically possible to have a 100% labour-value theory of price. Ricardo in his own day had failed to show this, and many others have tried, but we now have for the first time a fairly general proof."

fulfilment of one of the conditions (2) or (5) implies both full expenditure of national income and full employment.

On the other hand, the effective demand condition is broken in two when free trade is allowed: (2) becomes a condition for full employment and (5) becomes a condition for full expenditure of national income. If these two conditions are considered simultaneously[10], they express a new condition, which can be viewed as embodying a notion of trade balance equilibrium. From the point of view of country *U* this may be expressed as:

$$\sum_{i=1}^{n-1}(\xi\ a_{i\hat{n}} - a_{\hat{i}n})a_{ni} + \sum_{i=1}^{n-1}(\xi\ a_{ki,\hat{n}} - a_{\hat{ki},n})a_{nki} = 0 \qquad (8)$$

The basic intuition for this result can perhaps be grasped by observing that the labour coefficients a_{ni} and a_{nki} weight both the export and import demand coefficients for commodities *i* and *ki*, respectively. Hence, this condition requires that exported commodities expressed in terms of quantities of labour in country *U* must be equal to imported commodities also expressed in terms of quantities of labour in *U*[11].

From (6) and considering that the labour coefficients for sectors *i* and *ki* may be written respectively as $a_{ni} = \frac{p_i}{w} - \pi_i \frac{p_{ki}}{w}$ and $a_{nki} = \frac{p_{ki}}{w}$, condition (8) may be expressed in terms of prices as:

$$\sum_{i=1}^{n-1}(\xi\ a_{i\hat{n}} - a_{\hat{i}n})(p_i - \pi_i p_{ki}) + \sum_{i=1}^{n-1}(\xi\ a_{ki,\hat{n}} - a_{\hat{ki},n})\,p_{ki} = 0 \qquad (9)$$

[10] The fulfilment of conditions (2) and (5) imply the equilibrium in the trade balance but the reverse is not true. That is, this equilibrium does not imply full employment of the labour force and full expenditure of national income.

[11] Note that the trade balance equilibrium was not written in terms of prices as usual but in terms of labour coefficients. This shows why it is important to keep a theory in terms of vertically integrated labour.

Along with the full employment condition, expression (8) will play an important role when the circumstances in which free trade is beneficial are analysed.

3. Learning as the Primary Source of International Gains

Dealing with the 'principle of static comparative cost advantage' and its dynamic counterpart (diffusion of comparative productivity change), let us show that some conditions must hold to guarantee the benefits of trade: first, the structure of costs must be the best as possible. Second, the level of employment under free trade has to be larger or at least equal to this level under autarky. In addition, intertemporal trade balance equilibrium has to be reached. If such conditions do not hold, international trade fails to compensate losses in the level of employment and per-capita income.

In order to assess this point, let us assume that the number of commodities is the same in both countries. The medium of exchange (money) is anchored to gold, so that the exchange rate between the two currencies is fixed by the ratio of gold contents of the two monetary units. Suppose that all commodities (gold included) can be produced in *A* with 1/10th of the labour they require in *U*. Technical knowledge of the average person is such that per capita productivity in country *A* is ten times greater than in country *U*, for each single commodity, i.e. $a_{ni} = 10a_{\hat{n}\hat{i}}$ and $a_{nki} = 10a_{\hat{n}\hat{k}\hat{i}}$. From the previous section, the prices for capital goods *ki* in country *U* and *A* are respectively:

$$p_{ki} = a_{nki}w \tag{10}$$

$$p_{\hat{k}\hat{i}} = a_{\hat{n}\hat{k}\hat{i}}\hat{w} \tag{11}$$

As pointed out by Pasinetti (1981), one of the features of this hypothetical case is that all prices, in terms of gold, are exactly the same in both countries. Using this principle, it is possible to conclude from expressions (10) and (11) and from the relationship among the coefficients that:

$$\hat{w} = 10w \tag{12}$$

The real per capita income at the disposal of the average person is ten times greater in *A* than in *U*. Since the structure of costs is the same in both countries, international trade is not a source of gains.

It is worth to analyse the relationship among international prices considering that the average over-all productivity is ten times greater in *A* than in *U* but sectoral productivities differ according to a much wider range. The productivity in the gold sector is assumed to be equal to the average of the economy, i.e. it is ten times greater in *A* than in *U*. As the price of gold is the same in both countries, i.e. $p_{\hat{g}} = p_g$, and $p_{\hat{g}} = a_{\hat{n}\hat{g}}\hat{w}$ and $p_g = a_{ng}w$, then the relationship between the wage rates is given by: $\hat{w} = 10w$. The real per capita income is ten times greater in *A* than in *U*, as in the previous case. However, the prices of capital goods in both countries are related according to the following proposition.

Proposition:

Those capital goods for which differences in productivity are smaller than tenfold, i.e. $a_{nki} < 10a_{\hat{n}\hat{k}i}$, have a lower price in *U* than in *A*. Those capital goods for which differences in productivity are greater than tenfold, i.e. $a_{nki} > 10a_{\hat{n}\hat{k}i}$, have a lower price in *A* than in *U*.

Proof:

To verify this, let us focus on the capital goods for which differences in productivity are smaller than tenfold, i.e. $a_{nki} < 10a_{\hat{n}\hat{k}i}$. The prices in countries *A* and *U* may be written in terms of the gold price, respectively, as $p_{\hat{k}i} = \left(\frac{a_{\hat{n}\hat{k}i}}{a_{\hat{n}\hat{g}}}\right)p_{\hat{g}}$ and

$p_{ki} = \left(\frac{a_{nki}}{a_{ng}} \right) p_g$. Considering that the price of gold is the same in both countries and that $a_{ng} = 10a_{\hat{n}\hat{g}}$, the following relationship is obtained $p_{ki} = \frac{1}{10}\left(\frac{a_{nki}}{a_{\hat{n}\hat{k}i}} \right) p_{\hat{k}i}$. With the assumption that $a_{nki} < 10a_{\hat{n}\hat{k}i}$, this implies that $p_{ki} < p_{\hat{k}i}$, as we want to prove. Adopting the same procedure in relation to capital goods for which differences in productivity are greater than tenfold, i.e. $a_{nki} > 10a_{\hat{n}\hat{k}i}$, we have $p_{ki} > p_{\hat{k}i}$. QED

This proposition shows that if free trade is allowed, capital goods are induced to move between countries. Families in *A* would buy goods of the first type in *U*, where they are cheaper, and families in *U* would buy goods of the second type in *A*. Country *U* would be induced to specialise in producing, and then exporting, the first type of capital goods, while country *A* would be induced to specialise in producing, and then exporting, the second type of capital goods. This is nothing but the 'principle of comparative cost advantage' (Ricardo, 1817), which states that each country would be induced to concentrate on producing those commodities for which it is able to secure the highest levels of productivity.

In order to show that learning is the primary source of gains from international relations, it is necessary to focus on the problem of choice and change of technique[12]. As pointed out by Pasinetti (1981, p.189), the problem of choice of technique arises at the level of each single production unit, at a given point in time: a choice has to be made

[12] Meacci (1999) shows that this distinction, which is set out explicitly in Pasinetti's book (1981), lies implicitly at the roots of some crucial arguments developed within the modern theory of accumulation. In the present case we intend to show that this distinction is also essential to understand the benefits of the learning process when the economy is open to international relations.

between many alternative methods of production requiring well defined input of man-hours to produce a certain amount of final product.

Suppose that an arbitrary sector *i* has a number of firms or producers equal to L. Let Ω_l be the set of alternative techniques, $f_l^{\alpha},...,f_l^{\omega}$, available to the *l*-th firm, *l* = 1,...,L, to produce the quantity X_l i.e.:

$$\begin{cases} f_l^{\alpha}(K_l^{\alpha}, x_{nl}^{\alpha}) = X_l \\ \\ f_l^{\omega}(K_l^{\omega}, x_{nl}^{\omega}) = X_l \end{cases} \tag{13}$$

where the superscripts α,..., ω stand for the Ω_l alternative available technical methods. The quantities of labour required for each of these methods to produce the quantity X_l are represented by $x_{nl}^{\alpha},...,x_{nl}^{\omega}$. Each $K_l^k,(k=\alpha,...,\omega)$ stands for the vector of all inputs of physical machines and intermediate commodities required to produce the quantity X_l. The choice of technique by the *l*-th firm is made to minimise the costs of production according to the following *choice of technique function*:

$$\text{Cost of the chosen method} = Min\begin{cases} p_{ki}^{\alpha}K_l^{\alpha} + x_{nl}^{\alpha}w \\ \\ p_{ki}^{\omega}K_l^{\omega} + x_{nl}^{\omega}w \end{cases} \tag{14}$$

Being X_l the produced quantity of commodity *i* by firm *l*, x_{nl} the quantity of labour employed by firm *l*, *l* = 1,..., L, the labour coefficient of firm *l* is $a_{nl} = x_{nl} / X_l$. The sectoral labour coefficient a_{ni}, is reckoned as follows:

$$a_{ni} = \frac{\sum_{l=1}^{L} x_{nl}}{\sum_{l=1}^{L} X_l} = \frac{\sum_{l=1}^{L} a_{nl}X_l}{\sum_{l=1}^{L} X_l} \tag{15}$$

The coefficient a_{ni} refers to the quantity of labour socially necessary to produce one unit of commodity *i*. In the first volume of his *Capital*, Marx (1887, p. 47) states

that “the labour-time socially necessary is that required to produce an article under the normal conditions of production, and with the average degree of skill and intensity prevalent at the time”. Hence, it is reasonable to assume that some of the producers are able to produce a unit of commodity *i* with less labour than what is socially necessary. Others will require more labour.

Consider that, under free trade, the price of commodity *i* in country *U* is higher than in country *A*. There is an incentive for the families of country *U* to import commodity *i*. Let us assume that some of the domestic firms in *U* produce good *i* with less labour than what is socially required. For these firms, the challenging foreign price can probably be reached. In this case, they will survive, keeping its jobs. However, the jobs associated to those firms that are less productive and are unable to learn how to diminish costs will vanish. The result of this process over the labour coefficient a_{ni} is to decrease it. On average the quantity of labour socially necessary to produce commodity *i* will be less than before in country *U*. So there is a trend for equalisation of the prices of the commodity *i* in both countries. If this trend is confirmed, then complete international specialisation will not take place.

This illustration shows that only when all possible efforts to learn have been attempted may a country hope for further gains from free trade. In other words, all efforts of increasing productivity should have done before the process of full specialisation takes place. When the economy is open and firms face the cheaper foreign prices they try to adapt and establish a competitive price for their products. They either learn how to cut down costs or close down. Hopefully some of them may learn and survive. In this case a process of change of techniques is likely to take place and each of the *choice of technique function* is continually being enriched by new, previously unknown, technical methods. Such situation illustrates the case in which the possibility

of ‘learning quickly’ how to bring costs down to international level is the primary source of international gains[13]. From this possibility, a complete specialisation in some sectors can be a misleading policy if some firms can eventually reach the necessary level of productivity.

Before proceeding to the dynamic analysis of the benefits of international trade, it is important to establish a criterion to compare the eventual gains or losses from commerce. In a context of structural change analysis the comparison between the level of employment of the labour force under autarky and free trade arises as the natural criterion since this is the main focus of the investigation proposed by Pasinetti (1981, 1993).

Taking the derivative of the full employment condition (5) with respect to time allows to evaluate if gains in terms of jobs accruing from exportation compensate losses from importation in the short run. This happens if:

$$\begin{aligned}&\sum_{i=1}^{n-1}\left\{\left[\frac{da_{in}(t)}{dt}+\xi\frac{da_{i\hat{n}}(t)}{dt}\right]a_{ni}(t)+[a_{in}(t)+\xi\ a_{i\hat{n}}(t)]\frac{da_{ni}(t)}{dt}\right\}+\\&+\sum_{i=1}^{n-1}\left\{\left[\frac{da_{ki,n}(t)}{dt}+\xi\frac{da_{ki,\hat{n}}(t)}{dt}\right]a_{nki}(t)+[a_{k,in}(t)+\xi\ a_{ki\hat{n}}(t)]\frac{da_{nki}(t)}{dt}\right\}\geq 0\end{aligned} \tag{16}$$

As we intend to perform a structural dynamic analysis, the benefits of international trade should be evaluated mainly in the long run. In order to carry out this analysis let us assume that when $t = 0$ the employment condition is at the same level for the economy under free trade and autarky. So:

$$\sum_{i=1}^{n-1}(a_{in}+\xi\ a_{i\hat{n}})a_{ni}+\sum_{i=1}^{n-1}(a_{ki,n}+\xi\ a_{ki,\hat{n}})a_{nki}=\sum_{i=1}^{n-1}\alpha_{in}\alpha_{ni}+\sum_{i=1}^{n-1}\alpha_{ki,n}\alpha_{n,ki} \tag{17}$$

where the coefficients in the right hand side have the same meaning of section 2, corresponding to a free trade economy, and α_{in}’s are the demand coefficients, $\alpha_{ki,n}$’s

[13] The possibility of learning new techniques from abroad is considered by Oda (1999, p. 208).

are the investment coefficients, α_{ni}'s and α_{nki}'s are the labour coefficients, all of them for a closed economy. Central to this analysis of trade benefits in a context of structural change is the identification of the dynamical paths of the demand and technical coefficients[14]. For an autarky, they can be described as:

$$\alpha_{in}(t) = \alpha_{in}(0)e^{r_i t} \tag{18}$$

$$\alpha_{ki,n}(t) = (g + r_i)\alpha_{in}(t) \tag{19}$$

$$\alpha_{ni}(t) = \alpha_{ni}(0)e^{-\rho_i t} \tag{20}$$

$$\alpha_{nki}(t) = \alpha_{nki}(0)e^{-\rho_{ki} t} \tag{21}$$

where r_i is the growth rate of internal demand for commodity *i*. In the same vein, ρ_i is the rate of change of productivity for sector *i* while ρ_{ki} has the same meaning in relation to sector *ki*. For the case of a free trade economy, the dynamic paths of the coefficients are:

$$a_{in}(t) = a_{in}(0)e^{r_i t} \tag{22}$$

$$a_{i\hat{n}}(t) = a_{i\hat{n}}(0)e^{r_{\hat{i}} t} \tag{23}$$

$$a_{ki,n}(t) = (g + r_i)a_{in}(t) \tag{24}$$

$$a_{ki,\hat{n}}(t) = (g + r_{\hat{i}})a_{i\hat{n}}(t) \tag{25}$$[15]

$$a_{ni}(t) = a_{ni}(0)e^{-(\rho_i + \gamma_i \rho_{\hat{i}}) t} \tag{26}$$

$$a_{nki}(t) = a_{nki}(0)e^{-(\rho_{ki} + \gamma_{ki} \rho_{\hat{ki}}) t} \tag{27}$$

[14] Naturally, we are assuming that all coefficients and parameters are positive. The process of technological change through learning takes places through time according its impact on technical coefficients.

[15] Expressions (19), (24) and (25) represent capital accumulation conditions.

where $r_{\hat{i}}$ stands for the growth rate of foreign demand for good i. Besides, $\rho_{\hat{i}}$ and $\rho_{\hat{k}i}$ are the rate of change of productivity in the foreign sectors $\hat{\imath}$ and $\hat{k}_i$, respectively. The symbols γ_i and γ_{ki} stand for the fraction of foreign technological progress that is captured through international learning, $0 \leq \gamma_i \leq 1$ and $0 \leq \gamma_{ki} \leq 1$.

Note that the counterparts of expressions (20) and (21) are expressions (26) and (27), which consider the process of technological change due to learning from abroad. International trade is beneficial if:

$$\sum_{t=0}^{\infty}\sum_{i=1}^{n-1}\left[(a_{in} + \xi\ a_{i\hat{n}})a_{ni} + (a_{ki,n} + \xi\ a_{ki,\hat{n}})a_{nki}\right]e^{-\theta t} \geq \sum_{t=0}^{\infty}\sum_{i=1}^{n-1}\left[\alpha_{in}\alpha_{ni} + \alpha_{ki,n}\alpha_{n,ki}\right]e^{-\theta t} \qquad (28)$$

where $\theta > 0$ is the rate of social discount. Expression (28) is an intertemporal comparison between the employment condition under free trade and autarky. The level of employment under free trade has to be larger or at least equal to this level under autarky. In addition, the intertemporal equilibrium of the trade balance has to be kept:

$$\sum_{t=0}^{\infty}\left[\ \sum_{i=1}^{n-1}(\xi\ a_{i\hat{n}} - a_{\hat{i}n})a_{ni} + \sum_{i=1}^{n-1}(\xi\ a_{ki,\hat{n}} - a_{\hat{k}i,n})a_{nki}\ \right] = 0 \qquad (29)$$

Expressions (28) and (29) show that the benefits of trade are conditioned to the structural dynamics of the economy and to the intertemporal equilibrium of the trade balance[16]. Note that the economic system presented here is a growing one. The economic expansion arises as a consequence of technical progress particular to each sector and country. Furthermore, the demand coefficients that appear both in (28) and (29) have particular dynamic paths according to the inherent patterns of human needs

[16] See Appendix for numerical illustrations of the cases in which free trade is beneficial or not. Smith (1984) argues that we should be cautious of concluding that the introduction of capital destroys the main body of orthodox trade theory. This view deserves further inquire.

and preferences. They give rise to entirely different compositions of consumer demand, and therefore different structures of production and employment.[17]

4. Concluding Remarks

This paper extends Pasinetti's model of structural change (1981,1993) to international economic relations. The conditions for full employment of the labour force, full expenditure of national income and equilibrium of the trade balance are established along with solutions of the systems for prices and physical quantities for a free trade economy with capital goods. Structural change is then considered in an economic system that is faced with international flows of goods and possibilities of learning.

Under the assumptions of (i) proportionality of the rate of profit to the sectoral growth rate, and (ii) constant sectoral capital-output ratios, a dynamic labour theory of value holds. It follows that the results obtained by Pasinetti (1983, 1991), Araujo & Teixeira (2001, 2001b) and Teixeira & Sarquis (2001) concerning international trade are not confined to the restricted case of a pure labour economy, but remain valid when capital goods are introduced. To conclude: Pasinetti's statement that technical learning is the primary source of international economic gains, being the disparities of comparative costs and endowments only a secondary one, is rigorously proved here – confirming the validity of his insight.

[17] See Pasinetti (1981) for the dynamical path of production coefficients subject to sectoral technical progress. According to his approach, technical change, although taking place at a different pace in the various sectors, is exogenously determined. Reati (1998) goes a step further and introduces long waves in this model assuming that productivity growth is fundamentally driven by technological revolution, giving rise to a complex dynamic involving a set of prices, physical quantities and employment.

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Appendix

Consider an economic system facing two alternative situations characterised by different parameters. The illustrations, carried out from the point of view of country *U*, maintain that both countries *U* and *A* produce and consume the same two consumption and capital goods (i=1,2 and n=3) but with different structures of production and patterns of demand. In both illustrations, the employment conditions for the cases of free trade and autarky are compared. We assume, for simplicity, the same population size for *U* and *A* (ξ =1) and no population growth (g = 0). Besides, trade balance equilibrium and equal employment condition under free trade or autarky hold at t = 0.

In the first illustration, we assume non-homothetic tastes, that consumption good 1 is a simple kind of commodity and that consumption good 2 has a higher income elasticity of demand in country *A*. So there is expansion of per capita consumption for good 1 only in country *U*. The initial conditions are: $\alpha_{13}(0) = 0.8$, $\alpha_{23}(0) = 0.2$, $\alpha_{k1,3}(0) = 0.4$, $\alpha_{k2,3}(0) = 0.2$, $\alpha_{31}(0) = 0.5$, $\alpha_{32}(0) = 1$, $\alpha_{3k1}(0) = 0.5$, $\alpha_{3k2}(0) = 1$ and the parameters values are r_1=0.01, r_2=0.005, ρ_1=0.015, ρ_2=0.015, ρ_{k1}=0.015 and ρ_{k2}=0.015. Given the dynamic expressions (18)-(21), after t=5 periods, the employment condition in autarky (EC) corresponds to:

$$\sum_{t=0}^{5} \mathrm{EC}(t) = 1.0000 + 0.6075 + 0.6074 + 0.6075 + 0.6076 + 0.6077 = 4.0377$$

Consider now opening the economy to international trade. Assume that in country *A* the consumption pattern for good 1 is already in the upper part of Engel's curve (saturation). Suppose, moreover, that country *U* is not able to adopt the better technological conditions prevailing in country *A* ($\gamma_i = \gamma_{ki} = 0$):

The initial conditions[18] are $a_{13}(0) = 0.6$, $a_{1\hat{3}}(0) = 0.2$, $a_{23}(0) = 0.1$, $a_{2\hat{3}}(0) = 0.1$, $a_{k1,3}(0) = 0.3$, $a_{k1,\hat{3}}(0) = 0.1$, $a_{k2,3}(0) = 0.1$, $a_{k2,\hat{3}}(0) = 0.1$, $a_{31}(0) = 0.5$, $a_{32}(0) = 1$, $a_{3k1}(0) = 0.5$, $a_{3k2}(0) = 1$, $a_{\hat{1}3}(0) = 0.2$, $a_{\hat{2}3}(0) = 0.1$, $a_{\hat{k}1,3}(0) = 0.1$, $a_{\hat{k}2,3}(0) = 0.1$ and the values of additional parameters are $r_{\hat{1}} = 0$ and $r_{\hat{2}} = 0.005$. Given the dynamic expressions (22)-(27), after *t*=5 periods, the employment condition in free trade (ECT) corresponds to:

$$\sum_{t=0}^{5} \mathrm{ECT}(t) = 1.0000 + 0.5991 + 0.5946 + 0.5902 + 0.5859 + 0.5815 = 3.9513$$

Comparing the employment condition for open and closed economies, one gets: $\sum_{t=0}^{5} \mathrm{EC}(t) > \sum_{t=0}^{5} \mathrm{ECT}(t)$ [19]. Considering the behaviour of trade balance from *t* = 0 to *t* = 5, one gets:

$$\sum_{t=0}^{5} \mathrm{TB}(t) = 0 - 0.0020 - 0.0030 - 0.0039 - 0.0048 - 0.0057 = -0.0194$$

This is a situation where international trade will not bring gains for country *U*. The low income elasticity of the consumption good 1 in country *A* is the mechanism responsible for country *U* failing to obtain gains from trade.

In the second illustration, we assume that in country *U* the foreign growth rate of demand for commodity 2 is larger than the domestic one. The initial conditions are the

[18] Note that initial conditions are the same for the cases of autarky and free trade. This is expressed by the fact that $\alpha_{i3}(0) = a_{i3}(0) + a_{i\hat{3}}(0)$, $\alpha_{ki,3}(0) = a_{ki,3}(0) + a_{ki,\hat{3}}(0)$, $\alpha_{3i}(0) = a_{3i}(0)$ and $\alpha_{3ki}(0) = a_{3ki}(0)$, *i*=1,2. In this case the employment condition is the same for both cases when *t*=0, as required by expression (17).

[19] Note from expression (28) that it is not necessary to consider the rate of intertemporal discount since it affects both sides of inequality in the same way.

same of the previous illustration. The relevant parameters are, instead, r_1=0.01, r_2=0.005, ρ_1=0.015, ρ_{k1}=0.015, ρ_2=0.015 and ρ_{k2}=0.015. For t = 5, the employment condition in autarky corresponds to:

$$\sum_{t=0}^{5} \mathrm{EC}(t) = 1.0000 + 0.6075 + 0.6074 + 0.6075 + 0.6076 + 0.6077 = 4.0377$$

Setting $r_{\hat{1}} = 0.01$ and $r_{\hat{2}} = 0.065$, the employment condition in free trade corresponds to:

$$\sum_{t=0}^{5} \mathrm{ECT}(t) = 1.0000 + 0.6078 + 0.6103 + 0.6133 + 0.6167 + 0.6202 = 4.0683$$

Comparing the employment condition for both situations, one gets: $\sum_{t=0}^{5} \mathrm{EC}(t) < \sum_{t=0}^{5} \mathrm{ECT}(t)$. Considering the value of the trade balance equation from t=0 to t=5, one gets:

$$\sum_{t=0}^{5} \mathrm{TB}(t) = 0.0000 + 0.0125 + 0.0192 + 0.0262 + 0.0335 + 0.0412 = 0.1326$$

In this specific example, the superiority of free trade to autarky is established: international trade is beneficial for country *U*.