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Abstract

In this article the growth models of Feldman (1928) and Mahalanobis (1953) are extended to analyse the implications of the process of structural change on the decisions of investment allocation. By using the device of vertical integration, their constructions are shown to be a particular case of the Pasinetti's (1981) model of structural change. Their analysis is then carried out in a multi-sector framework, where both productivity and demand change at a particular rate in each of the sectors. A particular rate of investment allocation for each sector is established subject to the full employment of the labour force. Following these lines, we are able to put some of the Halevi's (1996) descriptive observations into a formal context and within this context to analyse and to extend his ideas about the role of demand on the decisions of investment allocation. Finally, an additional condition is added to the Pasinetti's model in order to fully characterise the equilibrium path in the most general version of his framework, where capital goods are needed to produce capital goods.

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1. Introduction

In this article we study the consequences of the structural change process on the decisions of investment allocation. For some authors, such as Halevi (1996) and Araujo, Teixeira & Araujo (1999), the Pasinetti's (1981,1993) model of structural change can provide new views on central questions of the theory of economic development. In particular, it can be useful to understand the role played by the investment allocation between capital and consumption good sectors on the process of economic growth. Our purpose on the subject is to put Halevi's descriptive contributions into a proper general context and, within that context, to analyse, to extend and to formalise his ideas. The insight to be gained from our paper is that it allows clarifications on the connections between the growth rate of productivity and the role played by evolving patterns of demand.

Feldman (1928) and Mahalanobis (1953) models, hereafter F-M model, are generally used as a benchmark to study the effects of the investment allocation on economic growth[1]. In order to introduce a normative criterion to these models, Bose (1968) and Weitzman (1971) established an optimum rate of investment allocation in a context of dynamic optimisation of consumption. However, these analyses did not take into account the composition of consumption demand since they were accomplished in a bi-sector

[1] Dutt (1990:120) considers that no discussion related to models with investment and consumption good sectors is complete without considering the contribution of Feldman-Mahalanobis.

model[2]. In this case, a change in the distribution of investment between the capital and consumption goods sectors only modifies once and for all the growth rate. As pointed out by Halevi (1996), "The Marx-Feldman-Mahalanobis two-sector model cannot possibly take into account the composition of consumption demand because it contains only one consumption good. Any increase in per capita income is transformed into a higher level of consumption of the same commodity."

In order to mitigate the limitations of the F-M model in relation to the passive role of per capita consumption demand, we show in section 2 that it can be treated as a particular case of Pasinetti's model of structural change. This is accomplished by using the correspondence between the concepts of vertical integration, as used by Pasinetti (1981, 1990), and sub-systems, as defined by Sraffa (1960). An important characteristic of Pasinetti's approach is its focus on the analysis of structural dynamics, which is performed in a multi-sector model. In the same vein, by showing that the F-M model is a particular case of Pasinetti, it is possible to carry out the analysis of investment allocation in a multi-sector framework where demand and productivity change at particular rates in each of the sectors.

From this study, it is possible to show that an additional condition must be introduced in the most general version of the Pasinetti's model, where capital goods are required to produce capital goods. This new condition is referred here as the investment allocation condition and it is required to ensure the fulfilment of the capital accumulation

[2] In order to carry on the structural change analysis properly; it is necessary to consider an economy disaggregated in a number of sectors. This is the reason why traditional models of economic growth, such as those developed by Harrod (1939), Domar (1946), Solow-Swan (1956) and Uzawa (1961) are not appropriate to perform this analysis.

condition in the next period. It is also a normative criterion for the F-M model since it refers to the rate of investment allocation subject to the full employment of labour force and to the evolving pattern of demand. The existence of such condition imposes more difficulties in ensuring an equilibrium path to the economy since it has to be satisfied simultaneously in each of the sectors.

As an analytical device, we consider firstly, in section 3, a simple case of economic growth, where growth is driven solely by the expansion of the working force. This step allows a better comprehension of the most interesting case where technical progress is introduced. In both cases the complete characterization of the equilibrium path is derived and the possibility for the economy to follow this trajectory are analysed. Section 4 concludes.

2. Feldman-Mahalanobis as a Particular Case of Pasinetti

The Pasinetti's model of structural change and economic dynamics is carried out, not in terms of input-output relations, as has become usual in multi-sector models, but rather in terms of vertically integrated sectors. This device is used to focus on final commodities rather than on industries. In this case, it is possible to associate each commodity to its final inputs - a flow of working services and a stock of capital goods - thus eliminating all intermediate inputs. From this point of view, this model may be viewed as a particular case of the input-output model submitted to the process of vertical integration.

As pointed out by Pasinetti (1981, p.111): "The difference, in other words, lies only in the classification, and we can pass from the one to the other simply by an algebraic re-arrangement, corresponding to a process of solving a system of linear equations: the coefficients of a vertically integrated model turn out to be a linear combination of the

production coefficients of the corresponding input-output model." This makes it possible to analyse the economic growth process in terms of the structural dynamics of production, of prices and employment.

Pasinetti considers that the Sraffa's (1960) 'production of commodities system' is, along with the input-output model, the logical static counterpart of his dynamic analysis. The connection among these models can be grasped by focusing on what Sraffa called sub-systems, i.e., parts of the economic system formed by smaller and self-reproducible systems, which has as final output only one kind of commodity. As can be verified this concept of sub-systems can be fully applied to describe vertically integrated sectors since they have these same characteristics. Quoting Pasinetti (1990, p.232): "I am referring to what Sraffa, by looking at the economic system from an inter-industry point of view, has called the method of the 'subsystems'; and what I myself, by looking at it from a final demand point of view, have called the method of 'vertically integrated sectors'."

Following these lines it is possible to show that F-M is a particular case of Pasinetti, but let us present firstly the main results of the former, adjusting conveniently the notation[3]. This model considers two sectors: one produces a consumption good and is denoted by subscript 1 and the other is the corresponding capital good sector, denoted by k1. The capital goods are used by both sectors but once installed, they cannot be transferred from one sector to the other (non-shiftability assumption). A proportion λ of the current production of the investment sector is allocated to itself while the remaining, $1-\lambda$, is allocated to sector 1. $(1 \geq \lambda \geq 0)$

Besides, in both sectors, technology is described by Leontief production functions

[3] We use here a continuous version of the F-M model. For a discrete version see Sengupta, Fox and Thorbecke (1966).

and the limiting factor of production in both sectors is the stock of capital goods. So we can write:

$$X_1 = min\ [K_1/v_1; L_1/u_1] \Rightarrow X_1 = K_1/v_1 \quad (1)$$

$$X_{k1} = min\ [K_{k1}/v_{k1}; L_{k1}/u_{k1}] \Rightarrow K_{k1} = K_{k1}/v_{k1} \quad (2)$$

where X_1 and X_{k1} stand for, respectively, to the production of sectors 1 and k1. K_1 and K_{k1} refer to the stock of investment goods available in each of the sectors; v_1 and v_{k1} represent the capital-output ratio in each sector; L_1 and L_{k1} are the quantity of employed working force and u_1 and u_{k1} are labour coefficients.

For the sake of convenience only, it is assumed that there is no depreciation of capital goods. The investment goods cannot be imported and the production of capital goods does not depend on the production of consumption goods sector. Now it is possible to establish the growth rate of investment. The change in investment is given by:

$$\dot{X}_{k1} = \dot{K}_{k1} / v_{k1} \quad (3)$$

But the variation in stock of capital in sector kl, that is K_{k1}, depends only on the proportion of the total output of this sector that is allocated to itself:

$$\dot{K}_{K1} = \lambda X_{k1} \quad (4)$$

Substituting (4) in (3) leads to the growth rate of the investment sector:

$$\frac{\dot{X}_{k1}}{X_{k1}} = \frac{\lambda}{v_{k1}} \quad (5)$$

Adopting the same procedure in relation to the consumption sector and considering that $\dot{K}_1 = (1-\lambda)X_{k1}$,we establish the growth rate of this sector:

$$\frac{\dot{X}_1}{X_1} = \frac{(1-\lambda)X_{k1}}{v_1 X_1} \quad (6)$$

The main results of the F-M model are well known and may be briefly summarised

as follows: the growth rate of consumption depends on the growth rate of investment and, in the long run, the former converges to the later, which will be the growth rate of the economy as a whole. Notice that the F-M two-sector model cannot possibly take into account the composition of consumption demand since it contains only one consumption good. Thus any increase in per capita income is transformed into a higher level of consumption of the same kind, as pointed out by Halevi (1996, p.169). The emphasis on demand brings out an important qualitative improvement in relation to the F-M approach.

As can be noted, despite the fact that we are dealing with a bi-sector model, the economic system described by F-M has the same characteristics of what Sraffa (1960, appendix A) has called sub-systems, i.e, it is self-reproducible, it uses no intermediate goods to produce only one kind of commodity. Therefore, it represents an economy in which sectors are vertically integrated.

A striking difference between F-M and Pasinetti refers to the level of utilisation of vertical integration. While in the former this device was used almost to the limit, thus generating a model with only one consumption good, in the later it was used to generate a multi-sector framework. In this sense, F-M can be obtained from Pasinetti considering the existence of only one commodity in the later, and consequently, of one consumption good sector and its correspondent capital good sector.

In fact the concept of vertical integration has been widely used in macroeconomics. As pointed out by Lavoie (1995), "the concept of vertical integration, although extensively but implicitly used in macroeconomic analysis, has always been difficult to seize intuitively". What is behind this affirmation is that models that are aggregated in one or two sector are based on the device of vertical integration.

From this point of view, the technical coefficients in F-M are nothing but a linear

combination of the vertically integrated coefficients of Pasinetti and it is possible to go from one model to the other without difficulty. Therefore, the analysis of investment allocation can be extended to a multi-sector economy in which the sectors are vertically integrated in a pasinettian sense. In this case, according to the pasinettian approach, given the hierarchical order in which the production of consumption goods ought to proceed, the composition of investment should reflect on the input side, the very same order of priorities. This presents a vision linking productivity growth with the rise in per capita income , i.e., the dynamics of effective demand intimately connected to variations in the coefficients of per capita demand. Needles to say that mainstream theories of economic growth have neglected to a large extent such connection.

Hence, the sraffian concept of sub-systems is what allows us to go from one model to the other. The input-output model represents an extreme case, where there is no vertical integration. The pasinettian model corresponds to an intermediate case, where the device of vertical integration was used to generate a multi-sector model. And, finally, the F-M model is another extreme case, where vertical integration was used to generate a one consumption good model. We also show in the next section that the analysis of investment allocation is enriched when carried out in a multi-sector model.

3. The Investment Allocation Condition

An important characteristic of the Pasinetti's approach is its focus on structural dynamics. According to Hishiyama (1966, p.198): "that is to say, the structural dynamics of prices, production and employment that evolve through time in a different way from one particular period to the next". As a first approach to this section, let us consider a simple economic growth, where there is no technical progress and the expansion is due only to

increases in the labour force. As we have indicated, this is clearly an uninteresting case when analysing the effects of structural change since all sectors grow at the same rate. But it is valuable as a initial step to the important situation, where technical progress plays a central role, submitting the economic system to the process of structural change.

As in Pasinetti (1983), we introduce some hypothesis about the dynamic path of exogenous variables and study the implications on the equilibrium. The initial conditions are characterised by the full employment of the labour force and full utilisation of capital, i.e., respectively, the effective demand condition and the capital accumulation condition particular to each sector are fulfilled. The fundamental equations of "Structural Change and Economic Dynamics" concerning the static equilibrium run as follows:

$$\sum_{i=1}^{n-1} a_{in} a_{ni} + \sum_{i=1}^{n-1} a_{nki} a_{kin} = 1 \qquad (7)^4$$

$$X_i\,(0) = K_i\,(0) \text{ and } X_{ki}\,(0) = \gamma_i\, K_{ki}\,(0) \qquad (8)$$

where a_{in} and a_{kin} are demand and investment coefficients and a_{ni} and a_{nki} are labour coefficients to sectors i and ki respectively. X_i and X_{ki} stand for the production and K_i and K_{ki} represent the stock of capital[5] in sectors i and ki, respectively. In addition, γ_i is the ratio of one physical unit of capital goods expressed in terms of productive capacity for the consumption goods sector to one unit of capital goods expressed in terms of productive capacity for the capital goods sector. Population X_n grows at a rate g>0, i.e.:

$$X_n\,(t) = X_n\,(0)\, e^{gt} \qquad (9)$$

There is no technical progress and the consumer's demand does not change over time. The equilibrium path is characterised by the fulfilment of the effective demand and capital accumulation conditions over time. They can be expressed by:

[4] From now on, we assume that i=1,...,n-1 for all expressions.
[5] As in F-M model, we are assuming that the life span of capital goods in each sector is ilimited.

$$\sum a_{in}(t)\, a_{ni}\,(t) + \sum a_{nki}(t) a_{kin}\,(t) = 1 \tag{10}$$

$$a_{kin}(t) = [g/(1 - g\gamma_i)]\; a_{in}\,(t) \tag{11}$$

The effective demand condition is immediately satisfied for all t since it holds when t = 0 and all coefficients are constant over time. These two conditions are enough to characterise the pasinettian equilibrium in the case where capital goods are produced by labour alone. However, when dealing with the most general case, in which capital goods are required to produce capital goods, there is an additional condition to be observed, that is the investment allocation condition. This condition is also a normative criterion to the F-M model since it is nothing but a requirement on the rate of investment allocation in order to keep the full employment over time. It can be derived as follows. According to the Pasinetti's model, the growth rate of the ki-th sector is given by:

$$\frac{\dot{X}_{k1}}{X_{k1}} = g \tag{12}$$

This is the growth rate that has to be observed in order to fulfil the demand requirements. But from F-M model, we know that the possible growth rate of capital good production is given by:

$$\frac{\dot{X}_{k1}}{X_{k1}} = \frac{\lambda_i}{v_{ki}} \tag{13}$$

By equalising these two expressions, the value of λ_i that warrants the growing of investment at a rate compatible to the path of a growing of demand is determined:

$$\lambda_i = g v_{ki} \tag{14}$$

Such expression introduces a normative criterion for the F-M model: capital goods have to be allocated according to (14) in every period in order to allow the fulfilment of the correspondent capital accumulation condition in the next one. Since we are assuming that

capital goods are non-shiftable, if the investment allocation condition does not hold in one period, it will not be possible to fulfil the capital accumulation condition from there on.

Now we are ready to analyse the most general case where the economic growth is also due to technical progress. The procedure here is similar to the previous case but now the evolving pattern of demand and technical coefficients are introduced.

As before, the initial conditions are characterised, *ex-hipothesi*, by the full employment and full capacity utilisation. This means that:

(i) The stock of capital goods in each sector corresponds exactly to the quantity required by demand.

(ii) The investment coefficients are such that they fulfil the dynamical requirements of the system (these requirements will be detailed later).

(iii) The technical and demand coefficients satisfy jointly the effective demand condition.

The dynamical path of exogenous variables can be described as follows. The population continues to grow at a constant rate g>0 and the productivity changes at a particular rate ρ_i and ρ_{ki} in each sector of consumption goods and capital goods, respectively. These rates are considered different from one sector to the others but does not change over time in the same sector This means that:

$$a_{ni}(t) = a_{ni}(0)\, exp\,(-\rho_i t) \tag{15}$$

$$a_{nki}(t) = a_{nki}(0)\, exp(-\rho_{ki} t) \tag{16}$$

The per capita demand changes at a particular rate r_i to each commodity. These rates are constant in the time interval considered but they can change from the passage of one period to the other. Then the dynamical path of demand coefficients may be written as:

$$a_{in}(t) = a_{in}(0) exp(r_i t) \tag{17}$$

By using the same reasoning of the previous case, the equilibrium growth rate of

consumption and investment goods in each sector is given by:

$$\frac{\dot{X}_i}{X_i} = g + r_i \tag{18}$$

$$\frac{\dot{X}_{ki}}{X_{ki}} = g + r_i \tag{19}$$

Comparing (19) to the growth rate of investment goods obtained from the F-M model, it follows that:

$$\lambda_i / v_{ki} = (g + r_i) \tag{20}$$

The investment allocation condition in this case may be written as:

$$\lambda_i = (g + r_i)\, v_{ki} \tag{21}$$

Therefore the equilibrium path is characterised by the investment allocation condition and capital accumulation condition, both applied to each sector, and by the effective demand condition, which in the present case may be written as:

$$\sum a_{in}(0)\, a_{ni}(0) \exp[(r_i - \rho_i)t] + \sum (g + r_i) a_{nki}(0)\, a_{in}(0) \exp[(r_i - (-\rho_{ki})t)] = 1 \tag{22}$$

Notice that, the effective demand condition presents serious problems to be fulfilled since there is no warranty that the right hand side of expression (22) will be equal to the unit over time. Besides, the investment allocation condition presents a very particular sectoral term, that is r_i. This condition must then be fulfilled in each of the sectors in order to endow the economy with the required capital goods to maintain full employment.

Since the growth rate of demand r_i plays an important role in this analysis, it is important to focus on its determinants. Through this rate, it is possible to capture the changes in consumption due to the expansion on income, described by the Engel Law. This means that r_i depends not only on the consumers preferences but also on their evolution over time, given by the following expression.

$$r_i(t) = f_i\{a_{n1},....,a_{n,n-1},a_{nk1},....,a_{nkn-1}:d/dt[a_{n1},....a_{n,n-1},a_{nk1},....,....a_{nkn-1}]\} \quad (23)$$

As pointed out by Pasinetti (1981), the technical coefficients that appear in the expression above affect the growth rate of demand in two ways: through the level and the variation of real per capital income and through changes in the structure of prices. Therefore, as shown by expression (21), the rate of investment allocation derived here suggests the need to pay an special attention to the evolution of consumers demand.

From the perspective presented in this section, the limitations of the F-M model in relation to the passive role of per capita consumption demand are diminished. In the present case, the composition of investment will reflect, on the input side, the same order of priorities in which production of consumption goods is organised according to the consumer's preferences.

4. Concluding Remarks

In this article, it was shown that by introducing structural change in an extended version of the Feldman-Mahalanobis model of investment allocation it is possible to obtain a new result concerning a central question on the theory economic development. The standpoint of the analysis, following a pasinettian approach is in the interaction between the technical progress - which produces responses in per capita income - and the changes of per capita consumption which spreads unevenly among the different kinds of commodities, due to the Engel law.

The influence of these factors on the investment allocation between capital and consumption goods sectors were analysed in order to establish the rate of investment allocation subject to the maintenance of both full employment and full capacity utilisation. This rate is determined by taking into account the structure of consumer preferences.

It was also shown that when dealing with the most general version of the Pasinetti's model of structural change, where capital goods are needed for the production of capital goods, there is an additional condition to be verified in order to ensure the full employment of labour force. This condition was referred here as the investment allocation condition. So we were able to formalise some important descriptive ideas contained in Halevi's paper, and therefore to proceed to a more technical discussion of these matters.

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