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# Monetary Policy and Endogenous Market Structure in a Schumpeterian Economy

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## Abstract

In this study, we develop a monetary Schumpeterian growth model with endogenous market structure (EMS) to explore the effects of monetary policy on the number of firms, firm size, economic growth and social welfare. EMS leads to different results from previous studies in which market structure is exogenous and richer implications on transitional dynamics. In the short run, a higher nominal interest rate reduces the growth rates of innovation, output and consumption and also decreases firm size due to a reduction in labor supply. In the long run, an increase in the nominal interest rate reduces the equilibrium number of firms but has no steady-state effect on economic growth and firm size due to EMS. Although monetary policy has no long-run effect on economic growth, an increase in the nominal interest rate permanently reduces the levels of output, consumption and employment. Taking into account transition dynamics, we find that social welfare is decreasing in the nominal interest rate. Given that a zero nominal interest rate maximizes welfare, the Friedman rule is optimal in this economy.

*JEL classification:* O30, O40, E41

*Keywords:* monetary policy, economic growth, R&D, endogenous market structure

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# 1 Introduction

In this study, we develop a monetary Schumpeterian growth model to explore the effects of monetary policy on economic growth, social welfare and endogenous market structure (EMS). In contrast to previous studies with exogenous market structure, we find that monetary policy has only transitory, not permanent, effects on the rate of economic growth. Specifically, we find that although an increase in the nominal interest rate, accompanied by an increase in the money growth rate, reduces the level of output on the balanced growth path, it has no effect on the steady-state growth rate. In other words, money is superneutral with respect to the *growth rate* of output but not to the *level* of output in accordance with empirical evidence; see for example Fisher and Seater (1993), King and Watson (1997) and Bullard (1999). The reason for this result is that the economy's market structure responds endogenously to changes in labor supply induced by monetary policy. In other words, market structure, measured by the market size of each firm, is endogenously determined through the entry and exit of firms in response to macroeconomic conditions. More importantly, each firm's incentives to invest in R&D depend on the size of its market, which is determined by market structure but not aggregate market size.

To capture EMS and R&D in a dynamic framework, we use a variant of the second-generation R&D-based growth model, pioneered by Peretto (1998), Young (1998), Howitt (1999) and Segerstrom (2000). To our knowledge, this is the first analysis of monetary policy in the second-generation R&D-based growth model. The second-generation R&D-based growth model realistically features two dimensions of technological progress: variety expansion (i.e., horizontal innovation) and quality improvement (i.e., vertical innovation). In the horizontal dimension, entrepreneurs create new firms by introducing new products, and the number of firms in equilibrium determines two important elements of market structure: market concentration and firm size. In the vertical dimension, each incumbent firm performs in-house R&D to improve the quality of its products, and the return to in-house R&D is determined by the market size of the firm. In this economy, technological progress and market structure are jointly determined in equilibrium: market structure is measured by firms' market size, whereas technological progress is determined by the growth rate of vertical innovation. One advantage of the second-generation R&D-based growth model is that it is consistent with empirical facts in the industrial organization (IO) literature. For example, the return to R&D depends on firm size rather than aggregate market size; see Cohen and Klepper (1996a,b). Furthermore, theoretical implications of the second-generation R&D-based growth model with EMS are supported by empirical studies, such as Laincz and Peretto (2006) and Ha and Howitt (2007).

In this growth-theoretic framework, an increase in the nominal interest rate<sup>1</sup> reduces labor supply via a cash-in-advance (CIA) constraint on consumption and gives rise to interesting transitional dynamic effects. In the short run, the reduction in labor supply caused by a higher nominal interest rate reduces average firm size and the growth rates of innovation, output and consumption.<sup>2</sup> Intuitively, when the nominal interest rate increases, households

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<sup>1</sup>In this study, the nominal interest rate is a policy instrument chosen by the monetary authority. However, one could consider an alternative analysis by having the monetary authority choosing the money growth rate.

<sup>2</sup>For example, Evers *et al.* (2007) provide empirical evidence for a negative effect of inflation and the nominal interest rate on total factor productivity growth.

decrease consumption and increase leisure due to an extra cost of consumption imposed by the CIA constraint. As a result, the reduced supply of labor causes lower employment per firm on the transition path, which in turn reduces economic growth temporarily. In the long run, an increase in the nominal interest rate reduces the steady-state equilibrium number of firms but has no effect on economic growth and firm size due to the endogeneity of market structure. Intuitively, some firms exit the market as a result of the smaller aggregate market size measured by the supply of labor, and the number of firms adjusts such that employment per firm in the steady state returns to the initial level. Therefore, long-run economic growth is independent of the nominal interest rate. Although monetary policy has no long-run effect on economic growth, an increase in the nominal interest rate permanently reduces the levels of output, consumption and employment. Furthermore, taking into account transition dynamics, we find that social welfare is decreasing in the nominal interest rate. Intuitively, the supply of labor is suboptimally low in equilibrium, so that a positive nominal interest rate that reduces labor supply is suboptimal. Given the zero lower bound on the nominal interest rate, a zero nominal interest rate maximizes social welfare, and hence, the Friedman rule is optimal in this economy.<sup>3</sup> To our knowledge, this is the first analytical derivation of optimal monetary policy that takes into account transition dynamics in the R&D-based growth model.

This study relates to the literature on inflation and economic growth;<sup>4</sup> see Tobin (1965) and Stockman (1981) for seminal studies and Wang and Yip (1992) for a discussion on different approaches of modelling money demand. A common approach of modelling money demand in this literature is through a CIA constraint on consumption; see for example Gomme (1993), Dotsey and Ireland (1996) and Mino (1997). In this study, we follow this approach to model money demand. Studies in this literature often analyze the growth and welfare effects of monetary policy in variants of the overlapping generations model or the Neoclassical growth model. For example, Wu and Zhang (2001) also analyze the effects of inflation on the number of firms and firm size in a Neoclassical growth model; however, they do not consider R&D-driven economic growth and transition dynamics. Our study takes into consideration these elements and relates to a more recent subbranch of the literature that analyzes the growth and welfare effects of monetary policy in R&D-based growth models; see for example, Marquis and Reffett (1994), Funk and Kromen (2010), Chu and Cozzi (2013), Chu and Lai (2013) and Chu *et al.* (2012). These studies consider either the variety-expanding model or the quality-ladder model. The present study differs from them by analyzing the effects of monetary policy in a second-generation R&D-based growth model in which market structure is endogenous and responds to monetary policy. In other words, we consider a Schumpeterian growth model with EMS; see Peretto (1996, 1999) for seminal studies in R&D-based growth models with EMS and Etro (2012) for an excellent textbook treatment. This study contributes to the literature with a novel analysis of monetary policy on economic growth and market structure in an R&D-based growth model and also provides a novel result that the long-run effects of monetary policy in an R&D-based growth model with EMS are reflected in the economy's market structure and the level of output rather than the growth rate of output. This theoretical result has an important empirical implication

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<sup>3</sup>See Mulligan and Sala-i-Martin (1997) for a discussion of the Friedman rule.

<sup>4</sup>Gillman and Kejak (2005) provide a survey of this literature.

that money is superneutral with respect to the *growth rate* of output but not to the *level* of output in accordance with the empirical evidence discussed above.

The rest of this study is organized as follows. Section 2 presents the monetary Schumpeterian growth model with EMS. Section 3 analyzes the effects of monetary policy on economic growth and social welfare. The final section concludes.

## 2 A monetary Schumpeterian growth model with EMS

Our growth-theoretic framework is based on the Schumpeterian model with in-house R&D and EMS in Peretto (2007, 2011). We introduce money demand into the model via the Lucasian approach of a CIA constraint on consumption as in Lucas (1980).<sup>5</sup> As in standard CIA models, monetary policy affects the economy by distorting households' tradeoff between consumption and leisure. In our analysis, we provide a complete closed-form solution for the economy's transition dynamics as well as its balanced growth path.

### 2.1 Households

There is a representative household, who has the following lifetime utility function:

$$U = \int_0^{\infty} e^{-\rho t} \ln u_t dt = \int_0^{\infty} e^{-\rho t} [\ln c_t + \gamma \ln(L - l_t)] dt, \quad (1)$$

where  $c_t$  denotes consumption of final goods (numeraire) at time  $t$  and  $l_t$  denotes labor supply. The parameters  $\rho > 0$  and  $\gamma > 0$  determine respectively subjective discounting and leisure preference. Each household maximizes (1) subject to the following asset-accumulation equation:<sup>6</sup>

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_t l_t + \tau_t - c_t - \pi_t m_t. \quad (2)$$

Monopolistic intermediate goods firms are owned by the household, and the value of firms' shares is  $a_t$ .<sup>7</sup> The real rate of return on  $a_t$  is  $r_t$ , which we will refer to as the real interest rate.<sup>8</sup> The household has a labor endowment of  $L$  units and elastically supplies  $l_t$  units to earn a real wage rate  $w_t$ . The household also faces a lump-sum transfer (or tax)  $\tau_t$  from the government. The household carries real balances  $m_t$  to facilitate purchases of consumption goods.<sup>9</sup> The cost of holding money is the inflation rate  $\pi_t$ . The CIA constraint is given by

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<sup>5</sup>We do not consider other CIA constraints in this study in order to focus on the consumption-leisure tradeoff, which is an important channel through which monetary policy affects economic growth in the literature. See Chu and Cozzi (2013) for an analysis of CIA constraints on production and R&D investment.

<sup>6</sup>We also impose the usual no-Ponzi game condition that requires the household's lifetime budget constraint to be satisfied.

<sup>7</sup>Final goods firms make zero profit, so their ownership is not reflected in the household's budget.

<sup>8</sup>In the presence of real bonds (in zero net supply), the real interest rate on these bonds would equal the real rate of return on  $a_t$ .

<sup>9</sup>In this study, we focus on a single type of money, namely currency. See for example Santomero and Seater (1996) for an analysis of an economy with several types of money.

$\xi c_t \leq m_t$ , where the parameter  $\xi > 0$  determines the importance of the CIA constraint. In the limiting case  $\xi \rightarrow 0$ , monetary policy would have no effect on the real economy.

The optimality condition for consumption is

$$\frac{1}{c_t} = \eta_t(1 + \xi i_t), \quad (3)$$

where  $i_t = r_t + \pi_t$  is the nominal interest rate and  $\eta_t$  is the Hamiltonian co-state variable on (2).<sup>10</sup> The optimality condition for labor supply is

$$w_t(L - l_t) = \gamma c_t(1 + \xi i_t). \quad (4)$$

The intertemporal optimality condition is

$$-\frac{\dot{\eta}_t}{\eta_t} = r_t - \rho. \quad (5)$$

In the case of a constant nominal interest rate  $i$ ,<sup>11</sup> combining (3) and (5) yields the familiar Euler equation  $\dot{c}_t/c_t = r_t - \rho$ .

## 2.2 Final goods

Following Aghion and Howitt (2005, 2008) and Peretto (2007, 2011), we assume that final goods  $Y_t$  are produced by competitive firms using the following production function:<sup>12</sup>

$$Y_t = \int_0^{N_t} X_t^\theta(j) [Z_t^\alpha(j) Z_t^{1-\alpha} l_t/N_t]^{1-\theta} dj, \quad (6)$$

where  $\theta, \alpha \in (0, 1)$  and  $X_t(j)$  denotes intermediate goods  $j \in [0, N_t]$ .<sup>13</sup> The productivity of intermediate good  $X_t(j)$  depends on its quality  $Z_t(j)$  and also on the average quality  $Z_t \equiv \frac{1}{N_t} \int_0^{N_t} Z_t(j) dj$  of all intermediate goods capturing R&D spillovers.<sup>14</sup> From profit maximization, the equilibrium wage rate is determined by

$$w_t = (1 - \theta)Y_t/l_t, \quad (7)$$

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<sup>10</sup>There is also a co-state variable on the CIA constraint, and we have substituted out this co-state variable using the first-order conditions in order to derive (3).

<sup>11</sup>Given that the nominal interest rate is exogenously chosen by the monetary authority, the inflation rate endogenously responds to changes in the real interest rate.

<sup>12</sup>Peretto (2007, 2011) consider a slightly different production function that replaces  $l_t/N_t$  by  $l_{x,t}(j)$ , which denotes labor that uses intermediate goods  $X_t(j)$ . Given that  $l_{x,t}(j) = l_t/N_t$  in equilibrium, we follow Aghion and Howitt (2005, 2008) to use the more direct specification  $l_t/N_t$ , which has the advantage of being generalizable. Peretto (2013) considers a more general specification  $l_t/N_t^\sigma$ , where  $\sigma \in (0, 1)$  inversely measures the social return to varieties. Our result of superneutrality of monetary policy with respect to economic growth is robust to this generalization. Derivations are available upon request.

<sup>13</sup>There is no capital in the production function. Instead, one can treat intermediate goods as capital that depreciates instantaneously, which is a common treatment in this type of models; see Peretto (2007, 2011).

<sup>14</sup>Here the average quality  $Z_t$  captures in a simple way R&D spillovers across firms; see for example Jaffe (1986) and Bernstein and Nadiri (1988, 1989) for empirical evidence.

and the conditional demand function for  $X_t(j)$  is

$$X_t(j) = \left( \frac{\theta}{p_t(j)} \right)^{1/(1-\theta)} Z_t^\alpha(j) Z_t^{1-\alpha} l_t / N_t, \quad (8)$$

where  $p_t(j)$  denotes the price of  $X_t(j)$  denominated in units of  $Y_t$ . The demand for type- $j$  intermediate goods depends on the market size of each firm measured by  $l_t/N_t$ . The number of firms and the market size of each firm are endogenously determined in equilibrium. Perfect competition implies that final goods producers pay  $\theta Y_t = \int_0^{N_t} p_t(j) X_t(j) dj$  to intermediate goods firms.

## 2.3 Intermediate goods

There is a continuum of industries producing differentiated intermediate goods  $X_t(j)$  for  $j \in [0, N_t]$ . Each type of intermediate goods is produced by a single monopolistic firm that has price-setting power. Thus, the number of intermediate goods  $N_t$  is the same as the number of firms that produce them. There are two types of R&D, vertical and horizontal. Vertical R&D is quality improvement, carried out by incumbent firms in an attempt to increase the demand for their products. This formulation is consistent with the empirical facts in the IO literature that most R&D is done by incumbents; see for example Dosi (1988) for a survey. Horizontal R&D is the invention of new products, carried out by entrepreneurs who enter the market as new firms producing the newly invented goods. Through the entry of firms, the number of firms and the market size of each firm are determined endogenously in equilibrium.

### 2.3.1 Incumbents

Existing intermediate goods firms produce differentiated goods with a technology that requires one unit of final goods to produce one unit of intermediate goods. Following Peretto (2007), we assume that the firm in industry  $j$  incurs  $\phi Z_t^\alpha(j) Z_t^{1-\alpha}$  units of final goods as a fixed operating cost. This specification implies that managing facilities are more expensive to operate in a technologically more advanced environment. To improve the quality of its products, the firm invests  $R_t(j)$  units of final goods in R&D. The innovation process is

$$\dot{Z}_t(j) = R_t(j). \quad (9)$$

The profit flow of firm  $j$  is

$$F_t(j) = [p_t(j) - 1] X_t(j) - \phi Z_t^\alpha(j) Z_t^{1-\alpha}, \quad (10)$$

and the dividend flow is

$$\Pi_t(j) = F_t(j) - R_t(j), \quad (11)$$

which is distributed to the household who owns the firm. The value of the monopolistic firm in industry  $j$  is

$$V_t(j) = \int_t^\infty \exp\left(-\int_t^u r_s ds\right) \Pi_u(j) du. \quad (12)$$

Taking the conditional demand function (8) as given, the firm sets its own price and devotes resources to in-house R&D to maximize  $V_t(j)$ . The current-value Hamiltonian for this optimization problem is<sup>15</sup>

$$H_t(j) = \Pi_t(j) + q_t(j)\dot{Z}_t(j). \quad (13)$$

Following the standard approach in this class of models, we consider a symmetric equilibrium in which  $Z_t(j) = Z_t$  for  $j \in [0, N_t]$ .<sup>16</sup> The return to in-house R&D is increasing in the market size of each firm measured by employment per firm  $l_t/N_t$ . This property is consistent with the empirical facts in the IO literature discussed in the introduction.

**Lemma 1** *The return to in-house R&D is given by*<sup>17</sup>

$$r_t^I = \alpha \left[ \theta^{(1+\theta)/(1-\theta)} (1-\theta) \frac{l_t}{N_t} - \phi \right]. \quad (14)$$

**Proof.** See the Appendix. ■

### 2.3.2 Entrants

A firm that is active at time  $t$  must have been born at some earlier date. A new firm pays a setup cost  $\beta X_t(j) > 0$  at time  $t$  to set up its operation and introduce a new variety of product.<sup>18</sup> Following the standard treatment in the literature, we assume that the new product comes into existence with the average level of quality as existing products. We refer to this process as entry. Suppose entry is positive (i.e.,  $\dot{N}_t > 0$ ). Then, the no-arbitrage condition is<sup>19</sup>

$$V_t(j) = \beta X_t(j). \quad (15)$$

Under symmetry,  $V_t(j) = V_t$ , and the familiar Bellman equation implies that the return to equity (i.e., entry) is

$$r_t^E = \frac{\Pi_t}{V_t} + \frac{\dot{V}_t}{V_t}, \quad (16)$$

which is the usual profit rate plus the capital gain. In equilibrium,  $r_t^E$  must equal the real interest rate  $r_t$ , which is determined by the Euler equation in Section 2.1.

<sup>15</sup>See the Appendix for the solution of this optimization problem.

<sup>16</sup>See Peretto (1998, 1999, 2007) for a discussion of the symmetric equilibrium being a reasonable equilibrium concept in this class of models.

<sup>17</sup>Given that monopolistic firms are owned by the household, the return to in-house R&D must equal the real interest rate  $r_t$ . However, following the usual presentation in this class of models, we label the rate of return to in-house R&D as  $r_t^I$  in order to distinguish it from the rate of return to entry  $r_t^E$ . In equilibrium, it must be the case that  $r_t^I = r_t^E = r_t$  because all assets are owned by the household and they must yield the same rate of return.

<sup>18</sup>The setup cost is proportional to the new firm's initial volume of output. This assumption captures the idea that the setup cost depends on the amount of productive assets required to start production. See Peretto (2007) for a discussion.

<sup>19</sup>It is useful to note that we have followed the standard approach in this class of models to treat entry and exit symmetrically (i.e., the scrap value of exiting an industry is also  $\beta X_t(j)$ ). Therefore,  $V_t(j) = \beta X_t(j)$  always holds. Otherwise, there would be an infinite number of entries or exits.



## 2.4 Monetary authority

The nominal money supply is denoted by  $M_t$ , and its growth rate is  $\mu_t \equiv \dot{M}_t/M_t$ . The real money balance is  $m_t = M_t/P_t$ , where  $P_t$  is the price of final goods. The monetary policy instrument that we consider is  $i_t$ . Given a nominal interest rate  $i_t$  exogenously chosen by the monetary authority, the inflation rate is endogenously determined according to  $\pi_t = i_t - r_t$ . Then, given  $\pi_t$ , the growth rate of the nominal money supply is endogenously determined according to  $\mu_t = \pi_t + \dot{m}_t/m_t$ . On the balanced growth path, the nominal interest rate is related to the money growth rate simply as  $i = r + \pi = \rho + \mu$ ; therefore, it is the *growth rate* of money supply that affects the real economy in this model.<sup>20</sup> The monetary authority distributes the newly printed money to the household via a lump-sum transfer, and this transfer has a real value of

$$\tau_t = \dot{M}_t/P_t = \dot{m}_t + \pi_t m_t. \quad (17)$$

## 2.5 General equilibrium

The equilibrium is a time path of allocations  $\{m_t, a_t, c_t, Y_t, l_t, X_t(j), R_t(j)\}$ , prices  $\{r_t, w_t, p_t(j), V_t\}$  and monetary policy  $\{i_t\}$  such that the following conditions are satisfied:

- the household chooses  $\{m_t, a_t, c_t, l_t\}$  to maximize utility taking  $\{r_t, w_t, \pi_t\}$  as given;
- competitive final goods firms choose  $\{l_t, X_t(j)\}$  to maximize profits taking  $\{w_t, p_t(j)\}$  as given;
- incumbents in the intermediate goods sector choose  $\{p_t(j), R_t(j)\}$  to maximize the present value of profits taking  $\{r_t\}$  as given;
- entrants make entry decisions taking  $\{V_t\}$  as given;
- the value of all existing monopolistic firms adds up to the value of the household's assets such that  $a_t = N_t V_t$ ;
- the market-clearing condition of labor holds; and
- the market-clearing condition of final goods holds.

The resource constraint on final goods is

$$Y_t = c_t + N_t(X_t + \phi Z_t + R_t) + \beta X_t \dot{N}_t. \quad (18)$$

Substituting (8) into (6) and imposing symmetry yield the aggregate production function

$$Y_t = \theta^{2\theta/(1-\theta)} Z_t l_t, \quad (19)$$

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<sup>20</sup>In contrast, a one-time change in the *level* of money supply affects the price level and has no effect on the real economy. This is the well-known distinction between the neutrality and superneutrality of money. The evidence generally favors neutrality and rejects superneutrality (with respect to the level of output), consistent with our model. See for example Fisher and Seater (1993), King and Watson (1997) and Bullard (1999) for a discussion on the neutrality and superneutrality of money.

which uses markup pricing  $p_t(j) = 1/\theta$ .

In the Appendix, we show that the consumption-output ratio  $c_t/Y_t$  jumps to a unique and stable steady-state value, a property that greatly simplifies the analysis of the transition dynamics.

**Lemma 2** *The consumption-output ratio jumps to a unique and stable steady-state value*

$$(c/Y)^* = 1 - \theta + \rho\beta\theta^2. \quad (20)$$

**Proof.** See the Appendix. ■

Given a constant nominal interest rate  $i$  and a stationary consumption-output ratio, one can use (4) to show that the supply of labor  $l_t$  also jumps to its steady-state value given by

$$l^* = \left[ 1 + \gamma(1 + \xi i) \left( 1 + \frac{\rho\beta\theta^2}{1 - \theta} \right) \right]^{-1} L. \quad (21)$$

Equation (21) shows that the equilibrium supply of labor is decreasing in the nominal interest rate  $i$ . Intuitively, an increase in the nominal interest rate increases the cost of consumption relative to leisure because of the CIA constraint on consumption. As a result, the household reduces consumption and increases leisure. Given that labor supply is stationary for any given nominal interest rate  $i$ , (19) and (20) imply that

$$\frac{\dot{Z}_t}{Z_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho, \quad (22)$$

where the last equality uses the Euler equation. Setting  $r_t^I = r_t$ , one can then use (14) and (22) to derive the equilibrium growth rate given by

$$g_t \equiv \frac{\dot{Z}_t}{Z_t} = \max \left\{ \alpha \left[ \theta^{(1+\theta)/(1-\theta)} (1 - \theta) \frac{l^*}{N_t} - \phi \right] - \rho, 0 \right\}, \quad (23)$$

which is increasing in each firm's market size measured by employment per firm  $l^*/N_t$ .<sup>21</sup> The growth rate  $g_t$  is strictly positive if and only if

$$N_t < \bar{N} \equiv \frac{\theta^{(1+\theta)/(1-\theta)} (1 - \theta)}{\phi + \rho/\alpha} l^*.$$

This inequality means that if the number of firms is below a critical level  $\bar{N}$ , each firm's market size is large enough to make it profitable for firms to do in-house R&D. Otherwise, there are too many firms diluting the return to R&D. As a result, firms do not invest in R&D, and the growth rate of vertical innovation is zero. In the Appendix, we provide the derivations of the dynamics of  $N_t$ .<sup>22</sup>

<sup>21</sup>Laincz and Peretto (2006) provide empirical evidence that is consistent with the theoretical prediction from this class of models that economic growth is positively related to the average firm size.

<sup>22</sup>Given the positive entry cost  $\beta X_t(j) > 0$ , the number of firms  $N_t$  is a state variable. In other words, the positive entry cost acts a barrier to entry and bounds  $\dot{N}_t$  (i.e., the change in the number of firms).

**Lemma 3** *The growth rate of  $N_t$  is given by*

$$\frac{\dot{N}_t}{N_t} = \begin{cases} \frac{1-\theta}{\beta\theta} - \left(\phi + \frac{\dot{Z}_t}{Z_t}\right) \frac{N_t}{\beta\theta^{2/(1-\theta)}l^*} - \rho & \text{if } N_t < \bar{N} \\ \frac{1-\theta}{\beta\theta} - \phi \frac{N_t}{\beta\theta^{2/(1-\theta)}l^*} - \rho & \text{if } N_t > \bar{N} \end{cases}. \quad (24)$$

**Proof.** See the Appendix. ■

The following Lemma provides the steady-state values of  $N_t = N^*$  and  $g_t = g^*$  as well as the parameter restrictions that ensure  $N^* \in (0, \bar{N})$  and  $g^* > 0$ .<sup>23</sup>

**Lemma 4** *Under the parameter restrictions that  $\frac{1-\theta}{\beta\theta} - \alpha\phi < \rho < \frac{(1-\alpha)(1-\theta)}{\beta\theta}$ ,<sup>24</sup> the economy is stable and has a positive and unique steady-state value of  $N_t$  as well as a positive and unique steady-state growth rate given by*

$$N^* = \left[ \frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho \right] \frac{\beta\theta^{2/(1-\theta)}l^*}{\phi(1-\alpha) - \rho} > 0, \quad (25)$$

$$g^* = \alpha \left[ \theta^{(1+\theta)/(1-\theta)}(1-\theta) \frac{l^*}{N^*} - \phi \right] - \rho = \frac{(\rho + \alpha\phi)\beta\theta - (1-\theta)}{(1-\alpha)(1-\theta)/\rho - \beta\theta} > 0. \quad (26)$$

**Proof.** See the Appendix. ■

### 3 Growth and welfare effects of monetary policy

In this section, we analyze the effects of monetary policy on the number of firms, the market size of each firm, economic growth and social welfare. Specifically, we consider the effects of an unexpected permanent change in the nominal interest rate  $i$ . In Section 3.1, we analyze the effects of monetary policy on economic growth. In Section 3.2, we analyze the effects of monetary policy on social welfare.

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<sup>23</sup>In this model, we have assumed zero population growth, so that  $N_t$  converges to a steady-state value. If we assume positive population growth, it would be the number of firms per capita that converges to a steady state instead, and our main results would be unchanged.

<sup>24</sup>This parameter restriction would depend on a larger set of parameters if we parameterize R&D productivity in (9) and the productivity in producing intermediate goods from final goods. For simplicity, we have implicitly normalized these productivity parameters to unity.

### 3.1 Effects of monetary policy on economic growth

Proposition 1 provides our first main result: an increase in the nominal interest rate  $i$ , accompanied by an increase in the money growth rate  $\mu$ , reduces the steady-state equilibrium number of firms but does not affect the steady-state equilibrium growth rate. Intuitively, an increase in  $i$  reduces the supply of labor  $l^*$  in (21), which in turn leads to a decrease in the steady-state equilibrium number of firms  $N^*$ . A reduction in labor supply decreases the aggregate market size, which in turn induces some firms to exit the market such that the market size of each firm remains constant in the steady state. Due to this endogeneity of market structure, steady-state employment per firm  $l^*/N^*$  remains unchanged. As a result, the steady-state equilibrium growth rate in (26) is independent of the nominal interest rate  $i$  and the money growth rate  $\mu$ .

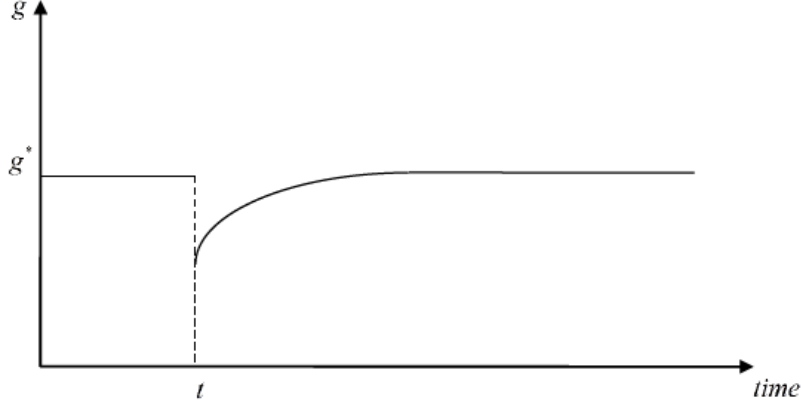
**Proposition 1** *The steady-state equilibrium number of firms is decreasing in the nominal interest rate, but the steady-state equilibrium growth rate and firm market size are independent of the nominal interest rate and the money growth rate.*

**Proof.** Use (21), (25) and (26). Also, recall that  $i = \rho + \mu$  in the steady state. ■

The above result of monetary superneutrality with respect to economic growth differs from previous studies, such as Chu and Lai (2013) and Chu *et al.* (2012) who find that an increase in the money growth rate reduces the steady-state equilibrium growth rate of innovation. This difference is due to the fact that the earlier literature uses a monetary R&D-based growth model with an exogenous market structure. In contrast, the market structure in our model is endogenous. Entry and exit of firms in response to profit opportunities imply that the number of firms increases or decreases with aggregate market size. This mechanism implies that the number of firms changes in response to endogenous changes in labor supply leading to our result that monetary policy has no steady-state effect on economic growth and the market size of each firm.

We now show the effects of monetary policy on economic growth along the transition path. The model features transition dynamics because  $N_t$  is a state variable that gradually converges to its state-state value  $N^*$ . When the monetary authority increases the nominal interest rate, the equilibrium supply of labor  $l^*$  adjusts instantly, but the equilibrium number of firms adjusts slowly. Given that the equilibrium growth rate is determined by firm market size  $l^*/N_t$ , monetary policy can have an effect on economic growth during the transition to the steady state. Proposition 2 shows that an increase in the nominal interest rate reduces the growth rates of vertical innovation, output and consumption on the transition path. Figure 1 illustrates the transitional effects of a permanent increase in the nominal interest rate at time  $t$ .

**Figure 1: Transitional effects of the nominal interest rate on economic growth**



**Proposition 2** *An increase in the nominal interest rate reduces the growth rates of vertical innovation, output and consumption on the transition path.*

**Proof.** Use (21), (22) and (23). Also, recall that  $N_t$  is a state variable. ■

Intuitively, an increase in the nominal interest rate reduces labor supply, which adjusts instantly and leads to a temporary decrease in firm market size  $l^*/N_t$ . The smaller firm market size reduces the returns to R&D in (14) and the equilibrium growth rate in (23). Over time, the smaller aggregate market size determined by  $l^*$  induces some firms to leave the market. As a result, firm market size  $l^*/N_t$  gradually increases and returns to the initial level at which point, the equilibrium growth rate also returns to the initial level as shown in Figure 1.

This transitional dynamic analysis of the effects of monetary policy is novel relative to previous studies, such as Marquis and Reffett (1994), Funk and Kromen (2010), Chu and Cozzi (2013) and Chu and Lai (2013), which focus on the steady-state equilibrium growth rate. Given that real world data is affected by both transitional and steady-state effects, having theoretical results on the characteristics of transition dynamics could be helpful for formulating empirically testable hypotheses.

### 3.2 Effects of monetary policy on social welfare

In this subsection, we analyze the welfare effects of monetary policy. Specifically, we consider the effects of a permanent change in the nominal interest rate at time 0 on flow utility  $\ln u_t$  at any arbitrary time  $t \geq 0$ . We show that  $\partial \ln u_t / \partial i < 0$ , which is sufficient for  $\partial U / \partial i < 0$  because  $U = \int_0^\infty e^{-\rho t} \ln u_t dt$ . Taking the log of (19), we obtain

$$\ln Y_t = \frac{2\theta}{1-\theta} \ln \theta + \ln Z_t + \ln l_t = \frac{2\theta}{1-\theta} \ln \theta + \int_0^t g_s ds + \ln l^*, \quad (27)$$

where we have normalized  $Z_0 = 1$ . Taking the log of (20), we obtain

$$\ln c_t = \ln(1 - \theta + \rho\beta\theta^2) + \ln Y_t. \quad (28)$$

Therefore, an increase in the nominal interest rate at time 0 decreases the levels of output and consumption at any arbitrary time  $t > 0$  through two channels. First, it reduces the supply of labor  $l^*$ . Second, it temporarily reduces the growth rate of technology, which decreases the level of technology in the future.

**Proposition 3** *An increase in the nominal interest rate at time 0 decreases the levels of output and consumption at any arbitrary time  $t > 0$ .*

**Proof.** Use Proposition 2 and (21) in (27) and (28). ■

Substituting (27) and (28) into flow utility  $\ln u_t$  in (1) and then differentiating it with respect to  $i$  yield

$$\frac{\partial \ln u_t}{\partial i} = \underbrace{\int_0^t \frac{\partial g_s}{\partial i} ds}_{-} + \underbrace{\frac{\partial \ln l^*}{\partial i}}_{-} + \underbrace{\gamma \frac{\partial \ln(L - l^*)}{\partial i}}_{+}. \quad (29)$$

An increase in the nominal interest rate  $i$  thus has three effects on social welfare. First, it reduces welfare by temporarily decreasing the growth rates of vertical innovation, output and consumption. Second, it reduces welfare by decreasing the levels of output and consumption through a decrease in labor supply  $l^*$ . Third, it improves welfare by increasing leisure  $L - l^*$ . The first two effects dominate the third effect because the loss of consumption dominates the gain in leisure so that  $\partial \ln u_t / \partial i < 0$ . Intuitively, the supply of labor is suboptimally low in equilibrium partly because the CIA constraint imposes an extra cost on consumption relative to leisure. To see this result,

$$\frac{\partial \ln l^*}{\partial l^*} + \gamma \frac{\partial \ln(L - l^*)}{\partial l^*} = \frac{L - (1 + \gamma)l^*}{l^*(L - l^*)} > 0 \quad (30)$$

because  $L/(1 + \gamma) > l^*$  in (21). As a result, a positive nominal interest rate that reduces labor supply is suboptimal. We summarize these welfare implications in the following proposition.

**Proposition 4** *Social welfare is decreasing in the nominal interest rate; therefore, the Friedman rule (i.e., a zero nominal interest rate) is socially optimal in this economy.*

**Proof.** Use (29) and (30). Also, recall from (21) that  $\partial l^* / \partial i < 0$ . ■

Previous studies, such as Marquis and Reffett (1994) and Chu and Lai (2013), also find that the Friedman rule is optimal in the R&D-based growth model.<sup>25</sup> However, these studies focus on steady-state welfare. To our knowledge, our result is the first analytical derivation of optimal monetary policy that takes into account the endogeneity of market structure and transition dynamics in the equilibrium growth rate of an R&D-based growth model.

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<sup>25</sup>See Chu and Cozzi (2013) for an analysis on the suboptimality of the Friedman rule in the Schumpeterian growth model with a CIA constraint on R&D investment.

## 4 Conclusion

In this study, we have analyzed the effects of monetary policy on economic growth, social welfare and market structure in a Schumpeterian growth model with endogenous market structure. Unlike previous studies that analyze the effects of monetary policy on economic growth either in an AK-type growth model or in the first-generation R&D-based growth model, this study analyzes the effects of monetary policy in a second-generation R&D-based growth model in which we have obtained novel results and richer implications. A novel result is that monetary policy has a negative effect on economic growth only in the short run; in the long run, monetary policy has no effect on the equilibrium growth rate because of the endogenous response of the economy's market structure to changes in labor supply induced by monetary policy. In other words, we find that money is superneutrality with respect to economic growth. This result highlights the importance of endogenous market structure and differs from previous studies that analyze the effects of monetary policy in R&D-based growth models with exogenous market structure. Furthermore, we analyze optimal monetary policy by analytically deriving the complete changes in welfare along the transition path and find that the Friedman rule is socially optimal in this economy.

A potential direction for future research is to investigate the effects of monetary policy on economic growth and social welfare in a growth-theoretic framework in which R&D endogenously alters the importance of labor as a factor of production. The behavior of labor is central to our results, so a model in which the importance of labor changes as a result of R&D might deliver interesting new insights into the relation between money and economic growth. See Peretto and Seater (2013) for the recent development of such a model without money.

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## Appendix

**Proof of Lemma 1.** Substituting (8), (10) and (11) into (13) yields

$$H_t(j) = \theta [Z_t^\alpha(j)Z_t^{1-\alpha}l_t/N_t]^{1-\theta} [X_t(j)]^\theta - X_t(j) - \phi Z_t^\alpha(j)Z_t^{1-\alpha} - R_t(j) + q_t(j)R_t(j). \quad (\text{A1})$$

The first-order conditions include

$$\frac{\partial H_t(j)}{\partial X_t(j)} = 0 \Leftrightarrow p_t(j) = \theta \left[ \frac{Z_t^\alpha(j)Z_t^{1-\alpha}l_t/N_t}{X_t(j)} \right]^{1-\theta} = \frac{1}{\theta}, \quad (\text{A2})$$

$$\frac{\partial H_t(j)}{\partial R_t(j)} = 0 \Leftrightarrow q_t(j) = 1, \quad (\text{A3})$$

$$\frac{\partial H_t(j)}{\partial Z_t(j)} = \alpha(1-\theta) \frac{\theta [Z_t^\alpha(j)Z_t^{1-\alpha}l_t/N_t]^{1-\theta} [X_t(j)]^\theta}{Z_t(j)} - \alpha\phi Z_t^{\alpha-1}(j)Z_t^{1-\alpha} = r_t^I q_t(j) - \dot{q}_t(j). \quad (\text{A4})$$

Substituting (A2) and (A3) into (A4) yields

$$r_t^I = \alpha \left[ (1-\theta)\theta^{(1+\theta)/(1-\theta)}l_t/N_t - \phi \right], \quad (\text{A5})$$

where we have applied  $Z_t(j) = Z_t$ . ■

**Proof of Lemma 2.** Substituting  $\tau_t = \dot{m}_t + \pi_t m_t$  into (2) yields

$$\dot{a}_t = r_t a_t + w_t l_t - c_t. \quad (\text{A6})$$

Then, substituting (15) into  $a_t = V_t N_t$  yields

$$a_t = \beta X_t N_t = \beta \frac{p_t X_t N_t}{p_t} = \beta \theta^2 Y_t, \quad (\text{A7})$$

where the last equality uses (A2) and  $p_t X_t N_t = \theta Y_t$ . Substituting (A7) into (A6) yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{a}_t}{a_t} = r_t + \frac{w_t l_t - c_t}{\beta \theta^2 Y_t}. \quad (\text{A8})$$

Substituting the Euler equation and  $w_t l_t = (1-\theta)Y_t$  into (A8) yields

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{Y}_t}{Y_t} = \frac{c_t/Y_t}{\beta \theta^2} - \left( \frac{1-\theta}{\beta \theta^2} + \rho \right). \quad (\text{A9})$$

Therefore, the dynamics of  $c_t/Y_t$  is characterized by saddle-point stability such that  $c_t/Y_t$  must jump to its steady-state value in (20). ■

**Proof of Lemma 3.** Substituting (10), (11), (15) and (A2) into (16) yields

$$r_t^E = \frac{1-\theta}{\beta \theta} - \frac{\phi Z_t + R_t}{\beta X_t} + \frac{\dot{X}_t}{X_t}, \quad (\text{A10})$$

where we have applied  $Z_t(j) = Z_t$  and  $\dot{V}_t/V_t = \dot{X}_t/X_t$ . Substituting (A2) into (8) yields

$$X_t = \theta^{2/(1-\theta)} Z_t \frac{l^*}{N_t}. \quad (\text{A11})$$

Substituting (9) and (A11) into (A10) yields

$$r_t^E = \frac{1-\theta}{\beta\theta} - \left( \phi + \frac{\dot{Z}_t}{Z_t} \right) \frac{N_t}{\beta\theta^{2/(1-\theta)} l^*} + \frac{\dot{Z}_t}{Z_t} - \frac{\dot{N}_t}{N_t}, \quad (\text{A12})$$

where we have used  $\dot{X}_t/X_t = \dot{Z}_t/Z_t - \dot{N}_t/N_t$ . Setting  $r_t^E = r_t$  and substituting (22) into (A12) yield the dynamics of  $N_t$  given by

$$\frac{\dot{N}_t}{N_t} = \frac{1-\theta}{\beta\theta} - \left( \phi + \frac{\dot{Z}_t}{Z_t} \right) \frac{N_t}{\beta\theta^{2/(1-\theta)} l^*} - \rho. \quad (\text{A13})$$

Equation (A13) describes the dynamics of  $N_t$  when  $N_t < \bar{N} \equiv \frac{\theta^{(1+\theta)/(1-\theta)}(1-\theta)}{\phi+\rho/\alpha} l^*$ . When  $N_t > \bar{N}$ ,  $\dot{Z}_t/Z_t = 0$  as shown in (23). ■

**Proof of Lemma 4.** This proof proceeds as follows. First, we prove that under  $\rho < \min \left\{ \phi(1-\alpha), \frac{(1-\alpha)(1-\theta)}{\beta\theta} \right\}$ , there exists a stable, unique and positive steady-state value of  $N_t$ . Then, we prove that under  $\rho > \frac{1-\theta}{\beta\theta} - \alpha\phi$ , the growth rate of vertical innovation is strictly positive. Finally, the above parameter conditions can be merged into  $\frac{1-\theta}{\beta\theta} - \alpha\phi < \rho < \frac{(1-\alpha)(1-\theta)}{\beta\theta}$ , which ensures  $\frac{(1-\alpha)(1-\theta)}{\beta\theta} < \phi(1-\alpha)$ . We consider the equilibrium under which there is positive in-house R&D. Substituting (23) into the first equation of (24) yields

$$\frac{\dot{N}_t}{N_t} = \frac{\rho - \phi(1-\alpha)}{\beta\theta^{2/(1-\theta)} l^*} N_t + \frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho. \quad (\text{A14})$$

Because  $N_t$  is a state variable, the dynamics of  $N_t$  is stable if and only if  $\rho < \phi(1-\alpha)$ . Solving  $\dot{N}_t = 0$ , we obtain the steady-state value of  $N_t$  in an economy with positive in-house R&D.

$$N^* = \left[ \frac{(1-\alpha)(1-\theta)}{\beta\theta} - \rho \right] \frac{\beta\theta^{2/(1-\theta)} l^*}{\phi(1-\alpha) - \rho}. \quad (\text{A15})$$

Given  $\rho < \phi(1-\alpha)$ , (A15) shows that  $N^* > 0$  if and only if

$$\rho < \frac{(1-\alpha)(1-\theta)}{\beta\theta}. \quad (\text{A16})$$

Combining  $\rho < \phi(1-\alpha)$  and (A16) yields

$$\rho < \min \left\{ \phi(1-\alpha), \frac{(1-\alpha)(1-\theta)}{\beta\theta} \right\}. \quad (\text{A17})$$

Substituting (A15) into (23) yields (26). Given (A16), (26) shows that  $g^* > 0$  if and only if  $\rho > \frac{1-\theta}{\beta\theta} - \alpha\phi$ . ■