Monetary part of Abenomics: a simplified model

Krouglov, Alexei

3 February 2014

Monetary Part of Abenomics: A Simplified Model

Alexei Krouglov

alexkrouglov@gmail.com

Monetary Part of Abenomics: A Simplified Model

ABSTRACT

Presented is a simplified mathematical model that describes dynamics developing on financial market after the liquidity pumping. The model is used to examine theoretical and practical implications of the monetary component of Abenomics. Based on the theoretical considerations, proposed is a somewhat practical suggestion how to increase the efficiency of Abenomics' monetary policy.

JEL Classification Numbers: E32, E44, C61

Keywords: monetary policy, Abenomics, mathematical model

1 Introduction

After becoming the Prime Minister of Japan at the end of 2012, Shinzō Abe announced an implementation of the fresh economic policy consisting of three parts (or "arrows"): expanding monetary policy, supportive fiscal policy, and growth oriented economic reforms. The policy was labeled by some intellectuals as *Abenomics*. The monetary policy part was implemented in concert with the Bank of Japan led by governor Haruhiko Kuroda since March 2013.

The Bank of Japan started aggressively buying Japanese government bonds. Since the demand for bonds increased the price of the bonds increased as well initially and correspondingly the bond yield decreased. However, a "weird" – from an economic point of view – outcome followed in May 2013. Despite the continuous purchase of bonds by the Bank of Japan their price significantly dropped and the yield increased.

This paradoxical event caused some confusion among the experts. One camp was saying that explanation was applicable to the anticipated recovery of real economy. Particularly, investors were anticipating an economic recovery, which would increase demand for the risky assets and decrease demand for the government bonds, which, in turn, would project an increase of the bond yield both in the future and in the present correspondingly. Personally, I found this explanation a bit convoluted. The other camp was saying that the event was associated with the financial market experience. Primarily, they said the cautious investors (where the cause of their nervousness was differently argued by different experts) were selling the Japanese government bonds in larger quantities, and overpowering the purchases of them by the Bank of Japan.

The author considered himself to be in the financial market camp.[1] Nevertheless, he decided to build a reduced model to check his "intuition" based on the framework of mathematical dynamics of economic systems (Krouglov, 2006; 2009) developed by the author earlier.

Below the author describes a mathematical model of financial market. The economic forces acting on the market represent both inherent demand and supply market forces and government interventions and are expressed through the system of ordinary differential equations.

For didactic purposes, the author develops the model in two steps. First, he simplifies the situation in order to show the underlying impact of fluctuations. Second, he brings back the damping force, which defines the inflection point in the system.

This is author's second article devoted to the lessons from the Great Recession. The previous article (Krouglov, 2013) was dedicated to the nature of financial crisis.

2 Basic Model of Liquidity Pumping into Financial Market

Concepts and methodology presented in this section are based on the framework of mathematical dynamics of economic systems developed in Krouglov, 2006; 2009.

When there are no disturbing economic forces, financial market is in equilibrium position, i.e., the supply of and demand for financial product are equal, they are developing with a constant rate and the financial product's price is fixed.

When the balance between the product supply and demand is broken, the financial market is experiencing an economic force, which acts to bring the market to a new equilibrium position.

[1] Quote: "My "theory": there is an optimum point. If you pump liquidity less than the optimum point then bond prices increase. If you pump liquidity more than the optimum point then bond price decrease," from the author's comment at "Worthwhile Canadian Initiative" blog.

I assume the market had been in an equilibrium until time $t = t_0$, volumes of financial product's supply $V_S(t)$ and demand $V_D(t)$ on market were equal, and they both were developing with a constant rate r_D^0. The financial product's price $P(t)$ at that time was fixed,

$$V_D(t) = r_D^0 (t - t_0) + V_D^0 \tag{1}$$

$$V_S(t) = V_D(t) \tag{2}$$

$$P(t) = P^0 \tag{3}$$

where $V_D(t_0) = V_D^0$.

According to the examined scenario, I assume that amount of injected liquidity $L(t)$ on the financial market increases since time $t = t_0$ according to the following formula,

$$L(t) = \begin{cases} 0, & t < t_0 \\ \delta_L (t - t_0), & t \geq t_0 \end{cases} \tag{4}$$

where $L(t) = 0$ for $t < t_0$ and $\delta_L > 0$.

Economic forces trying to bring financial market into a new equilibrium position are described by the following ordinary differential equations regarding to the financial product's supply $V_S(t)$, demand $V_D(t)$, and price $P(t)$ on financial market (see Krouglov, 2006; 2009),

$$\frac{dP(t)}{dt} = -\lambda_P (V_S(t) - V_D(t) - L(t)) \tag{5}$$

$$\frac{d^2 V_S(t)}{dt^2} = \lambda_S \frac{dP(t)}{dt} \tag{6}$$

$$\frac{d^2 V_D(t)}{dt^2} = -\lambda_D \frac{d^2 P(t)}{dt^2} \tag{7}$$

In Equations (5 – 7) above the values $\lambda_P, \lambda_S, \lambda_D \geq 0$ are constants.[2]

Let me first assume for the clarity of presentation the "demand damping" constant $\lambda_D = 0$. Also let me introduce a new variable $D(t) \equiv (V_S(t) - V_D(t) - L(t))$ representing the volume of financial product's surplus (or shortage) on the market.

Then behavior of $D(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 D(t)}{dt^2} + \lambda_P \lambda_S D(t) = 0 \tag{8}$$

with the following initial conditions, $D(t_0) = 0$, $\frac{dD(t_0)}{dt} = -\delta_L$.

Thus, the following solution for time $t \geq t_0$ may be obtained (Piskunov, 1965; Petrovski, 1966),

$$D(t) = -\frac{\delta_L}{\sqrt{\lambda_P \lambda_S}} \sin\left(\sqrt{\lambda_P \lambda_S}\,(t - t_0)\right) \tag{9}$$

Correspondingly, corresponding solutions for the financial product's supply $V_S(t)$, demand $V_D(t)$, and price $P(t)$ for time $t \geq t_0$ are following,

$$V_D(t) = r_D^0 (t - t_0) + V_D^0 \tag{10}$$

$$V_S(t) = (r_D^0 + \delta_L)(t - t_0) + V_D^0 - \frac{\delta_L}{\sqrt{\lambda_P \lambda_S}} \sin\left(\sqrt{\lambda_P \lambda_S}\,(t - t_0)\right) \tag{11}$$

$$P(t) = -\frac{\delta_L}{\lambda_S} \cos\left(\sqrt{\lambda_P \lambda_S}\,(t - t_0)\right) + P^0 + \frac{\delta_L}{\lambda_S} \tag{12}$$

[2] These constants practically characterize price inertance, supply inductance, and demand damping correspondingly.

Thus, in a system where a damping effect is not present the liquidity pumping into financial market causes the fluctuations of the financial product's supply and likewise the financial product's price on the market. Therefore, the financial product's price in such circumstances experiences a volatile behavior.

3 Model of Damping on Financial Product's Price Fluctuations

In this section I abandon the condition of annulment for damping constant λ_D. Therefore, here I assume that $\lambda_D \geq 0$. This fact will significantly change conclusions for the financial product's price fluctuations developed in the previous section.

For $\lambda_D \geq 0$ the behavior of the financial product's price $P(t)$ introduced earlier is described by following equation for $t > t_0$,

$$\frac{d^2 P(t)}{dt^2} + \lambda_P \lambda_D \frac{dP(t)}{dt} + \lambda_P \lambda_S \left(P(t) - P^0 - \frac{\delta_L}{\lambda_S} \right) = 0 \tag{13}$$

with the following initial conditions, $P(t_0) = P^0$, $\frac{dP(t_0)}{dt} = 0$.

Let me introduce another variable $P_1(t) \equiv P(t) - P^0 - \frac{\delta_L}{\lambda_S}$ to simplify an analysis of financial product's price behavior. Then behavior of the variable $P_1(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 P_1(t)}{dt^2} + \lambda_P \lambda_D \frac{dP_1(t)}{dt} + \lambda_P \lambda_S P_1(t) = 0 \tag{14}$$

with the following initial conditions, $P_1(t_0) = -\frac{\delta_L}{\lambda_S}$, $\frac{dP_1(t_0)}{dt} = 0$.

The behavior of solution for variable $P_1(t)$ described by Equation (14) depends on the roots of the corresponding characteristic equation (Piskunov, 1965; Petrovski, 1966).

When the roots of characteristic equation are complex-valued (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} < \lambda_P \lambda_S$) the solution $P_1(t)$ experiences damped oscillations for time $t \geq t_0$,

$$P_1(t) = \exp\left\{-\frac{\lambda_P \lambda_D}{2}(t - t_0)\right\} \times \left(\begin{array}{l} -\frac{\delta_L}{\lambda_S} \cos\left(\sqrt{\lambda_P \lambda_S - \frac{\lambda_P^2 \lambda_D^2}{4}}(t - t_0)\right) \\ -\frac{\delta_L}{\lambda_S} \frac{\lambda_P \lambda_D}{2\sqrt{\lambda_P \lambda_S - \frac{\lambda_P^2 \lambda_D^2}{4}}} \sin\left(\sqrt{\lambda_P \lambda_S - \frac{\lambda_P^2 \lambda_D^2}{4}}(t - t_0)\right) \end{array} \right) \tag{15}$$

Correspondingly, for the case of complex-valued roots the solution for the financial product's price $P(t)$ also experiences damped oscillations for time $t \geq t_0$,

$$P(t) = \exp\left\{-\frac{\lambda_P \lambda_D}{2}(t - t_0)\right\} \times \left(\begin{array}{l} -\frac{\delta_L}{\lambda_S} \cos\left(\sqrt{\lambda_P \lambda_S - \frac{\lambda_P^2 \lambda_D^2}{4}}(t - t_0)\right) \\ -\frac{\delta_L}{\lambda_S} \frac{\lambda_P \lambda_D}{2\sqrt{\lambda_P \lambda_S - \frac{\lambda_P^2 \lambda_D^2}{4}}} \sin\left(\sqrt{\lambda_P \lambda_S - \frac{\lambda_P^2 \lambda_D^2}{4}}(t - t_0)\right) \end{array} \right) + P^0 + \frac{\delta_L}{\lambda_S} \tag{16}$$

Since in Equation (16) the power of exponential function is negative for time $t \geq t_0$ (for the reason that $-\frac{\lambda_P \lambda_D}{2} \leq 0$), the exponential function vanishes for $t \to +\infty$, and it takes place $P(t) \to P^0 + \frac{\delta_L}{\lambda_S}$ for $t \to +\infty$.

When the roots of characteristic equation are real and different (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} > \lambda_P \lambda_S$) the financial product's price $P(t)$ doesn't oscillate for time $t \geq t_0$,

$$P(t) = -\frac{\delta_L}{\lambda_S} \left(\begin{array}{l} \dfrac{\dfrac{\lambda_P \lambda_D}{2} + \sqrt{\dfrac{\lambda_P^2 \lambda_D^2}{4} - \lambda_P \lambda_S}}{2\sqrt{\dfrac{\lambda_P^2 \lambda_D^2}{4} - \lambda_P \lambda_S}} \exp\left\{ \left(-\dfrac{\lambda_P \lambda_D}{2} + \sqrt{\dfrac{\lambda_P^2 \lambda_D^2}{4} - \lambda_P \lambda_S} \right) (t - t_0) \right\} \\ + \dfrac{-\dfrac{\lambda_P \lambda_D}{2} + \sqrt{\dfrac{\lambda_P^2 \lambda_D^2}{4} - \lambda_P \lambda_S}}{2\sqrt{\dfrac{\lambda_P^2 \lambda_D^2}{4} - \lambda_P \lambda_S}} \exp\left\{ \left(-\dfrac{\lambda_P \lambda_D}{2} - \sqrt{\dfrac{\lambda_P^2 \lambda_D^2}{4} - \lambda_P \lambda_S} \right) (t - t_0) \right\} \end{array} \right) \tag{17}$$

$$+ P^0 + \frac{\delta_L}{\lambda_S}$$

Since in Equation (17) the powers of exponential functions are negative for time $t \geq t_0$ (for the reason that $-\frac{\lambda_P \lambda_D}{2} \leq 0$ and $\lambda_P \lambda_S \geq 0$), the exponential functions vanish for $t \to +\infty$, and it takes place

$P(t) \to P^0 + \frac{\delta_L}{\lambda_S}$ for $t \to +\infty$.

When the roots of characteristic equation are real and equal (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} = \lambda_P \lambda_S$) the financial product's price $P(t)$ doesn't oscillate for time $t \geq t_0$ as well,

$$P(t) = \left(-\frac{\delta_L}{\lambda_S} - \frac{2\delta_L}{\lambda_D} (t - t_0) \right) \exp\left\{ -\frac{\lambda_P \lambda_D}{2} (t - t_0) \right\} + P^0 + \frac{\delta_L}{\lambda_S} \tag{18}$$

Since in Equation (18) the power of exponential function is negative for time $t \geq t_0$ (for the reason that $-\frac{\lambda_P \lambda_D}{2} \leq 0$), the exponential function vanishes for $t \to +\infty$, and it takes place $P(t) \to P^0 + \frac{\delta_L}{\lambda_S}$ for $t \to +\infty$.

Thus, we can clearly see that if liquidity is pumped into the financial market with a constant rate $\delta_L > 0$ then the quantitative value of that rate δ_L doesn't have an impact on the financial product's price volatility. This fact contradicts what the author had predicted in this regard earlier.[3] The volatility of the financial product's price is determined by the quantitative value of the damping constant $\lambda_D \geq 0$. If the damping constant's value is smaller than the critical value ($0 \leq \lambda_D < 2\sqrt{\frac{\lambda_S}{\lambda_P}}$) then the financial product's price fluctuates when liquidity is pumped into the market. If it is bigger than the critical value ($\lambda_D \geq 2\sqrt{\frac{\lambda_S}{\lambda_P}}$) then the financial product's price doesn't experience fluctuations when liquidity is pumped into the market.

Another observation that can be made is that eventual change of the financial product's price is constrained by the value $\frac{\delta_L}{\lambda_S} \geq 0$. Therefore, pumping liquidity into the financial market with a constant rate $\delta_L > 0$ doesn't produce a profound terminal effect on the value of financial product's price.

Nevertheless, the model swift shows that amount of eventual price adjustment $\frac{\delta_L}{\lambda_S} \geq 0$ is directly proportional to the rate of liquidity pumping $\delta_L > 0$ into a financial market (and, correspondingly, inversely proportional to the "supply inductance" constant $\lambda_S \geq 0$ inherent to an economic system).

[3] Nevertheless, the model provides an explanation of the phenomenon within financial markets. Primarily, the process of liquidity pumping is increasing the financial product's (combined) demand over the product's supply. That process increases the price of financial product (or decreases the yield). The price increase swiftly causes a rise of the product's supply. Yet, the process doesn't significantly decrease the product's (market) demand due to a small damping constant in Japan. The product's supply overshoots (i.e., exceeds the product's (combined) demand). Hence, the price fluctuates.

The author regards the following empirical observations as supporting the presented theoretical reflections.

First, the loyalty of Japanese society to the government reflected in a continuous purchase of the Japanese government bonds even when their yields become miniscule is consistent with the relatively insignificant damping constant $\lambda_D \geq 0$ for the Japanese economic system.

Second, the effect of petering out an initial enthusiasm on the financial market caused by the expansionary monetary policy of Abenomics is consistent with the theoretical result above that increase of the financial product's price is constrained by the finite value $\frac{\delta_L}{\lambda_S} \geq 0$.

Third, the amount of eventual price adjustment is directly proportional to the rate of liquidity pumping $\delta_L > 0$ into a financial market.

4 Model of Accelerated Liquidity Pumping

In this section I assume that the amount of injected liquidity $L(t)$ in the financial market increases since time $t = t_0$ according to following formula,

$$L(t) = \begin{cases} 0, & t < t_0 \\ \delta_L (t - t_0) + \frac{\varepsilon_L}{2} (t - t_0)^2, & t \geq t_0 \end{cases} \tag{19}$$

where $L(t) = 0$ for $t < t_0$, $\delta_L > 0$, and $\varepsilon_L > 0$.

Then the behavior of the financial product's price $P(t)$ introduced earlier is described by the following equation for $t > t_0$,

$$\frac{d^2P(t)}{dt^2} + \lambda_P\,\lambda_D\,\frac{dP(t)}{dt} + \lambda_P\,\lambda_S\left(P(t) - P^0 - \frac{\delta_L}{\lambda_S} - \frac{\varepsilon_L}{\lambda_S}(t - t_0)\right) = 0 \qquad (20)$$

with the following initial conditions, $P(t_0) = P^0$, $\frac{dP(t_0)}{dt} = 0$.

Let me introduce a new variable $P_2(t) \equiv P(t) - P_R^0 - \frac{\delta_L}{\lambda_S} - \frac{\varepsilon_L}{\lambda_S}(t - t_0) + \frac{\lambda_D}{\lambda_S^2}\varepsilon_L$ to simplify an analysis.

The behavior of variable $P_2(t)$ is then described by the following equation for $t > t_0$,

$$\frac{d^2P_2(t)}{dt^2} + \lambda_P\,\lambda_D\,\frac{dP_2(t)}{dt} + \lambda_P\,\lambda_S\,P_2(t) = 0 \qquad (21)$$

with the following initial conditions, $P_2(t_0) = -\frac{\delta_L}{\lambda_S} + \frac{\lambda_D}{\lambda_S^2}\varepsilon_L$, $\frac{dP_2(t_0)}{dt} = -\frac{\varepsilon_L}{\lambda_S}$.

It can be seen that Equations (14) and (21) are identical (though their initial conditions differ).

Consequently, conditions for fluctuations of the solutions of Equations (14) and (21) are also the same (i.e., the conditions for the financial product's price $P(t)$ fluctuations are analogous if the liquidity is pumped into financial market with a constant rate $\delta_L > 0$ or if it is pumped with a constant acceleration $\varepsilon_L > 0$).

As earlier, if the damping constant is smaller than the critical value ($0 \le \lambda_D < 2\sqrt{\frac{\lambda_S}{\lambda_P}}$), the variable $P_2(t)$ fluctuates and likewise fluctuates the financial product's price $P(t)$. If the damping constant is larger than the critical value ($\lambda_D \ge 2\sqrt{\frac{\lambda_S}{\lambda_P}}$) then the financial product's price $P(t)$ doesn't fluctuate.

However, the terminal values of the financial product's prices $P(t)$ are different for situations when the liquidity is pumped into financial market with a constant rate $\delta_L > 0$ or if it is pumped with a constant acceleration $\varepsilon_L > 0$.

Indeed, since the variable $P_2(t)$ vanishes with time $t \to +\infty$ due to the coefficient $\lambda_P \lambda_D > 0$ similar to $P_1(t)$ in the previous section, it takes place $P(t) \to \frac{\varepsilon_L}{\lambda_S}(t - t_0) + P_R^0 + \frac{\delta_L}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_L$ for $t \to +\infty$.

Therefore, when the liquidity is pumped into a financial market with a constant acceleration $\varepsilon_L > 0$ the financial product's price $P(t)$ is continuously increasing with the passage of time.[4] It is clearly different from the case of pumping the liquidity into a financial market with a constant rate $\delta_L > 0$, which produces a terminal change of the financial product's price constrained by the finite value $\frac{\delta_L}{\lambda_S} \geq 0$.

5 Conclusions

Presented here is a simplified mathematical model explaining the effect of the liquidity pumping into financial markets.

There were considered implications of the liquidity pumping into financial markets as a part of the monetary policy of so-called Abenomics introduced by Japanese Prime Minister Shinzō Abe. The author was able to make three following observations with regard to the monetary policy in question.

[4] Not to forget a noted exception of the short-term damped fluctuations when the damping constant is smaller than the critical value ($0 \leq \lambda_D < 2\sqrt{\frac{\lambda_S}{\lambda_P}}$).

First, the model has explained the drop of the Japanese government bond prices as a side effect of Japanese society's loyalty to the government policy in general expressed in the model in the form of a small damping constant.

Second, pumping the liquidity into financial markets with a constant rate has been able to produce only a constrained impact on the prices of financial products. Eventually this impact will disappear.

Third, the amount of a final price adjustment is directly proportional to the rate of liquidity pumping into financial market.

Fourth, in order to implement a long lasting impact on the prices of financial products the monetary policy has to be modified. It would require the pumping of liquidity into financial markets not with a constant rate but with a persistently accelerated rate.

References

Krouglov, Alexei, 2006, *Mathematical Dynamics of Economic Markets* (New York: Nova Science Publishers).

Krouglov, Alexei, 2009, "Mathematical Dynamics of Economic Growth as Effect of Internal Savings," *Finance India,* Vol. 23, No. 1, pp. 99-136.

Krouglov, Alexei, 2013, "Simplified Mathematical Model of Financial Crisis," *Journal of Advanced Studies in Finance,* Vol. IV, No. 2 (8), DOI:10.14505/jasf.v4.2(8).04.

Petrovski, Ivan G., 1966, *Ordinary Differential Equations* (Englewoods Cliffs, New Jersey: Prentice Hall).

Piskunov, Nikolai S, 1965, *Differential and Integral Calculus* (Groningen: P. Noordhoff).