Exchange in the Monetary Economy

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Abstract

It is clear by now that pure exchange models are useless. For two reasons. First, because exchange is the other side of specialization in production, and, second, because a direct exchange of goods does not take place in the monetary economy. The decisive drawback of conventional exchange models, though, is that they cannot explain profit. Standard economics rests on behavioral assumptions that are expressed as axioms. The ultimate reason for the failure of conventional exchange theory is that human behavior and axiomatization are disjunct. Notable progress can be made by replacing the subjective-behavioral axioms by objective-structural axioms.

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1 Out of order

> … *every* neoclassical discussion of production postdated the specification of the model of exchange. Such an order of inquiry would have been an anomaly in classical political economy. (Mirowski, 1995, p. 281), original emphasis

It is clear by now that pure exchange models are useless. For two reasons. First, because exchange is the other side of specialization in production, and, second, because a direct exchange of goods does not take place in the monetary economy. The decisive drawback of conventional exchange models, though, is that they cannot explain profit, that is, they leave us in utter darkness about the world we happen to live in.

> In the modern capitalist economy, the primary purpose of production is not to exchange for other consumables but to "make money" – that is, to sell at a profit. (Wray, 2002, p. 30)

The standard approach is committed to methodological individualism.

> Theories of exchange attempt to predict the terms of trade and the resulting transactions from the market structure and the agents' attributes, such as endowments, productive opportunities, preferences, and information. (Wilson, 2008, p. 2)

In purely formal terms this means that standard economics rests on behavioral assumptions that are expressed as axioms (Debreu, 1959; Arrow and Hahn, 1991; McKenzie, 2008). The ultimate reason for the failure of conventional exchange theory is that human behavior and axiomatization are disjunct. Conceptual consequence demands therefore to discard the subjective-behavioral axioms and to take objective-structural axioms as the formal point of departure.

Section 2 provides the new formal foundations with the set of three structural axioms. These represent the pure consumption economy as the most elementary economic configuration. Section 3 describes the emergence of exchange from initial autarky. Section 4 reformulates the working of demand and supply, including profit, within the comprehensive framework of the stuctural-axiomatic exchange formalism. Section 5 concludes.

2 The advanced structure

> … not all axiomatic theories need to be phrased in terms of set theory but much more conveniently and intelligibly rather in terms of some advanced mathematical structures. (Schmiechen, 2009, p. 367)

2.1 Axioms

The formal foundations of theoretical economics must be objective and epitomize the interdependence of the real and nominal variables that constitutes the monetary economy.

The first three structural axioms relate to income, production, and expenditure in a period of arbitrary length. The period length is conveniently assumed to be the calendar year. Simplicity demands that we have for the beginning one world economy, one firm, and one product. Axiomatization is about ascertaining the *minimum* number of premises.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N. Nothing is implied at this stage about who owns the shares.

$$Y = WL + DN \quad |t \tag{1}$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

The axioms represent the pure consumption economy, that is, no investment, no foreign trade, and no government.

The economic content of the first three axioms is plain. The point to stress is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. This distinction seems insignificant but makes all the difference between analytical success or failure.

2.2 Definitions

Income categories

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms. With (4) wage income Y_W and distributed profit Y_D is defined:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{4}$$

Definitions add no new content to the set of axioms but determine the logical context of concepts. New variables are introduced with new axioms.

Ratios

We define the sales ratio as:

$$\rho_X \equiv \frac{X}{O} \quad |t. \tag{5}$$

A sales ratio $\rho_X = 1$ indicates that the quantity bought/sold X and the quantity produced O are equal or, in other words, that the product market is cleared.

We define the expenditure ratio as:

$$\rho_E \equiv \frac{C}{Y} \quad |t. \tag{6}$$

An expenditure ratio $\rho_E = 1$ indicates that consumption expenditures C are equal to total income Y, in other words, that the household sector's budget is balanced.

Monetary profit

Total profit consists of monetary and nonmonetary profit. Here we are at first concerned with monetary profit. Nonmonetary profit is treated at length in (2012).

The business sector's monetary profit/loss in period t is defined with (7) as the difference between the sales revenues – for the economy as a whole identical with consumption expenditure C – and costs – here identical with wage income Y_W:

$$Q_m \equiv C - Y_W \quad |t. \tag{7}$$

Because of (3) and (4) this is identical with:

$$Q_m \equiv PX - WL \quad |t. \tag{8}$$

This form is well-known from the theory of the firm.

The Profit Law

From (7) and (1) follows:

$$Q_m \equiv C - Y + Y_D \quad |t \tag{9}$$

or, using the definitions (5) and (6),

$$\begin{aligned} Q_m &\equiv \left(\rho_E - \frac{1}{1+\rho_D}\right) Y \\ \text{with} \quad \rho_D &\equiv \frac{Y_D}{Y_W} \quad |t. \end{aligned} \tag{10}$$

The four equations (7) to (10) are formally equivalent and show profit under different perspectives. The Profit Law (10) asserts that total monetary profit in the pure consumption economy is zero if $\rho_E = 1$ and $\rho_D = 0$, i.e. profit or loss for the business sector as a whole depends on the expenditure and distributed profit ratio and nothing else (for details see 2013).

Individual monetary profit

For firm 1 individually eq. (8) reads in the case of market clearing:

$$\begin{aligned} Q_{m1} &\equiv P_1 X_1 - W_1 L_1 \\ Q_{m1} &\equiv P_1 R_1 L_1 \left(1 - \frac{W_1}{P_1 R_1}\right) \\ \text{if} \quad \rho_{X1} &= 1 \quad |t. \end{aligned} \tag{11}$$

Monetary profit of firm 1 is zero under the condition that the quotient of wage rate, price, and productivity is unity. This holds independently of the level of employment or the size of the firm. From the zero profit condition follows:

$$\begin{aligned} P_1 &= \frac{W_1}{R_1} \\ \text{if} \quad \rho_{X1} &= 1, \, Q_{m1} = 0 \quad |t. \end{aligned} \tag{12}$$

The price of product 1 is, in the simplest case, equal to unit wage costs.

Relative prices

In the same way one gets the individual profits and the zero profit market clearing prices for all other firms. With this, the structure of relative prices is determined for the most elementary case.

$$\frac{P_1}{P_2} = \frac{\frac{W_1}{R_1}}{\frac{W_2}{R_2}} = \frac{R_2}{R_1} \tag{13}$$

$$\text{if} \quad W_1 = W_2,\, \rho_{X1} = 1,\, \rho_{X2} = 1,\, Q_{m1} = 0,\, Q_{m2} = 0 \quad |t.$$

Under the zero profit condition, relative prices stand in the same relation as unit wage costs. With equal wage rates, relative prices stand in inverse relation to productivities.

This limiting case is the structural-axiomatic counterpart to Walras's zero profit general equilibrium. In the case of a non-zero profit economy the derivation of the market clearing price vector is a bit more involved. In the following we deal at first with the most elementary case.

3 The emergence of exchange from initial autarky

> Neoclassical theorists wanted to assert that value was increased by the act of exchange. (Mirowski, 1995, p. 290)

3.1 One agent, one product

From (3), (5), and (6) follows the price as dependent variable:

$$P = \frac{\rho_E}{\rho_X}\frac{W}{R}\left(1 + \frac{Y_D}{Y_W}\right) \quad |t. \tag{14}$$

This is the general structural axiomatic law of supply and demand for the pure consumption economy with one firm. In brief, the price equation states that the market clearing price is ultimately determined by the expenditure ratio, unit wage costs, and the income distribution. The structural axiomatic price formula is testable in principle.

Under the condition of market clearing and zero distributed profit follows:

$$P = \rho_E \frac{W}{R}$$
$$\text{if} \quad \rho_X = 1, Y_D = 0 \quad |t. \tag{15}$$

The market clearing price depends now alone on the expenditure ratio and unit wage costs. Under the additional conditions of budget balancing follows:

$$P = \frac{W}{R}$$
$$\text{if} \quad \rho_E = 1, \rho_X = 1, Y_D = 0 \quad |t. \tag{16}$$

The market clearing price is equal to unit wage costs if the expenditure ratio is unity and distributed profit is zero. In this elementary case, profit per unit is zero and by consequence total profit is zero. All changes of the wage rate and the productivity affect the market clearing price in the period under consideration. We refer to this formal property as *conditional* price flexibility because (16) involves no assumption about human behavior, only the purely formal condition $\rho_X = 1$.

With (16) the real wage $\frac{W}{P}$ is *uno actu* given; it is under the enumerated conditions invariably equal to the productivity R. The agent gets the whole product. The real wage is determined by the production conditions and *not* in the labor market.

3.2 One agent, two products

We now consider the agent splitting his labor time between two lines of production. Total income (1) remains unchanged:

$$Y = WL + \underbrace{DN}_{0} \quad |t. \tag{17}$$

The partitioning of agent 1's labor input is given by:

$$L \equiv L_{11} + L_{12} \quad |t. \tag{18}$$

With given productivities the respective outputs in the two lines of production follow from (2) as:

$$\begin{aligned} O_{11} &= R_{11} L_{11} \\ O_{12} &= R_{12} L_{12} \end{aligned} \quad |t. \tag{19}$$

From (3) follows for the respective consumption expenditures:

$$\begin{array}{l} C_{11} = P_{11} X_{11} \\ C_{12} = P_{12} X_{12} \end{array} \quad |t. \tag{20}$$

From (6) follow as corollaries:

$$\begin{array}{c} C_{11} = \rho_{E11} Y \\ C_{12} = \rho_{E12} Y \\ \text{if} \quad \rho_{E11}, \rho_{E12} \quad \text{are taken as independent} \quad |t. \end{array} \tag{21}$$

Under the condition of market clearing eqs. (21), (20) and (19) boil down to:

$$\begin{array}{c} \dfrac{P_{11}}{P_{12}} \dfrac{R_{11}}{R_{12}} \dfrac{L_{11}}{L_{12}} = \dfrac{\rho_{E11}}{\rho_{E12}} \\ \text{if} \quad \rho_{X11} = 1, \rho_{X12} = 1 \quad |t. \end{array} \tag{22}$$

So far we have without much interpretation translated the one-agent–two-products case into a well-arranged formula. In this formula the respective productivities are given by the actual production conditions, the other variables are at the agent's discretion.

Since there are no given market prices agent 1 has to make up his own mind how to fix relative prices. The decision would be simple if there were objective criteria. Let us turn for a moment to physics and reinterpret the human condition as a straightforward entropic process. The agents in their turn then need the products to counter entropic degradation to a certain degree. Let us assume that the anti-entropic properties of each product are known and let us call the measure ontropy. Hence, if product 1 had exactly double the ontropy of product 2 then the agent would fix the price relation in (22) from the consumer's perspective exactly at 2:1.

Of course, the ontropies of the respective products are not given. The objective physical concept has therefore to be replaced by a subjective concept. In other words, agent 1 has some leeway in the fixation of relative prices. It does, though, not really help much to assert that agent 1 sets the price according to his marginal utilities. If anything, the concept of marginal utility is even more woolly than ontropy.

The underlying idea of physical ontropy and psychological utility is that some products do something good to the agent. The price relation is an expression of the relative "goodness" with reference to the agent. There is, though, a second point to take into consideration.

Let us isolate the relationship between prices and productivities in (22) as follows:

$$\rho_h \equiv \frac{P_{11}}{P_{12}} \frac{R_{11}}{R_{12}} \quad |t. \tag{23}$$

Subjectively, relative prices are independent from relative productivities. Hence, as a matter of principle, the ratio ρ_h can assume any value. However, agent 1 can take the productivities as indicator of how hard it is to produce a specific output and translate relative "hardness" through the price relation. Then, if productivity in one line of production is comparatively low then the price is comparatively high. In the limiting case of $\rho_h = 1$ we have:

$$\frac{P_{11}}{P_{12}} = \frac{R_{12}}{R_{11}} \tag{24}$$

Relative prices are inverse to relative productivities. Relative prices are no longer subjective but now reflect objectively given productivities. The ratio ρ_h therefore expresses the degree of objectivity/subjectivity of the price relation, with unity signaling objectivity. A deviation from unity signals subjective over- or undervaluation. It is therefore appropriate to distinguish between subjective valuation prices B_{11}, B_{12} and objective prices P_{11}, P_{12} (for details see 2012, Sec. 4.2).

As can be seen from (13), the limiting case $\rho_h = 1$ in (23) corresponds exactly with zero profit in both firms. In other words, the zero profit condition completely eliminates the subjectivity from price setting.

On closer inspection, agent 1 plays three different roles. He provides the labor input, thereby earns a wage income, and is at the same time the owner of the firm. In the second faculty he has to take care that the firm is profitable or at least breaks even. His third role is that of a consumer. It makes a difference whether agent 1 looks at the price relation with the eyes of a consumer or of the firm's owner. Logically, the roles have to be separated. Since the firm is the price setter it is the objective zero profit condition that applies under the given circumstances. Physical ontropy or psychological utility do not matter.

After the fixation of relative prices eq. (22) can be rewritten as:

$$\begin{gathered}\rho_h \frac{L_{11}}{L_{12}} = \frac{\rho_{E11}}{\rho_{E12}} \\ \text{if} \quad \rho_{X11} = 1,\, \rho_{X12} = 1 \quad |t.\end{gathered} \tag{25}$$

If the ratio ρ_h is given the allocation of the labor input depends on the partitioning of consumption expenditures. The input in the first line of production is given by:

$$L_{11} = \frac{\rho_{E11}}{\rho_{E11} + \rho_h \rho_{E12}} L \quad |t. \tag{26}$$

In the limiting case of $\rho_h = 1$ and budget balancing the labor input in the first line of production is proportional to the expenditure ratio:

$$\begin{aligned} L_{11} &= \rho_{E11} L \\ \text{if} \quad \rho_h &= 1, \rho_{E12} + \rho_{E11} = 1 \quad |t. \end{aligned} \tag{27}$$

Mutatis mutandis for the second line of production

$$L_{12} = \frac{\rho_h \rho_{E12}}{\rho_{E11} + \rho_{E12}} L \quad |t \tag{28}$$

and as limiting case

$$\begin{aligned} L_{12} &= \rho_{E12} L \\ \text{if} \quad \rho_h &= 1, \rho_{E12} + \rho_{E11} = 1 \quad |t. \end{aligned} \tag{29}$$

Total employment L is in the limiting case allocated in exact proportion to consumption expenditures. We refer to this configuration as harmonic structure.

The partitioning of consumption expenditures is a purely subjective affair. Under the enumerated conditions the partitioning determines the allocation of labor input. If the agent strongly prefers product 1 he simply has to devote the greater part of his total labor time to the production of product 1. The objective valuation of the respective products is expressed by the price ratio, the allocation of labor depends on the partitioning of consumption expenditures expressed by the ratio of the expenditure ratios. If the subjective partitioning is in some sense optimal then the allocation is also optimal. This can always made to happen, at least verbally.

3.3 Making words yet saying nothing

Ask a middle-of-the-road economist when a blackberry picking boy will stop picking and eating, but do not tell him in advance that the answer should consist in a concrete number of minutes. He will come up with the following answer:

> Equilibrium is reached when at last his eagerness to play and the disinclination for the work of picking counterbalances the desire for eating. The satisfaction which he can get from picking fruit has arrived at its *maximum:* for up to that time every fresh picking has added more to his pleasure than it has taken away; ... (Marshall, 2009, p. 276), original emphasis

Marshall seems to have an explanation – the key words are equilibrium, satisfaction and maximum – but he cannot utter the number of minutes. Never. What he says in so many words is that the boy stops picking and eating when he stops. Nobody can ever prove this assertion wrong. Marshall's berry picker epitomizes the vacuousness of conventional economic explanation.

Since the marginal principle is empty it can be applied across-the-board. Hence, there is no formal problem to append it to the structural-axiomatic formalism. Eq. (22) is first rewritten as:

$$\frac{P_{11}}{P_{12}}\frac{X_{11}}{X_{12}} = \frac{\rho_{E11}}{\rho_{E12}} \quad \text{if} \quad \rho_{X11} = 1,\, \rho_{X12} = 1 \tag{30}$$

The familiar optimum condition says that the marginal rate of substitution MRS is equal to the price ratio. This condition is met at the tangential point in Figure 1.

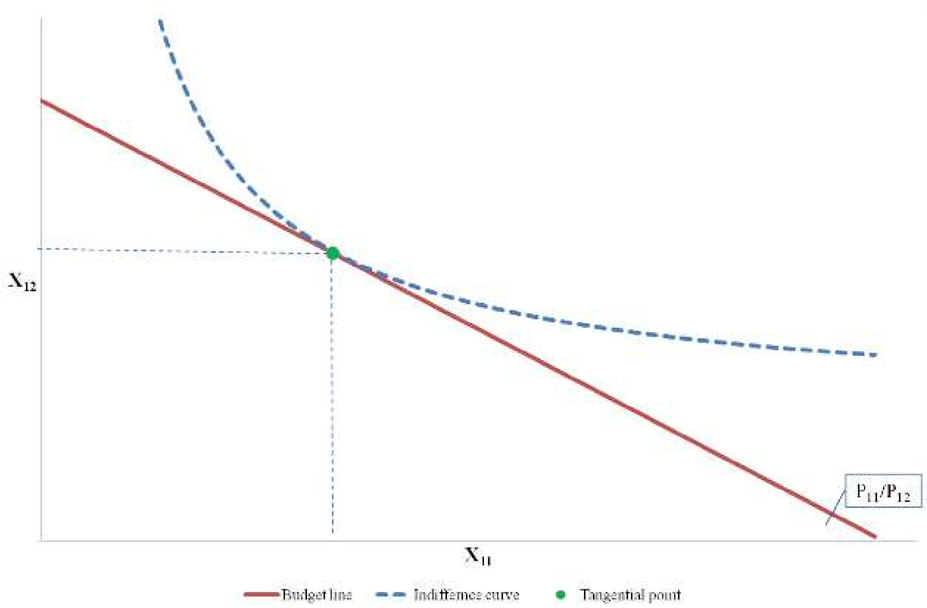

Figure 1: The familiar optimization rule determines the quantities X_{11}, X_{12} for given prices P_{11}, P_{12} and thereby the partitioning of the budget

The tangential point provides the respective quantities X_{11}, X_{12}. Together with the given price relation eq. (30) then delivers the optimal partitioning of the consumption expenditures C_1, C_2 or, what amounts to the same, the optimal breakup of the expenditure ratios ρ_{E11}, ρ_{E12}.

However, this marginalistic exercise does not really help much. Imagine that the two products are bread and wine then it can be argued that the partitioning of 80 money units for bread and 20 money units for wine is optimal because the MRS is equal to the ratio of bread price to wine price. By the same token, the combination of 20 money units for bread and 80 money units for wine is optimal depending on the shape of the nonentity called indifference curve. For an observer the partitioning is simply arbitrary. The attribution of an optimum is unwarranted. Seen from the structural-axiomatic viewpoint marginalism is a redundant add-on. It cannot, as a matter of principle, provide concrete numbers or anything else that deserves the attribute empirical.

> Much of the mystery surrounding the actual development of economic theory – its shift in formalism, its insulation from empirical assessment, its interest in proving purely formal, abstract possibilities, its unchanged character over a period of centuries, the controversies about its cognitive status – can be comprehended and properly appreciated if we give up on the notion that economics has any longer the aims or makes the claims of an empirical science of human behaviour. (Rosenberg, 1994, p. 230)

Note in passing that the optimal allocation of labor input has nothing to do with the price ratio which is the inverse of the given productivity ratio. If the agent's demand shifts from bread to wine this does *not* affect the price of bread or wine but only the allocation of labor between bread or wine production. Valuation and allocation are different things. This issue is considered in more detail in Section 4.

3.4 Two agents, two products

Now a second agent is introduced who resembles the first almost exactly. Total income doubles with double total labor input:

$$Y = WL + \underbrace{DN}_{0} \quad |t. \tag{31}$$

The partitioning of the respective labor inputs is given by:

$$L \equiv \underbrace{(L_{11} + L_{12})}_{L_{1\bullet}} + \underbrace{(L_{21} + L_{22})}_{L_{2\bullet}} \quad |t. \tag{32}$$

The labor inputs of both agents are equal:

$$L_{1\bullet} = L_{2\bullet} \quad |t. \tag{33}$$

All other things equal agent 1 produces more bread than wine and agent 2 more wine than bread. As a matter of fact both agents are complementary and perfectly satisfied with their respective pattern of consumption and allocation. Both agents now move from autarky to exchange:

- Agent 1 initially spends 80 money units on bread and 20 money units on wine, both self-produced.
- Agent 2 initially spends 20 money units on bread and 80 money units on wine, both self-produced.
- Agent 1 produces 100 money units bread and buys 20 money units wine from agent 2.
- Agent 2 produces 100 money units wine and buys 20 money units bread from agent 1.

The first agent's move reads:

$$\begin{array}{cccc} C_{11}=P_{\bullet 1}X_{11} & \text{bread} & \text{bread} & C_{11}=P_{\bullet 1}X_{11} \\ C_{12}=P_{\bullet 2}X_{12} & \text{wine} & \text{bread} & C_{21}=P_{\bullet 1}X_{21} \\ C_{1\bullet}=\rho_{E1}Y & & & \end{array} \tag{34}$$

The second agent's move reads:

$$\begin{array}{cccc} C_{21}=P_{\bullet 1}X_{21} & \text{bread} & \text{wine} & C_{12}=P_{\bullet 2}X_{12} \\ C_{22}=P_{\bullet 2}X_{22} & \text{wine} & \text{wine} & C_{22}=P_{\bullet 2}X_{22} \\ C_{2\bullet}=\rho_{E2}Y & & & \end{array} \tag{35}$$

Total consumption expenditures of both agents are equal.

$$\begin{gathered} C_{1\bullet}=C_{2\bullet} \\ P_{\bullet 1}\left(X_{11}+X_{21}\right)=P_{\bullet 2}\left(X_{22}+X_{12}\right) \quad |t. \end{gathered} \tag{36}$$

The amount agent 2 spends on product 1 is equal to the amount agent 1 spends on product 2.

$$P_{\bullet 1}X_{21}=P_{\bullet 2}X_{12} \quad |t. \tag{37}$$

The quantities exchanged are therefore given by:

$$\frac{X_{12}}{X_{21}}=\frac{P_{\bullet 1}}{P_{\bullet 2}} \quad |t. \tag{38}$$

Agent 1 buys X_{12} units of wine from agent 2, and sells X_{21} units of bread to agent 2. The real exchange boils down to X_{12} units of wine for X_{21} units of bread. Vice versa for agent 2.

Exchange presupposes the complementarity of wants. Without exact complementarity there is no complete specialization. Specialization and exchange go together.

After specialization the agents produce for themselves as before and exchange the respective surpluses. The move from autarky to exchange is indifferent with regard to the distribution of consumption goods but not with regard to the allocation of labor to the two lines of production. Exchange is the other side of specialization.

The final distribution of consumption goods is equal in the case of autarky and exchange. Profit is zero in both cases. There are neither real nor monetary gains from trade. The real benefit of exchange comes into existence as soon as specialization increases productivity. This is where Adam Smith started from in the *Wealth of Nations*. Both agents are proportionally better off in real terms if the productivity in both lines of production increases by the same percentage rate. Since prices decline inversely to the productivity improvements and wage costs remain unchanged profits in both firms are still zero. The productivity effect of specialization and exchange benefits the consumers in real terms. Exchange *per se* does not increase value.

3.5 Generalization

Whenever we write down the symbol for consumption expenditures C we are implicitly dealing with an exchange matrix in the general form of:

$$\begin{array}{cccc} C_{11} = P_{\bullet 1}X_{11} & C_{12} = P_{\bullet 2}X_{12} & C_{1j} = P_{\bullet j}X_{1j} & C_{1\bullet} \\ C_{21} = P_{\bullet 1}X_{21} & C_{22} = P_{\bullet 2}X_{22} & C_{2j} = P_{\bullet j}X_{2j} & C_{2\bullet} \\ C_{i1} = P_{\bullet 1}X_{i1} & C_{i2} = P_{\bullet 2}X_{i2} & C_{ij} = P_{\bullet j}X_{ij} & C_{i\bullet} \\ \\ C_{\bullet 1} & C_{\bullet 2} & C_{\bullet j} & C \quad |t. \end{array} \tag{39}$$

In the monetary economy there is no bi- or multilateral exchange of given real quantities. Barter models with fixed endowments are therefore inapplicable. The exchange consists of buying from other firms with the wage income out of the production of the i-th good. Exchange is, in a sense, maximized when nobody buys the good he helps to produce. This, then, is the counterpoint to individual autarky. Both limiting cases are covered by the structural axiom set and are made explicit through differentiation. The exchange matrix is integral part of the axiom set.

4 Modular interdependencies of demand and supply

> But in the act of exchange viewed as a whole, equals are in general always exchanged for equals, individual variations being canceled out. How then, are profits made, for, obviously, they are made? (Kirkenfeld, 1948, p. 35)

Modular interdependency means that any real world change can be approximated by a selected sequence of the following small modules. The minimum number of markets for the analysis of supply and demand variations is *two*. The Marshallian approach with supply-demand-equilibrium in one market has always been worse than useless.

4.1 Demand shift (I)

The differentiation of the 1st axiom reads for two firms:

$$Y = \underbrace{W_1 L_1}_{Y_{W1}} + \underbrace{W_2 L_2}_{Y_{W2}} + \underbrace{DN}_{0} \quad |t. \tag{40}$$

The differentiation of the 3rd axiom reads for two firms:

$$\begin{array}{ccc} C_{11} = P_{\bullet 1} X_{11} & C_{12} = P_{\bullet 2} X_{12} & C_{1\bullet} \\ C_{21} = P_{\bullet 1} X_{21} & C_{22} = P_{\bullet 2} X_{22} & C_{2\bullet} \\ C_{\bullet 1} & C_{\bullet 2} & C \\ C_1 = P_1 X_1 & C_2 = P_2 X_2 & C \\ C_1 = \rho_{E1} Y & C_2 = \rho_{E2} Y & C \quad |t. \end{array} \tag{41}$$

It is assumed that the household sector's demand shifts from firm 2 to firm 1, i.e. that ρ_{E1} goes up and ρ_{E2} goes down, such that the sum of the expenditure ratios is always unity. Prices remain unchanged, hence X_1 goes up and X_2 goes down according to (41). Productivities remain unchanged, hence employment L_1 goes up and L_2 goes down, such that total income Y remains constant in (40) with equal wage rates $W_1 = W_2$. Labor can move freely between the two firms. The budget is balanced, i.e. total income Y is equal to total consumption expenditures C; both variables remain unchanged.

The net effect of the demand shift is that the relation X_1/X_2 increases; the household sector buys more of product 1 and less of product 2 at *constant* prices. The households are perfectly free to realize any combination of the two goods with the given resources.

4.2 Demand shift (II)

It is assumed again that demand shifts from firm 2 to firm 1, i.e. that ρ_{E1} goes up and ρ_{E2} goes down, such that the sum of the expenditure ratios is always unity. Now quantities remain unchanged, hence P_1 goes up and P_2 goes down according to (41). Wage cost remains unchanged in both firms hence firm 1 makes a profit and firm 2 a loss according to (11). Profit for the business sector as a whole is zero according to (10).

The first round net effect of the demand shift is that profit and loss emerge. The situation of firm 2 becomes untenable.

To stabilize the situation it is assumed that the wage rate W_1 goes up and W_2 goes down such that profit and loss vanish again. Thus total income Y in (40) is not altered. Employment and output, too, remain unchanged in both firms.

The first and second round effect taken together make that the prices and wage rates move in parallel with the demand shift. Profits are again zero. The net effect is that the relations $^{P_1}/_{P_2}$ and $^{W_1}/_{W_2}$ both increase while the relation $^{X_1}/_{X_2}$ is not affected. To stop further adaptations it has to be assumed that the workers do not move from the firm with the lower wage rate to the firm with the higher wage rate. This would bring us back to Section 4.1.

4.3 Partial employment growth

It is assumed that labor cannot move freely between the firms and that the supply of firm 1 increases due to external factors. It is further assumed that the adaptation is hyperbolic, i.e. L_1 goes up and W_1 goes down, such that Y_{W1} in (40) remains constant. This in turn leaves total income Y unchanged.

With increasing labor input L_1 output increases and with it the quantity bought X_1 under the condition of market clearing. It is assumed that the price P_1 declines such that C_1 in (41) remains constant. The composition of nominal demand C and income Y does not change at all. Profit stays at zero in both firms.

The net effect is that the relation $^{X_1}/_{X_2}$ increases, that is, the household sector buys more of product 1 at a lower price P_1 and and the same quantity of product 2 at the same price.

We define the price level conveniently as:

$$P \equiv \frac{P_1 X_1}{X} + \frac{P_2 X_2}{X} \quad |t. \tag{42}$$

Since the compound quantity X increases in the denominator the price level declines as a consequence of partial employment growth.

4.4 General employment growth

It is assumed that the labor inputs L_1 and L_2 increase with the same percentage rate. With constant wage rates total income Y goes up and C, too, under the condition of budget balancing. Under the condition of market clearing the quantities bought X_1 and X_2 are up at constant prices.

The net effect is that the relation $^{X_1}/_{X_2}$ remains constant and profits stay at zero.

4.5 Partial wage rate increase

It is assumed that labor cannot move freely between the firms and that the wage rate W_1 in firm 1 goes up for whatever reasons. Employment L_1 falls hyperbolically and this leaves wage income Y_{W1} unchanged. On the other side the quantity X_1 goes down under the condition of market clearing and the price P_1 goes up such that C_1 remains unaltered. Total income and consumption expenditures stay put.

The net effect of the hyperbolic adaptation is that the relation X_1/X_2 decreases and the price level (42) rises. The unemployment rate increases. Profits stay at zero.

Note that the effect of a partial hyperbolic wage rate increase is different from a general wage rate increase as given by the elementary case of (16).

4.6 Partial productivity increase

It is assumed that the productivity R_1 in firm 1 increases. Under the condition of market clearing the quantity bought X_1 goes up. A hyperbolic price reduction leaves consumption expenditures C_1 at the previous level. Profit of firm 1 therefore stays at zero.

The net effect of the hyperbolic adaptation to a productivity boost is that the relation X_1/X_2 increases and the price level (42) recedes.

4.7 Increase of the overall expenditure ratio

The previous adaptations have taken place under the joint condition of market clearing, i.e. $\rho_X = 1$, and overall budget balancing, i.e. $\rho_E = \rho_{E1} + \rho_{E1} = 1$. Ignoring distributed profits at the elementary stage of inquiry, the latter condition makes, according to the Profit Law (10), that overall profit is zero. In the case of two or more firms that is to say that the sum of individual profits is equal to the sum of individual losses in the pure consumption economy.

It is assumed now that the household sector's total nominal demand C is greater than total income Y, i.e. $\rho_E > 1$. This is reflected in the exchange matrix as follows:

$$\begin{array}{ccc}
C_{11} = P_{\bullet 1}X_{11} & C_{12} = P_{\bullet 2}X_{12} & C_{1\bullet} \\
C_{21} = P_{\bullet 1}X_{21} & C_{22} = P_{\bullet 2}X_{22} & C_{2\bullet} \\
 & & \\
C_{\bullet 1} \geq W_1 L_1 & C_{\bullet 2} \geq W_2 L_2 & C \geq Y \\
Q_{m1} \geq 0 & Q_{m2} \geq 0 & Q_m \geq 0 \quad |t.
\end{array} \tag{43}$$

If the individual expenditure ratios ρ_{E1}, ρ_{E2} increase by the same percentage rate both firms make a profit. Under the condition that labor input and the wage rates remain unchanged the increase of nominal demand in both firms results, according to (43) in a proportional price increase. This is how profit emerges from exchange.

5 Conclusion

> The axiomatic method is indeed and remains the one suitable and indispensable aid to the spirit of every exact investigation no matter in what domain; ... To proceed axiomatically means in this sense nothing else than to think with knowledge of what one is about. (Hilbert, quoted in Kline, 1982, p. 193)

Axioms are indispensable to build up a theory that epitomizes formal and material consistency. The standard approach has not failed because of axiomatization but because of choosing the wrong axioms. The present paper replaces subjective-behavioral axioms by objective-structural axioms. The main results of the structural axiomatic analysis of exchange are:

- The move from autarky to exchange is indifferent with regard to the distribution of consumption goods but not with regard to the allocation of labor to the two lines of production. Exchange is the other side of specialization.
- The productivity effect of specialization and exchange benefits the consumers in real terms. Exchange *per se* does not increase value. There are neither real nor monetary gains from trade.
- The minimum number of markets for the analysis of supply and demand variations is two. The Marshallian approach with supply-demand-equilibrium in one market has always been worse than useless.
- If, starting from the zero profit baseline, the individual expenditure ratios increase by the same percentage rate all firms make a profit. Under the condition that labor input and the wage rates remain unchanged the increase of nominal demand in all firms results in a proportional price increase. This is how profit emerges from exchange.
- The exchange matrix is integral part of the structural axiom set.

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