A Theoretical Assessment on Optimal Asset Allocations in Insurance Industry

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Abstract

In recent years the financial markets known a rapid development and become more and more complex. So, many regulatory requirements, focused on banks as well as insurance sector, have been developed. These regulatory are concentrated essentially on business risk control and required capital to cover risks. These requirements have influenced the asset allocation issue in insurance industry. These requirements have influenced the asset allocation issue in insurance industry. This section is interested by this issue. In first time it highlights some research works in this issue. Then we will investigate the relation between Solvency and optimal asset allocation. Finally we will explore the principal used methods in modeling asset and in choosing the optimal portfolio composition.

Keywords: Portfolio investment; Optimal asset allocation; Solvency; Expected return; Expected utility; Assets modeling; Risky assets; Risk free asset; Insurance companies.

1. Introduction

Optimal asset allocation problems become an attractive research field in actuarial and financial literature. The key reasons of this field attraction are: the new regulatory developments and the enhancement of investment possibilities in financial markets.

The optimal asset allocation aims to seek the best wealth allocation between assets for a given period allowing the finance of future spending flows. This wealth allocation must be dynamically adjusted over time, that implying changes in the investment portfolio. The optimal choice of this portfolio in insurance industry is constrained by several conditions. First, there are environmental conditions under which operate the insurance company. These conditions are related essentially to the regulatory changes such as Solvency I and II in European countries. In the other hand we find constraints related to technical conditions. These conditions could be mainly attributed to the employed method to find the optimal investment portfolio and the method of modeling assets in which the insurance company can invest.

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This section is planned as follows. Sub-section 2 represents a theoretical background of optimal asset allocation in insurance industry. Sub-section 3 investigates the relation between Solvency and optimal asset allocation. Sub-section 4 explores the principal assets in which the company insurance can invest and the different used methods in modeling these assets. Finally the last sub-section is allocated to represent the principal used methods in the choice of the optimal asset allocation.

2. Theoretical Background

The problem of optimal asset allocation is usually based on the expected utility maximization of a portfolio value. The utility function is characterized by several properties, but the most important is the concavity property. Further using theoretical financial methods, like expected utility maximization of surplus, for insurance companies is too delicate due to the especial nature of these firms. Usually, most favored utility functions in previous research work are characterized by a strict floor in the terminal wealth that must be respected. Teplá (2001) investigates a related approach where he considers the solvency guarantee as a constraint. Although these solvency guarantees are suitable for some applications, but for insurance studies these guarantees may be unfeasible or financially undesirable. Detemple and Rindisbacher (2008) show that assets are not enough to cover with certainty all liabilities for an underfunded defined benefit pension plans. In addition, several studies in insurance industry show that claim processes characterized by probabilistic structures prove the uncertainty of liabilities, even in presence of guarantees, with a prefixed initial asset wealth. So, the floor uncertainty leads to an inappropriate optimization problem as a consequence solutions will be unreliable.

2.1. Optimal Asset Allocation in Life Insurance Business

The life length is associated by a potential uncertainty, for this reason the asset allocation problem in life insurance companies is more and more complicated. From a theoretical point of view, in a complete and perfect market, life annuities allow to hedge completely this risk. However, real world markets are characterized by the presence of frictions and imperfections that impede the insurer capacity to hedge this risk.

Indeed, several empirical investigations have been developed which try to find the optimal asset allocation for a life insurance company. At the beginning, studies focused on reserves distribution in life insurance companies have been developed from one policy to portfolios (Marceau and Gaillardetz (1999), Parker (1994), Dhaene (1989), Bellhouse and Panjer (1981), Panjer and Bellhouse (1980) and Waters (1978)). They use specific interest rate models, such as the models of AR (1) or ARCH (1), to conduct their studies. After that, research works have been oriented explicitly to study investment strategy for a single period asset allocation issue (Sherris (2006), Hurlimann (2002), Sherris (1992), Sharpe and Tint (1990), Wise (1987), Wilkie (1985), and Wise (1984)). On the other hand, studies are rarely focused on multiperiod asset allocation with discrete time, but researchers have been oriented more and more to investigate optimal investment strategy on continuous multiperiod time models using dynamic controls and other are based on martingale methods (Emms and Haberman (2007), Wang et al. (2007) and Chiu and Li (2006)). These new methods, take account of new generated information, but in discrete time the problem becomes too complex and intractable.

Dempster (1980) and Carino et al. (1994) propose another approach based on stochastic programming and provides numerical solution. This approach has advantages to be modeled easily and realistically, in addition it surmounts the theoretical solution problem. The key drawback of the stochastic programming that its asset return scenario is modeled by trees. A best market specification is function of nodes number at every decision point. The nodes number and time cost have an exponential growth. So, to fund solution in a reasonable time the nodes number must be restricted and consequently we cannot illustrate the real market in the model. In summary, the presence of error leads the researcher, of the optimal investment strategy in discrete time, to arbitrate between the solution's accuracy and the method's convenience.

2.2. Optimal Asset Allocation in Non-Life Insurance Business

The identification of optimal asset allocation strategy for non-life insurance companies is a research field rarely investigated in actuarial literature. Several studies show that stochastic control theory is the most used in the previous proposed models for non-life insurance companies, particularly to find either the optimal reinsurance contract or the optimal dividend payout (Hubalak and Schachermayer (2004), Paulsen (2003), Hipp and Vogt (2003), Schmidli (2002), and Hojgaard and Taksar (2002)). Hipp (2002) is a survey of the control theory that represents a particular field in ruin theory models. The author investigates optimal premium control, optimal reinsurance contract, optimal investment...

Several portfolio selection problems proposed in actuarial mathematics are focused principally on the choice of the optimal asset allocations for defined contribution pension plans or defined benefit (Haberman and Vigna (2002) and Cairns (2000)). This concentration is explained by the highest investment uncertainty in pension plans characterized by long-term contracts contrary to short-term contracts that concern nonlife insurance companies. Nevertheless, any financial institution seeks to improve its financial situation by taking advantage of investment opportunities in the financial market. Thus, nonlife insurance companies must negotiate assets on the market to make profit of available opportunities. Browne (1995) and Hipp and Plum (2000) are the principal papers confirming that non life insurance companies are concerned by the choice optimal investment on the stock market. Browne (1995) is based on final wealth to maximize expected utility and determine the optimal investment strategy. This strategy allows minimizing the ruin probability. Noting that, the author uses a Brownian motion process with drift in order to define insurer surplus. Referring to a classical collective risk model Hipp and Plum (2000) seek the optimal investment strategy, in an infinite time horizon, that minimizes ruin probability. Korn (2005) extends the research work of Browne (1995). He takes account of possible market crash while seeking optimal portfolio, this concept is called worst-case portfolio optimization. Based on the final wealth he maximizes the worst-case utility function.

3. Solvency and Asset Allocation

Solvency is a central issue for the insurance business. The supervisory authorities conduct a regular monitoring of this solvency for the insured behalf through a regulatory state. This latter gives the details of a solvency ratio calculation,

where the numerator is the real margin constituted from the balance sheet and the denominator represents the regulatory required solvency margin. This ratio must be imperatively greater than one.

The current regulatory solvency rules applied by European insurers are identical. These rules stem from the European directive implemented in 1973 and updated in 2002 to create rules, called Solvency I. the key rules of Solvency I are: First of all, there are requirements in assets composition and hedging liabilities by adequate assets. Second, the insurance company must annually present two principal reports regarding solvency and reinsurance. Third, each insurer has to achieve quarterly normalized simulations for judging asset-liability adequacy. Finally, the determination of the minimum equity level (i.e. the regulatory required solvency margin).

Currently, the required solvency margin of insurance companies is measured based on the business written amount (such as technical provision, premiums or claims), without taking account with an actuarial manner of actually incurred risks. The current European system of solvency measurement in insurance companies, Solvency I, has some weaknesses. The principal drawbacks are the absence of international financial convergence and the unsuitability of the weighing of risks borne by different agencies insurers in the calculation of the solvency margin. In addition, the calculation of capital requirements as defined by Solvency I doesn't take account of risk types and management quality. To overcome these weaknesses, several national legislations have been implemented, inducing a significant European heterogeneity.

To overcome the shortcomings of Solvency I a new directive is developed, called Solvency II. This new directive aims to modernize and harmonize solvency applicable rules in insurance companies to strengthen the policyholders' protection, to encourage companies to improve their risk management and ensure a harmonized application between European countries, using an approach based on risk assessment and using both quantitative and qualitative elements. Solvency II reform develops rules determining capital requirements in insurance companies, including the risk market issue. This reform is considered as an opportunity to deal risk and capital more efficiently. The Solvency Capital Requirement (SCR) developed in Solvency II is inspired from Risk Based Capital (RBC) regulations implemented since 1993 in the United States. This part is devoted to present the principal Characteristics and the three pillars of Solvency II.

3.1. Principal Characteristics of Solvency II

The new European Solvency system completely changes the vision of financial policy into insurance companies. Under Solvency II, the insurer will have to immobilize an amount of its capital, being used to cover the investment risk. The obligatory capital level is estimated via VaR models and it depends on the portfolio risk level. More the portfolio is risked more the insurer must enhance capital. This market risk covering method modifies objectives according to which the insurer guides his investment policy. He has to take account of the new criterion of assets allocation. So, the insurer has to take account, in addition to the traditional factors, of the immobilized amount in its own capital.

Under the previous European reform, Solvency I, the investment decisions were made primarily by financial direction. At the beginning, financial management is carried out only by the bottom up approach. So, the portfolio choice is only done at the higher hierarchical level of the financial direction. Against of Solvency II, the assets management must be analyzed under two prospects.

First, the bottom-up perspective: that takes account of the financial direction's vision which manages portfolio compared to the liabilities constraints and the allocated capital by the general direction to cover the investment risk.

Second, the top-down perspective: that takes account of the general direction vision which fixes a percentage of own capital dedicated to cover the market risk and establishes a new constraint of assets management by determining indirectly the investment portfolio.

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Involved Actors

In 2007 a European draft Directive is scheduled and its application will be in 2010 or 2012. To elaborate this directive the European commission has launched a broad consultation through the Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS). This committee is composed by delegates of controlling authorities of the European countries. After collection of consultation conclusions, carried out in the form of Call for advice from various actors in insurance industry, the CEIOPS advises the European Commission on the basis of opinion given by these actors (national controlling authorities, professional federations, insurance companies, etc). So, the delegates of each country have role of delivering a detailed technical opinion and checking the adequacy of the current model with their local problems.

Solvency II Goal

The new system Solvency II is intended for the European insurance and reinsurance companies. The principal goal of this system is to provide for the controlling authorities tools and evaluation capacity of the companies' general solvency through a prospective approach, based on:

On the one hand we find quantitative risks, like the reinsurance covering program or the adequacy between assets and liabilities, called Asset Liability Management.

in addition we finds qualitative risks, like systems reliability of internal audit and risk management, the management quality or the control of operational risk such as: failure of the information systems, defrauds, etc.

In this new solvency system there are three main references value that should be measured in addition to the control and supervision rules. The first concerns the technical provisions amount, the second defines the minimum required capital to carry on insurance business and the third concerns the own funds level (or target capital) that must be constituted in addition to the technical provisions. To measure target capital, called Solvency Capital Requirement (SCR), the European commission proposes a common standard model for all insurance companies in the European Union that will determine a prudent capital level. Moreover it allows setting up an internal model that takes account of some constraints in line with their economic reality.

3.2. The Three Pillars of Solvency II

Development Works of Solvency II are preceded in two principal stages:

> *Stage 1: this stage reflects the general form that must be taken by the European solvency system. This stage was completed in 2003. It led to a more suitable system consistent with the Lamfalussy approach and this system is based on three pillars, which make it consistent with banking system rules (Basel 2).*

> *Stage 2: This stage is focused on the methods identification that take account of different risks. It must lead to the establishment of methods to assess various risks faced by insurers.*

Pillar I: Quantitative Aspects

The quantitative capital requirements are related to the technical provisions amounts, target capital and internal risk management. This pillar aims to develop tools allowing the assessment of technical provision sufficiency and to formulate a harmonization of the calculation principles between different European insurance companies. Furthermore, these requirements must be distinct. So, the insurer capital should not be able to compensate a lack of technical provisions.

Pillar I is designed to measure risk quantitatively. It requires two capitalization sills:

> *The Solvency Capital Requirement (SCR) designates the desired capital level and it can be assessed by two methods. First, the standard approach is applicable to all type of insurance companies and it takes account of significant and quantifiable risks. Second, a model developed by the company and confirmed by the supervisory authorities.*

> *The Minimum Capital Requirement (MCR) represents the sill from which supervisory authorities can remove the signed agreement. To calculate MCR, several alternatives are currently being studied. While taking account of sanctions gravity (agreement remove and portfolio transfer), it must be led by a legally secure computation.*

The calculation of minimum and desired capital levels is based on the technical provisions evaluation. To make comparable the results of these assessments, it is necessary to harmonize provisions computation at European level.

Pillar II: Control Activities

While Pillar I represents the quantitative aspect of Solvency II, Pillar II defines the qualitative objectives, therefore it will complete Pillar I. it enables supervisory authorities to assess the internal control, Risk management and corporate governance of each insurance company. Indeed this pillar is based, simultaneously, on authorities control and internal control of companies. European authorities want to give supervisors resources to identify companies which have a significant financial or organizational risk. To this end, these supervisors must have possibility to enhance the capital requirement or to apply some procedures to reduce risks. Direct consequence of this pillar, supervisors will have power on the overall management and operation of an insurance agency, examining all risks including operational risk.

Pillar III: Information

Pillar III aims to redefine obligations of insurance companies publications with respect to their policyholders, investors and also some market authorities. The major discussion fields are the accessibility and transparency of information produced and its comparability at European level.

3.3. Expected Limits of Solvency II

Some participants on Solvency II highlight that the construction of the Solvency Capital Requirement (SCR), inspired by the current rules in the United States Risk-Based Capital (RBC), could have negative consequences on insurers investment. The American solvency standards establish the portfolio choice for insurers in the United States. These participants have worries regarding risky investments. So, the current American solvency system and future European regulations encourage riskier investments with higher capital. Thus, some academic researches claim that solvency II could encourage the substitution of risky investments by other classes of assets that require less of equity.

4. Assets Modeling Methods

Investments represent the most important part of assets in non life insurance companies, which serve to cover the whole provisions of the company. Indeed there are a large number of financial investments. Here is a non-exhaustive list of the most classical products: stocks, hedge funds, real estate, bonds, convertible bonds, the currency exchange, etc.

This part aims to explore the most important methods for modeling some assets used by the non-life insurance company.

4.1. Interest Rate Modeling

Rogers (1995) proposes a number of criteria to model an ideal rate such as:

The model must be realist. That taking account of empirical properties of the yield curve: non negative rates, rates are affected by mean reversion, no perfect correlation between rates evolutions, the short-term rates are more volatile than long rates…

Model parameters must be observable in the market, easily estimable, and also frequently re-adjustable.

The model has to be consistent with prices of different products available in the market.

It should be enough simple allowing quick and intuitive calculations.

The absence of arbitrage opportunities and the presence of the risk factor are primordial keys for the model.

In reality this ideal model does not exist and a variety of practical models are developed.

Vasicek Model

One of the first stochastic interest rate models is that of Vasicek (1977). This is a Gaussian process based on the process of Ornstein-Uhlenbeck to explain the mean reversion effect on the empirically observed yield curves. The proposed stochastic differential equation in this model has an explicit solution, so there is an exact discretisation of the process.

This model is characterized by a number of advantages. First of all, the model parameters are observable in the market, easily estimable, and can be adjusted frequently. So the model is too simple and its parameters are easily interpreted. As a second advantage of this model, it provides analytical expressions for the standard rate products like bonds. Finally, this model takes account of the mean reversion effect. So, the high rate levels tend to be more frequently followed by declines than increases. The opposite effect is also found for unusually low rate levels.

However this simple model has several disadvantages. First, the various parameters of the diffusion process are constant. So the instantaneous rate is supposed to be the original of yield curve deformations, which implies that rates are perfectly correlated. Second, the model is said inconsistent with the yield curve. So, it is possible to obtain most forms of the yield curve: increasing, decreasing and bumpy, but a curve does not take a hollow form (i.e. decreasing in the short term and increasing in the long term). Finally, this rate model follows a Gaussian process, so is negative with a non-zero probability. That makes it inconsistent with the assumption of no-arbitrage opportunity. Indeed, a rational economic agent will always prefer to keep their money rather than lend it to a negative rate.

Cox, Ingersoll, Ross (CIR) Model

Cox et al. (1985) propose another model, called CIR model, that respects the mean reversion property and it also eliminates the problem of negative rates modeling. This model is most often used by professionals as the Vasicek model.

CIR model is based on the concept of market equilibrium between supply and demand. There are no explicit solutions for the proposed stochastic differential equation of this model and the discretisation can be done only by

approximation. Several methods can be used such as Euler or Milstein schemes which are Taylor expansions for the stochastic differential equation characterized by greater or lesser important orders.

In addition to the advantages of Vasicek model, CIR model features a new highlight that is the non-negativity of the rates proposed by this model. Indeed, this rate model hasn't longer the Gaussian character. Also for disadvantages, CIR model has the same drawbacks of Vasicek except the problem of negative rate has been resolved.

Heath, Jarrow, Morton (HJM) Model

The model of Heath et al. (1990), called HJM model, is a generalization of the model developed by Ho and Lee (1986). It includes a whole class of forward rates models.

The HJM model doesn't based on the diffusion of instantaneous rate but on the diffusion of all instantaneous forward rate. It is a more general model since there is equivalence between the knowledge of instantaneous forward rates and the zero-coupon bond prices. This model is also based on the diffusion of the full spectrum of zero-coupons' prices. In addition it should be noted that the yield curve represents all rates, not only instantaneous rate.

This models class refers to an initial structure of forward rates and uses a set of parameters dependent of time and maturity. This makes possible to model the structure of rates in the long term while taking account of absence of arbitrage opportunities. This arbitrage assumption imposes that market risk premium is independent of maturity.

This class of models has a number of advantages. Principally, these models are compatible with the market yield curve. Indeed all curve shapes are attainable. In addition several models of this class are simple in their implementation and others are closer to ideal rate model. In other hands this class of models has some disadvantages. Some models of this class keep a Gaussian character. So, there is a probability to obtain negative rates. In other models the data-processing coding and parameterization are rather complicated because of the temporal parameters. In addition, they can depend on a curve of value in the beginning. However, the curve of the instantaneous forward rates is not easily observable on the market. Whereas the traditional models of VASICEK and CIR require only knowing the instantaneous rate.

4.1. Stock Market Modeling

Several methods have been developed for modeling stocks. There are two key methods to model stock prices: the referential model of Black and Schools (1973) and the jump-diffusion model originally developed by Merton (1976).

Black and Scholes Model

Black and Schools (1973) propose a model is based on solving a stochastic differential equation, assuming that stock price follows a geometric Brownian motion or a generalized Wiener process. This model always remains a reference in the stock price modeling.

Several assumptions have been explicitly imposed in using the Black and scholes model, such as:

> *The absence of arbitrage opportunities and riskless profit.*

There is a risk-free interest rate that allows both borrowing and lending cash.

There is no limit to buy or sell any fraction of stock amount.

Frictionless market. So, the model requires absence of any transaction fees or costs.

The stock price process is based on a Brownian motion and a normal distribution. In addition, drift and volatility should be constant.

No dividend distribution.

This model has the advantage of being simple, easy to calibrate and estimate since estimation of required parameters is based only on the stock price historic.

The imposed assumptions to this model make it fairly restrictive. So, these assumptions are contradicted while referring to the empirical observations. An important drawback is that stock prices aren't a strict stationary log-normal process. Therefore, the stock price is not necessarily continuous and may have discontinuities. Also, volatility is not really constant.

To overcome these disadvantages and to reflect the market reality, several other models have been developed such as: the model of Merton (1976) and that of Kou (2002).

Jump-diffusion Model

Since the publication of their paper, the financial markets have extensively used the Black-Scholes model. However, empirical observations based on market data contradict this model and show that implicit volatility is not constant. Its curve has in many cases a convexity compared to the stock prices, a phenomenon known conventionally as smile volatility. Moreover, empirical studies show that returns are skewed to the left and the distribution tails are thicker than a normal distribution Cont (2001) and Carr et al. (2002).

While taking for these empirical phenomena, several Lévy processes have been used. Among these models, the class of jump-diffusion model presented by the models of Merton (1976) and Kou (2002). Merton (1976) is the pioneer of this model. He proposes in addition of the geometric Brownian motion term, the stock price process is characterized by the presence of jumps. The risk of the latter is supposed to not be priced. No closed evaluation form has been obtained for this class of models except in marginal cases. Some studies have taking account of numerical solutions using the finite difference method (Andersen and Andreasen (2000), Cont and Voltchkova (2005)). Nevertheless, the Black-Scholes model is the most used and the most studied model.

5. Optimal Asset Allocation Methods

In general there are two principal used approaches to find the optimal investment portfolio: the mean-variance approach and the stochastic approach.

5.1. Mean-Variance Approach

Traditional research works that tried to resolve asset allocation problem, were mainly based on the mean-variance analysis of Markowitz (1952). Based on the return-risk tradeoff, this method allows finding the best portfolio, referring to an efficient frontier. This method represents the first optimal rule in a static setting, allowing investors to allocate wealth on risky assets. This approach has a great professional and practical impact because it is simple and intuitive. In addition to its presence in the recent research works, several financial intermediaries and financial planners refer to this approach while offering product or providing advices. According to Tobin (1958) an optimal portfolio contains only two funds such as risky assets and a risk free asset. The derived implications of this separation between portfolio funds on equilibrium prices are investigate by Sharpe (1964) and Lintner (1965). Though, Merton (1969) and Sumuelson (1969) inspect conditions allowing this portfolio to be optimal in a multi-period approach. Some Asset-liability management studies in financial institutions extend the mean-variance analysis while taking account of liability side (Chiu and Li (2006), Craft (2005), and Sharpe and Tint (1990)).

The above presented theoretical models are based on the estimation of their parameters, in order to implement the suggested portfolio policies. The needed parameters for a static portfolio models are the vector of expected returns and the variance-covariance matrix. However, the needed parameters for a dynamic portfolio are the expected returns, the volatilities and correlations for risky assets, and the risk free interest rate. These parameters can be estimated using traditional statistics such as ordinary least squares, maximum likelihood, and generalized methods of moments. These classical statistics methods suffer of several drawbacks, for this reason many recent research works have developed some new approaches to overcome these drawbacks (Jagannathan and Ma (2003), Michaud (1998), Chopra (1993), Black and Litterman (1992, 1990), and Frost and Savarino (1988)).

Brennan et al. (1997) suggest that mean-variance analysis has two key flaws: the use of a single period and the unsuitable utility function proposed for the investor.

5.1. Stochastic Approach

Merton (1971) proposes a new method to build an optimal portfolio in a stochastic investment environment. The effect of this method on the equilibrium asset prices has been developed by Merton (1973). recent investigations try to solve this problem numerically (Chacko and Viceira (2004), Brennan and Xia (2002), Wachter (2002), Campbell et al. (2001), Liu (2001), Xia (2001), Campbell and Viceira (1999), Skiadas and Schroder (1999), and Kim and Omberg (1996))or to reformulate it under an analytic expression form (Lynch (2001), Lynch and Balduzzi (2000), and Brennan et al. (1997)).

The literature that follows this approach considers the asset allocation problem as stochastic and the solutions are illustrated by Hamilton-Jacobi-Bellman partial differential equations (Yu et al. (2010)). Many drawbacks were detected in this approach. First, there are difficulties to solve the highly non-linear partial differential equations. Second, algebraic solutions aren't always feasible. Finally, more the number of state variables are higher more the stochastic problem is complicated.

Cox and Huang (1989) propose as solution to decrease the complexity problem in the stochastic dynamic programming, the applying of the martingale representation theory. Without taking account of the simplest cases, there are few applicable solutions to resolve the stochastic dynamic problem. There are two classes of technique global optimization and local optimization. The principal difference between these two techniques lies in the assumption regarding the shape of possible solution surface. Local optimization techniques are unsuitable to complex and multimodal simulation models (Yu et al. (2010)). To avoid deficiencies of local optimization techniques, researchers have developed the global optimization techniques such as: evolutionary algorithms, genetic algorithms, simulated annealing and swarms algorithms.

6. Conclusion

This paper is a literature review dealing the optimal asset allocation issue and its applications in the insurance industry. First of all, we have presented a theoretical overview of research work focused on the issue of optimal asset allocation in the insurance industry especially for life and non-life business.

Second we have investigated solvency regulations imposed on insurance companies and their key implications on the choice of optimal investment. As a first step we have introduced the current regulatory solvency rules applied by European insurers, Solvency I, and their principal requirements. Then we focused on the principal Characteristics of Solvency II, the three pillars (Quantitative Aspects, Control Activities, and Information). In a last step we summarize this part by expected limits of these new regulatory solvency rules.

Third we explore the principal assets in which the insurance company can invest and the different used methods in modeling these assets. In this part we have presented the principal used methods for modeling two key assets. The first is the interest rate asset, where the practical used methods for modeling is that of: Vasicek (1977), Cox et al. (1985), and this of Heath et al. (1990). The second is the stock market asset. There are two key methods to model stock prices: the referential model of Black and Schools (1973) and the jump-diffusion model originally developed by Merton (1976).

Finally, the last subsection is allocated to represent the principal used methods in the choice of the optimal asset allocation. There are two principal approaches that can be used in choosing the optimal asset allocation: the first is the mean-variance approach originally proposed by Markowitz (1952) and the second is the stochastic approach developed initially by Merton (1971). The used techniques in the last approach have known a considerable development and each technique has its own properties.

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