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FUZZY SETS FROM THE ETHICS OF SOCIAL PREFERENCES

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Abstract

We show that the problem of evaluating infinite sequences (or streams) of utilities by a unique utility (or social welfare function) can be stated in terms of fuzzy subsets of the set of infinite utility sequences. For each stream, its evaluation can be viewed as its degree of membership to the subset of 'ethically acceptable' streams within the set of possible sequences. Since the property 'being ethically acceptable' is not well defined and cannot be exactly measured, the fuzzy approach seems especially adequate.

Keywords: Fuzzy subset, Social welfare function, Ethical.

1 INTRODUCTION

The resolution of real-world intergenerational conflicts such as global warming has given place to many analyses of intergenerational social preferences over infinite streams of utilities or well-being. A very authoritative review of these contributions is covered by Asheim [2]. They raise the question: How should the streams of utilities be ranked from a social perspective, when the interests of all generations must be respected?

A practical way to perform this comparison consists of evaluating the relevant infinite streams of utilities by a unique utility, which is then called a social welfare function. In the present contribution we show that this problem can be stated in terms of fuzzy subsets of the relevant set of infinite utility streams. The idea is as follows: when these evaluations are valued in $[0,1]$ we can view them as membership functions that capture the degree of agreement with the imprecise statement 'the infinite stream is ethically acceptable'. This seems pretty appropriate because the property 'being ethically acceptable' is not well defined and cannot be exactly measured. In technical terms, any social welfare function $\mathbf{W}$ on a set of infinite streams $\mathbf{X}$ whose values lie in $[0,1]$ (and this is a matter of normalization) is identified with a fuzzy subset of $\mathbf{X}$. And for each $\mathbf{x}$, $\mathbf{W}(\mathbf{x})$ is its degree of membership to the subset of 'ethically acceptable' streams in $\mathbf{X}$.

Since social welfare functions are used to determine optimal policies in infinite horizon models, they are expected to verify an adequate combination of principles of two kinds: egalitarian treatment of the generations and sensitivity to the interests of each generation. Such requirement can be stated in terms of our alternative viewpoint, and then *ethical* fuzzy subsets appear naturally, with properties inherited from the literature on intergenerational equity. In fact the design of acceptable social welfare functions is controversial since many impossibility results show that an infinite number of generations cannot be treated equally while still being sensitive to the interests of each generation. We transfer that debate to the selection of ethical fuzzy subsets.

This work is organized as follows. In Section 2 we present the standard approch to the design of social welfare functions and some of its desirable properties in the context of intergenerational aggregation of the utilities. In Section 3 we state the corresponding problem in terms of *ethical* fuzzy sets, with prominent examples and the resolution of some particular statements. Other variations that lead to related approaches to the problem of constructing ethical fuzzy subsets are discussed too. We conclude in Section 4.

2 SOCIAL WELFARE FUNCTIONS: EQUITY AND EFFICIENCY PROPERTIES

Let $\mathbf{X} \subseteq \mathbb{R}^{\mathbb{N}}$ represent a domain of infinite-horizon utility sequences (henceforth, streams). We adopt the standard notation for infinite streams: $\mathbf{x} = (x_1, ..., x_n, ...) \in \mathbf{X}$. Each x_i can represent for example, either the allocation of utility to an agent from an infinite society or the welfare endowment of a generation or its consumption. For simplicity we use the terminology from intergeneral justice and say that each component is a generation. We write $\mathbf{x} \geqslant \mathbf{y}$ if $x_i \geqslant y_i$ for each $i = 1, 2, ...$; $\mathbf{x} \gg \mathbf{y}$ if $x_i > y_i$ for each $i = 1, 2, ...$; and $\mathbf{x} > \mathbf{y}$ if $\mathbf{x} \geqslant \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$.

Social welfare relations are binary relations on $\mathbf{X}$. They are interpreted as normative welfare criteria on the domain $\mathbf{X}$. A *social welfare function* (SWF) is a function $\mathbf{W} : \mathbf{X} \longrightarrow \mathbb{R}$, also regarded as a representable social welfare relation. The analysis of intergenerational aggregation by means of SWFs is usually called the Basu-Mitra approach. More generally, one can use binary relations in the comparison of utility streams.

Let $\mathbf{W}$ be a SWF. We first proceed to recall some efficiency properties (axioms) that we use along the paper. Then we mention the equity property under inspection, as well as other possible approaches.

2.1 EFFICIENCY PROPERTIES

The most standard version of the Pareto axiom is the very demanding principle that improving the allocation of at least one generation should increase the social evaluation:

Axiom SP (*Strong Pareto*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} > \mathbf{y}$ then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

The next axioms are all implied by Strong Pareto.

Axiom MON (*Monotonicity*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} \geqslant \mathbf{y}$ then $\mathbf{W}(\mathbf{x}) \geqslant \mathbf{W}(\mathbf{y})$.

MON is an undisputable property of efficiency. The next two properties are succesively weaker than SP:

Axiom IP (*Infinite Pareto*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and $x_i > y_i$ for an infinite number of indices i, then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

Axiom WP (*Weak Pareto*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} \gg \mathbf{y}$ then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

An independent weaker version of Strong Pareto is:

Axiom WD (*Weak Dominance*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, there is $j \in \mathbb{N}$ with $x_j > y_j$, and $x_i = y_i$ for all $i \neq j$, then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

2.2 EQUITY PROPERTIES

The Anonymity axiom (**Axiom AN**) demands that any finite permutation of a utility stream produces a socially indifferent utility stream. This is a *procedural* property of equal treatment of all generations. Such impartiality avoids biases towards particular generations and in particular, avoids dictatorships and impatiant behaviors.

Besides procedural equity, in order to implement various egalitarian principles the literature on intergenerational justice has provided a number of useful *consequentialist* equity properties. This term means that contrarily to the case of procedural equity requirements, some streams are declared as socially better than other ones on the basis of ethical principles. We return to this discussion in Subsection 3.4 below.

2.3 THE CODOMAIN OF SWFs

Observe that because there exist strictly increasing mappings $\rho : \mathbb{R} \longrightarrow [0, 1]$, every social welfare function $\mathbf{W} : \mathbf{X} \longrightarrow \mathbb{R}$ can be transformed into a mapping $\mathbf{W}' = \rho \circ \mathbf{W} : \mathbf{X} \longrightarrow [0, 1]$ in such way that $\mathbf{W}(\mathbf{x}) \geqslant \mathbf{W}(\mathbf{y})$ and $\mathbf{W}'(\mathbf{x}) \geqslant \mathbf{W}'(\mathbf{y})$ are equivalent, for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$.

The composition with ρ does not affect the fulfilment of the axioms above: $\mathbf{W}$ is SP, resp., MON, IP, WP, WD, AN, if and only if so is $\mathbf{W}' = \rho \circ \mathbf{W}$.

Therefore for the purpose of investigating the existence of SWFs with the axioms we have mentioned, we do not lose generality if the codomain is assumed to be $[0, 1]$.

3 SOCIAL WELFARE FUNCTIONS AND FUZZY SETS

SWFs are used to compare infinite streams of utilities, e.g. by a social planner that has to decide among distributions of well-being. Therefore SWFs are regarded as evaluations that must meet adequate properties of efficiency and equity in order to fulfil that role. Efficiency must be requested in some form, since deciding on allocations should be made consistently with certain sensitivity to the interest of the individuals or generations. Equity among generations is expected too, since this decision should be made with respect to commonly agreed egalitarian principles as well. In particular, the term *ethical* has been applied by prominent contributors to this literature to mean that both SP and AN are met (cf., Svensson [13]). Although we use it in this technical sense too, we also refer to the common meaning of the term 'ethical' in this context: a 'fair' combination of efficiency and equity properties.

From another perspective, when these evaluations or SWFs take values in $[0, 1]$ they can be regarded as membership functions that capture the degree of agreement with the statement 'the infinite stream is ethically acceptable'. Now the evaluation has an intrinsic value rather than being a mere way to compare among streams (in order to choose a maximally graded distribution of the welfare indicators, for example). In short, we can view any social welfare function $\mathbf{W}$ on $\mathbf{X}$ (whose values lie in $[0, 1]$) as a fuzzy subset of $\mathbf{X}$ and for each $\mathbf{x}$, $\mathbf{W}(\mathbf{x})$ is the degree of membership to the subset of 'ethically acceptable' streams in $\mathbf{X}$. To better fit these interpretations, in the usual case where $\mathbf{X} \subseteq [0, 1]^{\mathbb{N}}$ and both $\mathbf{1} = (1, 1, ..., 1, ...) \in \mathbf{X}$ and $\mathbf{0} = (0, 0, ..., 0, ...) \in \mathbf{X}$

hold true, it seems convenient to restrict our analysis to SWFs that verify $\mathbf{W}(\mathbf{1}) = 1$ and $\mathbf{W}(\mathbf{0}) = 0$, which due to MON is simply a matter of normalization.

Therefore we can view one of the topics of Social Choice as a matter of ellucidating the existence of fuzzy subsets of a fixed $\mathbf{X} \subseteq \mathbb{R}^{\mathbb{N}}$ (very often, $\mathbf{X} = [0,1]^{\mathbb{N}}$) that verify certain lists of properties. As argued above, we request that the degree of membership of the stream $\mathbf{1} = (1,1,...,1,...)$, resp. $\mathbf{0} = (0,0,...,0,...)$, must be 1, resp. 0. This means that whatever the interpretation of the term 'ethically acceptable distribution' of the infinite utilities, attaching the maximum value to all generations is absolutely acceptable, and attaching the null value to all generations is absolutely insupportable. Needless to say, one can reinterpret such fuzzy subsets as SWFs with the corresponding list of properties.

To illustrate this alternative position, we proceed to define some pertinent concepts of ethical fuzzy subsets (in the broad sense of the term) and then we give some results that concern their possible existence. Finally in this Section, we make a short digression on other related possibilities that are left unexplored in this first contribution.

3.1 PROMINENT EXAMPLES

A very direct definition of a fuzzy subset that is inspired in a well-known SWF is given in our first example:

Example 3.1. The *minimax* or Rawlsian fuzzy subset of $\mathbf{X} = [0,1]^{\mathbb{N}}$ is defined by the membership function $\mu_R : \mathbf{X} \longrightarrow [0,1]$ such that

$$\mu_R(\mathbf{x}) = \inf_i x_i, \forall \mathbf{x} \in \mathbf{X} \tag{1}$$

The most popular objective function used to determine optimal policies in infinite horizon models is the discounted sum of utilities, which depends on a discount factor $\delta \in (0,1)$. When $\delta > \frac{1}{2}$ the standard expression for such SWF produces evaluations of streams that are larger than 1 thus we need to adapt it in order to define our next prominent example of a fuzzy subset in this analysis:

Example 3.2. Given $\delta \in (0,1)$, the δ-discounted fuzzy subset of $\mathbf{X} = [0,1]^{\mathbb{N}}$ is $\mu_\delta : \mathbf{X} \longrightarrow [0,1]$ such that

$$\mu_\delta(\mathbf{x}) = (1-\delta)\sum_{i=1}^{\infty} \delta^{i-1} x_i, \forall \mathbf{x} \in \mathbf{X} \tag{2}$$

As requested by our definition, $\mu_R(\mathbf{1}) = \mu_\delta(\mathbf{1}) = 1$ and $\mu_R(\mathbf{0}) = \mu_\delta(\mathbf{0}) = 0$.

In the analysis of infinite horizon models, a recent proposal that has attracted much attention is the *Rank-discounted utilitarian* SWF (cf., Zuber and Asheim [14, Definition 1], where an *Extended* rank-discounted utilitarian SWF is defined too). This suggests the next example that supposes a variation of Example 3.2 above:

Example 3.3. Let $\bar{\mathbf{X}}$ be the set of allocations of $[0,1]^{\mathbb{N}}$ whose elements can be permuted to obtain non-decreasing streams. Given $\delta \in (0,1)$, the δ-rank-discounted fuzzy subset of $\bar{\mathbf{X}}$ is $\rho_\delta : \bar{\mathbf{X}} \longrightarrow [0,1]$ such that

$$\rho_\delta(\mathbf{x}) = (1-\delta)\sum_{i=1}^{\infty} \delta^{i-1} x_{\lfloor i \rfloor}, \forall \mathbf{x} \in \bar{\mathbf{X}} \tag{3}$$

where $(x_{\lfloor 1 \rfloor}, x_{\lfloor 2 \rfloor}, ...)$ is the non-decreasing infinite stream which is a permutation of $\mathbf{x}$.

3.2 ETHICAL FUZZY SETS: VARIATIONS OF A COMPREHENSIVE TERM

Depending on the combination of properties that we demand to our fuzzy subsets of $\mathbf{X}$, various concepts of ethical (in the comprehensive sense) fuzzy subsets come up.

Definition 3.1. A fuzzy subset of a domain of infinite utility streams $\mathbf{X} \subseteq [0,1]^{\mathbb{N}}$ such that the degree of membership of $\mathbf{1} \in \mathbf{X}$ is 1, resp. of $\mathbf{0} \in \mathbf{X}$ is 0, is called

1. **Ethical** if *a*) The degree of membership of any $\mathbf{x} \in \mathbf{X}$ does not change under finite permutations of its coordinates; and *b*) when $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x}$ allocates more than $\mathbf{y}$ to some generation, and $\mathbf{x}$ does not allocate less than $\mathbf{y}$ to any generation, then $\mathbf{x}$ has a higher degree of membership than $\mathbf{y}$.

2. **Pre-ethical** if *a*) above; and *b*) when $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x}$ allocates more than $\mathbf{y}$ to an infinite number of generations, and $\mathbf{x}$ does not allocate less than $\mathbf{y}$ to any generation, then $\mathbf{x}$ has a higher degree of membership than $\mathbf{y}$.

3. **Weakly ethical** if *a*) above; and *b*) when $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x}$ allocates more than $\mathbf{y}$ to all generations, then $\mathbf{x}$ has a higher degree of membership than $\mathbf{y}$.

4. **Quasi-ethical** if *a*) above; and *b*) when $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x}$ allocates more than $\mathbf{y}$ to a generation i, and $\mathbf{x}$ and $\mathbf{y}$ allocate the same amount to any generation other than i, then $\mathbf{x}$ has a higher degree of membership than $\mathbf{y}$.

5. **Basically ethical** if *a*) above; and *b*) when $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and $\mathbf{x}$ does not allocate less than $\mathbf{y}$ to any generation, then $\mathbf{y}$ does not have a higher degree of membership than $\mathbf{x}$.

Requirement *a*) is an anonymity prerequisite common to every property in the list. It implements the idea that all generations must be treated equally: when the allocations to any two generations are swapped, the degree of membership to the subset of 'ethically acceptable' streams does not vary. Fuzzy subsets that verify *a*) are called *anonymous*. The respective conditions *b*) implement efficiency concepts in the sense that improving the allocations to certain generations, the other generations not being worse-off, should increase the degree of membership to the subset of 'ethically acceptable' streams.

A handicap for accepting the discounted sum of utilities in the intergenerational analysis is that it discriminates future generations. In our approach, this translates into the fact that μ_δ does not meet requirement *a*), i.e., μ_δ is not anonymous. To overcome that drawback, one can refer to the corresponding rank-discounted sum ρ_δ: it coincides with μ_δ on non-decreasing streams, but it agrees with requirement *a*). We return to this point in subsection 3.3 below.

Remark 3.1. Any ethical fuzzy subset of $\mathbf{X}$ is pre-ethical, quasi-ethical, and basically ethical. Pre-ethical fuzzy subsets of $\mathbf{X}$ are weakly ethical. Furthermore, the concepts in Definition 3.1 are hereditary by (crisp) subsets of $\mathbf{X}$.

On the ground of our defense of the Monotonicity property of SWFs, it may appear that only basically ethical fuzzy subsets of our domain of infinite utility streams are worth considered when *a*) is imposed. However under such assumption some of the concepts above collapse into a single one, as the following simple lemma justifies:

Lemma 3.1. If a fuzzy subset of $[0,1]^{\mathbb{N}}$ is quasi-ethical and basically ethical then it is ethical.

3.3 RESULTS

The following questions arise: Do there exist ethical, resp., pre-ethical, weakly ethical, quasi-ethical, basically ethical, fuzzy subsets of $\mathbf{X}$? It seems intuitively natural that the answer to these questions can vary with the structure of $\mathbf{X}$. We proceed to examine all these questions separately.

3.3.1 Are there ethical fuzzy subsets?

The requirements on the set of infinite streams determines the answer to this question. Suppose first the case where every generation or individual has a common set of feasible allocations, i.e., $\mathbf{X} \supseteq Y^{\mathbb{N}}$ for some $Y \subseteq [0,1]$. Then Theorem 3.1 below states that the answer for this particular question is negative even if $\mathbf{X} = \{0,1\}^{\mathbb{N}}$. Such case is the simplest possible instance for analysis: it only distinguishes a 'good' state 1 and a 'bad' state 0 for each generation or agent, thus for any practical purpose we have impossibility of ethical fuzzy subsets (provided that there is a common set of feasible allocations across generations). This statement derives from a celebrated result by Basu and Mitra [6, Theorem 1], which establishes that there are no SP, AN social welfare functions on $\{0,1\}^{\mathbb{N}}$.

Theorem 3.1 (Basu and Mitra [6])**.** There do not exist ethical fuzzy subsets of $\mathbf{X} = \{0,1\}^{\mathbb{N}}$.

However we obtain a different conclusion when we refer our analysis to $\bar{\mathbf{X}}$ as defined in Example 3.3:

Theorem 3.2 (Zuber and Asheim [14], Prop. 5)**.** Example 3.3 defines an ethical fuzzy subset of $\bar{\mathbf{X}}$.

3.3.2 Are there pre-ethical fuzzy subsets?

Building on [6], Crespo, Núñez, and Rincón-Zapatero [8, Theorem 3.3] prove that there are not IP, AN social welfare functions on $\{0,1\}^{\mathbb{N}}$. From this we deduce:

Theorem 3.3 (Crespo, Núñez, Rincón-Zapatero [8])**.** There do not exist pre-ethical fuzzy subsets of $\mathbf{X} = \{0,1\}^{\mathbb{N}}$.

Remark 3.2. Theorem 3.1 is a trivial Corollary to Theorem 3.3, because ethical fuzzy subsets of $\mathbf{X} = \{0,1\}^{\mathbb{N}}$ are pre-ethical. Another reason for this redundance is that [8, Theorem 3.3] generalizes the aforementioned [6, Theorem 1] by proving that the incompatibility between SP and AN remains when the weaker IP replaces SP. However we believe that it is just fair to state Theorem 3.1 on its own right due to the key importance of [6] in the recent development of the problem of aggregating infinite utility streams.

However in view of Remark 3.1 and Theorem 3.2, Example 3.3 defines a pre-ethical fuzzy subset of $\bar{\mathbf{X}}$.

3.3.3 Are there weakly ethical fuzzy subsets?

This is a case where the choice of the domain of infinite utility streams provides a rich discussion. The motivation for the analysis of this case is Basu and Mitra [7]. Accordingly, we can state:

Theorem 3.4 (Basu and Mitra [7])**.** Let $\mathbf{X} = Y^{\mathbb{N}}$ be a domain of infinite utility streams.

1. If $Y = \mathbb{N}$ then there exist weakly ethical fuzzy subsets of $\mathbf{X}$.
2. If $Y = [0,1]$ then there do not exist weakly ethical fuzzy subsets of $\mathbf{X}$.

As in the previous impossibility results, the cardinality of the set of feasible utilities (namely, Y in Theorem 3.4 above) is key in the argument. In those impossibility results one simply 'runs out of numbers' when the constraints are imposed. This partially explains the different conclusion when Y changes from the countable (and not order-dense) $\mathbb{N}$ to $[0,1]$.

We emphasize that the appeal to discrete sets of feasible utilities like $\mathbf{X} = Y^{\mathbb{N}}$ with $Y = \mathbb{N}$ is supported by the recognition that human perception is not endlessly fine. It is a natural restriction e.g., when the utilities have a well-defined smallest unit (as happens when the endowments of the generations are monetary amounts).

Furthermore, Example 3.3 defines a weakly ethical fuzzy subset of $\bar{\mathbf{X}}$ as argued above.

3.3.4 Are there quasi-ethical fuzzy subsets?

Here we obtain a widespread affirmative answer. Of course, Example 3.3 defines a quasi-ethical fuzzy subset of $\bar{\mathbf{X}}$ as argued above. But now we can do better than this due to Basu and Mitra [7], who prove that there are WD, AN social welfare functions on $\mathbf{X} = [0,1]^{\mathbb{N}}$. Thus Theorem 3.5 below benefits from their result in order to state that the answer to our question for the quasi-ethical restriction on fuzzy subsets is affirmative for any $\mathbf{X} \subseteq [0,1]^{\mathbb{N}}$ too.

Theorem 3.5 (Basu and Mitra [7])**.** There exist quasi-ethical fuzzy subsets of any $\mathbf{X} \subseteq [0,1]^{\mathbb{N}}$.

3.3.5 Are there basically ethical fuzzy subsets?

The answer to the question if there are basically ethical fuzzy subsets of a suitable domain of utility streams is affirmative for any $\mathbf{X} \subseteq [0,1]^{\mathbb{N}}$. We just need to check that the *minimax* or Rawlsian fuzzy subset μ_R in Example 3.1 verifies the requested properties.

Although we have shown that there are quasi-ethical fuzzy subsets of $[0,1]^{\mathbb{N}}$ and also basically ethical fuzzy subsets of $[0,1]^{\mathbb{N}}$, it is remarkable that quasi-ethical fuzzy subsets of $[0,1]^{\mathbb{N}}$ cannot be basically ethical. This is due to Lemma 3.1 in combination with Theorem 3.1.

3.4 OTHER VARIATIONS AND APPROACHES TO THE CONCEPT

3.4.1 Equity in other forms

The 'ethical' concepts of fuzzy subsets of a (crisp) set of infinite utility streams in Definition 3.1 do not exhaust the possibilities in this regard. Other proposals can be imported from the extensive literature on ranking infinite utility streams. Suppose first that we are concerned with the spirit of anonymity as in subsection 3.3. Then authors like Kamaga and Kojima [9], Lauwers [10], Mitra and Basu [11], or Zuber and Asheim [14] among others have investigated the implications of stronger versions of our Anonymity property. And in order to attempt a positive alternative to the impossibilities that have arisen, other authors appeal to weakened versions of Anonymity. To name but a few proposals, the aforementioned Crespo et al. [8, Definition 4.1] or Asheim et al. [3]'s treatment of relative anonymity, strong anonymity or fixed-step relative anonymity. Sakai [12] provides a discussion of the controversial problem of selecting appropriate anonymity axioms in the context of aggregating infinite utility streams.

Besides these procedural properties there are other approaches to equity, and in relation with then one can define more variations of the general concept of an 'ethical' fuzzy subset of a domain of utility streams. The interested reader is addressed to the analysis of consequentialist equity properties like the Pigou-Dalton transfer principle or Hammond Equity in various forms. These are classical principles that originate in the analysis of allocations to a finite number of agents. As to egalitarian principles that are specifically designed for the analysis of infinite societies or societies with an infinite number of periods, the most relevant property may be Hammond Equity for the Future. As in the case of weakly ethical fuzzy subsets, its implications are very different depending on the structure of the set of feasible streams. This follows from a comparison between Alcantud and García-Sanz [1] and Banerjee [5].

3.4.2 Respect for specific rules

Besides the generic forms of equity or efficiency mentioned above, some authors have defended that certain incomplete criteria for comparing streams on the basis of their acceptability must be respected. This alternative approach can be adapted as the following explanatory example shows.

Definition 3.2. *Let μ be a fuzzy subset of a domain of infinite utility streams $\boldsymbol{X} \subseteq [0,1]^{\mathbb{N}}$ such that the degree of membership of $\boldsymbol{1} \in \boldsymbol{X}$ is* 1*, resp. of $\boldsymbol{0} \in \boldsymbol{X}$ is* 0*. We say that μ respects von Weizsäcker's criterion if $\mu(\boldsymbol{x}) > \mu(\boldsymbol{y})$ whenever there is $n_0 \in \mathbb{N}$ such that $\sum_{k=1}^{n} x_k > \sum_{k=1}^{n} y_k$ for every $n > n_0$.*

Definition 3.2 asks that when a stream **x** overtakes another stream **y**, the degree of membership of **x** must be higher than the degree of membership of **y**. It is simple to check that fuzzy subsets that verify anonymity and respect von Weizsäcker's criterion are ethical. Therefore in view of Theorem 3.1:

Corollary 3.1. *There do not exist anonymous fuzzy subsets of $\boldsymbol{X} = \{0,1\}^{\mathbb{N}}$ that respect von Weizsäcker's criterion.*

We omit the details of further developments in this direction. Let us just point out that Asheim et al. [3] present a new version of the overtaking criterion called generalized time-invariant overtaking. Other interesting rules that are arguably worth considering include various infinite extensions of the leximin rule (cf., e.g., Asheim and Tungodden [4], and Asheim et al. [3, Subsection 6.2]).

4 CONCLUSIONS

The concept of social welfare function (SWF) means an evaluation of the objects under consideration, that verifies certain properties making it efficient and egalitarian. We have concentrated on SWFs on domains of infinite-horizon utility distributions or streams. We have shown that the identification of SWFs with adequate lists of properties can be viewed as a problem in fuzzy set theory: the original problem is equivalent to the identification of fuzzy subsets of the set of feasible distributions with the corresponding list of properties. Results from the standard literature on SWFs can be imported to the new framework. Although here we have focused on the case of infinite utility streams, which is mathematically more challenging, the approach can be employed to study allocations to a finitely-lived –or finite– population instead.

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