The Typical Spectral Shape of an Economic Variable: A Visual Guide with 100 Examples

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Abstract

Granger (1966) describes how the spectral shape of an economic variable concentrates spectral mass at low frequencies, declining smoothly as frequency increases. Despite a discussion about how to assess robustness of his results, the empirical exercise focused on the evidence obtained from a handful of series. In this paper, I focus on a broad range of economic variables to investigate their spectral shape. Hence, through different examples taken from both actual and simulated series, I provide an intuition of the typical spectral shape of a wide range of economic variables and the impact of their typical treatments. After performing 100 different exercises, the results show that Granger's assertion holds more often than not. I also confirm that the basic shape holds for a number of transformations, time aggregations, series' anomalies, variables of the real economy, and also, but to a lesser extent, financial variables. Especially fuzzy cases are those that exhibit some degree of transition to a different regime, as are those estimated with a very short bandwidth.

JEL-Codes: A20, C02, C14, C18, E32.
Keywords: *Frequency domain*, *spectral analysis*, *nonparametric econometrics.*

Resumen

Granger (1966) describe cómo la forma espectral de una variable económica concentra masa espectral en bajas frecuencias, decayendo suavemente cuando aumenta la frecuencia. A pesar de presentar una discusión sobre el análisis de robustez de sus resultados, su ejercicio empírico se enfoca en la evidencia obtenida con un puñado de series. Este trabajo, por su parte, se enfoca en un amplio rango de variables económicas para investigar su forma espectral. Así, a través de diferentes ejemplos desarrollados con series efectivas y simuladas, se provee una intuición sobre la forma espectral típica de un amplio rango de variables económicas, así como el impacto de algunos tratamientos típicos en su forma espectral. Después de 100 ejercicios diferentes, los resultados muestran que la afirmación de Granger es válida para la mayoría de los casos. También se confirma que la forma básica se mantiene para una serie de transformaciones, agregaciones de tiempo, anomalías, variables reales, y también, aunque en menor medida, en variables financieras. Casos especialmente difusos son los que exhiben algún grado de transición a un régimen diferente, así como aquellos estimados con un ancho de banda estrecho.

Códigos JEL: A20, C02, C14, C18, E32.
Palabras clave: *Dominio de frecuencias, análisis espectral, econometría no paramétrica.*

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Resumen no técnico

En el influyente trabajo de Clive W.J. Granger, 1966, "The Typical Spectral Shape of an Economic Variable", *Econometrica* **34**(1): 150-161, se describe cómo la forma espectral de una variable económica, es decir, descompuesta a través de sus ciclos, concentra masa espectral en bajas frecuencias, decayendo suavemente a medida que aumenta la frecuencia. A pesar de presentar una interesante discusión sobre cómo es posible llevar a cabo un análisis de robustez de los resultados, el ejercicio empírico se enfoca en la evidencia obtenida con un puñado de series.

En este trabajo, el foco se extiende a un amplio rango de variables económicas para investigar su forma espectral. Así, a través de diferentes ejemplos desarrollados con series macroeconómicas efectivas de la economía estadounidense y otras series simuladas con características especiales, se pretende entregar una intuición sobre la forma espectral típica de una variable económica. También se analiza el impacto en la forma espectral de algunos tratamientos típicos dentro de la literatura de economía aplicada.

Después de 100 ejercicios diferentes, los resultados muestran que la afirmación de Granger es válida para la mayoría de los casos. También se confirma que la forma básica se mantiene para una serie de transformaciones típicamente utilizadas en investigaciones empíricas, agregaciones de las series a través del tiempo —cambios de frecuencia—, anomalías — como valores atípicos o cambios de base—, variables de la economía real, y también, aunque en menor medida, en variables financieras. Casos especialmente difusos son los que exhiben algún grado de transición a un régimen diferente—distinta media y varianza—, así como aquellos estimados con un ancho de banda angosto.

1 Introduction

Any economic time series can be analyzed from two points of view: *time domain* and *frequency domain.* The intuition behind frequency domain lies in the useful manner in which a variable can be plotted in terms of its cycles—measuring its strength in decibels—for any given frequency, without requiring new information. Thus, spectral analysis allows to analyzing the relationships between the frequencies with ease. This distinction is worthwhile because it allows estimating model parameters on different frequency bands. This implies that some set of parameters, estimated at a certain frequency band, casts for a better in-sample fit and/or forecast accuracy. Also, model estimation on frequency domain accounts better for dissimilar effects, say, rigidities or agents' habits. This paper is an attempt to map from time to frequency domain common cases and treatments typically applied in empirical economics.

Granger (1966) states that the spectral shape of an economic variable should concentrate spectral mass mostly at low frequencies, declining smoothly as frequency increases. The paper was an attempt to promote the use of *frequency domain* in economic time series analysis. It explains how the spectral shape of an economic variable measured in levels should look: *The long-term fluctuations in economic variables, if decomposed into frequency components, are such that the amplitude of the components decrease smoothly with decreasing period,* (Granger, 1966, p. 155).

Despite a discussion about how to assess the robustness of his results, the setting is mainly focused on the level of sporadic evidence obtained from a few series. As the purpose was to illustrate a typical economic variable, the result holds in a broad range of cases. In this paper, I focus on a wide range of economic variables, plus several typical treatments used in applied economics. This range covers real activity variables, interest rates, and soft indicators, all of the US economy, along with simulated series with key macroeconomical/statistical features. The treatments include sensitivity to sample span, bandwidth selection, transformations and time aggregations, among others. Hence, through different examples taken from both actual and simulated series, I will be able to provide an intuition of the typical spectral shape of a large set of economic variables with different statistical properties.

Few systematic attempts have been carried out in the literature on a comprehensive manner. See, for instance, Cunnyngham (1963), Hatanaka (1963), Granger and Morgenstern (1963), Granger and Hatanaka (1964), and Nerlove (1964).[1] Priestley (1981) develops a comprehensive analysis including issues regarding estimation in the frequency domain. As was stated before, the frequency analysis is important because spectral methods are useful to uncover key characteristics of economic time series, with relevant implications for model building (Granger, 1966). A recent example of this is the estimation on the frequency domain of a medium-scale dynamic stochastic general equilibrium (DSGE) model presented in Sala (2013). Remarkably, the author found that estimating under different frequency bands deliver significantly different parameters. This is an indication that a model built with variables with equally-weighted frequencies—as the time domain supposes—are unable to match all frequencies with one set of parameters.

The graph of frequencies versus decibel is called the *periodogram*—a special case of the spectrum. In a spectral graph, the low frequencies at the left correspond to slowly changing components—like a trend—while higher frequencies correspond to rapidly changing components—like a white noise variable. Peaks in the spectral plot at certain frequencies of actual data indicate the presence of regular patterns within the sample. For a covariance-stationary process $\{y_t\}_{t=t_0}^{t=T}$, the *sample periodogram* for frequency ω is defined as:

$$\widehat{s}(\omega) = \frac{1}{2\pi} \sum_{j=-T+1}^{T-1} \widehat{\gamma}_j \cdot e^{-i\omega j}, \tag{1}$$

where $\widehat{\gamma}_j = \frac{1}{T}\sum_t (y_t - \overline{y})(y_{t-j} - \overline{y})$ are sample autocovariances of order j of $\{y_t\}$, estimated until order m, $j = \{1, ..., m\}$, with $\widehat{\gamma}_{-j} = \widehat{\gamma}_j$ as $\{y_t\}$ is a covariance-stationary process. Note that i stands for $\sqrt{-1}$. An order m chosen with information criteria, such as the Akaike Information Criterion (AIC), tends to

[1] For a comprehensive use of the spectrum for economic time series analysis, see Granger and Watson (1984) and Hamilton (1994).

generate smoother spectral amplitudes as it exaggerates the dynamic of a series. Common strategies to select the order m include Bartlett, Tukey, and Parzen's window selection criteria. Since the parameters $\widehat{\gamma}_j$ are variance-corrected covariances between y_t and y_{t-j}, the informational content of the spectrum with respect to the time domain remains fixed (Hamilton, 1994).

Note that using De Moivre's theorem, the term $e^{-i\omega j}$ is equal to $\cos(\omega j) - i \cdot \sin(\omega j)$, and using the trigonometrical identities $\cos(0) \equiv 1$, $\sin(0) \equiv 0$, $\sin(-\theta) \equiv -\sin(\theta)$, and $\cos(-\theta) \equiv \cos(\theta)$, the expression can be written in terms of the cosine function.[2] Since $\cos(\omega j) = \cos[(l + 2\pi k)j]$ for any integers k and j, the spectrum is a periodic function of ω. Hence, it is necessary to know the values of $\widehat{s}(\omega)$ between 0 and π to infer the periodogram value for any ω.

Thirty-seven years after Granger's paper, Levy and Dezhbakhsh (2003) confirmed the shape of spectra by analyzing the Gross Domestic Product (GDP) of 58 countries with annual data. As the authors focused on the same variable—for different countries—it is not surprising that similar spectral shapes were found. In this line, the objective of this paper is twofold: first, to confirm if the Granger results hold for a variety of economic time series—and different to that used by Levy and Dezhbakhsh—such as interest rates or other financial variables. Secondly, to document a general visual context of spectral shapes by stressing its capabilities to provide intuition of its sensitiveness in different macroeconomical/statistical scenarios.

After performing 100 different exercises, the results show that Granger's assertion holds for the majority of the cases. This implies that *the same basic shape is found regardless of the length of data available, the size of the truncation point used in the estimations procedure, or the trend removal method used.* (Granger, 1966, p. 154). Besides Granger's statement I also confirm, by means of empirical exercises, that the basic shape holds for a number of transformations, time aggregations, series' anomalies such as outliers, variables of the real economy, and also, but to a lesser extent, financial variables. Especially fuzzy cases are those that exhibit some degree of transition to a different regime, as is the case of some financial variables of the US economy, and corroborated with simulated data. The use of a very short truncation point to estimate the spectrum (*i.e.* bandwidth) also deliver distorted spectral shapes, and therefore should be analyzed comprehensively.

Hence, the intuition of the spectral shape relies on the decomposition of a time series in terms of orthogonal components, each one associated to a specific frequency that contributes to the total variance of the series. This implies that spectral mass concentrated in a specific frequency—*e.g.* peaks—indicates that those movements dominate the dynamic of the series. As the typical spectral shape for an economic variable concentrates spectral mass mostly at lower frequencies, it implies that the long-run dynamics are those that govern the series' movements.

Alongside theoretical findings, the empirical exercise carried out in this investigation confirms that the finding of spectral shape is robust to a number of situations. For instance, a series with outliers, ramp, level shift, or other anomalies, does not interfere in the spectral shape. The sample span plays no role at spectra given that the series considered are covariance-stationary. Furthermore, the shape holds even if the trend is removed since it consists of an unbiased variance reduction. When the mean of the series relative to the standard deviation is large, several transformations—especially those reducing variance, such as logarithms—keep the spectral shape unaltered. Finally, more persistent series are closer to the typical spectral shape than those less persistent because they have a longer memory. This should be also the case with fractionally integrated series with an integration coefficient close to zero (Granger and Joyeux, 1980).

The rest of the paper proceeds as follows. Section 2 reviews general topics concerning spectral plots, based on an idealized example for illustrative purposes. Next, section 3 details the setup of the different exercises to be performed. Then, section 4 briefly discusses particular interesting results and concludes.

[2] Hence, becoming:

$$\widehat{s}(\omega) = \frac{1}{2\pi}\left[\widehat{\gamma}_0 + 2\sum_{j=1}^{T-1}\widehat{\gamma}_j \cdot \cos(\omega j)\right].$$

2 The frequency domain: a typical example

For illustrative purposes, I make use of a highly seasonal simulated series, one of the most common applications of the spectrum in economics. As Granger (1979) pointed out, the decomposition of a series into a trend plus a seasonal and a remaining irregular component, is relevant because seasonality explains the majority of the variance of a series, while economically insignificant.[3]

Note that a series has seasonality if its spectrum exhibits a peak at frequencies of $\frac{1}{2}$ or $\frac{1}{4}$ —typically measured in radians. As the spectrum is typically used to test the presence of seasonality and then the quality of the adjustment, it is estimated for original and final seasonally adjusted series. The key issue is to keep in mind the goal of the absence of *residual seasonality*; this is, absence of seasonality in series that theoretically should not have it. According to the well-known program X-12-ARIMA (Findley *et al.*, 1998), a series is called seasonal if it shows a peak in the original series spectrum at seasonal frequencies. A peak is called significant if it is above the median of $\widehat{s}(\omega_k)$ values (where ω_k stands for the frequency $\omega_k = \frac{k}{120}$, $0 \leq k \leq 60$), and must be larger than its neighboring (not including $\omega_{60} = \frac{1}{2}$) values $\widehat{s}(\omega_{k-1})$ and $\widehat{s}(\omega_{k+1})$ by at least $\frac{6}{52}$ times the range $\widehat{s}^{\max} - \widehat{s}^{\min}$, where $\widehat{s}^{\max} = \max_k \widehat{s}(\omega_k)$ and $\widehat{s}^{\min} = \min_k \widehat{s}(\omega_k)$. For this reason, X-12-ARIMA plots spectra with 52 frequencies. So, the unit of measure is standardized to "stars"—equivalent to $\frac{1}{52}$ unit of frequency—so a peak (six or more stars) is easy to detect visually.

As spectral analysis allows seeing the relationships between the frequencies, it is easy to quantify the importance of certain frequencies relative to the frequencies of other components. Thus, for a comparison between two or more adjustments for the same variable, the result is direct. A lower insignificant peak—or even better, the absence of it—in the seasonally adjusted series at specific frequencies reflects an adjustment of better quality. Always keep in mind that the smoothness pursued by seasonal adjustment is in spectral plots of seasonally adjusted series rather than in a filtered version of the original series.

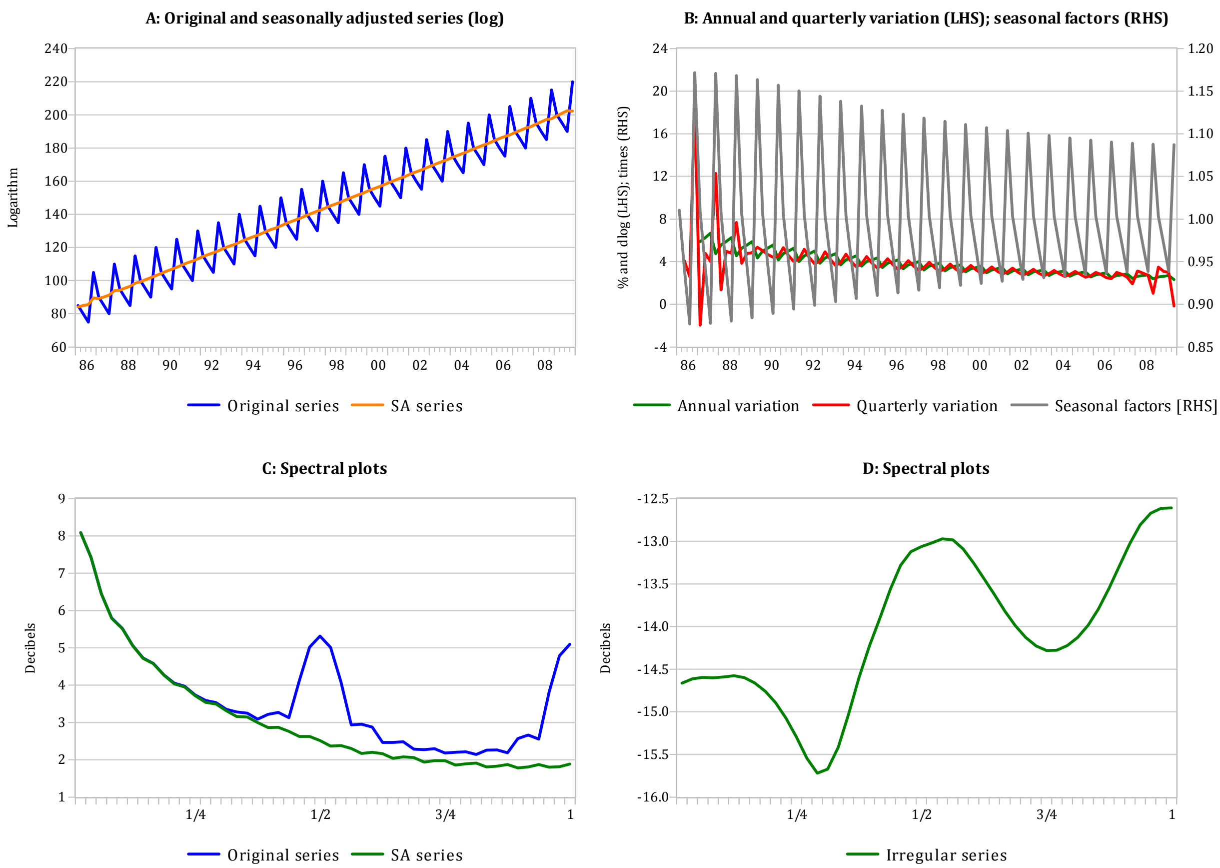
In figure 1, I plot the illustrative series. Some comments about the example: First, the illustrator series is built in "intrayear" cycles of 4 observations. Thus, it mimics a quarterly series. Second, it is composed of a (nonstochastic) trend divided by seasonal factors across "years" without outliers. These factors fluctuate by a little rate across years, leaving room for an irregular component. Third, by construction, the irregular component has a small variance relative to the variance of the original series, making the identification process easier. Fourth, the series is an index with a base year (1989) equal to 100, spanning from 1986 to 2009 (96 observations).

In this idealized example, the presence of seasonality is relatively obvious. But, in reality, economic time series are hit by a large variety of shocks and other perturbations that obscure the identification of seasonality. In these scenarios, spectral plots emerge as a powerful model-free tool for identification. Panel A shows the logarithm of the original and seasonally adjusted series. Notice that the adjusted series should not exactly coincide with the original trend because of the presence of the irregular component. Panel B shows the annual and quarterly variation of previous series. The repeating pattern across the years of the quarterly variation of the seasonally adjusted series reflects the presence of seasonal factors. The seasonal factors are also depicted in panel B. Note that as they shrink as sample increases, the absolute value of quarterly variation decreases.

Panels C and D depict three different spectral plots. The first two are those of the original and seasonally adjusted series, while the third is the irregular component. Notice in panel C the effect on spectrum caused by removing seasonality: a complete removal of peaks at seasonal frequencies. The resulting spectrum illustrates *the typical spectral shape of an economic variable.* Panel D shows the erratic cyclical behavior of the irregular series, which reflects a successful seasonal adjustment.

[3] The decomposition of a time series (y_t) to be seasonally adjusted includes a trend-cycle component (y_t^{τ}) plus (or times, depending on the kind of seasonality) a seasonally adjusted component (y_t^{sa}), plus (or times) a residual irregular component (y_t^{ir}; then $y_t = y_t^{\tau} + y_t^{sa} + y_t^{ir}$, or $y_t = y_t^{\tau} \times y_t^{sa} \times y_t^{ir}$). Hence, the two latter components, y_t^{sa} and y_t^{ir}, should not exhibit a cyclical behavior. Seasonal factors correspond to $y_t - y_t^{sa} = y_t^{\tau} + y_t^{ir}$ (or $\frac{y_t}{y_t^{sa}} = y_t^{\tau} \times y_t^{ir}$).

Figure 1: An example with a highly seasonal series



Source: Author's elaboration.

3 Exercises

The exercises considered are divided into eight categories, representing typical situations confronted when doing empirical economics. The aggregate series described within the *Basic estimations* category should be also considered as a sum of disaggregates; a common situation with real-economy variables, such as GDP. This category represents the most frequent model-free cases of economic variables, thus providing a valid insight prior to modeling. In this same line, several decisions have to be made to this end concerning *sample length*, *bandwidth* or *window lag selection*, *data transformation*, and *frequency setting*. Hence, all these issues constitute the four subsequent categories. Finally, effective macroeconomic variables of the US economy along with series pertaining to traditional ARMA models are also studied.

The preferred default bandwidth used is $m = 30$, following the suggestion given in Hood (2007). The elements of each category are the following (the number of plotted spectra is presented in [ ]):

1. *Basic estimations* [16]. This category includes a constant-slope trend series, a constant-slope trend line plus an outlier, a constant-slope trend plus two outliers in opposite directions, a series with level shift, a series with a ramp, a series with a stochastic trend, a series with multiplicative seasonality, a series with additive seasonality, a series with additive seasonality plus two opposite outliers, a deterministic cubic trend, a white noise, a set of independent $\mathcal{N}(0, 5)$ realizations, a series with three regimes (suggesting a successful gradual stabilization policy), a three-regime series with one outlier, an v-shaped line, and a w-shaped line. Finally, all the series have a length of 100 observations.

2. *Sensitivity to sample span* [16]. A common shortcoming to deal with in empirical economics is sample span. Often, methodological advances in time series econometrics lie in asymptotic properties for a certain framework. Nevertheless, few economic variables have been collected for a lengthy period of time. Several drawbacks may emerge as a consequence of parameter uncertainty, due to a short sample span. To tackle the impact of sample span on spectral shape, the following exercises are performed. A stochastic trend with $N = \{5000;1000;500;200;100\}$ observations, the monthly Industrial Production Index of the US (IPI, source: *FRED*) with $N = \{1137;500;200;100\}$, the monthly Fed Funds Rate (FFR, source: *FRED*) with $N = \{3096;2000;1000;200;50\}$, and the Box, Jenkins, and Reinsel's (1994) "*Series G. International Airline Passengers: Monthly Totals, 1949—1960*" (BJR-G) with $N = \{144;52\}$, being $n = 144$ its original length. In the cases of IPI an FFR, the span is shortened by dropping earlier observations.

3. *Bandwidth selection* [6]. Besides sample span, there are several other reasons why the spectral shape may differ for the same variable. One of these options, along with data transformations, is bandwidth selection. This relates to the selection/estimation of the integer m in equation (1). As was mentioned, there are several procedures to estimate it efficiently as with Bartlett, Tukey, and Parzen's window selection criteria.[4] However, the three criteria tend to deliver same quantitative results. The exercises analyzed in this paper considers different bandwidths for a given series and its sample span. These series are a stochastic trend with N =5000, the IPI with N =1137, the FFR with $N = \{3096;50\}$, and BJR-G with $N = \{144;52\}$. Thus, spectra are depicted using a bandwidth ranging from 141, 50, 30, 26, 30, 13, 10 to 2 lags.

4. *Sensitiveness to typical transformations* [14]. Major methodological issues in econometrics concern stationary series. Many economic variables, however, are not stationary. Thus, a transformation is required in order to meet methodological assumptions. The exercises considered are the following. Beginning from a nonstationary simulated stochastic trend series: logarithmic transformation, detrended series, and differenced series. Beginning from the US Consumer Price Index (CPI, source: *FRED*) denominated in levels (index 1982-84=100): logarithmic transformation, annual variation, first difference of logarithmic seasonally adjusted series (monthly variation; adjusted with X-12-ARIMA, whole sample), and the accumulated change in 3, 6, 9 and 24 months. Actual macroeconomic time series are used on their default frequency and sample availability.

5. *Time aggregation* [14]. It is a matter of fact that a set of series used for modeling does not necessarily match their original frequencies. Thus, a frequency (dis)aggregation is needed. There are basically two kinds of time aggregations: one for stock series and other for flow series. In the first case, the last value of the higher frequency matches those values of lower frequency. In the second case, averaging (or adding up to) higher frequency values cast for the lower frequency value. To illustrate the effect of these aggregations on their spectral shape, three exercises are performed. First, an aggregation of the inflation rate starting from its original monthly frequency, passing by quarterly to lastly, annual frequency (representing an annual rate time averaging). Second, the same treatment is made with the IPI, to illustrate the averaging of an index series. Finally, FFR (considered as stock) is transformed by taking its last value from weekly to monthly, quarterly, semi-annual and annual frequency. In the two first cases, the annual variation of their aggregates is also computed to illustrate the different behaviors.

6. *Macroeconomic variables 1: Real activity series* [16]. This category includes actual macroeconomic variables from the US, all of them related to real activity (source: *FRED*). The series are used in their default sample span, denomination, and frequency. All series are monthly except "Corporate Profits After Tax (without IVA and CCAdj)" that is released on a quarterly basis. The series are the following: Industrial Production Index, Capacity Utilization: Total Industry, Total Business Inventories, ISM Manufacturing: PMI Composite Index, Real Retail and Food Services Sales, Light Weight Vehicle Sales: Autos & Light Trucks, Manufacturers' New Orders: Nondefense Capital Goods Excluding Aircraft, Manufacturers' New Orders: Durable Goods, Commercial and Industrial Loans at All Commercial Banks, Total Consumer Credit Owned and Securitized, Outstanding, Corporate Profits After Tax

[4] See Bartlett (1955), Parzen (1961) and Tukey (1961) for details.

(without IVA and CCAdj), Housing Starts: Total: New Privately Owned Housing Units Started, New Private Housing Units Authorized by Building Permits, New Privately-Owned Housing Units Under Construction: Total, S&P Case-Shiller 20-City Home Price Index, and Civilian Unemployment Rate.

7. *Macroeconomic variables 2: Prices, financial, banking variables, and soft indicators* [13]. This category includes the following variables of the US economy (source: *FRED*): St. Louis Adjusted Monetary Base, Reserve Balances with Federal Reserve Banks, M1 Money Stock, S&P 500 Stock Price Index, Dow Jones Industrial Average, CBOE Volatility Index: VIX, St. Louis Fed Financial Stress Index, 3-Month Treasury Constant Maturity Rate, 1-Year Treasury Constant Maturity Rate, 10-Year Treasury Constant Maturity Rate, and Moody's Seasoned Aaa Corporate Bond Yield. Also included are (source: *Survey Research Center*, *University of Michigan*): Index of Consumer Sentiments (database symbol: `ics_all`) and Current Economic Conditions Index (`icc_all`). The series are used in their default frequency and denomination as well as sample availability.

8. *Univariate autoregression processes of finite order* [5]. This category analyzes several cases where the data generating process for a series $\{y_t\}_{t=1}^{t=T}$ corresponds to the $AR(p)$ model:

$$y_t = \alpha + \sum_{i=1}^{p} \rho_i y_{t-i} + \varepsilon_t,$$

   where $\{\alpha, \rho_1, ..., \rho_p\}$ are parameters to be estimated—fixed for the simulation exercises—and ε_t is a white noise $iid\mathcal{N}(0, \sigma_\varepsilon^2)$. The spectral plot of the original series—that of true model—is presented in this category along with the residuals of an $AR(p)$ model estimated with the true order p—neglecting model uncertainty—subject to parameter uncertainty. The following cases, with nonskipped terms from 1 to p, have been considered:

   (a) $p = 1$, $\alpha = 1$, $\sigma_\varepsilon^2 = 1$, $T = 5000$, and $\rho = 0.99$,
   (b) $p = 6$, $\alpha = 1$, $\sigma_\varepsilon^2 = 1$, $T = 5000$, $\sum_i^p \rho_i = 0.99$, with $\rho_i = \rho_{i-1} + 0.05$, $\rho_0 = 0.04$,
   (c) $p = 12$, $\alpha = 1$, $\sigma_\varepsilon^2 = 1$, $T = 5000$, $\sum_i^p \rho_i = 0.99$, with $\rho_i = \rho_{i-1} + 0.0025$, $\rho_0 = 0.0689$,
   (d) $p = 1$, $\alpha = 1$, $\sigma_\varepsilon^2 = 1$, $T = 5000$, and $\rho = 0.50$,
   (e) $p = 1$, $\alpha = 1$, $\sigma_\varepsilon^2 = 1$, $T = 50$, and $\rho = 0.99$.

4 Results and concluding remarks

The results for the eight categories are presented in figures 2 to 9. Original series are depicted in blue under the title *time domain*, while spectral plots in green under the title *frequency domain*. Spectra are estimated and plotted across the integer k, $0 \leq k \leq \frac{N}{2}$, corresponding to the frequency $\omega_k = \frac{k}{N}$. Information about spectral bandwidth and the sample length of original series is always provided. In cases with actual—nonsimulated—variables, the sample span is also reported.

The intuition behind the spectral shape relies on the decomposition of a time series in terms of orthogonal components, each one associated to a specific frequency that contributes to the total variance of the series. This implies that spectral mass concentrated in a specific frequency indicates that those movements dominate the dynamic of the series. As the typical spectral shape for an economic variable concentrates spectral mass mostly at lower frequencies, it implies that the long-run dynamics are those that govern series' movements.

As a conclusion, effects suchs as outliers, ramp, level shift, or other anomalies, do not interfere in the spectral shape. The sample span plays no role at spectra because the series considered are covariance-stationary. Furthermore, the shape holds even if the trend is removed since it consists in an unbiased variance reduction. Since the mean of the series relative to the standard deviation is large, several transformations kept the spectral shape unaltered. Finally, more persistent series are closer to the typical spectral shape rather than those less persistent because they have a longer memory. This should be also the case with series fractionally integrated with an integration coefficient close to zero.

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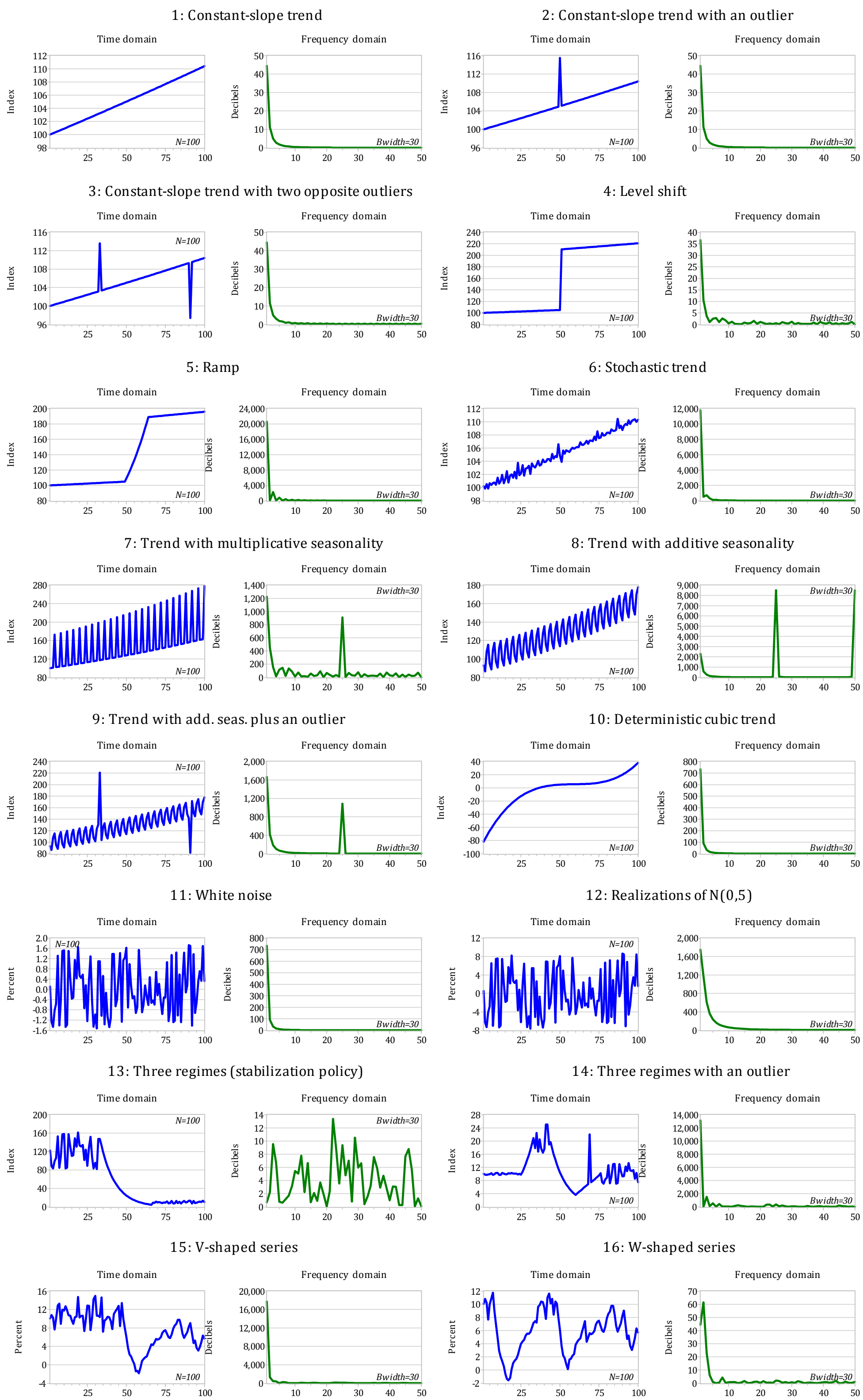
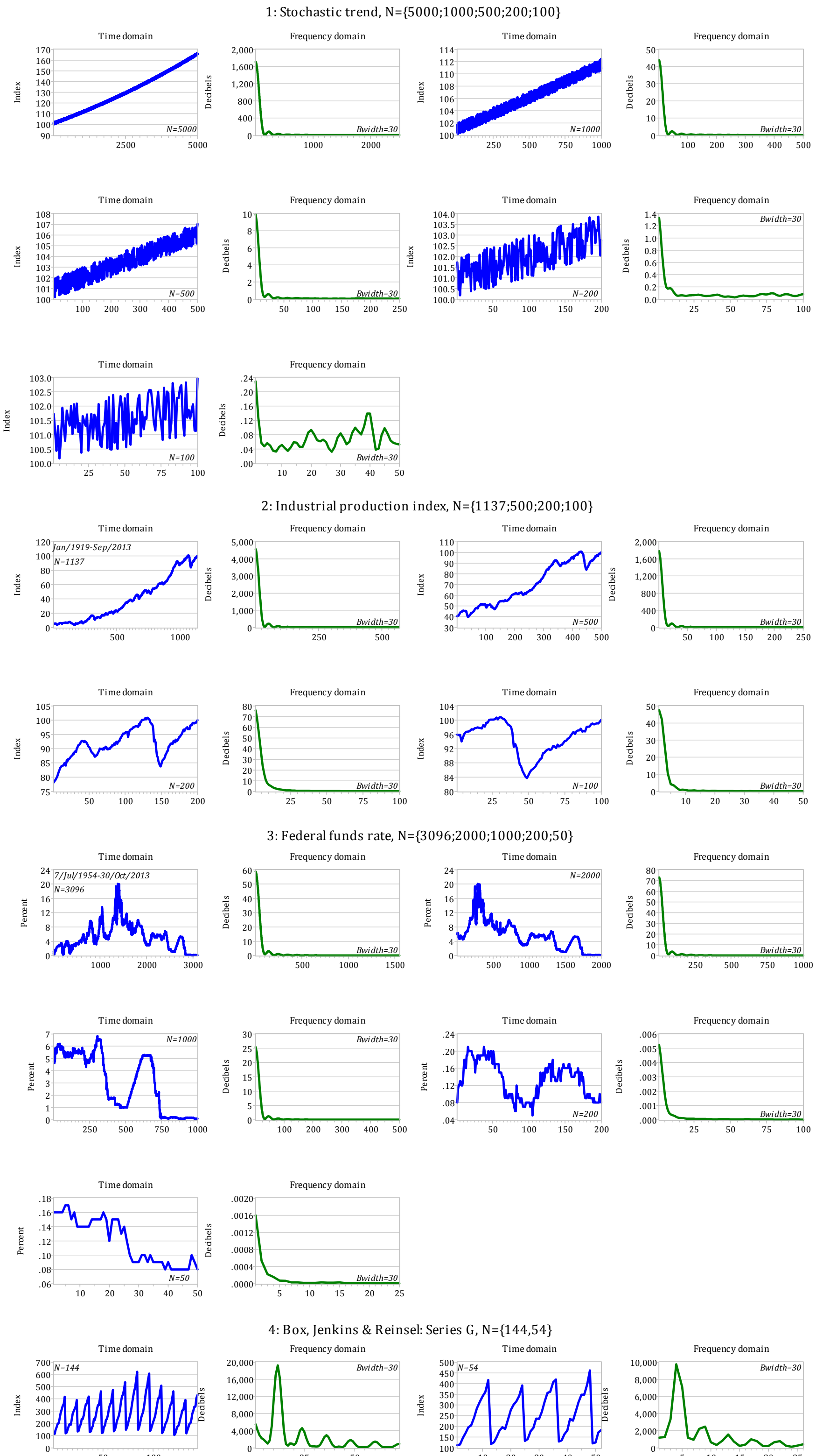
Figure 2: Basic estimations



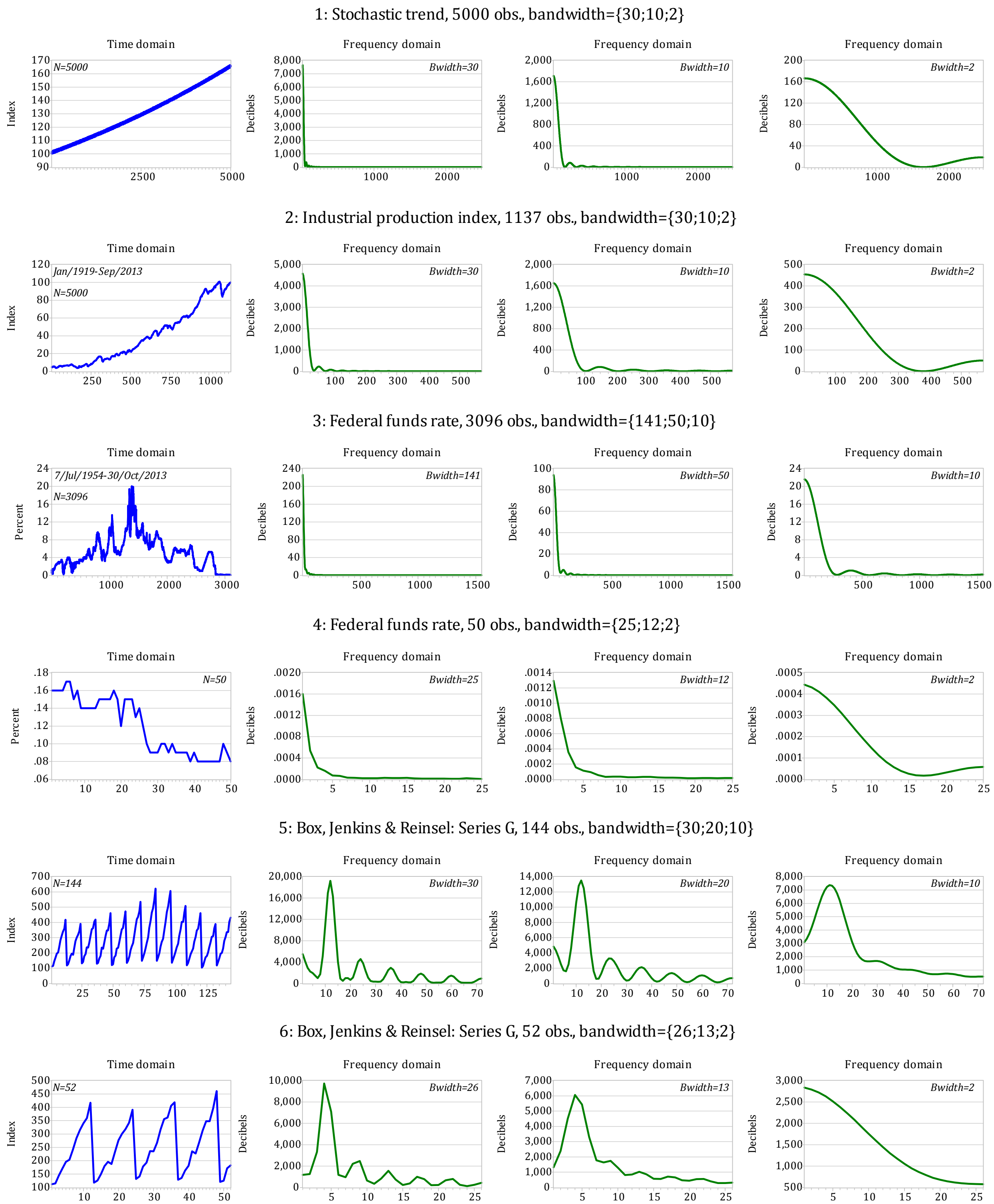
Source: Author's elaboration.

Figure 3: Sensitivity to sample span



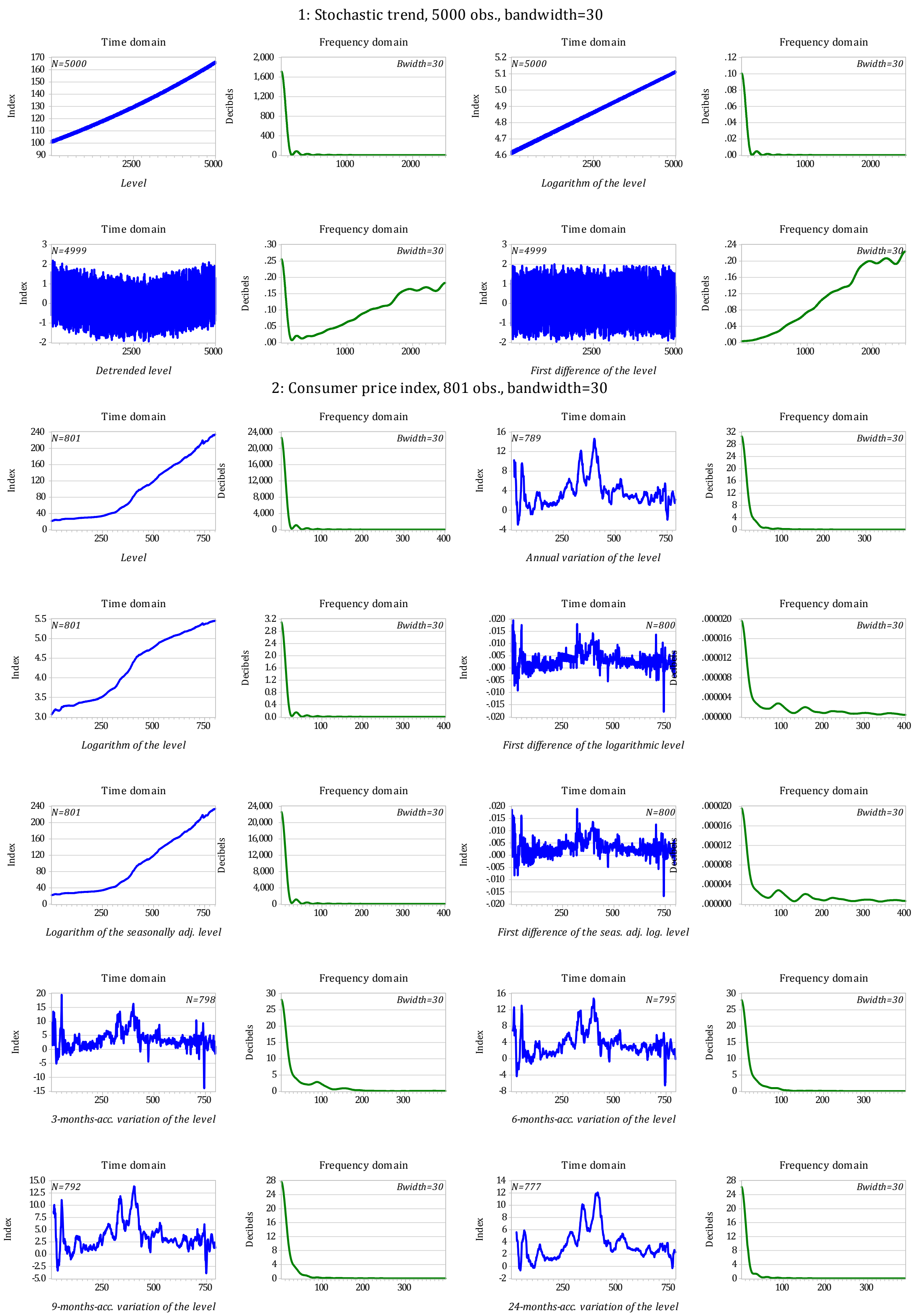
Source: Author's elaboration.

Figure 4: Bandwidth selection



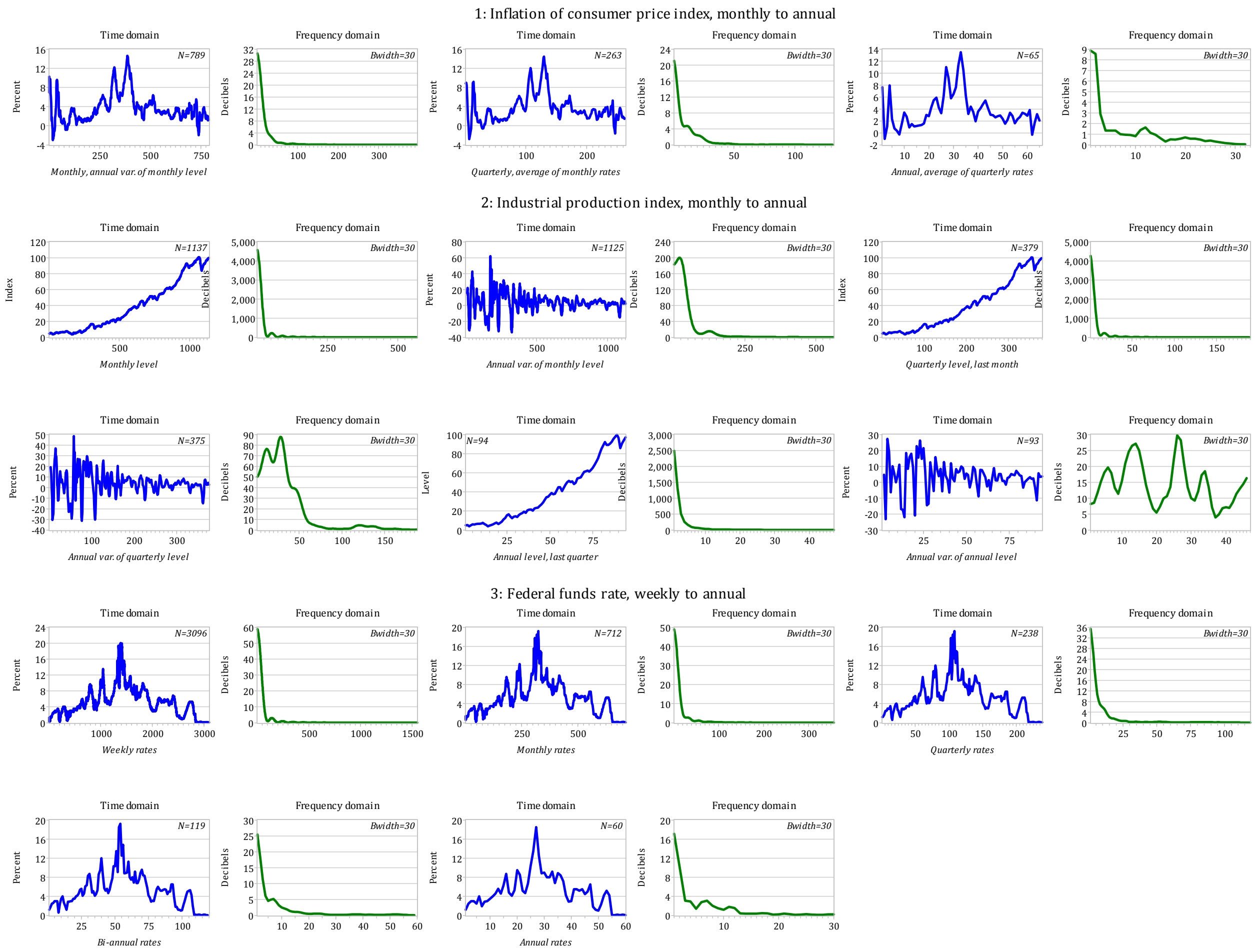
Source: Author's elaboration.

Figure 5: Sensitiveness to typical transformations



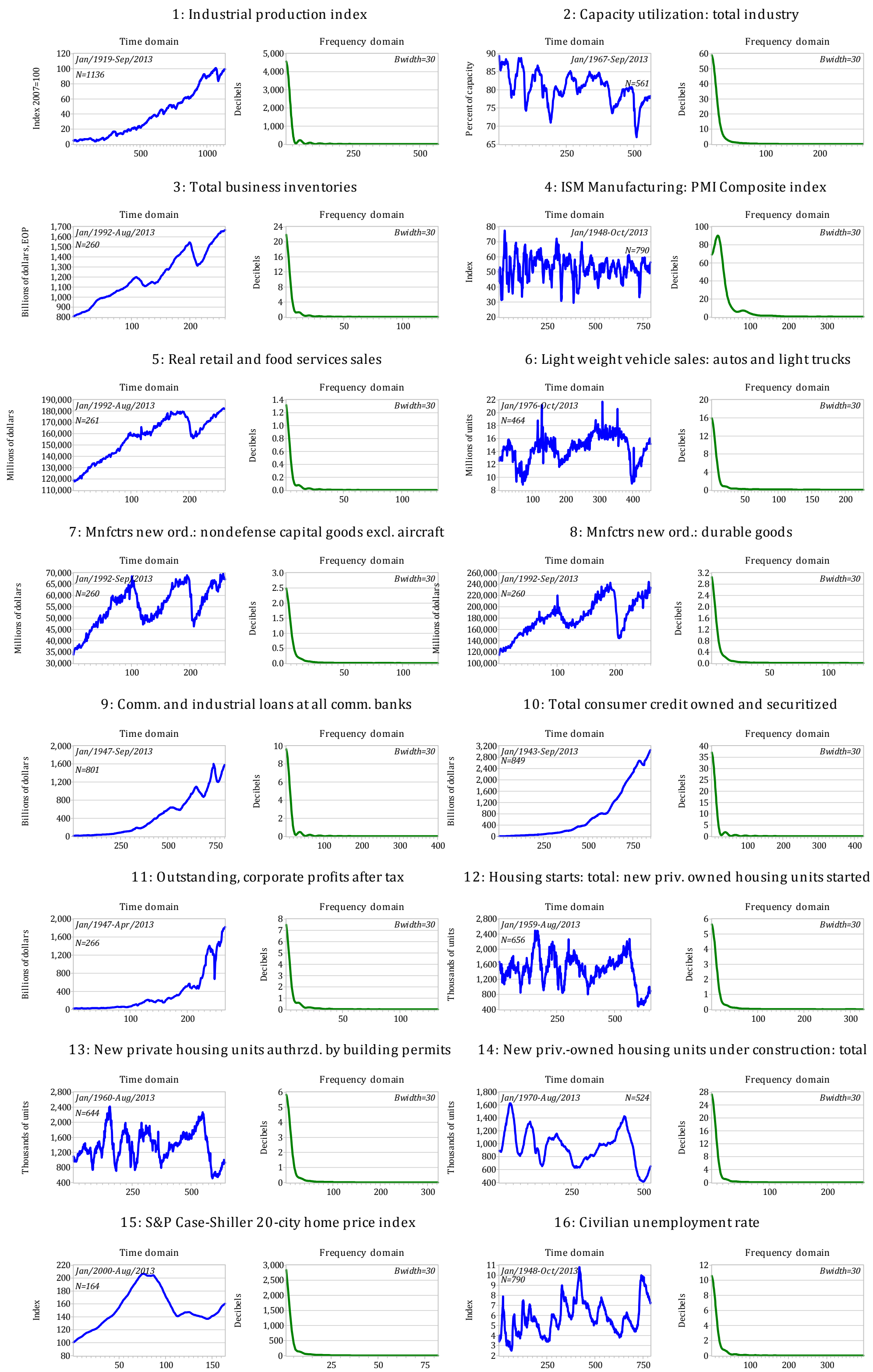
Source: Author's elaboration.

Figure 6: Time aggregation



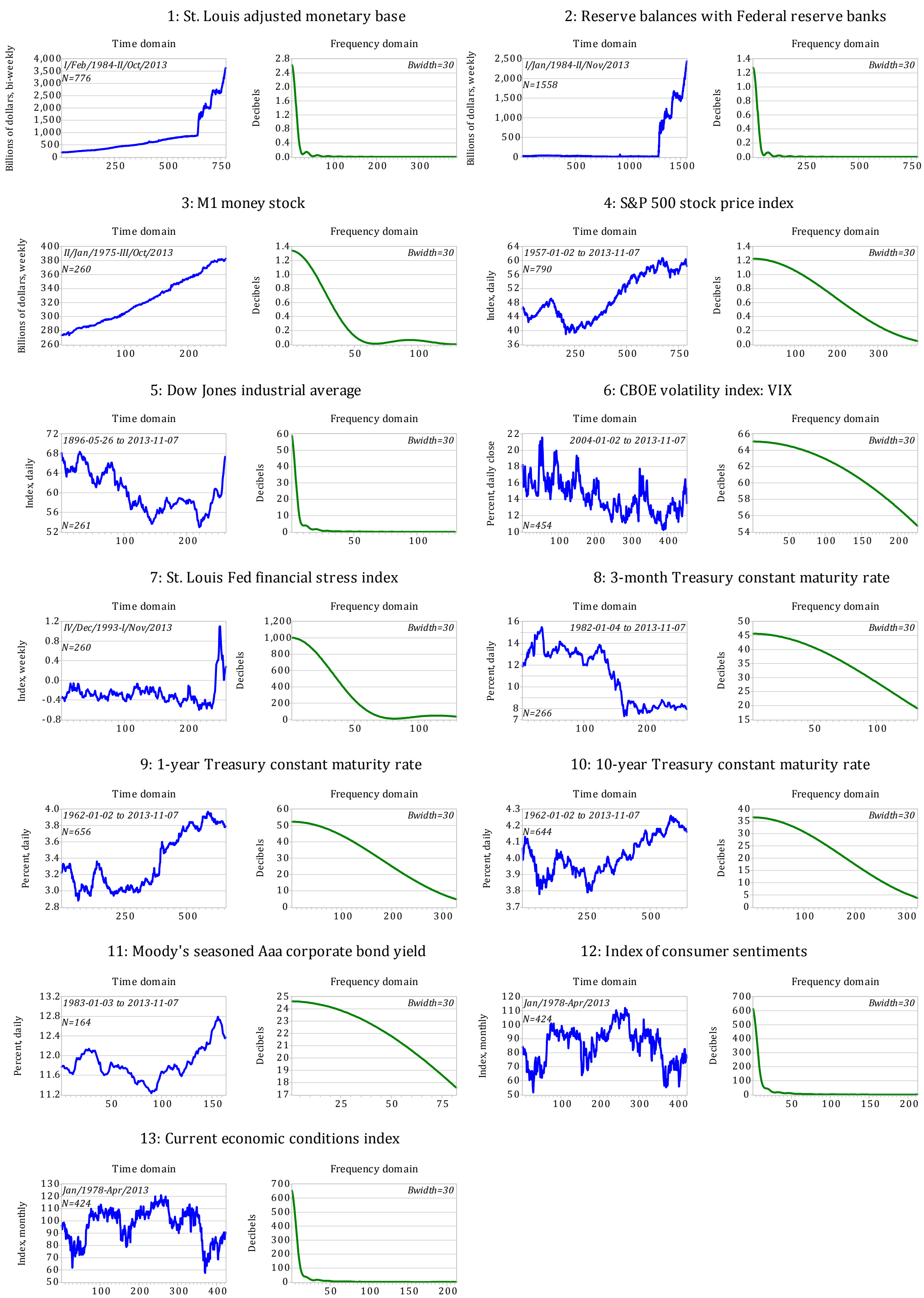
Source: Author's elaboration.

Figure 7: Macroeconomic variables 1: Real activity series



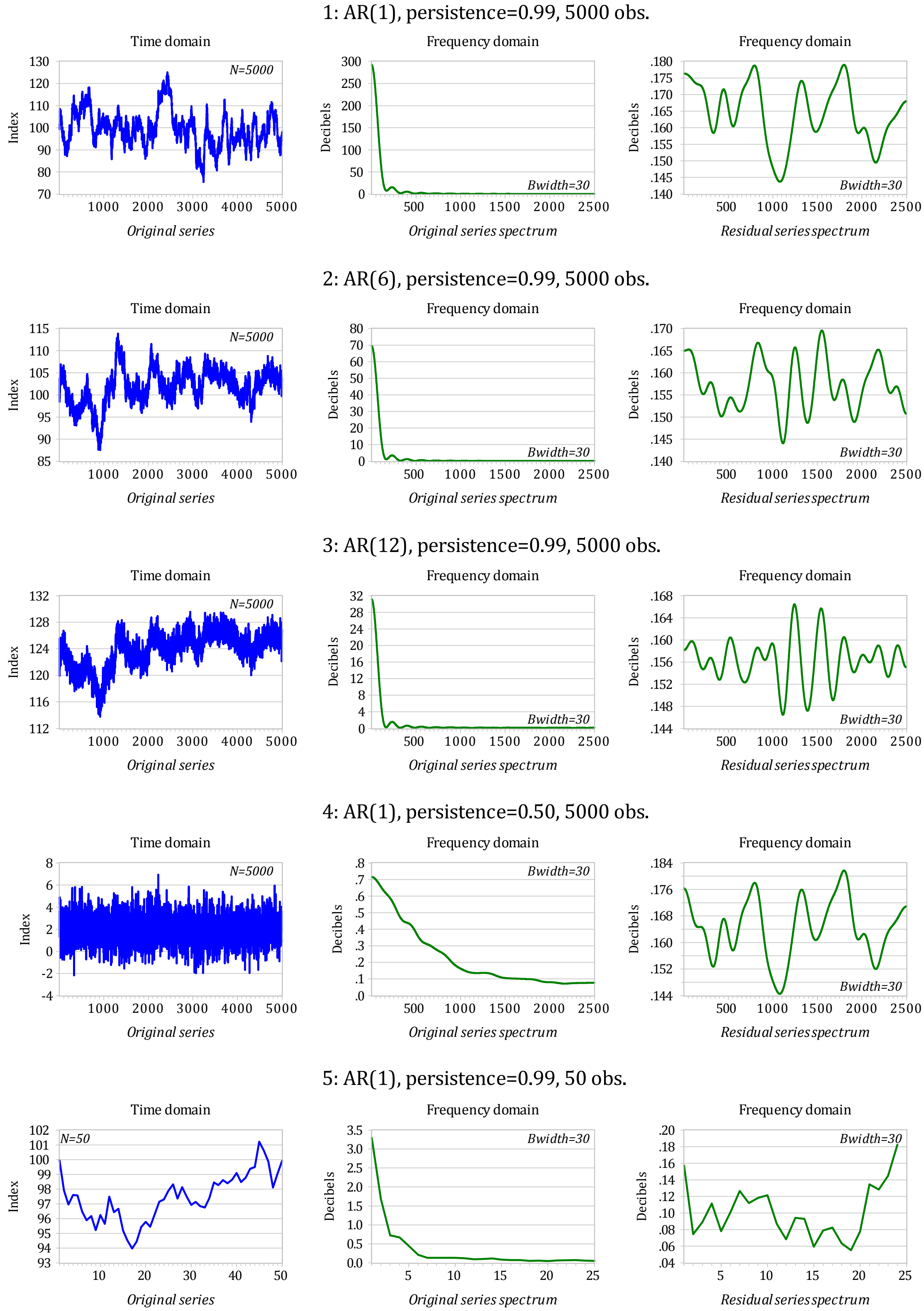
Source: Author's elaboration.

Figure 8: Macroeconomic variables 2: Prices, financial, banking variables, and soft indicators



Source: Author's elaboration.

Figure 9: Univariate autoregression processes of finite order



Source: Author's elaboration.