



Munich Personal RePEc Archive

**Is Agricultural Productivity Growth  
Good for Industrialization?  
Infrastructures and the Welfare  
Maximizing Tax Rate**

Kamei, Keita and Sasaki, Hiroaki

11 February 2014

Online at <https://mpra.ub.uni-muenchen.de/53606/>  
MPRA Paper No. 53606, posted 12 Feb 2014 14:55 UTC

# Is Agricultural Productivity Growth Good for Industrialization? Infrastructures and the Welfare Maximizing Tax Rate

Keita Kamei\*

Hiroaki Sasaki†

## Abstract

This paper develops a dynamic Ricardian trade model that incorporates productive infrastructures into the manufacturing sector financed by tax. We investigate the relationship between the timing of opening trade and total welfare. We also compare the two kinds of total welfare: the total welfare that a country obtains by closing international trade until it has a comparative advantage in manufacturing and then engaging in free trade and the total welfare that the country obtains by specializing in agriculture according to the law of comparative advantage from the beginning. The main results are as follows: (1) there is the optimal tax rate maximizing the total welfare; (2) an increase in agricultural productivity can accelerate the timing of opening trade, which, however, does not necessarily improve the total welfare; and (3) the total welfare under specialization in manufacturing can be higher than that under specialization in agriculture depending on conditions.

*Keywords:* productive infrastructure; industrialization; timing of opening trade; agricultural productivity

*JEL Classification:* F43; F10; O14

---

\*Research Fellow of the Japan Society for the Promotion of Science (JSPS). Graduate School of Economics, Kyoto University. Yoshida Honmachi, Sakyo-Ku, Kyoto 606-8501, Japan. E-mail: keita.kamei@gmail.com

†Graduate School of Economics, Kyoto University. Yoshida-Honmachi, Sakyo-ku, Kyoto 606-8501, Japan. E-mail: sasaki@econ.kyoto-u.ac.jp

# 1 Introduction

The importance of agricultural productivity growth for industrialization has long been recognized by economists. For example, Nurkse (1953) stated that “[e]veryone knows that the spectacular industrial revolution would not have been possible without the agricultural revolution that preceded it.” Rostow (1960) argued that “revolutionary changes in agricultural productivity are an essential condition for successful take-off.”

There are many theoretical studies on this issue. In particular, Matsuyama (1992) is a celebrated study that examines how an increase in agricultural productivity affects industrialization by using a two-sector growth model.<sup>1)</sup> He shows that an increase in agricultural productivity does not lead to industrialization in a small open economy because it promotes a comparative advantage in agriculture at the expense of manufacturing. In addition, he shows that if a developing country under free trade cannot industrialize, then the country begins to specialize in agriculture, and consequently, has a lower growth rate. He uses the Stone-Geary utility function of non-homothetic preferences, which implies that the income elasticity of demand for agricultural goods is less than unity. The engine of growth of the Matsuyama model is driven by learning by doing in the manufacturing sector.

Chang et al. (2006) investigate the relationship between public provision of infrastructure based on Barro (1990) and industrialization. They introduce the provision of infrastructure into Matsuyama’s (1992) model, and show that an increase in the public provision of infrastructure promotes a comparative advantage in manufacturing and an increase in agricultural productivity can promote industrialization.

Despite these attractive results, Chang et al. (2006) have the following two problems. First, their model assumes that infrastructures are automatically produced by tax revenues: no production factor is used for production of infrastructures. Hence, they do not consider the trade-off of labor between sectors: an increase in workers in the public sector decreases workers in the agricultural and manufacturing sectors. Second, they focus their attention on the analysis of the GDP growth rate. Therefore, they do not explicitly consider the relationship between the tax rate and welfare.

In this paper, we extend Chang et al. (2006) to a model that the government employs workers in the public sector and their wages are financed by the tax revenue. In addition, our model derives the total welfare and investigates the relationship between tax rates and the total welfare.

---

1) Lewis (1954) is the forerunner of two-sector models of industrialization in developing countries. For extensions of the Lewis model, see also Kirkpatrick and Barrientos (2004), Temple (2005), and Wang and Piesse (2013). Wong and Yip (1999) incorporate capital accumulation into the two-sector trade model of Matsuyama (1992).

Moreover, our model considers the concept of dynamic comparative advantage proposed by Redding (1999). In his model, it is assumed that the home country does not initially have a comparative advantage in the manufacturing sector. Therefore, if the home country engages in free trade at the initial time, then it specializes in the agricultural sector. However, it is assumed that the learning potential, that is, the efficiency of learning by doing in the manufacturing sector of the home country, is higher than that of the foreign country. Hence, if the home country continues to be an autarkic economy until it has a comparative advantage in the manufacturing sector, it will eventually be able to industrialize. This type of endogenous comparative advantage is called as “dynamic comparative advantage.” Note that Matsuyama (1992) and Chang et al. (2006) do not directly address dynamic comparative advantage.<sup>2)</sup>

Redding (1999) suggests that an increase in agricultural productivity would not promote a dynamic comparative advantage in the manufacturing sector because he assumes the Cobb-Douglas utility function of homothetic preference. He shows that a country can obtain a dynamic comparative advantage in the manufacturing sector as long as it has a high learning potential in the manufacturing sector. However, we believe that this is only applicable to a small number of developing countries that have a growth potential from the beginning.

Wong and Yip (2010) construct a two-sector small open economy model, which is similar to the model of Chang et al. (2006). They discuss the dynamic comparative advantage in the manufacturing sector as Redding (1999). Their model assumes that the lump-sum tax and the manufacturing sector subsidy attract labor from the agricultural sector to the manufacturing sector. In addition, they show that there is a tax rate maximizing the welfare during the autarkic period. In contrast, if the country can specialize in the manufacturing sector, then the government stops subsidizing.

In this paper, we compare the welfare under specialization in the agricultural sector with the welfare under specialization in the manufacturing sector from some point in time. In our model, the home country has a comparative advantage in the agricultural sector at initial time. However, if the home country delays engaging in trade until it has a comparative advantage in the manufacturing sector, it can industrialize and obtain a higher welfare. In addition, we investigate the effect of an increase in agricultural productivity on the total welfare and the timing of opening trade.

The results of our analysis are summarized as follows. (1) There is an optimal tax rate maximizing the total welfare. (2) If basic consumption is positive, that is, the preference is non-homothetic, then an increase in agricultural productivity can accelerate the timing of

---

2) The model of Ortiz (2004) is similar to that of Chang et al. (2006). He incorporates productive public expenditure based on the work of Barro (1990) into a two-sector small open economy model of Matsuyama (1992). However, he also does not investigate the dynamic comparative advantage à la Redding (1999).

opening trade. In addition, we show that the acceleration of the timing of opening trade does not necessarily improve the total welfare. (3) If there is no basic consumption, that is, the preference is homothetic, then an increase in agricultural productivity delays the timing of opening trade and necessarily decreases the total welfare. (4) An increase in the efficiency of the public sector accelerates the timing of opening trade irrespective of the existence of basic consumption. (5) At the very timing of opening trade, the instantaneous utility under industrialization is lower than that under specialization in agriculture. However, the growth rate of the instantaneous utility under industrialization is higher than that under specialization in agriculture. Hence, depending on conditions, the total welfare under industrialization can be higher than that under specialization in agriculture.

The rest of this paper is organized as follows. Section 2 explains our model and investigates it under autarky. Section 3 investigates the model under a small open economy and obtains the timing of opening trade. Section 4, by using both analytical and numerical methods, shows the existence of optimal tax rates maximizing the total welfare and investigates the relationship between agricultural productivity and welfare. Section 5 concludes the paper.

## 2 Model

### 2.1 Production

In the home country (Home, hereafter), there are two sectors: the agricultural sector ( $a$ ) and the manufacture sector ( $m$ ). Labor is the only production factor and total labor endowment is normalized to unity,  $L (= 1)$ . We neglect population growth and migration. We assume that labor is perfectly mobile between the two sectors and hence, wage  $w_t$  is identical across the two sectors. We specify production functions as follows:

$$X_{mt} = M_t L_{mt}, \quad (1)$$

$$X_{at} = A L_{at}. \quad (2)$$

where  $X_{it}$  denotes the total output in sector  $i (= a, m)$  at time  $t$ ,  $A$  is the agricultural productivity and a constant parameter, and  $M_t$  is the manufacturing productivity.<sup>3)</sup>

Through learning by doing and investment in infrastructures, the manufacturing productivity  $M_t$  continues to increase through time. Let  $G_t$  denote the quantity of infrastructures

---

3) The linear production functions in equations (1) and (2) are different from those used in Matsuyama (1992). This difference does not affect results of our study so much.

provided by the government.<sup>4)</sup> Furthermore, the government must employ labor to supply infrastructures. Following Chang et al. (2006), we specify the growth rate of manufacturing productivity as follows:

$$\dot{M}_t = G_t X_{mt} \implies \frac{\dot{M}_t}{M_t} = G_t L_{mt}. \quad (3)$$

The infrastructure production function is defined as follows:

$$G_t = \phi L_g, \quad (4)$$

where  $\phi > 0$  is the efficiency parameter in the public sector and  $L_g$  is the number of workers employed in the public sector.<sup>5)</sup>

We assume that the government imposes the same production tax rate  $\tau (> 0)$  on both sectors. This setting of the production tax rate follows Ortiz (2004) and Chang et al. (2006). Thus, the revenue of the government is given by

$$\tau(P_t X_{mt} + X_{at}) = \tau w_t, \quad (5)$$

where  $P_t$  is the relative price of manufactured goods in terms of agricultural goods. Furthermore, from equation (5), the government uses the tax revenue to pay wages for public workers.

$$w_t L_g = \tau w_t \iff L_g = \tau. \quad (6)$$

Accordingly, workers in the public sector are constant through time. From equations (4) and (6), we obtain  $G = \phi\tau$ .

The labor market clearing condition is given by

$$L_{at} + L_{mt} + L_g = 1. \quad (7)$$

The profit of each sector is as follows:

$$\pi_{mt} = P_t(1 - \tau)X_{mt} - w_t L_{mt}, \quad (8)$$

$$\pi_{at} = (1 - \tau)X_{at} - w_t L_{at}. \quad (9)$$

---

4) We can consider infrastructures as scientific research, transportation system, education, and so forth.

5) This specification is similar to the specification used in Sasaki (2008), who considers a skill producing sector and assumes that the acquisition of skills requires labor. His skill producing sector is similar to infrastructures in our model.

From equations (8) and (9), we derive the profit maximization conditions and rewrite them as follows:

$$P_t = \frac{w_t}{(1 - \tau)M_t}, \quad (10)$$

$$1 = \frac{w_t}{(1 - \tau)A}. \quad (11)$$

Hence, the wage is given by  $w = (1 - \tau)A$  if both goods are produced. From equations (10) and (11), the relative price is given by  $P_t = A/M_t$ . Thus, if the manufacturing productivity increases, then the relative price decreases.

## 2.2 Consumer behavior

Let us assume that consumers obtain utility from the consumption of manufactured and agricultural goods. All consumers in the Home share identical preferences. We adopt the Stone-Geary utility function of non-homothetic preference.<sup>6)</sup> The utility maximization problem is given by

$$\max_{c_{at}, c_{mt}} W = \int_0^\infty u_t e^{-\rho t} dt \quad \text{where } u_t = \beta \ln(c_{at} - \gamma) + \ln c_{mt}, \quad (12)$$

$$\text{s.t. } c_{at} + P_t c_{mt} = w_t, \quad (13)$$

where  $\gamma > 0$  denotes the subsistence level of agricultural consumption,  $\rho > 0$  is the constant discount rate,  $c_{it}$  is the per capita consumption of good  $i (= a, m)$ , and  $\beta > 0$  is the preference parameter. In addition,  $c_{at}$  must satisfy the condition  $c_{at} > \gamma$ , that is, the consumption of agricultural goods exceeds the subsistence level.

Solving the above utility maximization problem, we obtain the aggregated utility maximizing condition as follows:

$$C_{at} = \gamma + \beta P_t C_{mt}, \quad (14)$$

where  $C_{at}$  and  $C_{mt}$  denote the aggregated consumption of agricultural goods and that of manufactured goods, respectively.

From equations (13) and (14), we derive the demand functions as follows:

$$C_{mt} = \frac{w - \gamma}{P_t(1 + \beta)}, \quad (15)$$

---

<sup>6)</sup> In the context of trade and development, the Stone-Geary utility function is often used: Matsuyama (1992), Spilimbergo (2000), Kikuchi (2004), and Azarnert (2014).

$$C_{at} = \frac{\beta w + \gamma}{1 + \beta}. \quad (16)$$

For  $C_{mt}$  to be positive, we assume that  $w > \gamma$ .

Substituting equations (15) and (16) into equation (12), we obtain the indirect utility function at  $t$ ,  $\tilde{u}_t$  as follows.

$$\tilde{u}_t = J + (1 + \beta) \ln(w_t - \gamma) - \ln P_t, \quad (17)$$

where  $J \equiv \beta \ln \beta - (1 + \beta) \ln(1 + \beta)$  is constant. A decrease in the relative price increases the real income, and hence, improves the indirect utility.

### 2.3 Autarky

Consider the equilibrium under autarky. The market clearing conditions are given by

$$C_{at} = (1 - \tau)X_{at}, \quad (18)$$

$$C_{mt} = (1 - \tau)X_{mt}. \quad (19)$$

Hence, from equations (18) and (19), employment in each sector is given by

$$L_A = \left[ \frac{\beta(1 - \tau)A + \gamma}{A(1 + \beta)} \right], \quad (20)$$

$$L_m = \left[ \frac{(1 - \tau)A - \gamma}{A(1 + \beta)} \right], \quad (21)$$

$$L_g = \tau. \quad (22)$$

From equation (21), for  $L_m$  to be positive, we need  $\tau > (A - \gamma)/A$ . From equations (20) and (21), an increase in  $\tau$  decreases the number of workers in the agricultural and manufacturing sectors.<sup>7)</sup>

Using equations (3), (4), (21), and (22), we obtain the growth rate of the manufacturing productivity under autarky as follows:

$$\frac{\dot{M}_t}{M_t} = \phi L_g L_m = \frac{\phi \tau [(1 - \tau)A - \gamma]}{A(1 + \beta)} > 0. \quad (23)$$

Note that we have already imposed the condition  $(1 - \tau)A - \gamma > 0$ .

---

<sup>7)</sup> Partially differentiating equations (20) and (21) with respect to  $\tau$ , we obtain  $\partial L_A / \partial \tau = -\beta / (1 + \beta) < 0$  and  $\partial L_m / \partial \tau = -1 / (1 + \beta) < 0$ .



Using equations (10), (11), and (17), we obtain the indirect utility function under autarky  $\tilde{u}_t^c$  is as follows:

$$\tilde{u}_t^c = J + (1 + \beta) \ln[(1 - \tau)A - \gamma] - \ln A + \ln M_t. \quad (24)$$

Equation (24) shows that an increase in the agricultural productivity increases the indirect utility under autarky.

Finally, we obtain the total welfare under autarky during  $[0, \infty)$  from equation (24).

$$W^c = \int_0^{\infty} \tilde{u}_t^c e^{-\rho t} dt. \quad (25)$$

### 3 Small open economy

In this section, we consider the case of a small open economy. Namely, the small home country trades with the large rest of the world (ROW, hereafter). The ROW's variables are distinguished from the Home's variables by adding an asterisk “\*.” The Home takes the world price  $P^*$  as given. The structure of the ROW is the same as that of the Home except that the ROW does not provide infrastructures. Here, an increase in the ROW's manufacturing productivity  $M_t^*$ , is given by  $\dot{M}_t^* = \delta^* X_{mt}^*$ , where  $\delta^* > 0$  and its growth rate is given by

$$\frac{\dot{M}_t^*}{M_t^*} = \frac{\delta^*(A^* - \gamma)}{A^*(1 + \beta)}. \quad (26)$$

In addition, we assume that the Home (the ROW) has a comparative advantage in agriculture (manufacturing) at the initial time.

**Assumption 1.** *The following inequality holds:*

$$\frac{M_0^*}{A^*} > \frac{M_0}{A}. \quad (27)$$

Hence, if the Home begins trading at the initial time, it specializes in agriculture according to the law of comparative advantage.

However, the manufacturing productivities of the Home and the ROW evolve through time, and hence, the initial comparative advantage pattern can change through time. There are three possible specialization patterns in  $t \in (0, \infty)$ : (a) if  $M_t^*/A^* > M_t/A$ , the Home specializes in agriculture; (b) if  $M_t^*/A^* < M_t/A$ , the Home specializes in manufacturing; and (c) if  $M_t^*/A^* = M_t/A$ , the Home incompletely specializes, that is, produces both goods.

Moreover, we assume the following rule with regard to industrialization.

**Assumption 2.** *If the Home's government intends to industrialize, then it chooses to close the economy until it has a comparative advantage in manufacturing.*<sup>8)</sup>

We define  $t_1$  as the time when the Home has a comparative advantage in the manufacturing sector. Therefore, the Home begins to open the economy at  $t_1$ . We now consider the following two cases. In the first case, we consider a situation in which the Home specializes in the agricultural sector during  $t \in [0, \infty)$ . In this case, the Home's government does not impose tax, that is,  $\tau = 0$ . In the second case, we consider a situation in which the Home continues to be under autarky during  $t \in [0, t_1]$  and engages in trade with the ROW during  $t \in (t_1, \infty)$ .

### 3.1 Specialization in the agricultural sector

We consider the case in which the Home specializes in agriculture: the Home continues to specialize in the agricultural sector during  $t \in [0, \infty)$ . Hence, from the profit maximization condition, we obtain

$$1 = \frac{w_t}{A} \iff w_t = A. \quad (28)$$

From equation (28) and  $P_t^* = A^*/M_t^*$ , we obtain the indirect utility function at  $t$ ,  $\tilde{u}_{at}^f$ , as follows:

$$\tilde{u}_{at}^f = J + (1 + \beta) \ln(A - \gamma) - \ln A^* + \ln M_t^*. \quad (29)$$

Note that the ROW's growth rate of manufacturing productivity  $\dot{M}_t^*/M_t^*$  is positive, and hence,  $\tilde{u}_{at}^f$  continues to increase. From equation (29), the total welfare of the Home under specialization in agriculture,  $W_a^f$ , is given by

$$W_a^f = \int_0^{\infty} \tilde{u}_{at}^f e^{-\rho t} dt. \quad (30)$$

### 3.2 Industrialization

We consider the case in which the Home's government decides to industrialize. For this policy to be feasible, the Home must have a comparative advantage in the manufacturing

---

8) Note that if, at  $t = 0$ , the Home chooses to continue to be under autarky through time, the instantaneous utility under autarky will be larger than that under specialization in agriculture at some point in time. Accordingly, the total welfare under autarky can be larger than that under specialization in agriculture depending on the discount rate. However, if the Home's government chooses to adopt the industrialization rule, the total welfare under the rule necessarily exceeds that under autarky. For this issue, see Figure 4 that will be explained later.

sector at  $t_1 \in (0, \infty)$ . Hence, the condition for industrialization is given by

$$\frac{M_t}{A} \geq \frac{M_t^*}{A^*} \quad \text{for any } t \text{ such that } t_1 \leq t \quad (31)$$

Accordingly, the condition  $M_{t_1}^*/A^* = M_{t_1}/A$  gives the timing of opening trade. Therefore,  $t_1$  is given by

$$t_1 = \frac{(\ln M_0^* - \ln M_0) - (\ln A^* - \ln A)}{\frac{\phi\tau[(1-\tau)A - \gamma]}{A(1+\beta)} - \frac{\delta^*(A^* - \gamma)}{A^*(1+\beta)}}. \quad (32)$$

The numerator of equation (32) is positive because equation (27) holds. Next, the denominator of equation (32) shows the difference between the growth rate of manufacturing productivity of the Home under autarky and the growth rate of manufacturing productivity of the ROW. Unless this difference is positive, then the Home cannot industrialize. Hence, the Home cannot have a dynamic comparative advantage in the manufacturing sector through time: there is no timing of opening trade. In contrast, if the denominator of equation (32) is positive, then there is  $t_1 \in (0, \infty)$ . Accordingly, we obtain the following lemma with regard to the tax rate.

**Lemma 1.** *For the Home to have a comparative advantage in manufacturing at time  $t_1$ , the the Home's government must impose the tax rate that satisfies the following inequality:*

$$\frac{(A - \gamma) - \sqrt{(A - \gamma)^2 - \frac{4\delta^*A^2(A^* - \gamma)}{\phi A^*}}}{2A} < \tau < \frac{(A - \gamma) + \sqrt{(A - \gamma)^2 - \frac{4\delta^*A^2(A^* - \gamma)}{\phi A^*}}}{2A}. \quad (33)$$

*Proof.* For  $t_1$  to be positive, the denominator of the right-hand side of equation (32) must be positive. Note that the denominator is a quadratic function of  $\tau$ . Accordingly, solving the condition that the denominator is positive, we obtain Lemma 1. ■

When the Home specializes in manufacturing, the employment share of the manufacturing sector and that of the public sector are respectively given by

$$L_m = 1 - \tau, \quad (34)$$

$$L_g = \tau. \quad (35)$$

Hence, the growth rate of the manufacturing productivity leads to

$$\frac{\dot{M}_t}{M_t} = \phi L_g L_m = \phi\tau(1 - \tau), \quad \text{for } t \in [t_1, \infty). \quad (36)$$

From the above analysis, the Home's manufacturing productivities during  $t \in [0, t_1]$  and during  $t \in [t_1, \infty)$  are respectively given by

$$M_t = \begin{cases} M_0 \exp \left[ \frac{\phi \tau [(1 - \tau)A - \gamma]}{A(1 + \beta)} \cdot t \right], & \text{for } t \in [0, t_1], \\ M_{t_1} \exp [\phi \tau (1 - \tau) \cdot (t - t_1)], & \text{for } t \in [t_1, \infty), \end{cases} \quad (37)$$

where  $M_{t_1} \equiv M_0 \exp \left[ \frac{\phi \tau [(1 - \tau)A - \gamma]}{A(1 + \beta)} \cdot t_1 \right]$ .

[Figure 1 around here]

Figure 1 shows the time paths of the logarithms of the relative productivities of the Home and the ROW. The Home continues to be under autarky until  $t_1$ , and then, it continues to engage in free trade: from  $t_1$ , the Home specializes in the manufacturing sector.

We investigate the relationship between the tax rate and the timing of opening trade. From equation (32), we obtain

$$\frac{dt_1}{d\tau} = 0 \iff \tau = \frac{A - \gamma}{2A} > 0. \quad (38)$$

Hence, we obtain the following proposition:

**Proposition 1.** *There is the tax rate minimizing the autarkic period  $t \in [0, t_1]$ .*

*Proof.* The denominator of the right-hand side of equation (32) is a quadratic function of  $\tau$  that is convex upward. Then,  $t_1$  is minimized when the denominator is maximized. Solving  $dt_1/d\tau = 0$ , we obtain Proposition 1. ■

Equation (32) is clearly a decreasing function of  $\phi$ , and hence, we obtain the following proposition.

**Proposition 2.** *As the efficiency parameter of the public sector  $\phi$  gets larger, the timing of opening trade  $t_1$  gets faster.*

We explain the intuition of Proposition 2. If the efficiency of the public sector increases, the growth rate of the Home's manufacturing productivity increases. Hence, an increase in  $\phi$  accelerates the timing of opening trade.

Furthermore, we obtain the following proposition.

**Proposition 3.** *The effect of an increase in agricultural productivity on the timing of opening trade is ambiguous.*

*Proof.* Differentiating  $t_1$  with respect to  $A$ , we obtain the following expression:

$$\frac{dt_1}{dA} = \frac{\overbrace{\frac{\Omega}{A}}^{\text{ICA effect}} - \overbrace{\ln\left(\frac{A}{M_0} \cdot \frac{M_0^*}{A^*}\right) \cdot \frac{\phi\tau\gamma}{A^2(1+\beta)}}^{\text{LRA effect}}}{\Omega^2}, \quad \text{where } \Omega \equiv \frac{\phi\tau[(1-\tau)A - \gamma]}{A(1+\beta)} - \frac{\delta^*(A^* - \gamma)}{A^*(1+\beta)} > 0. \quad (39)$$

■

Note that the term  $\ln[(A/M_0) \cdot (M_0^*/A^*)]$  is positive because of Assumption 1. The effect of an increase in the agricultural productivity on the timing of opening trade is decomposed into two effects. The first term of the numerator on the right-hand side shows the “initial comparative advantage effect” (ICA effect) while the second term of the numerator shows the “labor reallocation effect” (LRA effect). The initial comparative advantage effect means that an increase in  $A$  increases the degree of the Home’s initial comparative advantage in agriculture, and consequently, makes it more difficult to have a comparative advantage in manufacturing. Accordingly, the initial comparative advantage effect delays the timing of opening trade. The labor reallocation effect means that an increase in  $A$  releases labor from the agricultural sector to manufacturing sector, and consequently, intensifies learning by doing in manufacturing from equation (3), thereby increasing the manufacturing productivity  $M_t$ . Accordingly, the labor reallocation effect accelerates the timing of opening trade. Therefore, if the labor reallocation effect dominates the initial comparative advantage effect, then an increase in the agricultural productivity accelerates the timing of opening trade. In contrast, if the initial comparative advantage effect dominates the labor reallocation effect, then an increase in the agricultural productivity delays the timing of opening trade.

Moreover, if  $\gamma = 0$ , an increase in the agricultural productivity delays the timing of opening trade from equation (39) because the labor reallocation effect vanishes.

**Lemma 2.** *If  $\gamma = 0$ , an increase in the agricultural productivity necessarily delays the timing of opening trade.*

Lemma 2 is similar to a result of Redding (1999). Therefore, Proposition 2 generalizes the result of Redding (1999). Our result depends on the non-homotheticity of preference, that is,  $\gamma > 0$ . Note that Matsuyama (1992) and Chang et al. (2006) use the Stone-Geary utility function but do not discuss the timing of opening trade.

[Figures 2 and 3 around here]

Figures 2 and 3 numerically reveals the relationship between the effect of an increase in the agricultural productivity and the timing of opening trade.<sup>9)</sup> If  $\gamma = 0.1$ , an increase in the agricultural productivity from  $A = 1$  to  $A = 1.5$  almost delays the opening time: the initial comparative advantage effect is larger than the labor reallocation effect. However, if  $\gamma = 0.4$ , an increase in the agricultural productivity accelerates and delays the timing of opening trade depending on the size of the tax rate: the initial comparative advantage effect is smaller than the labor reallocation effect.

The relative world price of manufactured goods determines  $w_t$ . When the Home specializes in the manufacturing sector, the profit maximization condition gives

$$P_t^* = \frac{w_t}{(1 - \tau)M_t}. \quad (40)$$

Since  $P_t^* = A^*/M_t^*$  under free trade, from equation (40), we obtain

$$w_t = \frac{(1 - \tau)A^*M_t}{M_t^*}. \quad (41)$$

Substituting equation (41) and  $P_t^* = A^*/M_t^*$  into equation (17), we obtain the instantaneous indirect utility function for  $t > t_1$  as follows:

$$\text{for } t \in [t_1, \infty) \quad \tilde{u}_{mt}^f = J + (1 + \beta) \ln \left[ \frac{(1 - \tau)A^*M_t}{M_t^*} - \gamma \right] - \ln A^* + \ln M_t^*. \quad (42)$$

We explain the intuition of equation (42). The effect of  $M_t^*$  is decomposed into the following two opposing effects. An increase in  $M_t^*$  lowers the relative world price of manufactured goods, and hence, the Home's instantaneous utility improves. In contrast, an increase in  $M_t^*$  reduces the Home's comparative advantage in the manufacturing sector, and hence, the Home's instantaneous utility declines. In addition, we know that an increase in the agricultural productivity does not affect the Home's instantaneous utility during  $t \in (t_1, \infty)$ .

The Home's welfare during  $[t_1, \infty)$ ,  $W_m^f$ , is given by

$$W_m^f = \int_{t_1}^{\infty} \tilde{u}_{mt}^f e^{-\rho t} dt. \quad (43)$$

Hence, the Home's total welfare through time,  $W_m$ , is given by

$$W_m = W^c + W_m^f = \int_0^{t_1} \tilde{u}_t^c e^{-\rho t} dt + \int_{t_1}^{\infty} \tilde{u}_{mt}^f e^{-\rho t} dt. \quad (44)$$

---

9) The parameters are set as follows:  $A = 1$ ,  $A^* = 1$ ,  $M_0 = 0.5$ ,  $M_0^* = 1$ ,  $\phi = 1$ , and  $\delta^* = 0.05$ .

## 4 Welfare analysis

### 4.1 Welfare

In this section, we compare  $W_a^f$  and  $W_m$ . Figure 4 shows the time paths of the instantaneous utility. In the case of industrialization, the Home specializes in the manufacturing sector at  $t_1$ . Note that the Home's instantaneous utility under specialization in manufacturing during  $t \in [t_1, t_2]$  is smaller than that under specialization in agriculture during  $t \in [t_1, t_2]$ . However, from  $t_2$  on, the growth rate of  $\tilde{u}_{mt}^f$  is larger than that of  $\tilde{u}_{at}^f$ , and accordingly, the level of  $\tilde{u}_{mt}^f$  becomes larger than that of  $\tilde{u}_{at}^f$ . Hence, depending on the size of the discount rate  $\rho$ ,  $W_m$  can be larger than  $W_a^f$ .

**Proposition 4.** *The total welfare under industrialization can be larger than the total welfare under specialization in agriculture depending on the size of the discount rate.*

[Figure 4 around here]

### 4.2 Optimal tax rate

In this section, we examine the existence of the optimal tax rate maximizing  $W_m$ . However, the computation of equation (44) and the partial derivative of the resultant expression with respect to  $\tau$ ,  $\partial W_m / \partial \tau$ , are very complicated, and accordingly, we use numerical simulations. The reason for the existence of the optimal tax rate will be explained below.

Figures 5 and 6 show the existences of the optimal tax rates in the cases of  $\gamma = 0.1$  and  $\gamma = 0.4$ , respectively.

[Figures 5 and 6 around here]

In addition, Figure 5 shows that an increase in the agricultural productivity can reduce the total welfare, and Figure 6 shows that an increase in the agricultural productivity improves the total welfare. The reason for this is as follows. From equation (17), an increase in the agricultural productivity directly increases instantaneous utility under autarky  $\tilde{u}_t^c$ :

$$\frac{\partial \tilde{u}_t^c}{\partial A} = \frac{\beta A(1 - \tau) + \gamma}{A[(1 - \tau)A - \gamma]} > 0. \quad (45)$$

We call this the direct effect of agricultural productivity growth. In addition, as stated above, an increase in the agricultural productivity has the two effects on the timing of opening trade: the initial comparative advantage effect and the labor reallocation effect. The initial comparative advantage effect decreases the total welfare because an increase in the agricultural

productivity directly delays the timing of opening trade. In contrast, the labor reallocation effect shows that an increase in the agricultural productivity reallocates labor from the agricultural sector to the manufacturing and public sectors, thereby improving the total welfare.

Therefore, in Figure 5, the negative initial comparative advantage effect dominates the positive direct effect and the positive labor reallocation effect while in Figure 6, the positive direct effect and the positive labor reallocation effect dominate the negative initial comparative advantage effect.<sup>10)</sup>

**Proposition 5.** *There is the optimal tax rate that maximizes the total welfare under industrialization.*

**Proposition 6.** *An increase in the agricultural productivity increases or decreases the total welfare under industrialization depending on the size of the subsistence level of agricultural consumption.*

In our model, the reason why instantaneous utility continues to increase through time is that the manufacturing productivity continues to increase through time. Note that the growth rate of the manufacturing productivity is a quadratic function convex upward with respect to the tax rate both before and after industrialization. Accordingly, both too high and low tax rates decrease the growth rate of the manufacturing productivity. That is, roughly speaking, a tax rate that maximizes the growth rate of the manufacturing productivity corresponds to a tax rate that maximizes the total welfare through time.

Note that a tax rate that maximizes the growth rate of the manufacturing productivity under autarky,  $\tau_a$ , is given by<sup>11)</sup>

$$\tau_a = \frac{A - \gamma}{2A}, \quad (46)$$

which is exactly the same as equation (38), and a tax rate that maximizes the growth rate of the manufacturing productivity under specialization in manufacturing,  $\tau_m$ , is given by<sup>12)</sup>

$$\tau_m = \frac{1}{2}. \quad (47)$$

From equations (46) and (47), we find that productivity growth maximizing tax rates do not depend on the discount rate  $\rho$ . In contrast, from equation (44), we find that the optimal tax rate that maximizes the total welfare depends on the discount rate. Hence, the welfare maximizing tax rate is not equal to a productivity growth maximizing tax rate.

10) Moreover, the labor reallocation effect does not exist in the case of  $\gamma = 0$ . Then, in this case, an increase in the agricultural productivity always reduces the total welfare.

11) Differentiating equation (23) with respect to  $\tau$  gives equation (46).

12) Differentiating equation (36) with respect to  $\tau$  gives equation (47).



Next, we investigate the relationship between the total welfare and timing of opening trade. From Figures 2, 3, 5, and 6, we know that the optimal tax rate maximizing  $W_m$  is different from the tax rate minimizing the timing of opening trade. The reason is that providing infrastructures sacrifices consumption, which contributes to lowering the total welfare. In addition, the loss of consumption under autarky is larger than that under specialization in the manufacturing sector. If the discount rate is large, the loss under autarky is also large, and hence, the optimal tax rate is smaller than the tax rate minimizing the timing of opening trade.

**Lemma 3.** *There is the optimal timing of opening trade that maximizes the total welfare and this timing is obtained from the optimal tax rate.*

Wong and Yip (2010) also construct a dynamic Ricardian trade model and show that by intentionally delaying the timing of opening trade, an economy succeeds in industrialization. They argue dynamic comparative advantage though they do not refer to Redding (1999). They assume that under autarky, the government subsidizes the manufacturing to promote industrialization. On the one hand, this policy attracts workers from agriculture to manufacturing, which intensifies learning by doing in the manufacturing sector, leading to strengthening the potential comparative advantage in manufacturing. On the other hand, after opening trade (i.e., industrialization), all workers are employed in the manufacturing sector, and hence, the optimal policy is not to subsidize.

In contrast, in our model, under both autarky and industrialization, when the tax rate is zero, the employment of the public sector is zero, and hence, the growth rate of the manufacturing sector is zero. Then, the government must impose a positive tax rate to sustain specialization in manufacturing after industrialization. To sum up, in Wong and Yip (2010), there is no government policy after industrialization whereas in our model there is government policy after industrialization.

## 5 Conclusion

We have constructed a dynamic Ricardian trade model that incorporates public provision of infrastructures. We have shown that an increase in the agricultural productivity plays an important role for industrialization. The results are summarized as follows.

First, there is the optimal tax rate maximizing the total welfare. Second, if the level of basic consumption is positive, an increase in the agricultural productivity can accelerate the timing of opening trade. Third, if the level of basic consumption is zero, an increase in the agricultural productivity delays the timing of opening trade and decreases the total welfare.

Fourth, an increase in the efficiency of public provision of infrastructures accelerates the timing of opening trade. This result does not depend on the existence of the basic consumption. Fifth, at the timing of opening trade, the instantaneous utility under industrialization is lower than that under specialization in agriculture. However, the growth rate of the instantaneous utility under industrialization is larger than that under specialization in agriculture. Therefore, depending on the size of the discount rate, the total welfare under industrialization can be larger than that under specialization in agriculture.

Is agricultural productivity growth good for industrialization? The effect of an increase in the agricultural productivity on the total welfare depends on the size of the subsistence level of consumption for agricultural goods. A large subsistence level means that the income elasticity of consumption for manufactured goods is high. In addition, according to our results, when the subsistence level is large, an increase in the agricultural productivity raises the total welfare. Therefore, we can say that if an economy produces high quality, sophisticated manufactured goods, agricultural productivity growth is good for industrialization, whereas if an economy produces low quality, less-sophisticated manufactured goods, agricultural productivity growth is not good for industrialization.

Some extensions will be left for future research.

To begin with, we can introduce capital accumulation. In our model, labor is the only factor of production in both agriculture and manufacturing. However, manufactured goods are more capital intensive than agricultural goods. Accordingly, the introduction of capital accumulation will be interesting.

Next, we can modify the tax rule. In our model, the tax rule is constant through time, that is, the government imposes a constant, same tax rate during both autarky and industrialization. However, as a more general tax rule, it is possible that the government imposes different tax rates during autarky and industrialization.

## Acknowledgements

We thank Akihisa Shibata, Akihiko Yanase, Katsuzo Yamamoto, Katsuyuki Naito, Noritugu Nakanishi, Hiroyuki Mizuta, Naoto Jinji, Daishin Yasui, seminar participants at Onomichi City University, Kyoto University, and Hokkaido University for their useful comments and helpful suggestions on earlier versions of this paper. For financial support, the first and second authors are grateful to Grant-in-Aid for JSPS Fellows (25·5882) and Grant-in-Aid for Scientific Research (KAKENHI 25380295), respectively.

## References

- Azarnert, L. V. (2014) "Agricultural Exports, Tariffs and Growth," *Open Ecoomies Review*, doi: 10.1007/s11079-013-9297-1.
- Barro, R. (1990) "Government Spending in a Simple model of Endogenous Growth," *Journal of Political Economy* 98 (5), pp. 103–125.
- Chang, J. J., B. L. Chen, and M. Hsu (2006) "Agricultural Productivity and Economic Growth: Role of Tax Revenues and Infrastructures," *Southern Economic Journal* 72 (4), pp. 891–914.
- Kikuchi, T. (2004) "Agricultural Productivity, Business Services, and Comparative Advantage," *Open Economies Review* 15 (4), pp. 375–383.
- Kirkpatrick, C. and A. Barrientos (2004) "The Lewis Model after 50 Years," *The Manchester School* 72 (6), pp. 679–690.
- Matsuyama, K. (1992) "Agricultural Productivity, Comparative Advantage, and Economic Growth," *Journal of Economic Theory* 58 (2), pp. 317–334.
- Nurkse, R. (1953) *Problems of Capital Formation in Underdeveloped Countries*, New York: Oxford University Press.
- Ortiz, C. (2004) "An Economic Growth Model Showing Government Spending with Reference to Colombia and Learning-by-Doing," *Colombian Economic Journal* 2 (1), pp. 156–188.
- Redding, S. (1999) "Dynamic Comparative Advantage and the Welfare Effects of Trade," *Oxford Economic Papers* 51 (1), pp. 15–39.
- Rostow, W. W. (1960) *The Stages of Economic Growth: A Non-Communist Manifesto*, Cambridge: Cambridge University Press.
- Sasaki, H. (2008) "International Trade and Industrialization with Capital Accumulation and Skill Acquisition," *The Manchester School* 76 (4), pp. 464–486.
- Spilimbergo, A. (2000) "Growth and Trade: The North Can Lose," *Journal of Economic Growth* 5 (2), pp. 131–146.
- Temple, J. (2005) "Dual Economy Models: A Primer for Growth Economists," *The Manchester School* 73 (4), pp. 435–478.
- Wang, X. and J. Piesse (2013) "The Micro-Foundations of Dual Economy Models," *The Manchester School* 81 (1), pp. 80–101.

Wong, K.-Y. and C. K. Yip (1999) “Industrialization, Economic Growth, and International Trade,” *Review of International Economics* 7 (3), pp. 522–540.

Wong, K.-Y. and C. K. Yip (2010) “On the Optimal Timing of Foreign Trade,” mimeo.

# Figures

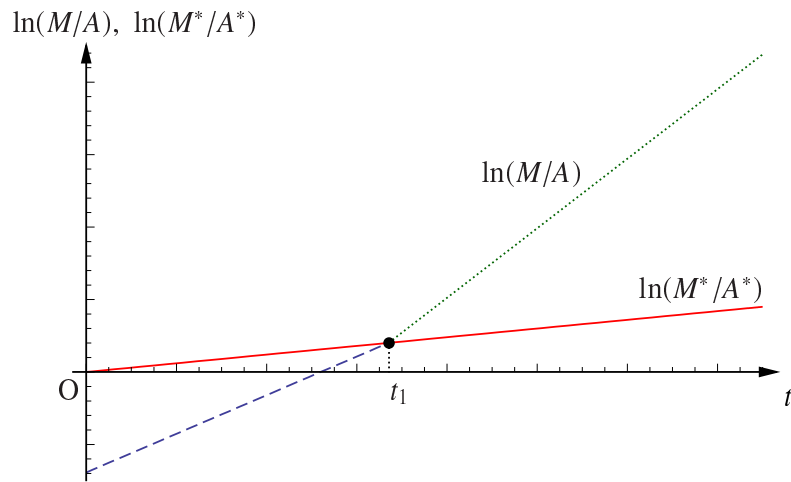


Figure 1: Time paths of relative productivities

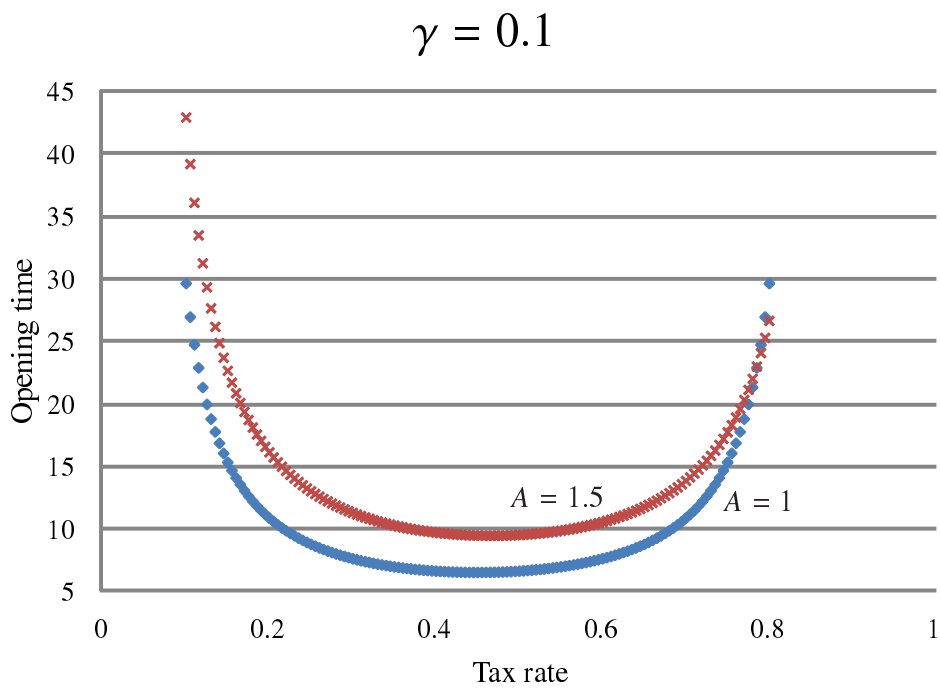


Figure 2: Relationship between agricultural productivity and timing of opening trade when  $\gamma = 0.1$

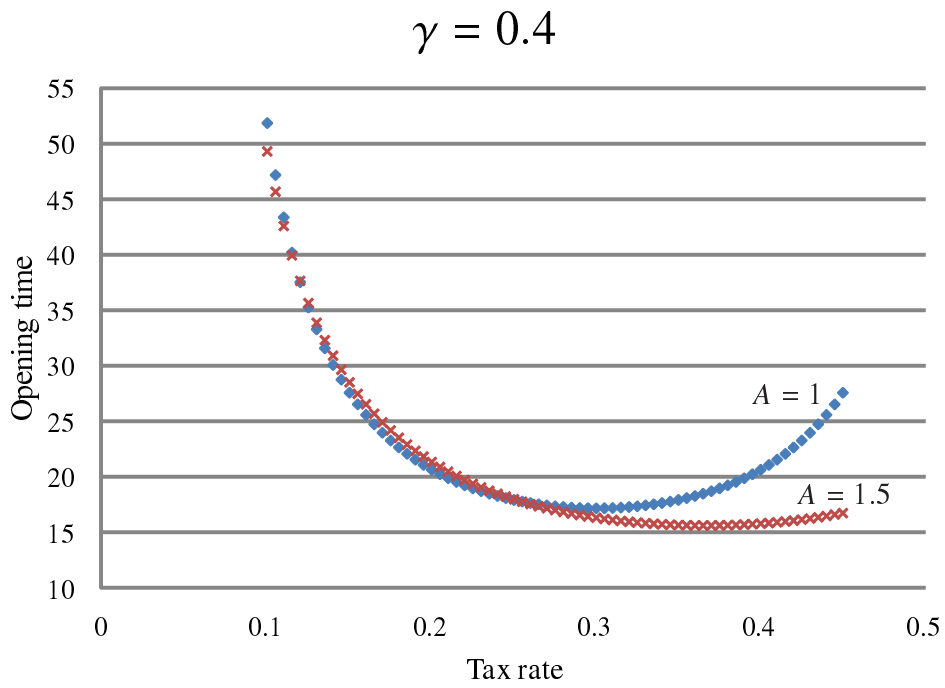


Figure 3: Relationship between agricultural productivity and timing of opening trade when  $\gamma = 0.4$

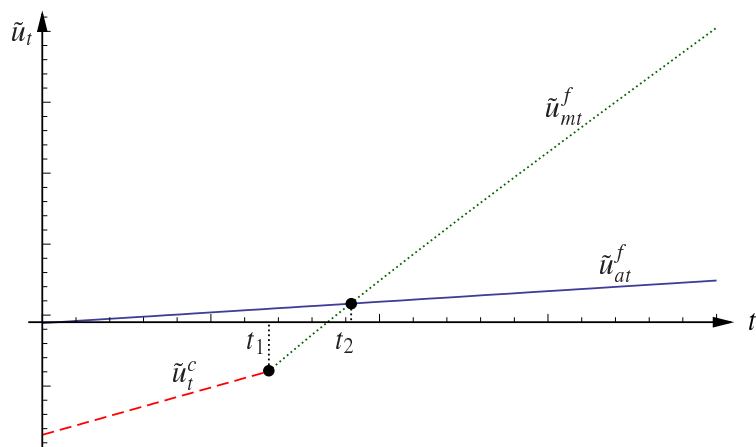


Figure 4: Time paths of instantaneous utilities

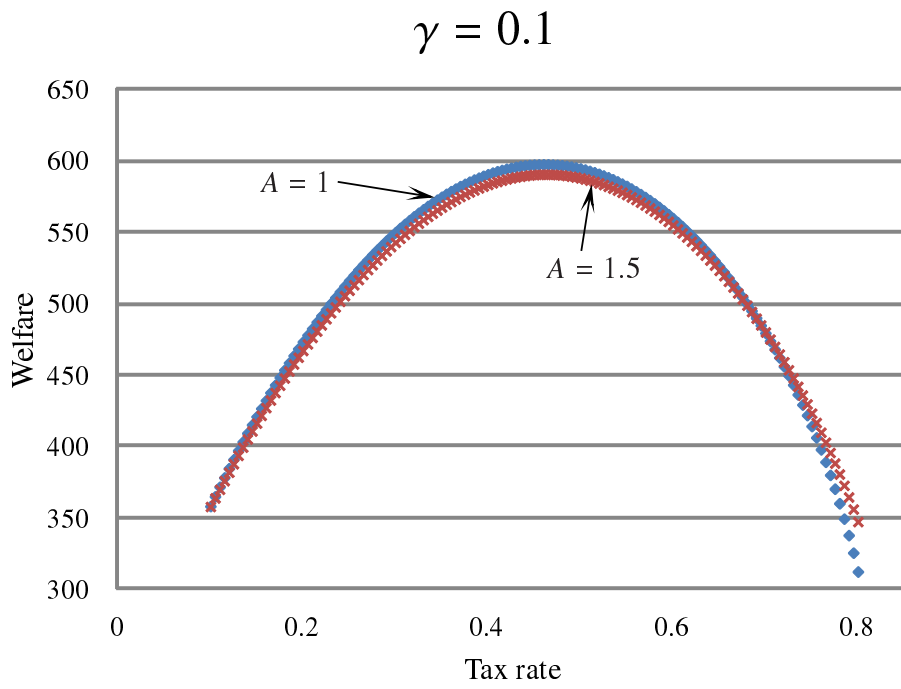


Figure 5: Optimal tax rate, welfare, and agricultural productivity when  $\gamma = 0.1$

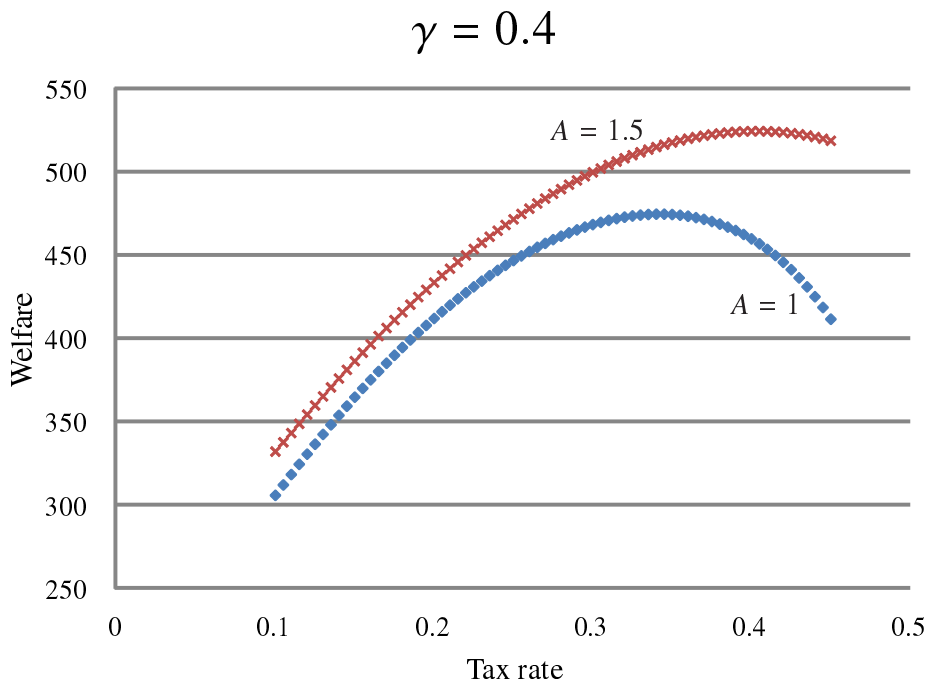


Figure 6: Optimal tax rate, welfare, and agricultural productivity when  $\gamma = 0.4$