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# Expectation-Driven Cycles: Time-varying Effects <sup>\*</sup>

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## Abstract

This paper provides new insights into expectation-driven cycles by estimating a structural VAR with time-varying coefficients and stochastic volatility. We use survey-based expectations of the unemployment rate to measure expectations of future developments in economic activity. We find that the effect of expectation shocks on the realized unemployment rate have been particularly large during the most recent recession. Unanticipated changes in expectations contributed to the gradual increase in the persistence of the unemployment rate and to the decline in the correlation between the inflation and the unemployment rate over time. Our results are robust to the introduction of financial variables in the model.

JEL classification: C32; E24; E32.

Keywords: Survey Expectations; Economic Fluctuations; Stochastic Volatility; Time Varying Vector Autoregression

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## Introduction

There has historically been a great deal of emphasis on changes in expectations as sources of macroeconomic fluctuations, beginning with Pigou (1927) and Keynes (1936). Yet it is only recently that the business cycle literature revived interest towards the importance of expectation-driven cycles. In an influential paper, Beaudry and Portier (2006) show that changes in expectations that are driven by news about future productivity growth are important sources of macroeconomic fluctuations. Since their contribution, several authors explored the importance of news-driven cycles in the context of VAR models.<sup>1</sup> More recently, changes in expectations of future developments in economic activity have been measured by directly introducing forward-looking survey data such as consumers' confidence (Barsky and Sims (2012)) and unemployment expectations (Leduc and Sill (2013)) into otherwise standard VAR models. The advantage of using survey data is that the econometrician does not need to impose any modelling assumptions to back out the expectations of the economic agents.

The aim of this paper is to assess the role of time variation in the macroeconomics effects of changes in expectations. The US economy experienced important changes over the last four decades and most macroeconomic variables exhibited marked time-variation. Several authors have stressed the importance of relaxing the constant parameters assumptions in macroeconomic models by allowing for time variation.<sup>2</sup> With this purpose, we estimate the effects of changes in expectations on the unemployment rate and other macroeconomic variables using a Time-Varying Coefficients VAR model with Stochastic Volatility (TV-VAR) as in Primiceri (2005). This approach allows for temporal changes in the size and correlation among forecast errors which can be due to changes in the size of exogenous shocks or their impact on macroeconomic variables, i.e. stochastic volatility. Further, it also allows for changes in the transmission of the shocks by means of time-varying coefficients.

The first objective of this paper is to provide new evidence on the quantitative importance of expectation shocks in shaping the dynamics of the unemployment rate and other macroeconomic variables. Following Leduc and Sill (2013) we use unemployment expectations, as compiled by the Survey of Professional Forecasters, to measure expectations of future developments in economic activity. In our baseline model, in addition to expectations, we use the unemployment rate, inflation rate and the short-term interest rate in order to take into account how changes in expectations interact with monetary policy. We aim to quantify the changing role of expectation shocks over time.

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<sup>1</sup>See Beaudry and Portier (2013) for a complete review of the theoretical and empirical literature on news-shocks-driven cycles.

<sup>2</sup>The great moderation and its causes received a great deal of attention (Clarida, Gali, and Gertler (2000); Cogley and Sargent (2005); Lubik and Schorfheide (2004)). A large literature explores the implications of changes in the conduct of monetary policy for macroeconomic volatility (Stock and Watson (2003); Primiceri, 2005; Boivin and Giannoni (2006); Canova and Gambetti (2009)). Few papers also investigate the importance of time-variations in the transmission of technology shocks (Galí and Gambetti (2009)). See also D'Agostino, Gambetti, and Giannone (2013) for the forecasting ability of such models.

Our results focus on the analysis of the impulse-responses of the endogenous variables to a positive shock to unemployment expectations over time and on the implied variance decomposition, i.e. the percentage of variance explained by such shock. Unanticipated downward revisions to expected unemployment generate a macroeconomic boom coupled with a monetary policy tightening as in Leduc and Sill (2013). Our results improve upon the previous findings in that:

- We detect significant changes in the evolution over time of the dynamic responses of the endogenous variables to shocks to expected future economic activity. In particular, the responses of the unemployment rate increase beginning in the early 2000s and are remarkably larger and more persistent during the most recent recession;
- Expectation shocks account for a sizable fraction of the forecast-error variance of the endogenous variables. The increase in the volatility of the unemployment rate over the second part of the sample can be largely explained by an increase in the variance share of unanticipated changes in expectations.

The second objective of the paper is to explore the impact of expectation shocks on selected key second moments of the unemployment rate and the related implications. Recent empirical evidence highlighted that the last two decades have been characterized by (i) a longer duration of high unemployment rates after the recessions with a consequent slowdown in the labor market recovery; (ii) a reduced sensitivity of inflation to changes in unemployment.<sup>3</sup> We relate our findings to these two main empirical facts.

We start by investigating how shifts in unemployment expectations affect the persistence of the unemployment rate. Our results highlight differences in the effects of expectation shocks across recessions. In particular, the responses of unemployment are increasingly large and persistent in the post-1990 recessions. Accordingly, unanticipated changes in expectations imply a gradual increase in the persistence of the unemployment rate.

We also explore the role of expectation shocks to study the correlation between the unemployment and the inflation rate. Our results point to a sizable decline in the correlation between inflation and unemployment after a shock to changes in expectations, since early 2000s. This is explained by the different impact that the expectation shock has on the two variables, i.e. larger and more persistent on unemployment, but smaller on inflation.

In addition we present a series of robustness to the inclusion of financial variables in our model. The last two business cycles have also been characterized by coincident booms in economic activity and asset prices, followed by sudden and remarkable falls in asset prices and economic recessions. In particular, during the late 1990s the US economy experienced a dramatic rise in stock prices. Similarly during the mid 2000s house prices displayed a sustained run-up. Both periods of expansion were followed by sudden falls in asset prices and economic downturns. We extend our analysis by

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<sup>3</sup>See, among others, Coibion and Gorodnichenko (2013) and Coibion, Gorodnichenko, and Koustas (2013).

including financial variables. We alternatively include in the VAR model stock prices, house prices and credit variables. The key findings of the paper are robust to using a specification which includes financial variables.

The rest of the paper is organized as follows: Section 2 describes the model and Section 3 describes the data used. Section 4 analyzes the time-varying effects of changes in expectations. Section 5 explores the implications of the model for the changing persistence of the unemployment rate and its correlation with the inflation rate. Section 6 shows the results of alternative model specification which include financial variables. Section 7 discusses the results and Section 8 concludes.

### The Time-Varying Vector Autoregressive Model

We investigate the effects of innovations to expected changes in the unemployment rate by means of a Time Varying coefficient Vector Autoregression (TV-VAR) with stochastic volatility. The model allows both the autoregressive coefficients and the elements of the innovation covariance matrix to drift over time. This statistical model allows us to investigate whether the link between the expectation shocks and the macroeconomy has been changing over time. The model can be summarized as:

$$Y_t = A_{0,t} + A(L)_t Y_{t-1} + \varepsilon_t \quad (1)$$

where  $Y_t$  is the vector of endogenous variables,  $A_{0,t}$  is the vector of time-varying intercepts,  $A(L)$  is a matrix polynomial in the lag operator  $L$  of time-varying coefficients, and  $\varepsilon_t$  is a vector of innovations.

Let  $A_t = [A_{0,t}, A_{1,t}, \dots, A_{l,t}]$  and  $\theta_t = \text{vec}(A'_t)$ , where  $\text{vec}(\cdot)$  is the column stacking operator. The law of motion for  $\theta_t$  is assumed to be:

$$\theta_t = \theta_{t-1} + \omega_t,$$

where  $\omega_t$  is a Gaussian white noise with zero mean and covariance  $\Omega$ .

The innovations in equation (1) are assumed to Gaussian white noises with zero mean and time-varying covariance  $\Sigma_t$  that is factorized as:

$$\Sigma_t = F_t D_t F'_t,$$

where  $F_t$  is lower triangular, with ones on the main diagonal and  $D_t$  a diagonal matrix. Let  $\sigma_t$  be the vector of the diagonal elements of  $D_t^{1/2}$  and the off-diagonal element of the matrix  $F_t^{-1}$ . We assume that the standard deviations,  $\sigma_t$ , evolve as geometric random walks, belonging to the class of models known as stochastic volatility. The contemporaneous relationships  $\phi_{it}$  in each equation of the VAR are assumed to evolve as an independent random walk, leading to the following specifications:

$$\begin{aligned} \log \sigma_t &= \log \sigma_{t-1} + \zeta_t \\ \phi_{it} &= \phi_{it-1} + \varphi_{it} \end{aligned}$$

where  $\zeta_t$  and  $\varphi_{it}$  are Gaussian white noise with zero mean and covariance  $\Xi$  and  $\Psi_i$ , respectively. We assume that  $\varepsilon_t, \omega_t, \zeta_t$ , and  $\varphi_{it}$  are mutually uncorrelated at all leads and lags and that  $\varphi_{it}$  is independent of  $\varphi_{jt}$  for  $i \neq j$ .

## Priors Specification

The model is estimated using Bayesian methods. In this section, we briefly discuss the specification of our priors. While the details of the posterior simulation are accurately described in the Appendix. Following Primiceri (2005), we make the following assumptions for the priors densities. First, the coefficients of the covariances of the log volatilities and the hyperparameters are assumed to be independent of each other. The priors for the initial states,  $\theta_0$ ,  $\phi_0$  and  $\log \sigma_0$ , are assumed to be normally distributed. The priors for the hyperparameters,  $\Omega$ ,  $\Xi$  and  $\Psi$  are assumed to be distributed as independent inverse-Wishart. More precisely, we have the following priors:

- Time varying coefficients:  $P(\theta_0) = N(\hat{\theta}, \hat{V}_\theta)$  and  $P(\Omega) = IW(\Omega_0^{-1}, \rho_1)$ ;
- Diagonal elements:  $P(\log \sigma_0) = N(\log \hat{\sigma}, I_n)$  and  $P(\Psi_i) = IW(\Psi_{0i}^{-1}, \rho_{3i})$ ;
- Off-diagonal elements:  $P(\phi_{i0}) = N(\hat{\phi}_i, \hat{V}_{\phi_i})$  and  $P(\Xi) = IW(\Xi_0^{-1}, \rho_2)$ ;

where the scale matrices are parametrized as follows  $\Omega_0^{-1} = \lambda_1 \rho_1 \hat{V}_\theta$ ,  $\Psi_{0i} = \lambda_{3i} \rho_{3i} \hat{V}_{\phi_i}$  and  $\Xi_0 = \lambda_2 \rho_2 I_n$ . The hyper-parameters are calibrated using a time invariant recursive VAR estimated using a sub-sample consisting of the first  $T_0 = 40$  observations. For the initial states  $\theta_0$  and the contemporaneous relations  $\phi_{i0}$ , we set the means,  $\hat{\theta}$  and  $\hat{\phi}_i$ , and the variances,  $\hat{V}_\theta$  and  $\hat{V}_{\phi_i}$ , at the maximum likelihood point estimates and four times its variance. For the initial states of the log volatilities,  $\log \sigma_0$ , the mean of the distribution is the logarithm of the residuals standard deviation, estimated in a time invariant VAR. The degrees of freedom for the covariance matrix of the drifting coefficient's innovations are set to be equal to  $T_0$ , the size of the initial-sample. The degrees of freedom for the priors on the covariance of the stochastic volatilities' innovations, are set to be equal to the minimum necessary to insure that the prior is proper. In particular,  $\rho_1$  and  $\rho_2$  are equal to the number of rows of  $\Xi_0^{-1}$  and  $\Psi_{0i}^{-1}$  plus one respectively.

The parameters  $\lambda_1$  is important since it controls the degree of time variation in the unobserved states. The smaller the parameter, the smoother and smaller are the changes in coefficients. The empirical literature has set the prior rather conservatively in terms of the amount of time variations. D'Agostino, Gambetti, and Giannone (2013) show that, in a three variables VAR (with unemployment rate, inflation and interest rate), small parameters deliver accurate forecasts.

In this paper, we fix these parameters differently and based on the in-sample accuracy of the fit. Given that the distribution of the fitted values is available at each point in time, we can compute percentiles at each date. Very loose values of  $\lambda_1$  would imply large variance of the coefficients' distribution, hence large variance in the distribution of the fitted values. In this case, the model

would tend to overfit the data: confidence bands around the fitted values would include a high percentage of observed data for any given percentile. The opposite would happen if the parameter  $\lambda_1$  is very tight. Ideally, we would like 1% of the observed data to lie outside 1% confidence bands, 2% to lie outside 2% confidence bands and so on. Therefore the percentage of points included in the bands should lie on the theoretical 45 degree line. Thus, we fix the parameter to the value for which the distance from the theoretical 45 degree line is minimized. The parameter  $\lambda_1$  is fixed to 0.001 in the baseline model which includes expected changes in unemployment rate, unemployment, CPI inflation and the interest rate.<sup>4</sup>

## Data and Identification

We follow Leduc and Sill (2013) in choosing macroeconomic and expectation variables. Our baseline VAR model includes a measure of unemployment expectations (EX-UR), the realized unemployment rate (UR), the inflation rate (CPI) and the short-term interest rate (IR). The model is estimated using quarterly data over the sample 1968:Q4 to 2012:Q3. As a measure of the short-term interest rate, we use the three-month Treasury bill. The unemployment rate is measured by the number of unemployed as a percent of the labor force. Inflation is measured by the annualized quarterly change in the consumer price. These variables have the advantage of not being revised over time with the exception of some minor revisions due to seasonal factors.

Unemployment forecasts are from the Survey of Professional Forecasters (SPF) and are used to measure the expectation formation process of the private sector. The survey, started in 1968, collects predictions from professional forecasters of the unemployment rate (and other variables) and it is conducted quarterly on about 40 to 50 participants.

SPF data are generally collected by the third week of the second month of the quarter, at which point survey respondents do not have information about the unemployment rate or the inflation rate of the same month. For example, in the first quarter of the year, the survey is collected within the first two weeks of February, when forecasters only know January's unemployment and inflation but not February's. Thus, presumably, concurrent economic conditions are not reflected in changes in unemployment expectations. Taking into account the timing of the survey, we redefine the quarters such that the first quarter starts in February, the second in May, the third in August and the fourth in November. Further, the other variables included in the model are aligned to reflect the fact that the information set of the forecasters includes only past values of the other macroeconomic variables. The timing of the survey is consistent with the choice of ordering of the survey variable first in a recursive (i.e. Cholesky) identification scheme, followed by the unemployment rate, CPI inflation and the interest rate as in Leduc and Sill (2013). Thus, innovations to other variables do not have a contemporaneous impact on the expected unemployment rate.

Figure 1 plots the four series of the baseline model. The 6-month-ahead expected unemploy-

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<sup>4</sup>Estimation is performed by discarding the explosive draws.

ment rate is used as a benchmark measure of expectations (first panel). The same figure reports the unemployment rate, the inflation rate and the interest rate. Both the expected and realized unemployment rate are persistent and tend to pick during the recessions. The inflation rate displays pronounced stabilization between the mid-1980s and mid-2000 and larger volatility at the beginning and at the end of the sample. The interest rate shows a flat path at the end of the sample and it is close to the zero lower-bound. The changes in the dynamics of the unemployment and the inflation series as well as the lack of volatility of the interest rate at the end of the sample are fully accounted in the time varying model, which is suitable to describe such situations.

## Baseline Model Results

In this section, we report the main results from the TV-VAR model. However, in order to develop intuition about the dynamics of the model, we first present the response to the expectation shocks implied by the constant parameters VAR. Figure 2 shows the impulse-responses to a negative shock to EX-UR. A negative shock to unemployment expectations leads to a decline in the unemployment rate, an increase in inflation and an increase in the interest rate. Unanticipated downward revisions to expected unemployment generate a macroeconomic boom coupled with monetary policy tightening, as in Leduc and Sill (2013). In the following, we comment on the IRFs and the variance decomposition implied by the TV-VAR described in Section 2.

## Impulse-Responses and Variance Decomposition

Figure 3 (panel A) shows the posterior median of the evolution over time of the dynamic responses of the realized unemployment rate, the inflation rate and the short-term interest rate to an expectation shock. In each plot of the figure, the x-axis ranges over time and the y-axis ranges over the 1 to 10 quarters horizon. The z-axis reports the responses of each variable to the expectation shock at different periods in time. The IRFs are constructed such that the initial shock has a size of one standard deviation at each point in time.

The responses of realized unemployment to an expectation shock display significant time variation. In the first part of the sample, the unemployment rate falls on impact and, after the second quarter, displays a maximum response below 1 per cent. Afterwards, it slowly reverts back to its pre-shock level. The impact and the persistence of the shock increase starting at the beginning of 2000s. The effect of the expectation shocks on the realized unemployment rate is substantially larger during the recent recession: around 2009, shocks to unemployment expectations generate a maximum response in unemployment of above 1.5 per cent.

A shift in expected unemployment leads to hump-shaped responses of the inflation and the interest rate. The responses of the inflation rate to unanticipated downward revisions in expected unemployment also exhibit substantial time variation. At the beginning of the sample, the inflation rate attains a maximum increase. Both the persistence and the impact of the shock decrease during



the 1980s before increasing again through to the mid 2000s. Afterwards, the effect of the shock on inflation is less sizable. Overall, the maximum response ranges from 3 percentage points at the beginning of the sample to about 2 percentage points both at the end of the 1980s and during the most recent recession. In early 2000s, the maximum response is about 2.5 percentage points. The response of the interest rate has much larger persistence than the response of inflation. The interest rate increases on impact and peaks after about 2 quarters to then revert back to its pre-shock level after about one year. The largest responses of the IR to the shock are displayed around year 2000.

To further explore the time-varying effects of shocks to expectations, we compute the contribution of this shock to the overall variance of the other variables in the system. Figure 3 (Panel B) reports such (posterior median) percentage.

Innovations to EXP-UR account for an increasing fraction of the variance of unemployment over time. Over the first part of the sample period, this shock explains about 40 per cent in the short run and 70 per cent in the long run of movements in realized unemployment. At the end of the sample, the same shock accounts for about 50 per cent of the volatility of UR in the short run and above 80 per cent over the long run.

The fraction of the forecast error variance of inflation due to expectation shocks is negligible in the short run. Over the long run, this shock explains about 10 per cent of the variance of inflation until the early 2000s. Afterwards, shocks to EXP-UR account for a less sizable fraction of variations in inflation. The variance share of the interest rate explained by this shock is similar for all horizons larger than 2-quarters ahead. The contribution of the expectation shock to the volatility of the IR is negligible at the beginning of the sample. However, it increases and stabilizes in a range of 50-60 per cent between the beginning of the 1990s and the mid 2000s. Afterwards, it declines and reaches a contribution of about 20 per cent during the recent recession. The contribution of unanticipated shifts in expectations to the IR increases again after year 2009. These results confirm remarkable time variation in the role of unemployment expectation shocks as sources of economic fluctuations.

## **Second Moments**

We now explore the implications of expectation shocks for selected key second order moments of the unemployment rate. We start by looking at the link between unanticipated shifts in unemployment expectations and the rising persistence of unemployment. Further, we explore the implications of unemployment expectation shocks for changes in the relationship between inflation and unemployment.

### **Unemployment Persistence**

As reported in Figure 1, the unemployment rate displayed substantial fluctuations over the last decades. By visual inspection, it is possible to detect longer cycles in the second part of the sample. The unemployment rate has been rising continuously since the end of 2006. It doubled since the

beginning of the recession dated in 2007 Q4 and reached about 9.8 per cent by the end of 2009. After the end of the most recent recession in 2009 Q2, the unemployment rate continued increasing, suggesting a lagging and persistent dynamic. This rising trend continued well after the end of the recession and only rebounded in the beginning of 2010. The recent pattern of US unemployment has been compared by many researchers to the 1980s European experience of high and persistent unemployment as described by Blanchard and Summers (1987) among others.

Turning to the differences in the pattern of unemployment around episodes of economic downturns, Figure 4 focuses on the dynamics of unemployment starting from 4 quarters before the end of the five most recent recessions. We define the recessionary periods as those determined by the NBER dating committee. The pattern of unemployment across different recessions suggests that the pre-1990s recessions featured a sharp increase in unemployment during the downturns and an immediate decline soon after the end of the recession. Indeed, during the recession of the early 1980s, the unemployment rate increased from slightly below 6 percent to about 7.6 per cent, then began declining following the end of the recession. In the 1981-1982 recession, unemployment increased by more than 2 percentage points and reached values above 10 per cent. As in the previous recession, as soon as other indicators of economic activity began to improve, the unemployment rate declined. One year after the end of the recession, the rate returned to levels displayed in the pre-recession period. In contrast, in the most recent recessions, unemployment displays a very slow recovery and a progressive increase in its persistence. The pattern of unemployment during the recession of the early 1990s and early 2000s displays features similar to those experienced during and after the most recent recession. In particular, the unemployment rate stayed high even after the end of the recession and took several quarters to rebound to pre-recession levels.

A variety of factors could have contributed to the rise in the US unemployment persistence over time. In this paper, we also assess to which extent shocks to unemployment expectations have contributed to the increase of unemployment persistence. Figure 5 provides a more structural interpretation of the dynamics of the unemployment rate after a shock to unemployment rate expectations during the five recession periods included in our sample. It reports the average response of unemployment over each recession period (solid line) and the 16<sup>th</sup> and 84<sup>th</sup> percentiles (dotted line). No differences can be detected in the responses of the unemployment rate during the twin recessions of the early 1980s. In contrast, our findings suggest that expectation shocks generated longer-lived effects in unemployment in the post-1990 recessions. The differences are particularly pronounced in the recent recession. In line with the previous findings, the response of unemployment to an expectation shock is remarkably larger and more persistent during the most recent recession, when compared with previous recessionary periods.

As a final step of our analysis we quantify the impact of expectation shocks on unemployment persistence. Similar to Gambetti and Gali (2009), we decompose the time-varying VAR as a time-varying distributed lag model and introduce a new measure of conditional persistence, which can be interpreted as the persistence of unemployment implied by expectation shocks. Essentially, it

is a measure of time-varying autocorrelation conditional to the shock on expectations. Appendix I provides the details of such computations, which are summarized in Equation 15. Figure 6 (left panel) plots the autocorrelation of the unemployment rate implied by the shock to unemployment expectations. The results are in line with the evidence of higher unemployment persistence in the post-1990s recessions. The figure also confirms that starting from the end of the 1980s, the expectation shock implies a gradually increase in the persistence of the unemployment rate, which peaks during the most recent recession. These results, coupled with the increasing importance of shocks to unemployment expectations over time, confirm that shifts in expectations strongly contributed to the rising persistence of unemployment.

All in all, the results in this section provide further evidence of the time-varying nature of the effects of expectation shocks. In particular, we show that such shocks generate higher post-1990 persistence dynamics in the unemployment rate.

### **Inflation-Unemployment Correlation**

Several authors have highlighted that the most recent recession also featured a decline in inflation that was small in comparison with the large and persistent increase in unemployment. Several explanations have been suggested for these changes in inflation developments over time, such as structural changes in the economy and improved monetary policy (Stock and Watson (2010)), a flattening of the slope of the Phillips curve and downward wage rigidity (Ball and Mazumder (2011)), globalization (Borio and Filardo (2007)) and better ‘anchored’ inflation expectations to central bank targets (IMF (2013), Gambetti and Gali (2009)).

In the following section, we analyze if, in response to expectation shocks, the sensitivity of inflation to unemployment has changed over time and, most importantly, if it declined during the most recent recession. To address this issue, we compute measures of conditional correlation and conditional covariance as described in the Appendix I, equations 16 and 12 respectively. Figures 6 (right panel) and Figure 7 plot these statistics; in particular Figure 6 (right panel) displays the correlation between the two variables conditional on a shock to expectation, whereas Figure 7 displays the conditional standard deviation of unemployment rate (left panel), the conditional standard deviation of CPI inflation (mid panel) and the conditional covariance between the two variables (right panel).

All moments display substantial changes over time. In addition, our results confirm a decline in the sensitivity of inflation to unemployment over time. As shown in Figure 6, the conditional correlation between the inflation and the unemployment rate starts declining in the early 1990s and reaches a minimum during the most recent recession. A similar pattern is observed in the standard deviations of the two variables as reported in Figure 7. After an initial reduction and a low volatility period beginning in the early 1980s (the “great moderation”), the volatility of both the unemployment rate and the inflation rate increases again at the beginning of the 2000s and it peaks in 2009.

Remarkably, changes in the standard deviations of the unemployment and the inflation rate are reflected in the decline of the correlation between the two variables. Notice that, the conditional covariance between the two series has been relatively low and stable for many years, as many authors have pointed out, and it increases only in the most recent part of the sample. Thus, the reduction in the correlation between inflation and unemployment is largely due to changes in the volatility of the two variables.

Notice that the increase in the standard deviations of the unemployment and the inflation rate mimic the changes over time in the pattern of the responses of the two variables to expectation shocks. In particular, as reported in Section 4, unemployment expectation shocks account for an increasing fraction of the volatility of the unemployment rate over time. Our results suggest that changes in the effects of shocks to unemployment expectations also contributed to the decline in the correlation between inflation and unemployment.

### **Robustness: Adding Financial Variables**

According to the results presented in Section 4, shifts in expectations are relevant sources of macroeconomic fluctuations. The role of expectation shocks in moving realized unemployment increases in the second part of the sample and is particularly large in the recent recession. The last two business cycles in the U.S. have been characterized by coincident booms in economic activity and asset prices. In particular, during the late 1990s, the US economy experienced a dramatic rise in stock prices, whereas during the mid 2000s, a sustained run-up was displayed in house prices. Both periods of expansions were followed by sudden falls in asset prices and economic downturns.

In order to provide robust evidence regarding the real effects of expectation shocks, we estimate VAR models with measures of asset prices. The first and second rows of Figure 8 report the responses to downward shifts in unemployment expectations for the model that includes stock returns as measured by the S&P 500 and real house prices. In response to unanticipated downward revisions to unemployment expectations, both measures of asset prices rise on impact and then rapidly decline. In particular, the first period response of stock prices sizably increases over time.

As a final check, we also take into account the evolution of the credit market. Figure 8 (bottom panel) reports the results of a VAR model which includes mortgage credit. The response on impact is somewhat negative, though not significant, whereas the maximum response of the variable to downward revisions in unemployment expectations is positive and increasing over time. These findings confirm time-variation in the effects of expectation shocks starting from the early 2000s. It is important to notice that the effects of downward revisions in the expected unemployment rate on the realized unemployment rate and the interest rate are largely unchanged by the introduction of financial variables.

## Discussion

The evidence reported in this paper suggests that unanticipated changes in unemployment expectations significantly contribute to economic fluctuations in the US. Changes in expected future economic activity display substantial variation over time with larger effects beginning in the early 2000s and particularly marked during the most recent recession.

Several authors have highlighted the important changes in the dynamics of unemployment and inflation over the last decade.<sup>5</sup> A variety of explanations have been proposed to rationalize these changes. In recent papers, Coibion and Gorodnichenko (2013) and Coibion, Gorodnichenko, and Koustas (2013) study the two phenomena separately and conclude that commonly suggested explanations do not fully account for changes in the inflation and unemployment rate. In particular, Coibion, Gorodnichenko, and Koustas (2013) test a wide range of economic, demographic and cultural factors that could have influenced the dynamics of unemployment. Contrary to the conventional wisdom, they find that financial shocks and wage stickiness do not contribute to the rising persistence of U.S. unemployment. The authors also argue that monetary and fiscal policies explain only part of the developments in unemployment during the most recent recession. Changes in U.S. labor mobility and demographic factors predict a decline in the persistence of unemployment, whereas the decline in "trust among Americans" has a statistically significant, although limited, impact on the persistence of unemployment. Regarding the missing disinflation during the most recent recession, Coibion and Gorodnichenko (2013) consider three explanations in the context of the Phillips curve: changes in the natural rate of unemployment, unusual wage dynamics and marginal costs, and changes in the slope of the Phillips curve. They conclude that none of these channels can fully account for the changes in the relationship between inflation and unemployment, while changes in consumer expectations are able to rationalize the missing disinflation. The results presented by the authors suggest that more attention should be paid to expectations. Overall, the message of these papers is that traditional channels may have had a limited role in explaining changes in inflation and unemployment over the last decade.

In this paper, we propose an alternative explanation of the changing dynamics of unemployment and inflation in the US. Our results show that changes in the second moments of unemployment and CPI inflation are linked to a shift in the response of the economy to expectation shocks. Indeed, shocks to unemployment expectations display a gradually larger and more persistent effect on the unemployment rate during the most recent recessions. Accordingly, changes in the autocorrelation of the unemployment rate, conditional to expectation shocks, are in line with evidence of higher unemployment persistence over time. Changes in the macroeconomic effects of expectation shocks also have nontrivial implications on the correlation between inflation and unemployment.

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<sup>5</sup>Other authors also highlighted interesting changes in other variables such as the cyclical of labor productivity (Garin, Pries, and Sims (2013)), the share of labor income (Karabarbounis and Neiman (2013)), and the demand for skills (Jaimovich and Siu (2012)). Due to the limited number of observables in the TV-VAR model, we restrict our analysis to changes in the behaviour of inflation and the unemployment rate.

Our findings capture the effects of shifts in expectations linked to the recent cycles in economic activity. The "Dot-com" boom of the late 1990s is put forth by many as an example of expectations-driven cycles. Developments in the satellite industry and the booming of the IT economy generated expectations of prosperous future economic prospects. We confirm that changes in expectations about future economic activity might have contributed to the economic boom observed in that period and to its sudden bust. See e.g. Beaudry and Portier (2013).

We find that the effects of unanticipated shifts in expectations on the unemployment rate are particularly large and long-lived during the 2007-2009 economic downturn. The most recent recession differs from previous ones mainly due to the fact that it followed the incipit of the global financial crises. The occurrence of a deep financial crises and the resulting increase in uncertainty could have raised the risk of pessimism and, thus, larger changes in unemployment expectations. This could have originated self-reinforcing feedback loop between unemployment expectations and economic activity that contributed to exacerbate the recession and further dampened the following recovery. In general, our results confirm that changes in the macroeconomic performance of the U.S. economy cannot be fully accounted for by factors that abstract from the role of changes in expectations.

## Conclusion

This paper provides new evidence on expectation-driven cycles by estimating a structural VAR with time-varying coefficients and stochastic volatility. We use unemployment expectations as compiled by the survey of professional forecasters to measure expectations of future developments in economic activity. Given the changes in the unemployment rate dynamics over the last decades, allowing for time variation in quantifying the role of expectation shocks in shaping the dynamics of the unemployment rate seems a reasonable choice. To the best of our knowledge, no other studies have investigated the time-varying effects of changes in expectations as a source of business cycle fluctuations.

Our results indicate that unanticipated shifts to expected unemployment are relevant sources of economic fluctuations. We detect significant changes to unemployment in response to expectation shocks beginning in early 2000s. The effects of the expectation shocks on economic activity are particularly pronounced around the time of the most recent recession. Unanticipated changes in expectations contributed to the gradual increase in the persistence of the unemployment rate and to the decline in the correlation between inflation and unemployment since the 2000s. Our results are robust to the introduction of financial variables in the model.

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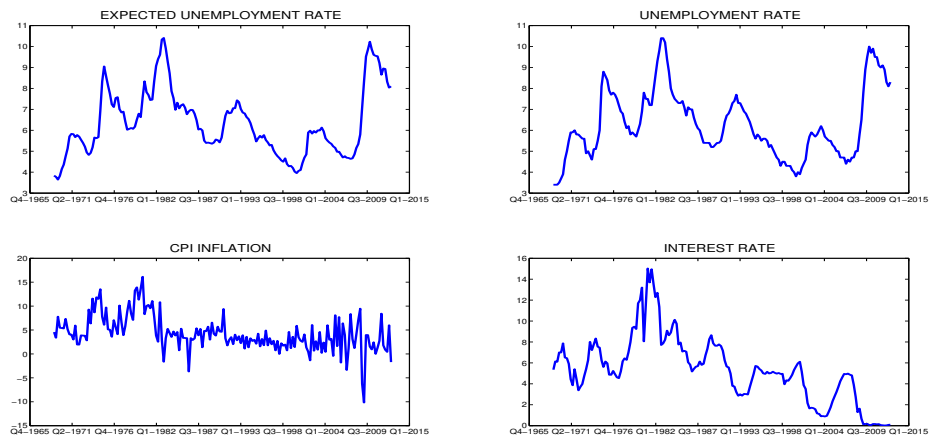
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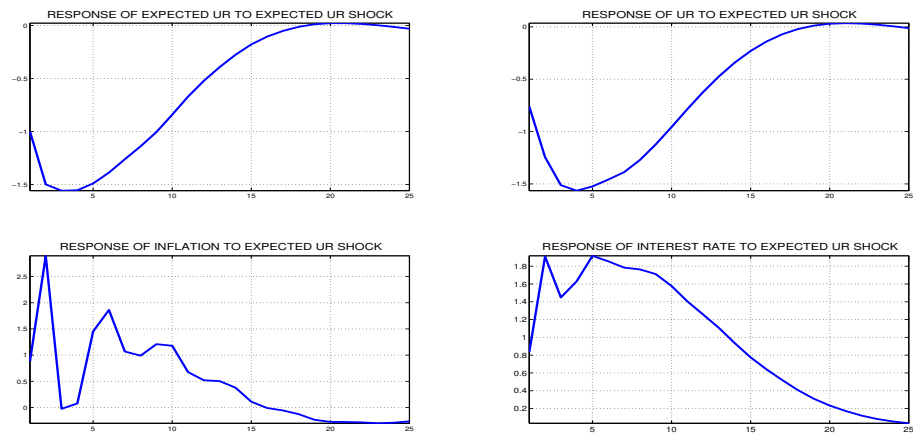
# Figures

Figure 1: *Expected UR, UR, CPI-Inflation 3-months Treasury Bills*



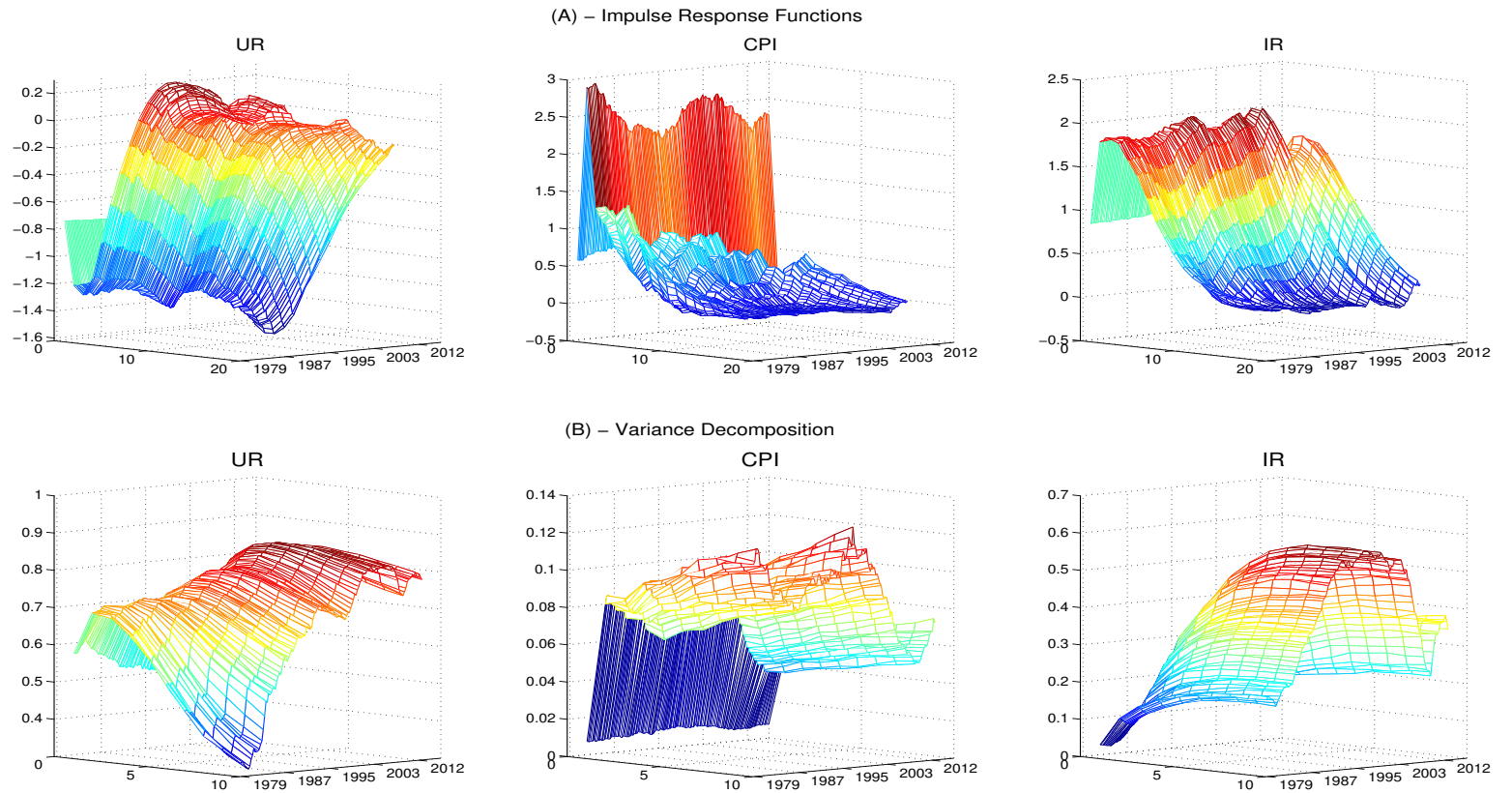
Note - The four variables of the baseline VAR.

Figure 2: *IRF constant parameters VAR; Negative shock to unemployment expectations. Baseline four variables VAR model.*



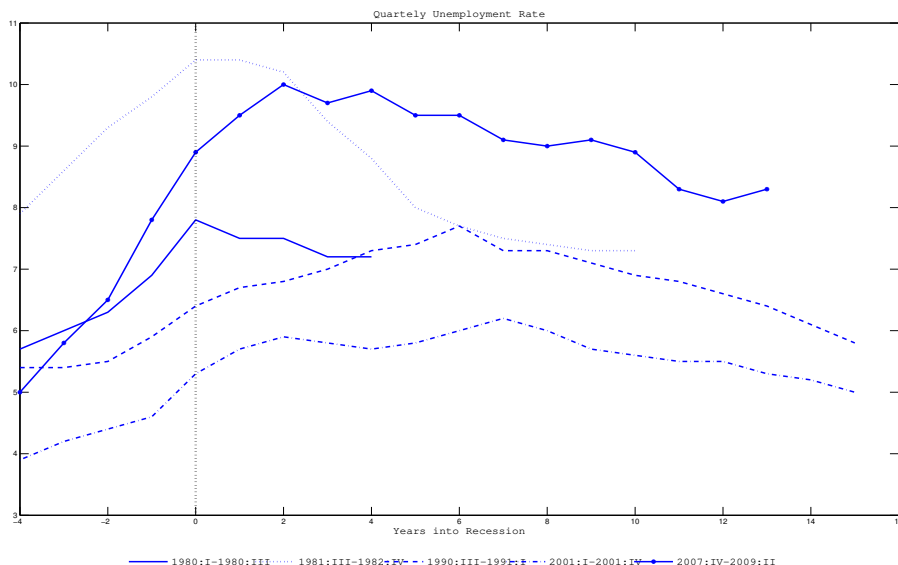
Note - Impulse response function from a VAR model with constant coefficients. Shock to unemployment expectations.

Figure 3: *IRF (panel A) and variance decomposition (panel B): shock to unemployment expectations - baseline VAR*



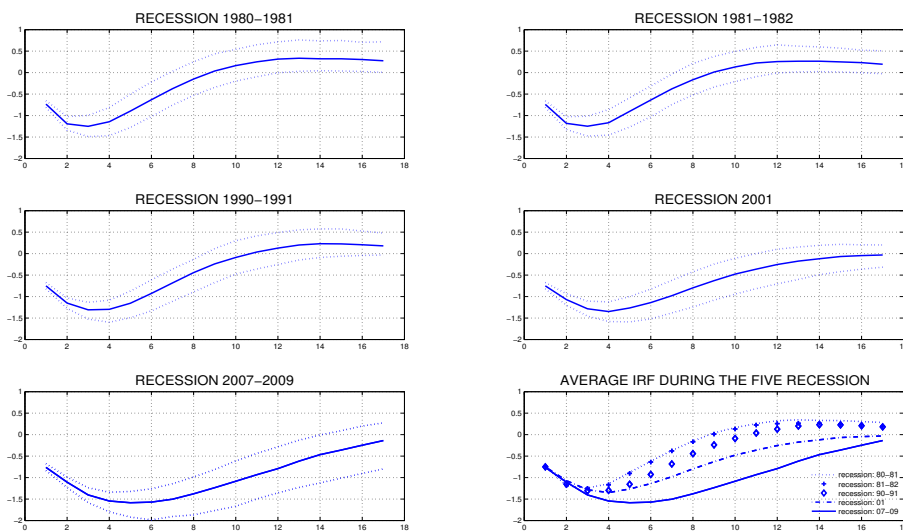
Note - Panel A: Negative shock to unemployment expectations. Impulse response functions at different horizons (y-axis) and over time (x-axis). Panel B: percentage of variance explained by the unemployment expectation shock at different horizons (y-axis) and over time (x-axis). Results are from the baseline four variables VAR model (UR expectations, UR, CPI inflation and IR).

Figure 4: *The dynamic of unemployment rate during and after recessions*



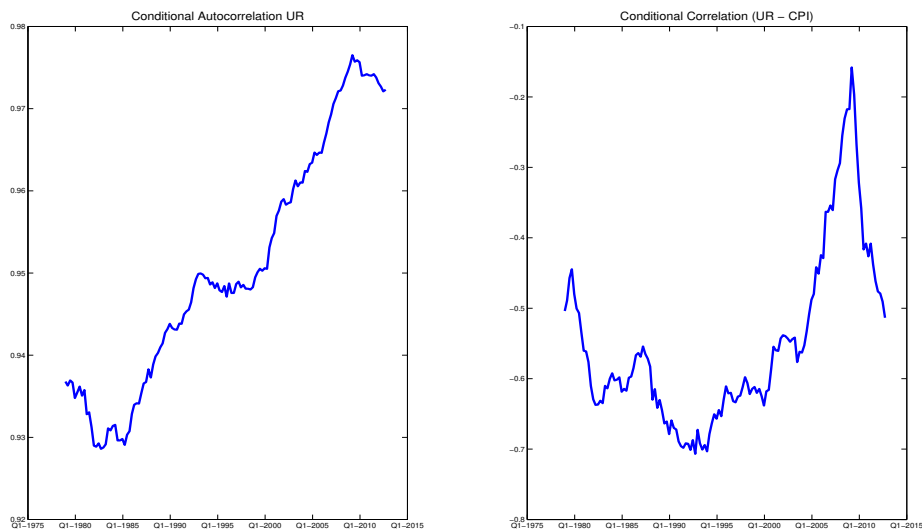
Note - The figure shows the evolution of unemployment rate from one year before the end of the recession (vertical line crossin zero) to some quarters after the end of the recessions. Unemployment rate dynamics are reported for the five recessions from 1980.

Figure 5: *Average IRFs during five recessions*



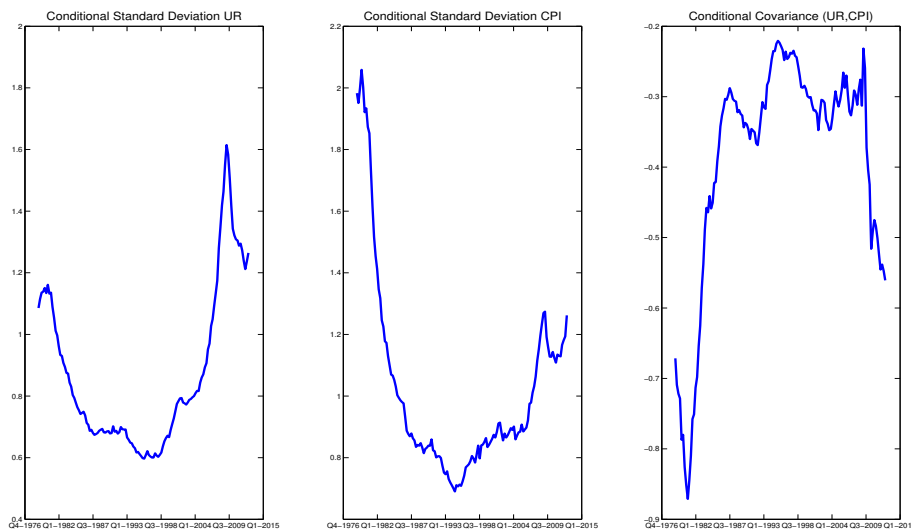
Note - Mean response of unemployment rate to a shock to unemployment rate expectations during five recessions and respective bands (16th and 84th percentile).

Figure 6: *Conditional moments: autocorrelation and correlation*



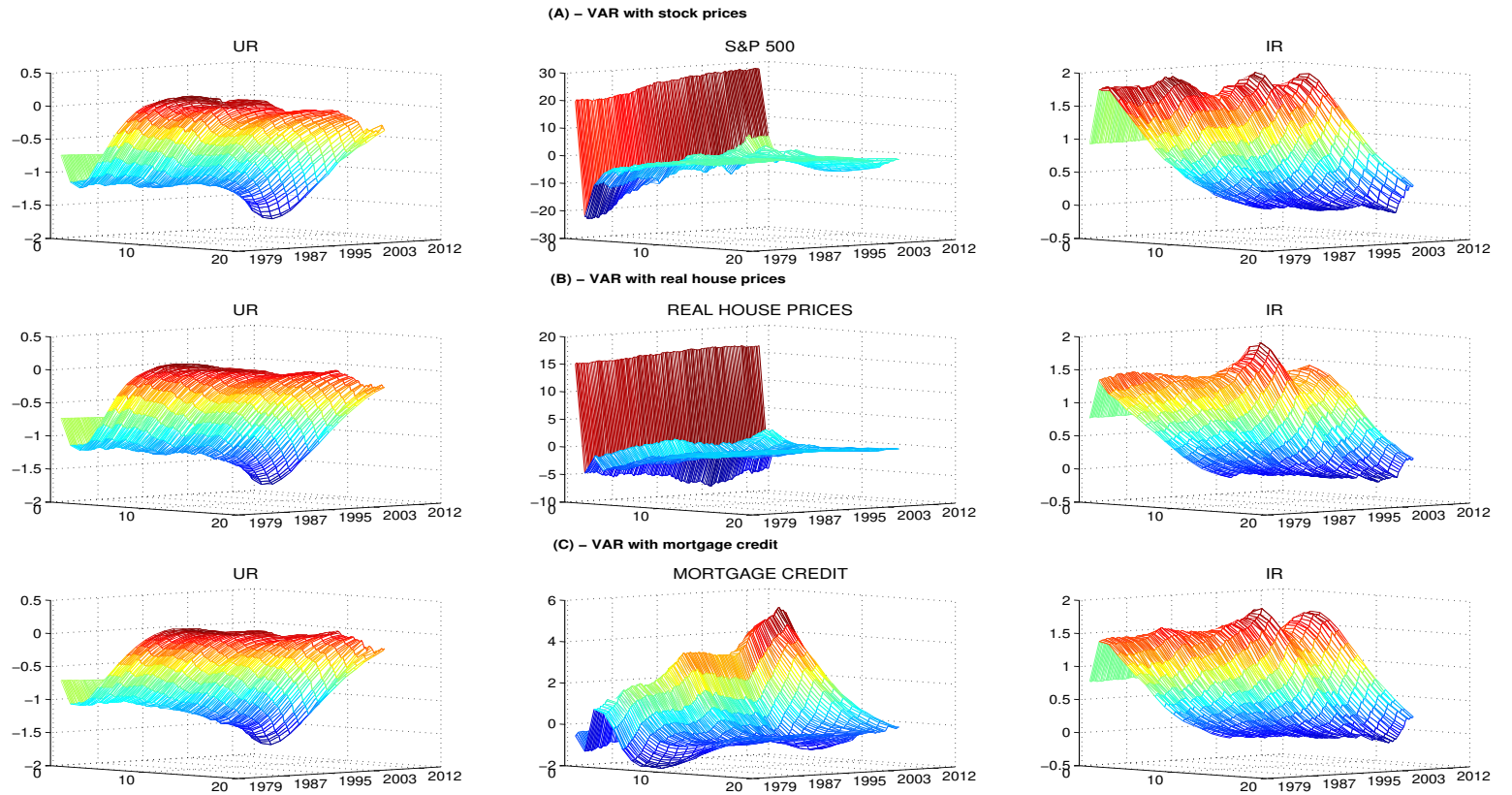
Note - Left panel: autocorrelation of UR conditional to the expectation shock. Right panel: correlation between UR and CPI inflation conditional to the expectation shock.

Figure 7: *Conditional moments: standard deviation and covariance*



Note - Left panel: standard deviation of UR conditional to the expectation shock. Middle panel: standard deviation of CPI inflation conditional to the expectation shock. Right panel: covariance between UR and CPI inflation conditional to the expectation shock.

Figure 8: *Impulse Response Functions in different VAR models: shock to UR expectations - response of UR*



Note - Alternative VAR specifications. Panel (A) Shock to unemployment expectations. Impulse response functions of UR, S&P 500 and IR. Panel (B) Shock to unemployment expectations. Impulse response functions of UR, Real House Prices and IR. Panel (C) Shock to unemployment expectations. Impulse response functions of UR, Mortgage Credit and IR.

## Appendix 1: Conditional Statistics

To derive the conditional statistics we rewrite equation(1) in companion form:

$$\mathbf{x}_t = \mu_t + \mathbf{A}_t \mathbf{x}_{t-1} + \epsilon_t \quad (2)$$

where  $\mathbf{x}_t \equiv [\mathbf{x}'_t, \mathbf{x}'_{t-1}, \dots, \mathbf{x}'_{t-p+1}]'$ ,  $\epsilon_t \equiv [\epsilon'_t, 0, \dots, 0]'$ ,  $\mu_t \equiv [\mathbf{A}'_{0,t}, 0, \dots, 0]'$  and  $\mathbf{A}_t$  is the companion matrix. Equation(2) can be rewritten as:

$$(I - \mathbf{A}_t L) \mathbf{x}_t = \mu_t + \epsilon_t \quad (3)$$

by inverting the term  $(I - \mathbf{A}_t L)$  on the left-hand side, we can derive the corresponding moving average representation:

$$\mathbf{x}_t = \eta_t + F_{t,0} \epsilon_t + F_{t,1} \epsilon_{t-1} + F_{t,2} \epsilon_{t-2} + \dots \quad (4)$$

where  $\eta_t = (I - \mathbf{A}_t L)^{-1} \mu_t$ ,  $\sum_{i=0}^{\infty} F_{t,i} \epsilon_{t-i} = (I - \mathbf{A}_t L)^{-1} \epsilon_t$  and  $F_{t,0} = I$ . We assume that the innovations  $\epsilon_t$  are a linear combination of orthogonal structural disturbances  $u_t$ , i.e.

$$\epsilon_t = K u_t \quad (5)$$

Equation 4 can be written in terms of orthogonal structural shocks as:

$$\mathbf{x}_t = \eta_t + \sum_{i=0}^{\infty} C_{t,i} u_{t-i} \quad (6)$$

where  $F_{t,i} K \equiv C_{t,i}$  for  $i = 0, 1, 2, \dots$ . For a single variable  $j$  and in particular for the variables baseline VAR it is:

$$x_{j,t} = \eta_{j,t} + \sum_{i=0}^{\infty} C_{t,i}^{j,ex} u_{t-i}^{ex} + \sum_{k=2}^4 \sum_{i=0}^{\infty} C_{t,i}^{j,k} u_{t-i}^k \quad (7)$$

Variable  $x_{j,t}$  is then written as a time-varying distributed lag model in four orthogonal shocks. The first one,  $u_t^{ex}$ , is the structural expectation shock, while the remaining three,  $u_t^k$   $k = 2, 3, 4$ , are orthogonal non-identified shocks. Given equation (7) it is straightforward to define second and cross-moments:

### Variance

$$var(x_{j,t}) = \sum_{i=0}^{\infty} (C_{t,i}^{j,ex})^2 + \sum_{k=2}^4 \sum_{i=0}^{\infty} (C_{t,i}^{j,k})^2 \quad (8)$$

### Covariance

$$cov(x_{j,t}, x_{s,t}) = \sum_{i=0}^{\infty} C_{t,i}^{j,ex} C_{t,i}^{s,ex} + \sum_{k=2}^4 \sum_{i=0}^{\infty} C_{t,i}^{j,k} C_{t,i}^{s,k} \quad (9)$$

## Autocovariance

$$cov(x_{j,t}x_{j,t-1}) = \sum_{i=0}^{\infty} C_{t,i+1}^{j,ex} C_{t-1,i}^{j,ex} + \sum_{k=2}^4 \sum_{i=0}^{\infty} C_{t,i+1}^{j,k} C_{t,i}^{j,k} \quad (10)$$

Similarly, other moments conditional to the expectation shock can be defined as:

## Conditional Variance

$$var(x_{j,t}|u_t^{ex}) = \sum_{i=0}^{\infty} (C_{t,i}^{j,ex})^2 \quad (11)$$

## Conditional Covariance

$$cov(x_{j,t}, x_{s,t}|u_t^{ex}) = \sum_{i=0}^{\infty} C_{t,i}^{j,ex} C_{t,i}^{s,ex} \quad (12)$$

## Conditional Autocovariance

$$cov(x_{j,t}x_{j,t-1}|u_t^{ex}) = \sum_{i=0}^{\infty} C_{t,i+1}^{j,ex} C_{t,i}^{j,ex} \quad (13)$$

Autocorrelation and conditional autocorrelation are defined respectively as:

$$corr(x_{j,t}) = \frac{cov(x_{j,t}x_{j,t-1})}{var(x_{j,t})} \quad (14)$$

and

$$corr(x_{j,t}|u_t^{ex}) = \frac{cov(x_{j,t}x_{j,t-1}|u_t^{ex})}{var(x_{j,t}|u_t^{ex})} \quad (15)$$

while correlation and conditional correlation as:

$$corr(x_{j,t}x_{s,t}) = \frac{cov(x_{j,t}x_{s,t})}{var(x_{j,t})^{(\frac{1}{2})} var(x_{s,t})^{(\frac{1}{2})}} \quad (16)$$

and

$$corr(x_{j,t}x_{s,t}|u_t^{ex}) = \frac{cov(x_{j,t}x_{s,t}|u_t^{ex})}{var(x_{j,t}|u_t^{ex})^{(\frac{1}{2})} var(x_{s,t}|u_t^{ex})^{(\frac{1}{2})}} \quad (17)$$

## Appendix 2: the bayesian algorithm

Estimation is done using Bayesian methods. To draw from the joint posterior distribution of model parameters we use a Gibbs sampling algorithm along the lines described in Primiceri (2005). The basic idea of the algorithm is to draw sets of coefficients from known conditional posterior distributions. The algorithm is initialized at some values and, under some regularity conditions, the draws converge to a draw from the joint posterior after a burn in period. Let  $z$  be  $(q \times 1)$  vector, we denote  $z^T$  the sequence  $[z'_1, \dots, z'_T]'$ . Each repetition is composed of the following steps:

1.  $p(s^T | y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)^6$
2.  $p(\sigma^T | y^T, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T)$
3.  $p(\phi^T | y^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi, s^T)$
4.  $p(\theta^T | y^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi, s^T)$
5.  $p(\Omega | y^T, \theta^T, \sigma^T, \phi^T, \Xi, \Psi, s^T)$
6.  $p(\Xi | y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Psi, s^T)$
7.  $p(\Psi | y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, s^T)$

### Gibbs sampling algorithm

- Step 1: sample from  $p(s^T | y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$

Conditional on  $y_{i,t}^{**}$  and  $r^T$ , we independently sample each  $s_{i,t}$  from the discrete density defined by  $Pr(s_{i,t} = j | y_{i,t}^{**}, r_{i,t}) \propto f_N(y_{i,t}^{**} | 2r_{i,t} + m_j - 1.2704, v_j^2)$ , where  $f_N(y | \mu, \sigma^2)$  denotes a normal density with mean  $\mu$  and variance  $\sigma^2$ .

- Step 2: sample from  $p(\sigma^T | y^T, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T)$

To draw  $\sigma^T$  we use the algorithm of Kim, Shephard and Chibb (KSC) (1998). Consider the system of equations  $y_t^* \equiv F_t^{-1}(y_t - X_t' \theta_t) = D_t^{1/2} u_t$ , where  $u_t \sim N(0, I)$ ,  $X_t = (I_n \otimes x_t')$ , and  $x_t = [1_n, y_{t-1} \dots y_{t-p}]$ . Conditional on  $y^T, \theta^T$ , and  $\phi^T$ ,  $y_t^*$  is observable. Squaring and taking the logarithm, we obtain

$$y_t^{**} = 2r_t + v_t \tag{18}$$

$$r_t = r_{t-1} + \xi_t \tag{19}$$

where  $y_{i,t}^{**} = \log((y_{i,t}^*)^2 + 0.001)$  - the constant (0.001) is added to make estimation more robust -  $v_{i,t} = \log(u_{i,t}^2)$  and  $r_t = \log \sigma_{i,t}$ . Since, the innovation in (18) is distributed as  $\log \chi^2(1)$ , we use, following KSC, a mixture of 7 normal densities with component probabilities  $q_j$ , means  $m_j - 1.2704$ , and variances  $v_j^2$  ( $j=1, \dots, 7$ ) to transform the system in a Gaussian one, where  $\{q_j, m_j, v_j^2\}$  are chosen to match the moments of the  $\log \chi^2(1)$  distribution. The values are:

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<sup>6</sup>See below the definition of  $s^T$ .



Table A1: Parameters Specification

$j$	$q_j$	$m_j$	$v_j^2$
1.0000	0.0073	-10.1300	5.7960
2.0000	0.1056	-3.9728	2.6137
3.0000	0.0000	-8.5669	5.1795
4.0000	0.0440	2.7779	0.1674
5.0000	0.3400	0.6194	0.6401
6.0000	0.2457	1.7952	0.3402
7.0000	0.2575	-1.0882	1.2626

Let  $s^T = [s_1, \dots, s_T]'$  be a matrix of indicators selecting the member of the mixture to be used for each element of  $v_t$  at each point in time. Conditional on  $s^T$ ,  $(v_{i,t}|s_{i,t} = j) \sim N(m_j - 1.2704, v_j^2)$ . Therefore we can use the algorithm of Carter and R.Kohn (1994) to draw  $r_t$  ( $t=1, \dots, T$ ) from  $N(r_t|r_{t+1}, R_t|r_{t+1})$ , where  $r_t|r_{t+1} = E(r_t|r_{t+1}, y^t, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T, )$  and  $R_t|r_{t+1} = Var(r_t|r_{t+1}, y^t, \theta^T, \phi^T, \Omega, \Xi, \Psi, s^T)$ .

- Step 3: sample from  $p(\phi^T|y^T, \theta^T, \sigma^T, \Omega, \Xi, \Psi, s^T)$

Consider again the system of equations  $F_t^{-1}(y_t - X_t'\theta_t) = F_t^{-1}\hat{y}_t = D_t^{1/2}u_t$ . Conditional on  $\theta^T$ ,  $\hat{y}_t$  is observable. Since  $F_t^{-1}$  is lower triangular with ones in the main diagonal, each equation in the above system can be written as

$$\hat{y}_{1,t} = \sigma_{1,t}u_{1,t} \quad (20)$$

$$\hat{y}_{i,t} = -\hat{y}_{[1,i-1],t}\phi_{i,t} + \sigma_{i,t}u_{i,t} \quad i = 2, \dots, n \quad (21)$$

where  $\sigma_{i,t}$  and  $u_{i,t}$  are the  $i$ th elements of  $\sigma_t$  and  $u_t$  respectively,  $\hat{y}_{[1,i-1],t} = [\hat{y}_{1,t}, \dots, \hat{y}_{i-1,t}]$ . Under the block diagonality of  $\Psi$ , the algorithm of Carter and R.Kohn (1994) can be applied equation by equation, obtaining draws for  $\phi_{i,t}$  from a  $N(\phi_{i,t}|t+1, \Phi_{i,t}|t+1)$ , where  $\phi_{i,t}|t+1 = E(\phi_{i,t}|\phi_{i,t+1}, y^t, \theta^T, \sigma^T, \Omega, \Xi, \Psi)$  and  $\Phi_{i,t}|t+1 = Var(\phi_{i,t}|\phi_{i,t+1}, y^t, \theta^T, \sigma^T, \Omega, \Xi, \Psi)$ .

- Step 4: sample from  $p(\theta^T|y^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi, s^T)$

Conditional on all other parameters and the observables we have

$$y_t = X_t'\theta_t + \varepsilon_t \quad (22)$$

$$\theta_t = \theta_{t-1} + \omega_t \quad (23)$$

Draws for  $\theta_t$  can be obtained from a  $N(\theta_t|t+1, P_t|t+1)$ , where  $\theta_t|t+1 = E(\theta_t|\theta_{t+1}, y^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$  and  $P_t|t+1 = Var(\theta_t|\theta_{t+1}, y^T, \sigma^T, \phi^T, \Omega, \Xi, \Psi)$  are obtained with the algorithm of Carter and R.Kohn (1994).

- Step 5: sample from  $p(\Omega|y^T, \theta^T, \sigma^T, \phi^T, \Xi, \Psi, s^T)$

Conditional on the other coefficients and the data,  $\Omega$  has an Inverse-Wishart posterior density with scale matrix  $\Omega_1^{-1} = (\Omega_0 + \sum_{t=1}^T \Delta\theta_t(\Delta\theta_t)')^{-1}$  and degrees of freedom  $df_{\Omega_1} = df_{\Omega_0} + T$ , where  $\Omega_0^{-1}$  is the prior scale matrix,  $df_{\Omega_0}$  are the prior degrees of freedom and  $T$  is length of the sample

use for estimation. To draw a realization for  $\Omega$  make  $df_{\Omega_1}$  independent draws  $z_i$  ( $i=1, \dots, df_{\Omega_1}$ ) from  $N(0, \Omega_1^{-1})$  and compute  $\Omega = (\sum_{i=1}^{df_{\Omega_1}} z_i z_i')^{-1}$  (see Gelman et. al., 1995).

- Step 6: sample from  $p(\Xi_{i,i}|y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Psi, s^T)$

Conditional the other coefficients and the data,  $\Xi$  has an Inverse-Wishart posterior density with scale matrix  $\Xi_1^{-1} = (\Xi_0 + \sum_{t=1}^T \Delta \log \sigma_t (\Delta \log \sigma_t)')^{-1}$  and degrees of freedom  $df_{\Xi_1} = df_{\Xi_0} + T$  where  $\Xi_0^{-1}$  is the prior scale matrix and  $df_{\Xi_0}$  the prior degrees of freedom. Draws are obtained as in step 5.

- Step 7: sample from  $p(\Psi|y^T, \theta^T, \sigma^T, \phi^T, \Omega, \Xi, s^T)$ .

Conditional on the other coefficients and the data,  $\Psi_i$  has an Inverse-Wishart posterior density with scale matrix  $\Psi_{i,1}^{-1} = (\Psi_{i,0} + \sum_{t=1}^T \Delta \phi_{i,t} (\Delta \phi_{i,t})')^{-1}$  and degrees of freedom  $df_{\Psi_{i,1}} = df_{\Psi_{i,0}} + T$  where  $\Psi_{i,0}^{-1}$  is the prior scale matrix and  $df_{\Psi_{i,0}}$  the prior degrees of freedom. Draws are obtained as in step 5 for all  $i$ .

The estimations are performed with 12000 repetitions discarding the first 10000 and collecting one out of five draws.