Optimal Unemployment Insurance in Labor Market Equilibrium when Workers can Self-Insure

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Abstract

I develop an equilibrium matching model in which workers have preferences over consumption and hours of work and are able to self-insure against unemployment risks by accumulating precautionary wealth. Wages and working hours are the outcomes of Nash bargaining between workers and firms. I focus on an unemployment insurance (UI) system with constant benefits of indefinite duration financed through a constant labor income tax. Low-wealth individuals work unusually long hours to quickly accumulate precautionary wealth. The Frisch elasticity of labor supply governs a worker’s utility cost of supplying labor and hence the cost of accumulating precautionary wealth. A lower elasticity implies a higher utility cost of adjusting hours. I take Frisch elasticities from recent research using household data and find that the optimal level of UI benefits is between 34 and 40 percent of average compensation. The potential welfare gains from moving from current 34 percent to the optimal policy are as large as 0.13 percent of lifetime consumption. The optimal replacement rate is decreasing in the Frisch elasticity of labor supply.

*Email: felixr [at] gmail.com. A web Appendix is available at [http://felixr.googlepages.com/webapp.pdf](http://felixr.googlepages.com/webapp.pdf). I am grateful to Bob Hall for his generous support and guidance throughout this project. I have also benefitted from discussions with Pete Klenow, Michèle Tertilt, Anders Frederiksen, Sri Nagavarapu, Masaki Nakabayashi, Alejandro Ponce-Rodriguez, Todd Schoellman, and participants of the Stanford Macro Lunch and the Stanford Labor Reading Group. I thank Ken Judd and Ben Malin for sharing their knowledge of computational methods.
1 Introduction

Workers can self-insure to avoid a sharp drop in consumption during spells of unemployment by accumulating precautionary wealth when they work. Self-insurance is an important, largely unexplored, determinant of the welfare consequences of government-provided unemployment insurance (UI). The level of precautionary wealth balances the benefits of consumption-smoothing against the loss of satisfaction from working extra hours to accumulate the wealth. I determine the optimal UI replacement rate in a dynamic equilibrium matching model in which hours of work are determined through bilateral bargaining between workers and firms, and borrowing-constrained workers may save a risk-free asset. I find that the optimal UI replacement rate is increasing in workers’ dislike for work as measured by the Frisch elasticity of labor supply and in their degree of impatience as measured by their personal discount rate.

My approach advances earlier research in several ways. I build on Shimer and Werning’s (2005) analysis and assume that the optimal UI policy consists of a constant benefit payment of indefinite duration and a constant tax rate upon reemployment that is independent of the duration of the previous unemployment spell. Most earlier analysis of UI is in a partial equilibrium framework. I build a model based on the Mortensen and Pissarides (1994) (MP) matching model which describes a full equilibrium of the labor market. One advantage of using the MP model is that the behavior of the employed is explicitly modeled, unlike, for example, in McCall’s (1970) partial equilibrium model. This is important because the level of the UI replacement rate will not only affect the welfare of the unemployed, but it also affects employed workers’ needs and incentives to self-insure. Another appealing feature of the MP model is that wages are determined through bilateral bargaining between a worker and a firm. This provides a convenient framework for the determination of working hours, but it is also important because workers’ wealth levels determine their outside option during the bargain, the value of being unemployed. Thus, as workers accumulate wealth, they bargain for higher wages and lower hours of work. Third, the role of individuals’ costs to self-insure in the determination of the optimal policy has been largely unexplored. To my knowledge, Lentz (2005) was the first to note that the optimal replacement rate is decreasing in the rate of return workers can achieve on their savings.

When I calibrate my model to match certain facts of the U.S. labor market, I find that the optimal replacement rate is between 34 and 40 percent of average after-tax compensation, depending on the social welfare function I consider. These results suggest that about half of the states in the U.S. have replacement rates that are close to optimal.

Interestingly, when I do not allow workers to adjust hours, the optimal replacement rates are lower than when hours are set efficiently. Low-wealth workers’ utility increases considerably with
higher UI benefits because they are able to increase consumption and decrease hours of work considerably. When workers are not allowed to adjust hours, the benefit of a higher replacement rate decreases, while the cost, an increase in the unemployment rate, stays the same. Hence the optimal replacement rate is lower to balance the marginal cost with the marginal benefit to the worker.

Another interesting result is that the optimal replacement rate is increasing in the average duration of unemployment. This suggests that the higher observed benefit rates in Europe may not only be the cause of their unemployment problem, but may also be a result of their unemployment problem.

My results suggest that not only do labor market characteristics and risk-aversion play important roles in determining the optimal replacement rate, but that preferences over working hours and properties of the asset market also play important roles.

2 Related literature

The rise in unemployment during the 1970s increased economists’ interest to study the interaction between UI and labor market outcomes. While earlier research focused mostly on the income support aspects of UI, much of the empirical and theoretical work since the 1970s has focused on the incentive aspects of UI. A large empirical literature aimed at estimating the elasticity of unemployment duration with respect to UI benefits, a measure of the degree to which UI benefit levels affect workers’ incentives to search for jobs.

The theory that motivated this empirical work goes back to at least Ehrenberg and Oaxaca (1976) and Mortensen (1977). This line of research established that UI policy is tightly linked to reservation wages and unemployment durations. Atkinson (1987) provides a thorough review of the early empirical studies on the incentive effects of UI, while Atkinson and Micklewright (1991) and Krueger and Meyer (2002) review the later literature. Although the early research produced mixed results, more recent research indicates that the incentive effects are modest. Krueger and Meyer argue that an elasticity of unemployment duration with respect to benefits of 0.5 is a reasonable summary of the literature. Fredriksson and Holmlund (2006) provide the most recent survey on unemployment insurance and incentive effects.

In recent papers, Chetty (2005) and Card, Chetty and Weber (2006) argue that the effect of UI benefits on durations may be largely due to a non-distortionary income effect for individuals who face borrowing constraints. Because UI benefits are transitory, Chetty (2005) argues, they affect search behavior mostly through income effects, so that the efficiency costs of unemployment
insurance are smaller than widely believed.

Much of the literature on the normative issues of UI design can be grouped into two categories according to how the labor market is modeled. The two most popular modeling choices include variations of McCall’s (1970) partial equilibrium search model and Mortensen and Pissarides’s (1994) (MP) equilibrium matching model. McCall focuses on the worker’s job search strategy in a partial equilibrium setting where firms post wages and workers make acceptance decisions. Employment is an absorbing state so that once a worker has found a job she stays in it forever. The probability of job finding is partially under the control of the job seeker through her choice of reservation wage, and in extensions of this model, her choice of search effort.

A potential disadvantage of wage posting models in general is the strong assumption that workers and firms commit to the posted terms of trade. This point is especially important in the economy I consider, because the joint surplus of a matched worker-firm pair, as well as the worker’s outside option, change over time and depend on the worker’s dynamic saving decisions. Hence, workers would want to change the terms of trade as they become wealthier and their outside option changes. The MP matching model circumvents this criticism of wage-posting models. Job seekers and firms trying to fill vacancies randomly find each other in an aggregate matching market and determine the wage through bilateral bargaining. Arguably, this is a more appealing model of the labor market, especially when workers’ saving decisions affect the bargaining position of one of the parties as is the case in my model. In addition, I am not only interested in the determination of wages, but also in the choice of working hours, which makes the bargaining approach even more appealing. See Rogerson, Shimer and Wright (2005) for an excellent survey of the literature on search-theoretic models of the labor market.

The seminal papers on the normative issues of UI design were published in the late 1970s and include Baily (1978), Flemming (1978), and Shavell and Weiss (1979). A general theme of these papers is that more generous benefits decrease search effort and lead to longer unemployment spells. Shavell and Weiss presented the first analysis of the optimal time path of UI benefits. Much of the more recent literature extends their work by adding additional policy instruments or by using equilibrium models.

Shavell and Weiss use McCall’s (1970) model to investigate the optimal time path of benefit payments when job search effort is unobservable by the UI administrator. They derive two important results. First, when workers are not allowed to borrow or save and if there is no moral hazard, then the optimal benefit level should be constant during the entire unemployment spell. Second, when there is moral hazard this result is overturned and the optimal benefit level must decline over the unemployment spell in order to give the unemployed appropriate incentives to search. Shavell
and Weiss also show that if the unemployed begin their spell with positive wealth, or if they are allowed to borrow, and if they cannot influence the probability of getting a job, then the optimal benefit level should be zero at first and then rise to a constant level. However, they were unable to characterize the optimal benefit profile in the general case with moral hazard and initial wealth or the ability to borrow. Shimer and Werning (2005) were the first to accomplish this.

Building on Shavell and Weiss’s analysis, Hopenhayn and Nicolini (1997) increase the number of policy instruments available and allow the UI administrator to impose a wage tax after reemployment that may depend on the duration of a worker’s unemployment spell. They find that benefits should decrease throughout the unemployment spell and that the wage tax upon reemployment should increase with the length of the unemployment spell. Compared to Shavell and Weiss, this two-instrument policy has the advantage of improving intertemporal consumption smoothing as well as intertemporal incentives.

In a more recent paper, Shimer and Werning (2005) build on the analysis in Hopenhayn and Nicolini (1997) and allow workers access to capital markets. As in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), Shimer and Werning focus on the decision problem of the unemployed and find that when workers have constant absolute risk aversion (CARA) preferences and sufficiently good access to capital markets, the optimal policy involves a constant benefit schedule of unlimited duration combined with a constant tax rate during employment that is independent of the duration of a worker’s previous unemployment spell. Although this result breaks down with constant relative risk aversion (CRRA) preferences, they find that constant UI benefits combined with a constant tax upon reemployment are approximately optimal with CRRA preferences. The intuition for this result is simple: As unemployment spells continue, workers deplete their assets in order to buy consumption goods. With a fixed benefit schedule, their consumption declines over time as their wealth decreases. As a result, workers’ marginal utilities of consumption increase during unemployment spells which increases their incentives to search. As in my model, UI benefits play the dual role of providing insurance against the uncertain duration of unemployment spells and ensures that workers have sufficient liquidity to smooth their consumption. One disadvantage of using McCall’s (1970) partial equilibrium model is that Shimer and Werning are unable to investigate how their choice of benefit timing affects the saving decision of employed workers, which in turn determines the wealth level of the unemployed.

The result that benefits should decrease with unemployment duration has been questioned by several other authors. Werning (2002) and Kocherlakota (2004), for example, study the optimal UI design problem with unobservable savings and find that once the direct link between income and consumption is broken, the optimal income path during unemployment may be constant (see
Kocherlakota, 2004), or in some cases even upward sloping (Werning, 2002). In an equilibrium search model without savings, Davidson and Woodbury (1997) find that the optimal UI policy consists of indefinite benefit payments and a replacement rate of 0.66. Their numerical examples suggest that the optimal replacement rate is as high as 1.3 when benefits are paid only for 26 weeks as currently in the U.S.

In contrast to this line of research, I do not investigate the optimal timing of benefits. As will become clear in Section [7], Shimer and Werning’s (2005) line of argument also applies to my model and, to simplify an already complicated model, I assume that the optimal UI benefit path is constant with an indefinite duration. The focus of my work is on the interaction between the optimal replacement rate and the cost of self-insuring. Instead of using a partial equilibrium model, I use an equilibrium matching model and allow workers and firms to bargain over wages and hours of work. I also pay closer attention to the choice and calibration of preferences, which previous authors usually chose for analytical convenience. For example, while my reading of micro studies on the intertemporal substitution of consumption suggests that individuals are quite risk-averse, Hopenhayn and Nicolini (1997) and Shimer and Werning (2005) choose low levels of risk-aversion with CRRA coefficients of 0.5 and 1.5, respectively.

Several other authors consider the effects of UI benefits on asset accumulation. Hansen and İmrohoroğlu (1992) develop a quantitative dynamic general equilibrium model in which workers are employed and work a fixed number of hours at a fixed wage, or unemployed and receive UI benefits and enjoy leisure. In addition to government-provided insurance, workers may self-insure by saving a non-interest-bearing asset. Each period, workers face an exogenously given probability of receiving an employment opportunity that is identical for all workers and across time. The source of moral hazard stems from the UI administration’s inability to perfectly monitor program applicants. Workers may decline job opportunities and still receive benefits with a positive probability. This probability determines the degree of the moral hazard problem. Because workers receive utility from leisure, they will decline to work if they have sufficiently high assets. The role of unemployment insurance then is not only to help individuals smooth consumption, but also to subsidize leisure of the wealthy. Hansen and İmrohoroğlu find potentially large welfare benefits from introducing UI and that saving drops to zero with the optimal replacement rate and no moral hazard. They determine the optimal UI replacement rate to be 65 percent with no moral hazard and as low as 5 percent with extreme moral hazard.

Wang and Williamson (2002) develop a model that combines aspects from Hansen and İmrohoroğlu (1992) and Wang and Williamson (1996). As in Hansen and İmrohoroğlu (1992), employed workers receive a fixed wage when employed, capital markets are incomplete, and workers may self-
insure by saving a non-interest-bearing asset. As in Wang and Williamson (1996), moral hazard arises from a worker’s unobserved effort decision, which not only determines the transitions into employment, but also those into unemployment. Their analysis focuses on the effects that experience rating and changes in the level and duration of UI benefits have on search behavior, shirking by the employed, unemployment, and welfare. They find that the optimal benefit schedule is U-shaped. Benefits are low in the beginning, then drop to zero before they rise again above initial levels. They find that welfare gains from changing the current system are small.

My work differs from these papers along several dimensions. First, I focus on the interaction between the cost of self-insuring and the optimal replacement rate, a point these authors do not speak to. Second, I use an equilibrium matching model in which wages and working hours are set efficiently. Hansen and İmrohoroğlu (1992) assume that employed workers have to spend an exogenously given amount of time at work, while Wang and Williamson (2002) do not model the intensive margin of labor supply. Using a bargaining approach is important because workers’ wealth affect their bargaining positions and hence influence the efficient choices of hours and the compensation they receive. Workers in my model not only make ample use of savings to smooth consumption, but the efficient determination of hours implies that they also make considerable use of adjustments in hours of work to smooth consumption. The employed work a lot when consumption is relatively low. Hansen and İmrohoroğlu (1992) and Wang and Williamson (2002) also choose low levels of risk-aversion with coefficients of relative risk aversion of 1.5 and 1.0, respectively.

Several other papers have considered the optimal UI design using search models in which workers may save. For example, Gomes, Greenwood and Rebelo (2001) study the general equilibrium effects of UI in an incomplete markets environment with job search, while Lentz (2005) determines the optimal UI benefit level in an estimated job search model using Danish micro data.

Several recent papers have assessed optimal UI design in matching models. Cahuc and Lehmann (2000) and Fredriksson and Holmlund (2001) investigate the optimal time path of UI benefits with endogenous search effort of the unemployed. The two papers differ in how wages are determined and in the exact specifications of the policy instruments they consider. Cahuc and Lehmann assume that wages are set by union-firm bargaining that gives rise to an insider/outsider problem. Because wages strongly respond to the timing of UI benefits, the authors argue, the advantages associated with a declining time path decrease. In Fredriksson and Holmlund (2001) wages are the outcome of bilateral bargaining between workers and firms. The authors find that the optimal time path is
declining over the unemployment spell. Coles and Masters (2006) analyze UI in a matching model with strategic bargaining and highlight the potentially welfare-increasing effects of job creation subsidies in conjunction with optimal UI benefit levels.

My work differs from these studies in that I focus on workers’ ability to self-insure. A major technical difficulty these authors disregarded arises from the dynamic saving decision of workers. An individual’s wealth level affects the quality of her outside option and hence influences the bargain over hours and compensation. Moreover, the aggregate wealth distribution affects firms’ incentives to post vacancies. I also pay much closer attention to the calibration of preferences and of other parameter choices.

3 Model

I extend a variation of the MP matching model along three important dimensions. First, workers are risk-averse and have preferences over consumption and hours of work. Second, I model the intensive margin of labor supply by assuming that hours of work are determined through bilateral bargaining between workers and firms. Third, workers may self-insure through savings but face liquidity constraints.

Time is discrete, a period is equal to one month, and the economy is populated by a unit measure of infinitely lived individuals who may either be employed or unemployed. Employed workers receive compensation $wh$, where $w$ is the hourly wage and $h$ is the number of working hours, and face an exogenously fixed hazard $s$ of job loss. The unemployed receive government-provided benefits $b$ and face a probability $f$ of finding a suitable job. Individuals do not have access to insurance markets and cannot borrow against future income, but they are able to hold a risk-free asset at the exogenously given interest rate $r$. The government provides a UI system that pays benefits $b$ during all periods of unemployment and spends an exogenously given amount $\chi$ on programs that individuals do not derive utility from. The government is required to balance the budget and levies a labor income tax $\tau$ on workers’ compensation to finance its expenditures.

Workers are homogeneous with respect to productivity but differ in their asset holdings because of different employment histories. I assume that firms are able to fully observe workers’ assets after they match. This assumption is reasonable because for the majority of the population an individual’s employment history is the main determinant of wealth. A job applicant’s employment history in form of a resumé is usually accessible to an employer. Furthermore, this is a simplifying assumption important for the worker-firm bargain over wages and hours. With asymmetric information, bilateral bargaining may result in a continuum of equilibria, an issue beyond
the scope of this paper (see, for example, Ausubel and Deneckere, 1989).

Another assumption I make is that the government is unable to observe a worker’s wealth level and is thus unable to design a policy \( b(a) \) where the benefit level would depend on assets. Allowing this would be an interesting project but is beyond the scope of this paper. See Hubbard, Skinner and Zeldes (1995) for a discussion of the effects of means tested social insurance on precautionary wealth.

There are two state variables in my model. One is the discrete labor market state of individuals \( i \in \{e, u\} \), and the other is the continuous variable asset holdings \( a \).

### 3.1 Aggregate matching market

An important feature of the MP model is the existence of search frictions in the labor market. It takes time for unemployed workers to meet suitable firms with unfilled vacancies. The number of new matches is a function of the number of unemployed workers and the number of firms posting vacancies. The aggregate matching market is characterized by the standard Cobb-Douglas matching function

\[
M(u, v) = \zeta u^\alpha v^{1-\alpha},
\]

where \( M \) denotes the number of successful matches, \( u \) the number of unemployed workers searching for a job, and \( v \) the number of available vacancies. The parameter \( \zeta > 0 \) controls the efficiency of the matching process and \( \alpha \in (0, 1) \) is the elasticity of the matching function with respect to the number of unemployed workers \( u \). Let \( \theta = v/u \) be the ratio of vacancies to unemployment, a measure of labor market tightness. Each period, an unemployed worker finds a new job with probability \( f(\theta) = M(u, v)/u \), while a firm with a vacancy hires a new worker with per-period probability \( f(\theta)/\theta \).

In steady-state, the flows into and out of unemployment must equal, so that the steady-state unemployment rate is given by

\[
u = \frac{s}{s + f(\theta)}.
\]

### 3.2 Individuals

Individual workers are either employed or unemployed. It is convenient to express the model in terms of Bellman value-transition equations. Let \( U(a) \) be the value a worker associates with being unemployed and searching for a new job when her asset level is \( a \). Similarly, \( E(a) \) is the value of an employed worker, and \( J(a) \) is the value a firm associates with employing a worker whose asset level is \( a \). Employed workers choose today’s consumption \( c_e \) to maximize the value of employment
subject to the intertemporal budget constraint and the exogenously given borrowing constraint

\[ E(a) = \max_{c_e} \left\{ u(c_e(a), h(a)) + \frac{1}{1 + \rho} \left[ sU(a'_e) + (1 - s)E(a'_e) \right] \right\} \]  

subject to

\[ c_e(a) = (1 - \tau)w(a)h(a) + (1 + r)a - a'_e(a) \]  

\[ a'_e(a) \geq a \]  

where \( u(c, h) \) is the momentary utility function, \( \rho \) is the subjective discount rate, \( s \) is the separation rate, \( r \) is the exogenously given risk-free interest rate, \( w \) is the hourly wage, \( h \) is the number of working hours, and \( a' \) are next period’s assets. \( \tau \) is a labor income tax the government uses to finance its budget. I index consumption and next period’s assets by the current labor market state \( e \). Because this is an incomplete market model in which workers lack full insurance, consumption depends on a worker’s current labor market state and asset level. Employed workers always find it optimal to increase their savings relative to the borrowing limit, up to some upper threshold, so that this constraint will never bind for them.

Unemployed workers solve a similar problem. They choose \( c_u \) to maximize the value of unemployment \( U(a) \) subject to the intertemporal budget constraint and the borrowing constraint

\[ U(a) = \max_{c_u} \left\{ u(c_u(a), 0) + \frac{1}{1 + \rho} \left[ f(\theta)E(a'_u) + (1 - f(\theta))U(a'_u) \right] \right\} \]  

subject to

\[ c_u(a) = b + (1 + r)a - a'_u(a) \]  

\[ a'_u(a) \geq a \]  

where \( b \) represents the monetary value of government-provided unemployment insurance benefits. I abstract from other sources of income such as severance payments, spousal support, etc. By definition, the unemployed do not spend any time working so that \( h = 0 \). I define the replacement rate \( \delta \) as

\[ \delta = \frac{b}{\mathbb{E}[(1 - \tau)w(a)h(a)]}, \]  

where \( \mathbb{E}[(1 - \tau)w(a)h(a)] = \int_{a}^{\infty} (1 - \tau)w(a)h(a) \, dG_e(a) \) is the average after-tax compensation, and \( G_e(a) \) the steady-state wealth distribution of the employed.

Note that I do not consider job-search decisions by the unemployed. Empirical evidence suggests that workers spend a minuscule amount of time on job-search activities. According to data from the American Time Use Survey (2004), the average unemployed worker in 2004 spent 3
minutes per day searching for a job, or only about 0.2 percent of available time. The average unemployed worker spent three times more on religious and spiritual activities, four times more on volunteer activities, and 124 times more on socializing, relaxing, and leisure than on job search.

I assume that workers do not think strategically when making their consumption/saving decisions. Since workers’ asset levels affect their bargaining position, strategic workers would want to choose consumption and next period’s assets to smooth consumption and to better their bargaining position in the next period. Because I want to focus on precautionary saving behavior, I assume that workers do not behave strategically. I discuss in web Appendix D how strategic behavior would affect my calculations.

Given this assumption and assuming an interior solution, the workers’ decision problem can be characterized by two Euler equations, one for each employment state. The optimal consumption choices of the employed satisfy

\[
uc_e(c_e(a), h(a)) = \frac{1 + r}{1 + \rho} \left[ s uc_u(c_e(a'), 0) + (1 - s) uc_e(c_e(a'), h(a')) \right],
\]

while the optimal consumption choices of the unemployed satisfy

\[
uc_u(c_u(a), 0) = \frac{1 + r}{1 + \rho} \left[ f(\theta) uc_e(c_e(a'), h(a'))(1 - f(\theta)) uc_u(c_u(a'), 0) \right].
\]

3.3 Firms

A firm’s value of a filled job \( J(a) \) is given by the flow profits it receives from employing a worker with assets \( a \) plus the expected present value of continuing the employment relationship

\[
J(a) = (m - w(a))h(a) + \frac{1 - s}{1 + r} J(a').
\]

I assume that firms are homogeneous, so that the marginal revenue product \( m \) is constant across all matches.

Firms expand recruiting efforts to the point where the cost \( k \) of posting a vacancy equals the expected value of a filled job, so that the value of a vacancy is zero. The corresponding Bellman equation is

\[
k = \frac{f(\theta)/\theta}{1 + r} \mathbb{E}[J(a')],
\]

where \( f(\theta)/\theta \) is the probability of hiring a worker, and \( \mathbb{E}[J(a')] = \int_0^\infty J(a') \, dG_u(a') \) is the expected value of a filled job conditional on having hired a worker. \( G_u(a) \) is the steady-state wealth distribution of the unemployed, induced by the workers’ consumption/saving decisions.

\[1\]The web Appendix is available at [http://felixr.googlepages.com/webapp.pdf](http://felixr.googlepages.com/webapp.pdf)
3.4 Wages and hours

As is standard in matching models, I assume that wages are determined by Nash bargaining. I further assume that working hours are also determined by bilateral bargaining. Bargaining over working hours has previously been discussed by Earle and Pencavel (1990) and Auray and Danthine (2005). Ham and Reilly (2002) reject the hypothesis that workers face hours constraints, which is consistent with my assumption of efficient bargaining over working hours. Both wages and hours are renegotiated every period.

The matched worker-firm pair chooses wages and hours by solving

$$\max_{w(a), h(a)} (E(a) - U(a))^\phi J(a)^{1-\phi}$$

subject to

$$c_e(a) = (1 - \tau)w(a)h(a) + (1 + r)a - a'(a),$$

where $E(a) - U(a)$ is the surplus a worker with asset level $a$ enjoys from employment, and $J(a)$ is the associated surplus of the firm. The firm’s outside option, the value of posting a vacancy, is zero. The worker’s bargaining weight is $\phi$ (see Binmore, Rubinstein and Wolinsky (1986) for an interpretation of this parameter).

The solution to the wage problem is given by

$$\frac{\phi}{(1 - \phi)} J(a) = \frac{E(a) - U(a)}{u_c(c_e(a), h(a))(1 - \tau)},$$

which can be rewritten as

$$\phi \frac{\partial (E(a) - U(a))}{\partial w(a)} J(a) + (1 - \phi) \frac{\partial J(a)}{\partial w(a)} (E(a) - U(a)) = 0,$$

where $(E(a) - U(a))/u_c$ is the worker’s surplus in units of consumption. This equation is a generalization of the standard surplus sharing rule with linear utility when $\partial E/\partial w = 1$. Note that equation (17) is also the solution to a bargain over total compensation $wh$. I focus on wages and hours separately, because changes in the benefit level have clear predictions for those variables, while the effect on compensation depends on the product of the effects on wages and hours and is not necessarily monotone.

Similarly, the solution to the hours problem is given by

$$\frac{\phi}{(1 - \phi)} J(a) + (1 - \phi) \frac{\partial J(a)}{\partial h(a)} (E(a) - U(a)) = 0$$

The optimal choice of hours weighs the benefits of an additional hour to the firm against the benefits to the worker. Intuitively, the worker-firm pair chooses working hours to maximize the
joint surplus and chooses compensation \(wh\) to split the surplus. Because of my assumption that a firm’s surplus depends linearly on hours, it is a worker’s preferences over hours that determine working hours and the size of the total surplus.

Combining equations (16) and (18), I can write

\[
-\frac{u_b(c_e(a), h(a))}{u_e(c_e(a), h(a))(1 - \tau)} = m
\]

This equation is the standard bilateral efficiency condition and states that the marginal rate of substitution between consumption and leisure must equal the marginal rate of transformation. See web Appendix E for a derivation of these results. The choice of hours makes sure that the worker-firm pair is on the contract curve, while the choice of compensation determines the location on the contract curve.

Because the surplus of a worker \(E(a) - U(a)\) is a decreasing function of wealth, the joint surplus of a matched worker-firm pair is also a decreasing function of a worker’s wealth level. In terms of units of consumption, it is given by

\[
S(a) = \frac{E(a) - U(a)}{u_e(c_e(a), h(a))(1 - \tau)} + J(a)
\]

### 3.5 Government

The main function of the government is to provide insurance that is not available in the market. It raises revenues by levying the labor income tax \(\tau\) and uses the revenues to pay benefits \(b\) to the \(u\) unemployed workers and to finance general expenditures \(\chi\). I assume that the government has to run a balanced budget, a reasonable assumption in a steady-state model, and that individuals do not derive utility from expenditures \(\chi\). Budget balance requires that

\[
u b + \chi = (1 - u)\tau \int_a^\infty w(a)h(a) dG_e(a),
\]

where \(G_e(a)\) is the endogenous steady-state wealth distribution of the employed, induced by the workers’ consumption/saving decisions.

### 3.6 Equilibrium

**Definition** Given the parameters of the model, a *stationary equilibrium* is characterized by the pair of consumption policy functions \(c_e(a)\) and \(c_u(a)\), the wage function \(w(a)\), the hours function \(h(a)\), the three value functions \(E(a)\), \(U(a)\), and \(J(a)\), the steady-state distributions of assets for the employed and the unemployed, \(G_e(a)\) and \(G_u(a)\), the vacancy-unemployment ratio \(\theta\), and the tax rate \(\tau\) such that
1. Given $w(a)$, $h(a)$, $\theta$, and $\tau$, the consumption policy functions $c_e(a)$ and $c_u(a)$ solve equations (10) and (11).

2. The policy functions $c_e(a)$ and $c_u(a)$ induce the stationary wealth distributions for the employed, $G_e(a)$, and the unemployed, $G_u(a)$.

3. Given $c_e(a)$, $c_u(a)$, $w(a)$, $h(a)$, $\theta$, and $\tau$, the value functions $E(a)$, $U(a)$, and $J(a)$ solve equations (3), (6), and (12).

4. Given $c_e(a)$, $c_u(a)$, $E(a)$, $U(a)$, $J(a)$, $G_e(a)$, and $G_u(a)$
   
   (a) The wage function $w(a)$ and the hours function $h(a)$ satisfy equations (16) and (18).
   
   (b) The vacancy-unemployment ratio $\theta$ solves equation (13).
   
   (c) The income tax $\tau$ satisfies the budget balance condition of equation (21).
   
   (d) The intertemporal budget constraints, equations (4) and (7), and the borrowing constraints, equations (5) and (8) are satisfied.

4 Preferences

The specification and calibration of preferences is of central importance when modeling individual decision making. I follow Hall (2006b), who uses evidence from the large literature on labor supply, the intertemporal elasticity of consumption, and consumption-hours cross-effects to calibrate a utility function with a fairly general functional form. In particular, Hall suggests working with Frisch systems. Frisch elasticities keep the marginal utility of wealth constant, are a convenient way to characterize preferences, and are commonly used in modern labor economics. Because of the lack of full insurance markets in my model, an individual’s marginal utility of consumption varies over time. However, the majority of workers in my model are close to being fully self-insured. See Browning, Deaton and Irish (1985) for a discussion of Frisch systems.

The Frisch (or $\lambda$-constant) labor supply and consumption demand functions satisfy

\[
\begin{align*}
  u_h(c, h, \lambda) &= -\lambda w \\
  u_c(c, h, \lambda) &= \lambda p
\end{align*}
\]

where $\lambda$, the marginal utility of wealth, is the Lagrange multiplier on the budget constraint. Using these two equations, I can solve for the Frisch elasticities of labor supply $\eta$, and consumption demand $\phi$, as

\[
\eta(c, h) \equiv \left. \frac{\partial h}{\partial w} \right|_{\lambda} = \left. \frac{u_h u_{cc}}{u_{cc} u_{hh} - u_{hc} u_{ch}} \right| h \frac{1}{h}
\]
Both elasticities are functions of current consumption $c$ and working hours $h$. Note that the Frisch elasticity of labor supply is not defined for unemployed individuals and that the Frisch elasticity of consumption demand for unemployed individuals is given by $\varphi(c, 0) = u_c / u_{cc} c$, which is the negative of the elasticity of intertemporal substitution. I derive both elasticities in web Appendix F.

4.1 Functional form

I assume the period utility function proposed by Malin (2006) and used by Hall (2006b)

$$u(c, h) = \frac{1}{1 - \mu} \left[ \frac{c^{-(1/\sigma - 1)} - c^{-(1/\sigma - 1)}}{1/\sigma - 1} - \frac{\gamma}{1 + 1/\psi} h^{1 + 1/\psi} \right]^{1 - \mu}$$

(26)

where $\sigma$ is the curvature parameter for consumption, $\psi$ is the curvature parameter for work, and $\mu$ determines the degree to which consumption and hours of work are complements or substitutes. If $\mu$ is positive, consumption and hours of work are complements so that $u_{ch} > 0$. The parameter $c$ determines the point at which the kernel inside the brackets is zero and only matters for specifications with $\mu \neq 0$. For individuals who do not work ($h = 0$), the kernel inside the brackets is zero when $c = c$. The parameter $\gamma$ governs the distaste for work, or alternatively can be thought of as the efficiency of home production (see Becker, 1965). As Malin discusses, this functional form nests several of the specifications commonly used in the literature.

There are a total of six parameters to choose. I normalize the product price $p = 1$, assume $c = 0.2$, and set $\gamma = 1$. To choose the remaining three parameters, $\sigma$, $\psi$, and $\mu$, I draw upon micro studies on the intertemporal substitution in consumption, the Frisch elasticity of labor supply, and the complementarity between consumption and working hours, which I discuss in web Appendix B.

5 Parameters

My model operates at a monthly frequency and has a total of 17 parameters, 6 of which are preference parameters. I aim to match the relevant characteristics of the post-World War II period U.S. labor market and choose the preference parameters to match the findings discussed in web Appendix B.
5.1 Preferences

Given my normalization of the product price $p = 1$ and my choices of $\zeta = 0.2$ and $\gamma = 1$, there are three remaining preference parameters, $\sigma$, $\psi$, and $\mu$. I choose to calibrate to an average Frisch elasticity of consumption demand of $\varphi = -0.35$, an average Frisch elasticity of labor supply of $\eta = 0.69$, and an average consumption drop of $\omega = 0.1$ when becoming unemployed. The resulting parameters are provided in Table 1.

These calibration choices imply that workers are on average quite risk-averse. I calculate risk aversion as $-u_{cc}/u_c$, which averages about 3.1 at the calibration point. See Chetty (2006) for a discussion of risk aversion when hours of work are included in preferences.

5.2 Labor market

Several authors have estimated separation rates for the U.S. labor market. The evidence presented by Shimer (2005a, 2005b), Nagypál (2004), and Abowd and Zellner (1985) suggests that the separation rate $s$ is quite constant over time with an average value of about 0.034. Hall (2005b) surveys the evidence on job-finding and separation rates. According to the Bureau of Labor Statistics (BLS) the average unemployment rate between January 1948 and July 2006 was 5.62 percent. Using $s = 0.034$ and $u = 0.056$, the steady-state unemployment equation (2) implies that the average monthly job-finding rate is $f = 0.57$. This estimate is slightly higher than the 0.45 that Shimer (2005a) finds. These values imply that the average job lasts approximately 29 months and the average unemployment spell about 7.5 weeks. For the period between 1967 and 2006, the BLS reports that the average unemployment spell lasted about 14 weeks, while the median unemployment spell lasted about 7 weeks.

Another important characteristic of the labor market is captured by the parameter $\alpha$. In the context of the model this parameter governs how much the unemployment rate changes in response to changes in UI benefits. Shimer (2005a) calculates the elasticity of the matching function $\alpha$ to be 0.72, while Hall (2005a) finds a value of 0.245. Petrongolo and Pissarides (2001) report values ranging from about 0.3 to 0.7. In light of this mixed evidence, I choose $\alpha = 0.5$, a value also used by Cahuc and Lehmann (2000) and Fredriksson and Holmlund (2001). I investigate how this choice affects the optimal policy in web Appendix C.3.

I normalize the vacancy-unemployment ratio to $\theta = 0.5$. Given my choice of matching function, the vacancy-unemployment ratio is inherently meaningless at the calibration point. I could simply adjust the value of the efficiency parameter $\zeta$ to accommodate other values for the vacancy-unemployment ratio. Using equation (1), these values imply the matching efficiency parameter
\[ \zeta = 0.81. \] From equation (13) I calculate the flow cost of posting a vacancy to be \( k = 1.39, \) which is equivalent to about five weeks of average compensation.

I set the marginal revenue product to \( m = 2 \) and choose the bargaining weight \( \phi = 0.5. \) The choice of \( \phi \) is not completely innocuous as it does effect the optimal policy, but is standard in the literature. I will investigate the implications of other values for \( \phi \) in web Appendix C.4.

The last labor market parameter to choose is the UI benefit \( b, \) or alternatively the UI replacement rate \( \delta. \) The Department of Labor (2006) reports an average replacement rate of 36 percent in the first quarter of 2006, with a high of 53 percent for Hawaii and a low of 24 percent for the District of Columbia (see Table 16 in web Appendix I). According to the Congressional Budget Office (2004), the median replacement rate of the long-term unemployed (more than four consecutive months of unemployment) was 40 percent and ranged from 24 to 64 percent, depending on previous earnings. While only 40 percent of all unemployed received UI benefits, approximately 80 percent of all job losers did. The difference is due to those who either exhausted their benefits or were ineligible for other reasons (for example, entrants into the labor market). Engen and Gruber (2001) report an average benefit replacement for the U.S. of 45 percent of covered workers’ last earnings, while Martin (1996) reports a replacement rate of 34 percent for the U.S. The level of the replacement rate at the calibration point also affects the optimal policy. I choose to calibrate to Martin’s value of \( \delta = 0.34 \) and investigate other values in web Appendix C.5.

I set the exogenous government spending to 20 percent of aggregate output, or \( \chi = 0.32. \) Together with the spending on UI benefits, this implies a tax rate of \( \tau = 0.221, \) which lies between the aggregate mean U.S. marginal and average tax rates. See Table 15 in web Appendix I for NBER data on marginal and average tax rates in the U.S. Since the marginal and the average tax rates in my model are the same, these numbers seem reasonable.

### 5.3 Asset market

The interest rate \( r \) and the subjective discount rate \( \rho \) also play an important role in determining optimal policy. The larger the difference between \( r \) and \( \rho \) the more costly it is for workers to hold assets. Lentz (2005) shows that the optimal UI benefit level is decreasing in the interest rate. I set the annual interest rate to 3 percent and the annual subjective discount rate to 5 percent, resulting in \( r = 0.00247 \) and \( \rho = 0.00407. \)

I set \( r < \rho \) for two reasons. First, I believe this to be a feature of reality. Most U.S. households hold very few assets and are unlikely to have the ability to save at a risk-free rate close to their individual discount rate. Data on individual wealth is consistent with this claim. Budría Rodríguez, Díaz-Giménez, Quadrini and Ríos-Rull (2002) provide details about the U.S. wealth distribution.
and find that households in the top 1 percent of the wealth distribution own about 35 percent of the total wealth, while households in the top 20 percent hold 82 percent of total wealth. On the other hand, about 10 percent of households have either no wealth or negative net worth and households in the bottom 40 percent of the wealth distribution own only 1.0 percent of the total wealth. According to Deaton (1991), the median household wealth, excluding pension rights and housing, is about $1,000. Deaton also discusses why assuming $r < \rho$ is appealing when modeling average consumers.

The second reason is technical. Aiyagari (1994) shows that in economies with uninsurable idiosyncratic shocks and an infinite horizon the interest rate must be strictly smaller than the subjective discount rate, $r < \rho$. If $r \geq \rho$, then workers would want to accumulate an infinite amount of assets so that the wealth distribution would not be bounded from above. Intuitively, the presence of precautionary motives increases the returns to assets. Thus, for a stationary equilibrium to exist it must be that $r < \rho$.

Given $r < \rho$, workers only want to accumulate enough assets to smooth consumption in the chance of becoming unemployed. That is, workers only save for precautionary reasons (see Schechtman and Escudero, 1977). In a general equilibrium model the choices of $r$ and $\rho$ would imply an empirically observable value for the median wealth to income ratio, $W/Y$ in the literature. Carroll, Dynan and Krane (2003) use data from the Survey of Consumer Finances and report a median $W/Y$ of 1.54 for 1989 and 1.37 for 1992. Since the only reason individuals save in my model is for precautionary reasons, my model is not able to replicate the U.S. wealth distribution. The relevant statistic to consider is the ratio of precautionary wealth to income and not the ratio of total wealth to income.

Carroll and Samwick (1998) estimate precautionary savings to amount to between 32 and 50 percent of wealth in their sample from the PSID, while Kennickell and Lusardi (2005) find that precautionary wealth amounts to approximately 8 to 20 percent of total wealth. Hurst, Luoh, Stafford and Gale (1998) report that the average wealth to income ratio of individuals aged 24 to 34 was 0.31 in 1989. At this age, individuals are unlikely to have accumulated much life-cycle related wealth and hence correspond most closely to individuals in my model.

My choices for $r$ and $\rho$ imply a median precautionary wealth-to-income ratio of 0.30, which is equivalent to a total wealth-to-income ratio of 1.54 if precautionary wealth is 19 percent of total wealth and close to the $W/Y$ ratio of the 24 to 34 old individuals in Hurst et al.’s sample. Following a similar strategy but using different data, Domeij and Flodén (2006) set $r = 0.02$ and $\rho = 0.053$.

The last parameter to choose is the borrowing constraint $a$. The exact borrowing constraint only matters marginally in my model as long as it is of a reasonable size, although it does affect the
<table>
<thead>
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<td>Exogenous government spending</td>
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<td>Calibration</td>
</tr>
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Table 1: Calibration targets and parameter choices

median wealth-to-income ratio. Since workers only save for precautionary reasons, they mostly care about being sufficiently far away from the constraint and less about the level of their assets.

I assume that workers may save but not borrow, so that the constraint is $\bar{a} = 0$. There is ample evidence that a large fraction of the population is credit constrained. According to Deaton (1991), “approximately one fifth of total consumption is accounted for by households who not only possess no stocks or bonds, but who have neither a checking nor a saving account” (p.1222). It is hard to imagine that these individuals are able to borrow money from others than relatives. Jappelli (1990) reports more direct evidence for liquidity constraints and Rendon (2006) finds evidence for very tight borrowing constraints. Using data from the youth cohort of the National Longitudinal Survey of Labor Market Experience, Rendon finds that individuals can only borrow 14 percent of
the present value of their risk-free income. Nirei (2006) finds support for a borrowing constraint worth three months of average wage income. I experimented with constraints equivalent to two years worth of average income without any significant changes in the results.

Throughout the paper, I will refer to this calibrated economy as the baseline economy. I provide a summary of the model parameters in Table 1.

6 Solution method

My model has two state variables. In addition to the binary variable describing an individual’s labor market state \( i \in \{e, u\} \), the model has the continuous state variable asset holdings \( a \). Using the methods laid out in chapters 6 and 11 in Judd (1998) I solve for the model’s steady-state equilibrium numerically, using projection methods to simultaneously solve for the four non-linear policy and three non-linear value functions of my model.

The unknown policy functions I solve for are the consumption choices for the employed and unemployed, \( \hat{c}_e(a) \) and \( \hat{c}_u(a) \), and the wage and hours functions, \( \hat{h}(a) \) and \( \hat{w}(a) \). These policy functions satisfy equations (10), (11), (16), and (18). The unknown value functions I solve for are the value of employment, \( \hat{E}(a) \), the value of unemployment, \( \hat{U}(a) \), and the value of the firm, \( \hat{J}(a) \). These functions correspond to equations (5), (6), and (12). Following Judd’s recommendations, I represent the policy and value functions as Chebyshev polynomials. In order to better estimate these functions close to the borrowing constraint, where their curvature is highest, I choose Chebyshev polynomials of degree 10 and perform a non-linear change of variables in assets.

The consumption functions, together with the budget constraints (equations (4) and (7)), induce the two stationary wealth distributions \( G_e \) and \( G_u \), which I approximate as \( \hat{G}_e \) and \( \hat{G}_u \). Following the procedure outlined in Hall (2006a), I use the continuous policy functions \( \hat{c}_e(a) \) and \( \hat{c}_u(a) \), and the budget constraints to calculate a 1500 \( \times \) 1500 Markov transition matrix that describes the workers’ transitions between employment states and wealth levels. Using this Markov transition matrix I calculate the joint distribution of labor market states and wealth holdings \( G \). I then calculate the marginal distributions of \( G \). These calculations result in distributions that are indistinguishable from those obtained by simulating employment histories over hundreds of thousands of months.

The interested reader should refer to Appendix A and the web Appendix G for more information about the solution procedure.
Figure 1: Monthly saving of the employed and unemployed as functions of current wealth. Saving and current wealth are measured in units of average monthly consumption of the employed.

7 Properties of the baseline economy

In this section I describe the properties of the baseline economy with the parameters set to those discussed in Section 5. Understanding the saving decisions of workers is key to understanding the properties of the model. Figure 1 shows the saving behavior of the employed and the unemployed as functions of current wealth, where wealth is measured in units of average monthly consumption of the employed. Employed workers save a positive fraction of monthly income up to a certain threshold, equivalent to approximately 4 months of average consumption. As employed workers accumulate wealth, their need for more precautionary savings declines. Workers who start out with zero assets reach the upper threshold after about 10 years of continuous employment at which point they will stop accumulating more wealth. The separation rate of 3.4 percent per month implies that the average job lasts approximately 30 months and the probability of being continually employed for at least 10 years is only 1.6 percent. The average employed worker saves about 1.5 percent of after-tax compensation and holds assets worth 3.5 months of average monthly consumption.
Unemployed workers always want to dissave in order to keep a smooth consumption profile. The average after-tax compensation of the employed is about 1 unit of average monthly consumption. The government subsidizes the unemployed with only 34 percent of average compensation, so that the consumption drop would be large if workers were unable to self-insure through savings. The average unemployed worker dissaves assets worth half a unit of average monthly consumption and consumes about 86 percent of average monthly consumption. This implies an average uncompensated consumption drop of 12.4 percent when a worker becomes unemployed.

While the average worker generally does a good job of self-insuring and avoiding low asset levels, few individuals are close to the borrowing constraint. Only 0.2 percent of the total population and only 1.1 percent of the unemployed own assets worth less than 1 month of average consumption. The proportion of individuals with assets less than 0.5 is close to zero: 0.04 percent for the total population and 0.27 percent for the unemployed.
As Figure 2 shows, individuals’ wealth levels directly affect their consumption. As the employed accumulate more savings, their saving rate decreases and consumption, measured relative to the average consumption, increases from a low 0.89 at the borrowing constraint to a high 1.01 at the maximum wealth level, a 13 percent difference. Consumption of the unemployed decreases with unemployment duration and is lowest for those closest to the borrowing constraint. Their consumption, also measured relative to average consumption of the employed, decreases from a high 0.89 at the highest wealth level to a low 0.34 at the borrowing constraint. The consumption of a no-wealth worker jumps from 0.34 to 0.89 when finding employment, a 160 percent increase.

Figure 2 also shows the steady-state distributions over wealth for both worker groups. From the shape of the distribution it is clear that my model is unable to replicate the observed wealth distribution of the U.S. with its large skewness to the right (see, for example, Budría Rodríguez et al., 2002, for data on the U.S. wealth distribution) and instead is skewed to the left. This should not come as a surprise as my model abstracts from worker heterogeneity, life-cycle motives, and inheritances. The only reason why individuals accumulate and hold wealth in my model is for precautionary reasons.

To understand the shapes of the wealth distributions, it is instructive to look at the Euler equations (10) and (11). These equations show that the employed plan consumption with the expectation of becoming unemployed with probability $s$ while the unemployed plan consumption with the expectation of becoming employed with probability $f$. At the calibration point, the job-finding rate is 0.57 so that the average unemployment spell lasts about 7.5 weeks. Only about 8 percent of the unemployed remain jobless for longer than 3 months. In comparison, the average employment spell lasts about 30 months. Individuals spend much more time employed and building up precautionary wealth than unemployed and dissaving.

However, once unemployed, individuals move quickly towards the borrowing constraint. While it takes an employed worker about 10 years to reach the upper bound of savings, conditional on no job-loss, it takes the unemployed who start out at the highest wealth level only 10 months of continuous unemployment to reach the borrowing constraint. The high job-finding rate makes this an extremely unlikely event: the probability of remaining unemployed for 10 months is only 0.01 percent. As a comparison, an unemployed worker with maximum wealth experiences a 10 percent decrease in consumption after about 5 months of continuous unemployment, while an unemployed worker with average wealth experiences the same decrease after only 3 months of continuous unemployment. Both events are quite unlikely. The first carries a probability of 1.4 percent while the latter carries an 8 percent probability of realizing. These statistics highlight another unrealistic part of my highly stylized model. All workers are equally productive and face the same stochastic
probability of job loss and of finding employment.

Note that not all of the observed differences in consumption between employed and unemployed workers is due to a lack of insurance markets. As I argue in web Appendix B.3, consumption and hours of work are complements. Even with full insurance, the unemployed would choose to consume less. While consumption for the wealthiest workers only drops by 12 percent, it drops by 60 percent for workers with no wealth. Hence, lack of sufficient funds and insurance accounts for the majority of the consumption difference for low-wealth workers.

Given the consumption patterns described above, it should not be surprising that low-wealth unemployed workers benefit the most from finding employment. The joint surplus of a matched worker-firm pair, given by equation (20), is a decreasing function of workers’ asset holdings. An employed worker with no wealth has a high marginal utility of consumption relative to wealthier workers and thus gains a lot more from finding employment. Equation (19) then requires that the efficient choice of working hours must be relatively higher for a no-wealth worker. In fact, as Figure 3 shows, an employed worker with no wealth works almost 40 percent more than the average
worker. Because the outside option of no-wealth workers is extremely bad (that is, returning to unemployment with very low consumption and hence a high marginal utility of consumption), her bargaining position is much worse than that of the average worker. As a result, the hourly wage of a no-wealth worker is 23 percent lower than that of the average worker. However, because no-wealth workers work longer hours, their after-tax compensation is about 6 percent higher than that of the average worker. As workers become wealthier, their wage increase and their labor supply decreases. Firms prefer to employ poor workers. The combination of lower wages and longer working hours means that a firm’s profit from employing a no-wealth worker is more than 12 times higher than employing the average worker.

The fact that working hours decrease as wages increase indicates that much of the labor supply of the poor is for consumption-smoothing purposes and for precautionary reasons, which is exactly what equation (19) indicates. When consumption is relatively low, hours of work must be relatively high. This result is not unique to my model. Pijoan-Mas (2006) analyzes a growth model with idiosyncratic labor market risk and compares a complete market economy with an incomplete market economy. He finds that individuals make ample use of labor supply as a consumption smoothing mechanism. In particular, low-wealth workers with low wage realizations work long hours to keep consumption high, while wealthy workers with high wage realizations work relatively little because they already enjoy high consumption. In a complete markets economy workers’ base their hours decision entirely on the labor-leisure trade-off. Idiosyncratic wage shocks do not carry any wealth effects and the variation in working hours are only determined by the substitution effect. This is no longer true in incomplete market economies, where idiosyncratic shocks directly affect consumption.

Using synthetic data Domeij and Flodén (2006) show that, conditional on the wage rate, low-wealth workers work considerably more than high-wealth workers (see their Table 1). They find that the existence of borrowing constraints biases labor supply elasticities downwards and confirm their results using data from the PSID. In my model, hours of work decrease with increases in wealth and wages. Although this seems counterintuitive at first, it is the result of an increase in the wealth of borrowing constrained workers.

Some interesting implications arise in my model. Consider a young person entering this economy with no wealth and starting in unemployment. It will take her about 2 months to find a job after which she will likely be continuously employed for the next 2.5 years. During her tenure, she will experience an average wage growth of 0.8 percent per month, a decline in hours worked of 0.96 percent per month and an increase in consumption by 0.36 percent per month. After 2.5 years, she will have accumulated savings worth more than 2.5 months of average consumption, which is
72 percent of the wealth that an average worker holds.

The Frisch elasticities of consumption demand and labor supply are both functions of consumption and working hours. The no-wealth unemployed have an elasticity of consumption demand of about -0.15 while unemployed workers at the maximum wealth level have an elasticity of -0.31. This compares to an average Frisch elasticity of consumption demand of -0.35 for the aggregate economy. These values imply high risk aversion, ranging from a high 6.5 to a low 3.2 for the unemployed. The Frisch elasticity of all employed is very close to -0.35, implying a risk aversion of about 3.1. Workers become more risk-averse the closer they move towards the borrowing constraint, and their risk aversion increases when they become unemployed. Note that this is consistent with the findings discussed in Attanasio, Banks and Tanner (2002). Workers with low wealth are more risk-averse.

Similarly, workers closest to the borrowing constraint have the lowest Frisch elasticity of labor supply. The elasticity for no-wealth workers is about 0.52, while that of the wealthiest individual is only slightly higher than the 0.69 of the aggregate economy. This is consistent with the findings of Domeij and Flodén (2006).

8 Optimal unemployment insurance

I focus on an unemployment insurance scheme with constant benefits and an indefinite duration. The scheme is financed through a constant labor income tax. The results in Shimer and Werning (2005) suggest that such a scheme is close to optimal when workers are able to self-insure through savings. My goal is to determine the optimal replacement rate and to understand the role workers’ cost of self-insurance plays in its determination.

I consider two social welfare functions discussed by Rawls (1971), the “Veil of Ignorance” and the minimax functions. The first one maximizes the expected lifetime utility of a worker who does not know her labor market state or wealth level, that is, the worker is behind a “Veil of Ignorance.” The second welfare function maximizes the expected lifetime utility of the worker who has the lowest utility. Although the individual with the lowest utility in my economy is an unemployed worker with no assets, I choose to focus on an employed worker with no wealth. This has the additional interpretation of maximizing the welfare of a new labor market entrant with no wealth. As discussed in Section 5 most individuals, and especially new labor market entrants, hold very little wealth. In addition, under current law workers only become eligible to receive UI benefits after being employed for some amount of time.
Definition (Optimal replacement rate under the “Veil of Ignorance”) Under the “Veil of Ignorance,” the optimal UI policy is the replacement rate \( \delta \) that maximizes the expected lifetime utility of workers before they realize their labor market state and wealth level, or

\[
\delta = \arg \max (1 - u) \int_0^\infty E(a) dG_e(a) + u \int_0^\infty U(a) dG_u(a)
\]

(27)

Definition (Optimal replacement rate under the minimax criterion) Under the minimax criterion, the optimal UI policy is the replacement rate \( \delta \) that maximizes the expected lifetime utility of an employed worker with no wealth, \( a = 0 \), or

\[
\delta = \arg \max E(0)
\]

(28)

I find the optimal policy by solving the model for different replacement rates, keeping all other parameters constant. I then calculate the expected lifetime utility of a worker under both criteria and measure the welfare gain as a percentage change in the consumption equivalent of expected lifetime utility (see web Appendix H for more details). The optimal replacement rate under the “Veil of Ignorance” is 34 percent of average after-tax compensation and is associated with a 22.2 percent tax on labor income. The optimal replacement rate under the minimax criterion is 40 percent of average compensation and is associated with a 22.8 percent tax on labor income. Note that under the “Veil of Ignorance” the optimal replacement rate equals the value to which I calibrated this economy.

My results suggest that replacement rates in the U.S. are close to optimal. Among the 50 U.S. states plus the District of Columbia, 24 states have replacement rates between 34 and 40 percent, 15 states have replacement rates below 34 percent and 12 states have replacement rates above 40 percent (see Table 16 in web Appendix I). Table 2 shows a comparison of the current policy with the optimal policy under the minimax criterion.

The optimal replacement rate weighs the benefits of higher insurance provision and higher wages against increases in the unemployment and tax rates. A higher replacement rate provides more insurance by directly increasing the consumption of unemployed workers with few assets. This makes prolonged unemployment a less scary event and increases workers’ outside option when bargaining with the firm over wages and hours. As a result hourly wages increase with higher replacement rates.

As Table 2 reports, low-wealth workers benefit the most from a higher replacement rate. While average wages increase by only 0.4 percent, hourly wages of no-wealth workers increase by 10.3 percent. This large increase in wages allows low-wealth workers to increase consumption. As a result, their marginal utility of consumption decreases and, to satisfy the efficiency condition of
<table>
<thead>
<tr>
<th>Optimality Criterion</th>
<th>Veil of Ignorance</th>
<th>Minimax</th>
<th>Difference (%)</th>
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<td>Optimal Replacement Rate (%)</td>
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<td>Wage</td>
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<td></td>
<td>Hours</td>
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<td></td>
<td>After-tax Compensation</td>
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<td>1.40</td>
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<td>0.40</td>
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<td>1.26</td>
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<tr>
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<td>Wage</td>
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<td>1.96</td>
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<tr>
<td></td>
<td>Hours</td>
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<td></td>
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<td>Unemployment Rate (%)</td>
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<td></td>
<td>Average Unemployment Duration (months)</td>
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<td>Welfare Gain**</td>
<td>Minimax criterion</td>
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<td>0.13</td>
</tr>
<tr>
<td></td>
<td>&quot;Veil of Ignorance&quot; criterion</td>
<td>0.00</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

Note: Assets and saving are measured relative to average consumption of the employed.
* Wage, hours, and after-tax compensation are aggregate averages for the employed.
** Welfare gain is the percentage change in lifetime utility measured in units of consumption associated with moving from the current 34 percent replacement rate to the optimal policy.

Table 2: Optimal replacement rates in the baseline economy

equation (19), they work fewer hours. At higher UI replacement rates, no-wealth workers receive higher compensation, enjoy more leisure, and are able to accumulate precautionary wealth more quickly.

As Figure 4 shows, an increase in the replacement rate increases wages for all workers, although the increase is largest for low-wealth workers. The change in working hours is also largest for low-wealth workers. For workers with savings worth more than one month of average consumption, more than 99 percent of the population in my model, a higher UI replacement rate affects working hours very little. Although low-wealth workers decrease their hours by up to 6.3 percent, average hours increase slightly by 0.1 percent.

While consumption of no-wealth individuals unambiguously increases with higher replacement rates, this is not true for wealthier individuals. Higher replacement rates are associated with higher
unemployment rates and higher labor income taxes. As workers bargain for higher wages, firms’ flow profits \((m - w)h\) decrease, lowering the expected value of hiring a worker (see equation (13)). This decreases firms’ incentives to post vacancies and results in a lower job-finding rate, a longer average unemployment duration, and a higher unemployment rate. At a replacement rate of 40 percent, unemployment is 16 percent higher than in the baseline economy. The combination of higher benefits and a higher unemployment rate requires the government to increase the labor income tax \(\tau\) to balance the budget. With a 17 percent increase in unemployment duration, workers accumulate 2.3 percent more wealth. Compared to the baseline economy, aggregate consumption is slightly lower in the economy with a replacement rate of 40 percent.

The high levels of the optimal replacement rates are surprising, given that workers may self-insure through savings. Although it is difficult to compare my results to those in Shimer and Werning (2005), they conjecture that the optimal replacement rate is low and consistent with findings in Gruber (1997) that the optimal replacement rate is in the range of 0 to 10 percent of income.
if individuals do not face liquidity constraints. In the more realistic case where individuals are borrowing constrained, they conjecture that optimal replacement rates may be much higher. There are several reasons why the optimal replacement rate lies between 34 and 40 percent of compensation in my model economy.

First, individuals in my economy are quite risk-averse. While Shimer and Werning (2005) choose a low level of risk-aversion with a CRRA coefficient of 1.5, my choice of parameters implies an average risk-aversion of 3.1. However, even if individuals in my economy were much less risk-averse, the optimal replacement rate would still be quite high. For an average risk aversion of 1.5, the optimal policy is still 20 percent of average compensation, see web Appendix C.1.

Second, accumulating and holding precautionary wealth is costly. Accumulating precautionary wealth is costly because individuals dislike work. In order to increase consumption and to quickly accumulate wealth, low-wealth workers work unusually long hours, about 40 percent more than the average worker. Holding precautionary wealth in my model is costly because the individual discount rate is higher than the return on assets, so that individuals would rather consume today instead of saving for the future. Shimer and Werning (2005) focus on the consumption/saving and search behavior of the unemployed and do not model the savings behavior of the employed. The results of my model indicate that the replacement rate not only influences the behavior and well-being of the unemployed, but that it also has important effects on the consumption, saving, and labor supply behavior of the employed.

Under the “Veil of Ignorance” social welfare function, the current policy with a replacement rate of 34 percent is optimal. The welfare losses from deviating from this policy are potentially large. Decreasing the replacement rate by 10 percentage points would result in a welfare loss of 0.57 percent. Although average consumption would increase by 0.5 percent, the consumption of low-wealth individuals would decrease by up to 40 percent. This suggests that this large decrease in welfare is mostly due to a decrease in insurance provision. A similar-sized increase in the replacement rate would result in a welfare loss of approximately 0.4 percent. Although consumption of low-wealth individuals would increase by up to 30 percent, aggregate consumption would decrease by 1 percent, mostly because of an increase in the unemployment rate from 5.6 percent to 7.3 percent.

Under the minimax welfare criterion, the potential welfare gain from moving from the current replacement rate of 34 percent to the optimal replacement rate of 40 percent is significant: 0.13 percent of expected lifetime utility. As I will discuss in the next subsection, this large increase is due to an increase in insurance provision and the associated decrease in the need for accumulating wealth quickly and working long hours. To put this number in perspective, consider that a typical
Figure 5: Consumption, wages, hours, and saving of employed no-wealth workers. Consumption and saving are measured in units of average monthly consumption of the employed.

A no-wealth worker who spends 40 years in the labor force spends on average only a little more than two years in unemployment. On top of paying a 3 percent higher income tax to finance the higher benefit level, a no-wealth worker would be willing to give up 0.13 percent of lifetime consumption to move to the optimal policy.

Figure 5 shows consumption, wages, hours worked, and saving of an employed no-wealth worker as a function of the UI replacement rate. Increasing UI benefits has the predicted effects on consumption. At the optimal policy according to the minimax criterion, wages are 10.3 percent higher, hours are 6.3 percent lower, and after-tax compensation is 2.5 percent higher. Of this 2.5 percent increase in compensation, a no-wealth worker uses 67 percent to increase consumption and 33 percent to increase saving.

The reason why the optimal policy is lower under the “Veil of Ignorance” criterion is that the unemployment rate plays a more important role. Under the minimax criterion, a lower job-finding rate only enters the social welfare function through the value of an unemployed worker $U(a)$.
Under the “Veil of Ignorance” criterion, however, the unemployment rate acts as a welfare weight. The higher the unemployment rate, the more weight is put on the value of being unemployed, which is always lower than the value of being employed.

### 8.1 Isolating the insurance effect of UI benefits

Increasing UI benefits not only provides workers with better insurance, it also increases their outside option during bargaining and hence increases their wages. Higher wages, however, reduce firms’ flow profits, the expected value of hiring a new worker, and hence their recruiting efforts. As a result, the job-finding rate decreases and the unemployment rate increases.

To isolate the insurance effects of UI benefits I restrict wages to equal the aggregate average wage in the baseline economy, so that changes in the benefit level no longer affect wages and unemployment. I then perform the same policy experiment as before with the exception that I keep wages constant. Working hours are still determined by Nash bargaining, but I replace equation \[17\] with \(w_{\text{fixed}} = \mathbb{E}[w_{\text{Nash}}(a)]\).

With fixed wages, the benefit level no longer has an effect on the unemployment rate. Figure 6 shows the main results of this exercise. As unemployment benefits increase, workers’ self-insurance needs decrease. As a result, saving slowly decreases until it reaches zero at a replacement rate of 87 percent. The number of working hours decreases as well, from 1.0 at the calibration point to a low of 0.85, a decline of 15 percent. As the saving rate decreases, consumption slowly increases. A replacement rate of 0.87 percent implies perfect insurance.

The welfare gain from moving from the current policy with a replacement rate of 34 percent to the optimal policy under the minimax criterion is 0.17 percent of expected lifetime utility. This is larger than the welfare gain of 0.13 percent achieved from moving to the optimal policy in the full equilibrium model reported in the previous section. This suggests that the optimal replacement rate under the minimax criterion is largely determined by low-wealth workers’ need for additional insurance. The reason why the welfare gain in the equilibrium model with endogenous wage determination is lower is that the job-finding rate decreases and the unemployment rate increases. The associated net welfare loss of an increase in the unemployment rate from 5.6 percent in the baseline economy to 6.5 percent at a replacement rate of 40 percent and an increase in the wage rate of 10 percent is 0.04 percent (0.17% − 0.13% = 0.04%).
Figure 6: Consumption, hours, and saving of employed no-wealth worker with fixed wages. Consumption and saving are measured in units of average monthly consumption of the employed with 34 percent replacement rate.

8.2 The role of adjustments in hours

To explore the role of adjustments in hours, I assume that working hours are fixed at the aggregate average of the baseline economy with $\delta = 0.34$. I replace equation (18) with the constant $h = 0.85$ but leave all other equations and parameters unchanged. The results are reported in Table 3.

In the economy with fixed hours and a replacement rate of 34 percent, a no-wealth worker spends 27 percent less time working than in the baseline economy with efficient determination of hours, although not by choice. Because workers do not have the ability to adjust hours, their expected lifetime utility is lower than in the economy where hours are determined efficiently. Although the bargained wage of a no-wealth worker is 11 percent higher than in the baseline economy, total compensation is almost 20 percent lower. As a result, consumption is 17 percent lower and saving is 31 percent lower compared to the baseline economy with efficient determination of hours.
Interestingly, average consumption, wages, and income differ only very slightly, all by less then 0.8 percent. While saving of the no-wealth worker is lower in the fixed-hours economy, aggregate wealth is 22 percent higher. Since no-wealth workers are unable to quickly increase their precautionary balances when needed (for example, after an unemployment spell), they choose to accumulate more wealth to have an extra buffer when becoming unemployed. This large increase in precautionary wealth is surprising given that very few workers ever get close to the borrowing constraint.

When working hours are adjustable, low-wealth workers spend more time on the job, have higher incomes, and save more. Yet, aggregate wealth is highest in the fixed hours economy. This strongly suggests that low-wealth workers work longer hours in order to accumulate precautionary wealth more quickly. If they are unable to adjust hours, workers choose to accumulate considerably more wealth to ensure that they will have the means to keep a smooth consumption profile even in the unlikely case of multiple unemployment spells with only short employment durations in between.

Surprisingly, when I do not allow workers to adjust hours, the optimal replacement rate is lower
than when hours are set efficiently. As I discussed in the previous section, low-wealth workers
decrease working hours considerably with increases in the replacement rate. The decrease in hours
is associated with an increase in utility. When workers are not allowed to adjust hours, the benefit
of a higher replacement rate decreases, while the cost, an increase in the unemployment rate,
stays the same. Hence the optimal replacement rate is lower to balance the marginal cost with the
marginal benefit to the worker.

These results suggest that preferences over hours are a potentially important determinant of the
optimal UI policy. When working longer hours is relatively costly (in this extreme example it is
infinitely costly) compared to accumulating and holding wealth, workers will substitute saving for
working longer hours. When hours of work do not respond to changes in the benefit level, as in
this example, the optimal replacement rate is lower.

9 The cost of self-insuring

As discussed before, self-insuring in my model is costly for two reasons. First, accumulating
precautionary wealth is costly because workers dislike work. Low-wealth workers work usually
long hours to increase consumption and to quickly build up wealth. However, this carries a high
utility cost. Higher UI benefits increase consumption of the constrained workers directly and
increases their marginal utility of consumption. For equation (19) to hold, hours must decrease.
Thus, higher benefits not only increase insurance, but also reduce the need for constrained workers
to work unusually long hours.

The second reason why self-insuring is costly is that workers are unable to save at an interest
rate $r$ equal to their personal discount rate $\rho$. As I discussed in detail in Section 5.3, data on
household wealth suggests that the annual personal discount rate is $\rho^a = 0.05$ when the annual
risk-free interest rate is $r^a = 0.03$. Since $r^a < \rho^a$, workers would rather consume income today
than save for tomorrow. The cost of holding precautionary wealth is increasing in the difference
$\rho^a - r^a$.

9.1 Implications of the Frisch elasticity of labor supply

The Frisch elasticity of labor supply is one determinant of the utility cost of working (the other is
the distaste parameter $\gamma$). To assess its effect on labor supply, saving, and the optimal UI policy,
I repeat my earlier calibration for a range of elasticities. I keep the average Frisch elasticity of
consumption demand at $\varphi(h) = -0.35$ and the average consumption drop at $\omega = 0.10$. I calibrate
with different values for the Frisch elasticity of labor supply $\eta(h)$ to get new values for the
parameters $\sigma$, $\mu$, and $\psi$. Table 13 in web Appendix I contains the new parameter values for the range of labor supply elasticities I consider. The parameter $\psi$ changes proportionally with the Frisch elasticity, while changes in the other parameters are modest. The only other parameter that changes is the vacancy creation cost $k$. It has to change with the Frisch elasticity to maintain an unemployment rate of 5.6 percent at the calibration point.

The level of the Frisch elasticity has large effects on labor supply, income, saving and thus the wealth distribution. The higher the Frisch elasticity, the more workers adjust hours in response to changes in the wage, holding marginal utility of consumption constant. Figure 7 shows working hours and wealth distributions as functions of current wealth for three levels of the Frisch elasticity. The choices of Frisch elasticities I consider are consistent with estimates provided by MaCurdy (1981), Browning et al. (1985), and Altonji (1986) (Frisch elasticity of $\eta(c, h) = 0.30$), and Mulligan (1995, 1998) and Kimball and Shapiro (2003) (Frisch elasticity of $\eta(c, h) = 1.10$).
Table 4: Implications of the Frisch elasticity of labor supply $\eta$

The replacement and unemployment rates are the same as in the baseline economy, that is $\delta = 0.34$ and $u = 0.056$. The number of working hours are measured relative to the mean number of hours in each economy.

Workers close to the borrowing constraint work considerably harder than the average worker. This difference is increasing in the Frisch elasticity of labor supply. With a low Frisch elasticity of $\eta = 0.30$, a constrained worker spends 17 percent more time working than the average worker. With a high elasticity of $\eta = 1.10$, a constrained worker spends 51 percent more time on the job than the average worker. The slope of the marginal disutility of working, $u_{hh}$, is decreasing in the Frisch elasticity of labor supply. To satisfy equation (19), this means that hours of work must increase more as the marginal utility of consumption increases close to the borrowing constraint.
Intuitively, the higher the Frisch elasticity, the less costly it is for workers to adjust hours.

As Table 4 shows, workers with no assets but a high Frisch elasticity of $\eta = 1.10$ work 13 percent more than their low-elasticity counterparts with $\eta = 0.30$. Although hourly wages are 7 percent lower for the high-elasticity workers, after-tax compensation is 6 percent, consumption 1.2 percent, and saving 50 percent higher. The difference in consumption between a constrained worker and the average worker is largest for workers with a low labor-supply elasticity. Constrained workers with a Frisch elasticity of $\eta = 0.30$ consume 17 percent less than the average worker, while a constrained worker with a Frisch elasticity of $\eta = 1.10$ consumes only 7 percent less than the average worker.

As Figure 8 shows, workers with high Frisch elasticities save more initially to build up wealth quickly. It takes a worker with a high Frisch elasticity about 17 months of continuous employment to accumulate assets worth two months of average consumption, while it takes a low-elasticity worker about 24 months. Since adjusting hours is more costly for workers with low Frisch elastic-
ities, they accumulate wealth more slowly than their high elasticity counterparts.

Getting close to the borrowing constraint is worse for workers with a low Frisch elasticity than for workers with a high Frisch elasticity of consumption. The marginal utility of consumption and the marginal disutility of working at the borrowing constraint (relative to the respective average marginal utilities) are considerably higher for workers with low elasticities of labor supply. Because being close to the borrowing constraint is worse for low-elasticity workers, they accumulate considerably more wealth than high-elasticity workers.

Given these results, it should not be surprising that the optimal replacement rate is decreasing in the Frisch elasticity. Because the cost of adjusting hours is lower for workers with a high Frisch elasticity, they are better able to self-insure and benefit less from a higher replacement rate. The welfare gains associated with moving from the current policy to the optimal policy are as large as 0.33 percent of lifetime utility for workers with low Frisch elasticities.

9.2 Implications of the cost of holding wealth

The cost of holding wealth is determined by the difference between the annual interest rate $r^a$ at which workers may save, and the annual personal discount rate of workers, $\rho^a$. To investigate the role that the cost of holding wealth plays in the determination of the optimal replacement rate, I solve the model with different values for $\rho^a$, leaving the remaining calibration unchanged. I report the results in Table 5.

Higher discount rates have almost no effect on most variables, except for aggregate assets. With a personal discount rate very close to the interest rate, workers accumulate substantially more wealth than with higher discount rates. With a discount rate of 3.3 percent, average assets equal 7.3 months of average consumption, or close to twice as much as in the baseline economy with a discount rate of 5 percent. Assuming that precautionary wealth represents about 20 percent of total wealth, this number would imply a median wealth-to-income ratio of 3.2, about twice as high as the one measured in the data (see Section 5.3). With a discount rate of 10 percent, average assets equal only 2.2 months of average consumption and imply a median wealth-to-income ratio of 1.

As the cost of holding wealth increases, so does the optimal replacement rate. Lentz (2005) appears to have been the first to point out the relationship between the optimal replacement rate and the spread between the interest rate and the personal discount rate in a search model with savings. He fixes the subjective time discount rate and then varies the interest rate. My analysis here simply shows that the difference between the interest rate and the personal discount rates of workers matters for the optimal policy.
<table>
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<tr>
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<td>Optimal Replacement Rate (%)</td>
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<td>35</td>
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<td>Welfare Gain (%)*</td>
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<td>0.06</td>
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</table>

All results are with replacement rate $\delta=0.34$.  
Note: Assets are measured relative to average consumption of the employed.  
* Welfare gain is the percentage change in lifetime utility measured in units of consumption associated with moving from the current 34 percent replacement rate to the optimal policy.

Table 5: Implications of the cost of holding wealth

10 Concluding remarks

I analyzed how individuals’ costs of self-insuring affect the determination of the optimal UI replacement rate. I conducted my analysis within a standard equilibrium matching model that I extended along three important lines. First, workers are risk-averse and have preferences over consumption and hours of work, which I calibrated to match results from micro studies on labor supply, the intertemporal substitution of consumption, and the complementarity between hours of work and consumption. Second, I explicitly model the intensive margin of labor supply as the outcome of bilaterally efficient bargaining between workers and firms. Third, I allow workers to self-insure by saving a riskless asset at an exogenously given interest rate.

My results suggest that the cost of accumulating and holding precautionary wealth is an important determinant of the optimal UI policy. Accumulating precautionary wealth is costly because working higher than usual hours imposes a utility cost on workers. Holding precautionary wealth is expensive because the interest rate at which individuals may save is lower than their personal discount rate. A high UI benefit level alleviates these costs to some degree, in addition to providing workers insurance against unusually long unemployment spells that would result in consumption disasters.

There are a few issues future research should address. First, I assumed that Shimer and Werning’s (2005) result that the optimal UI policy involves constant benefits of indefinite duration and a constant tax rate upon reemployment also holds in my model economy. This assumption needs further investigation. On the one hand, the fact that the marginal utility of consumption increases considerably the closer workers get to the borrowing constraint suggests that an increasing benefit
schedule might be optimal. On the other hand, an increasing benefit schedule will most likely distort the savings behavior of the employed considerably. Why save if you are only rewarded when you have few assets?

Future research should pay closer attention to generating the earnings, income, and wealth distributions observed in the U.S. economy. Heterogeneity among works and/or firms will obviously be important to achieve such a goal. Investigating the efficiency of means-tested UI benefits would be an interesting topic, as would be analyzing the role of insurance provision by family members, such as spouses or parents, and the role of life-cycle savings in providing an additional source of liquidity to otherwise constrained workers.

From a more theoretical perspective, investigating the bargaining between the worker and the firm would be of interest. In my model the interests of workers and firms are orthogonal. Workers want to accumulate assets to self-insure and to increase their bargaining position, while firms would prefer workers to remain poor indefinitely. How would the results change if workers were to save strategically? Could the firm offer a contract that aligns their interests with that of the workers?
Appendix

A  Details of the solution method

I represent the unknown functions as Chebyshev polynomials of degree 10 and use projection methods as described in Judd (1992) and chapter 11 in Judd (1998) to solve for the 70 Chebyshev coefficients. I solve the following system of equations by finding the zeros of the 7 residual functions $R_j(d_j; a)$, where $d_j$ is the 1x10 vector of Chebyshev coefficients associated with function $j$.

\[
R_E(d_E; a) = \tilde{E}(a) - u(c_v(a), \tilde{h}(a)) + \frac{1}{1 + \rho} \left[ s\tilde{U}(a') + (1 - s)\tilde{E}(a') \right]
\]

\[
R_U(d_U; a) = \tilde{U}(a) - u(c_u(a), 0) + \frac{1}{1 + \rho} \left[ f(\theta)\tilde{E}(a') + (1 - f(\theta))\tilde{U}(a') \right]
\]

\[
R_J(d_J; a) = \tilde{J}(a) - [m - \tilde{w}(a)\tilde{h}(a)] + \frac{1}{1 + r} \left[ \tilde{J}(a') \right]
\]

\[
R_w(d_w; a) = u_c(c_v(a), \tilde{h}(a)) - \frac{1}{1 + \rho} \left[ su_c(c_u(a'), 0) + (1 - s)u_c(c_v(a'), \tilde{h}(a')) \right]
\]

\[
R_u(d_u; a) = u_c(c_u(a), 0) - \frac{1 + r}{1 + \rho} \left[ f(\theta)u_c(c_v(a'), \tilde{h}(a')) + (1 - f(\theta))u_c(c_u(a'), 0) \right]
\]

\[
R_h(d_h; a) = \frac{\phi}{1 - \phi} \tilde{J}(a) - \frac{\tilde{E}(a) - \tilde{U}(a)}{u_c(c_v(a), \tilde{h}(a))(1 - \tau)}
\]

\[
R_b(d_b; a) = \frac{-u_b(c_v(a), \tilde{h}(a))}{u_c(c_v(a), \tilde{h}(a))} - (1 - \tau)m
\]

where

\[
c_u(a) = b + (1 + r)a - a'_u(a)
\]

\[
c_v(a) = (1 - \tau)w(a)\tilde{h}(a) + (1 + r)a - a'_v(a)
\]

A solution to this system of equations is given by the 70 coefficients $d_j$ that solve the 7 equations $R_j(d_j; a) = 0$ at the 10 Chebyshev collocation nodes chosen from $a \in [a, \bar{a}]$. Because the job-finding rate depends on the wealth distribution (see equation 13), I use the following iterative algorithm to solve the equilibrium model.

1. Given the parameters of the model, make a guess for the job-finding rate (for the calibration this is simply the calibration target)

2. Solve the the model by finding the zeros of the 7 residual functions $R_j(d_j; a)$
3. Calculate the steady-state wealth distributions $G_e(a)$ and $G_u(a)$ following Hall (2006a).

4. Approximate $\mathbb{E}[J(a')]$ in equation (13) by calculating $\mathbb{E}[J(a')] \approx \sum \tilde{J}(a) \tilde{G}_u(a)$.

5. Update the vacancy/unemployment ratio $\theta$ by calculating

$$\theta = \left( \frac{(1 + r)k}{m_0 \mathbb{E}[J(a')]} \right)^{-1/\alpha}$$

6. Calculate the new job-finding rate as $f(\theta) = \zeta \theta^{1-\alpha}$.

7. Update the tax rate $\tau$ by solving equation (21) as

$$\tau \approx \frac{u b + \chi}{(1 - u) \sum w(a) h(a) G_e(a)}$$

8. Exit the iteration if the proportional change in $\mathbb{E}[J(a')]$ and $f(\theta)$ is sufficiently small; I use $10^{-6}$ as my stopping rule. Otherwise, start a new iteration at number 1 above with the new guess for the job-finding rate being the result of this iteration.

When solving the system of equations above, it is important to have a good initial guess for the 70 coefficients of the 7 unknown functions. Without a good guess, even very good non-linear equation solvers such as NPSOL are unable to find a solution. To generate a sufficiently good guess for all 70 coefficients, I proceed as follows.

1. Make a guess for the 20 coefficients $d_{ue}$ and $d_{uu}$ and, treating $w$ and $h$ as parameters, solve the system

$$R_{ue}(d_{ue}; a) = 0$$
$$R_{uu}(d_{uu}; a) = 0$$

2. Make a guess for the 20 coefficients $d_E$ and $d_U$ and use the result for $d_{ue}$ and $d_{uu}$ as an initial guess to solve

$$R_{ue}(d_{ue}; a) = 0$$
$$R_{uu}(d_{uu}; a) = 0$$
$$R_E(d_E; a) = 0$$
$$R_U(d_U; a) = 0,$$

again treating $w$ and $h$ as parameters.
3. Make a guess for $d_J, d_w,$ and $d_h$ and use the results for $d_E, d_U, d_{u^e},$ and $d_{u^u}$ as an initial guess to solve the full system of 70 equations in 70 unknowns.

For discussions of the parameters choices, the strategic consumption choices, the Nash bargain solution, a derivation of the Frisch elasticities, more information on the solution method, and an explanation of how to calculate the welfare gains, please see the web Appendix at http://felixr.googlepages.com/webapp.pdf.
References


