Environment and Growth

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Abstract

This paper examines the implications of the mutual causality between environmental quality and economic growth. While economic growth deteriorates the environment through increasing amounts of pollution, the deteriorated environment in turn limits the possibility of further economic growth. In a less developed country, this link, which we call “limits to growth,” emerges as the “poverty-environment trap,” which explains the persistent international inequality both in terms of income and environment. This link also threatens the sustainability of the world’s economic growth, particularly when the emission of greenhouse gases raises the risk of natural disasters. Stronger environmental policies are required to overcome this link. While there is a trade-off between the environment and growth in the short run, we show that an appropriate policy can improve both in the long run.

Keywords: Environmental Kuznets Curve, Limits to Growth, Poverty-Environment Trap, Sustainability, Natural Disasters

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Figure 1: Long-term evolution of per capita GDP in the U.S. and Asian countries (in 1990 international dollars). Data source: Bolt and van Zanden (2013).

1 Introduction

One of the most important and challenging questions for economists has been how to harmonize economic growth with the natural world. Since the industrial revolution, the growth rate of income per capita has been fairly stable in the United States. As shown in Figure 1, the measured per capita real GDP in the U.S. has been expanding exponentially, with its growth rate after the mid-19th century being around 2%. Figure 1 also shows that a number of Asian countries are in the process of catching up to the U.S. income level. Although they differ in the timing when modern economic growth took off (e.g., Japan’s modern growth started relatively earlier, while China’s rapid growth is a much more recent phenomenon), their growth rates were typically higher than the U.S. after the second half of the 20th century. As long as this trend continues, the per capita income of successful countries will converge to the exponentially expanding U.S. per capita GDP level.

However, given that the world’s economic growth means the exponential expansion of output, especially if it requires ever increasing inputs of natural resources, it is obvious that this process cannot be continued for a very long time. This was the theme investigated by Meadows et al. (1972) under the title of the "Limits to Growth," which subsequently led to a large body of literature that examined the possibility of economic growth under resource scarcity (seminal studies include Dasgupta and Heal,
In addition to resource scarcity, the pollution that accompanies the production or use of particular kinds of inputs poses another constraint for economic growth. Although the literature on pollution and growth has largely been disjointed from that on the resource scarcity, the fundamental root of the problem is the same: the finiteness of the natural environment. Suppose that the aggregate production function has constant returns to scale and that all inputs are reproducible or non-exhaustible. In such a setting, long-term growth is typically achieved by a homothetic expansion of all inputs and outputs. However, if the production or use of some types of inputs involves pollution, such an expansion will result in an increasingly deteriorating environment. Given that nature itself cannot be expanded along with other inputs, the intensity of pollution (i.e., the ratio of pollution to environmental capacity) will increase with the growing production. The deteriorated environment in turn makes sustained economic growth difficult for a number of reasons, such as health problems and frequent natural disasters caused by global warming. In the paper “Are there limits to growth?”, Stokey (1998) considered this type of problem using an AK growth model with pollution and showed that it is not optimal to pursue sustained growth as long as the technology level is constant.

In this paper, we explain the implications of the interrelation between the environment and economic growth. In particular, we focus on two issues. The first is the feasibility of economic development in stagnant poor countries that are suffering both from low income and environmental degradation. Second, at the global scale we consider the sustainability of world economic growth in the future. While these two issues have so far been treated in two separate bodies of literature, we show that the key to understanding both issues is the same: the mutual causality between the envi-

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1 Theoretical models of economic growth that examine the relationship between pollution and growth often assumed away the finiteness of pollution-generating inputs. Besides the analytical tractability, one substantial reason for this is that when the emission of pollutants binds the possibility of economic growth, the constraint of resource scarcity becomes slack and will not affect the equilibrium or optimal outcome. Similarly, those that focus on the resource scarcity typically assume away pollution, because if the resource constraint is stricter, pollution will only have secondary effects on the possibility of economic growth. Nonetheless, some recent studies numerically examine the intricate interaction of pollution and the finiteness of resources and obtained quantitative implications of the interaction (for example, see Acemoglu et al. 2012).

2 In the endogenous growth literature, this type of model is called the “AK” growth model because, in the simplest form, the production function can be written as \( Y = AK \), where \( A \) is a constant technology parameter and \( K \) is the reproducible input, which is usually called capital (e.g., Rebelo 1991).
Figure 2: The relationship between income and pollution.

As we discussed in the introduction, we will inevitably face the “limits to growth” problem if the environment continues degrading as the economy develops. The consequences of “limits to growth” are illustrated in Figure 2, which depicts the mutual relationship between pollution and the income level in one phase diagram. In the figure, the $\dot{Y} = 0$ curve reflects the causality from pollution to long-term income: for a given intensity of pollution $P_t$, the output can grow up to the $\dot{Y} = 0$ curve in the long run.\(^3\) The downward slope of this curve means that the potential for economic growth is adversely affected by environmental degradation. For example, when air pollution harms human health (WHO, 2006), it not only lowers the productivity of workers but also reduces life expectancy and, hence, the return on education, which in turn lowers the incentives for parents to provide their children with higher education. Without sufficient educated workers, (foreign) firms with advanced technologies will be reluc-

\(^3\)The income (output) of the economy can grow only up to this downward sloping curve. For a given level of pollution, if $Y$ is smaller than this long-term level, the growth rate of output is positive, while the growth rate is negative if $Y$ is already larger the long-term level.
tant to invest in such regions. These considerations imply that higher pollution (i.e., environmental degradation) will adversely affect the long-term income.

What, then, determines environmental quality? We may think of economic growth as a determinant of pollution. At the initial stage of economic growth, the scale of production is small, and thus, both income and pollution would be small. In the figure, this means that the economy starts from a point near the origin. Then, as the economy develops, the scale of production increases. As long as the economy operates under the same technology and the same relative factor prices, the pollution $P$ would increase proportionally with output. In the figure, this means that the economy moves to the upper right direction and will eventually reach the $Y = 0$ curve, beyond which the economy cannot grow (denoted by path $a$).

While this seems a pessimistic result, in reality the technology level is not constant but improves as income grows. If improved technologies cause less pollution for a given amount of production, economic growth could mitigate the environmental problem through technological change. This consideration leads to the Environmental Kuznets Curve (EKC), a hypothesis that there should be an inverted U-shaped relationship between per capita income and various pollutants or environmental indicators. If this hypothesis is correct, environmental degradation continues until the income per capita reaches a certain level, but beyond it environmental quality will improve as the economy grows. In Figure 2, the path denoted as $b$ shows the movement of the economy following this hypothetical EKC. If pollution begins to decrease before the economy hits the $Y = 0$ locus, it might be possible that the economy can grow beyond the “limits to growth.” In fact, many studies, including seminal studies by Grossman and Krueger (1991, 1995) and Selden and Song (1995), confirm the existence of the EKC for local air pollutants, including sulfur dioxide ($SO_2$), suspended particulate

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4 More precisely, there are both supply-side and demand-side factors behind the effect of economic growth on the environmental quality. Grossman and Krueger (1991) assessed three supply-side determinants for the EKC: scale effects, composition effects, and technological effects. The scale effect simply means that pollution increases with the level of economic activity, as discussed so far in the main text. Second, the composition effect reflects a general structural change from pollution-intensive production to less pollution-intensive production in the development process. Third, the technological effect refers to an improvement in environmental quality through the introduction of cleaner technology resulting from economic growth. The shape of the EKC will reflect the aggregate magnitudes of these three effects (see a review by Brock and Taylor, 2005). Our models in Sections 3 and 4 formally consider the composition and technological effects and show that EKC emerges endogenously. On the demand side, the people’s demand for environmental quality tends to increase as their income grows because they can afford to devote more resources to abatement. This mechanism critically depends on the income elasticity of environmental quality (see John and Pecchenino, 1994; McConnell, 1997; Andreoni and Levinson, 2001; Lieb, 2002, 2004).
matter (SPM), and oxides of nitrogen (NO\textsubscript{x}).

Note, however, that the existence of the EKC does not always mean that every economy can overcome the “limits to growth.” Because of the differences in the characteristics of countries, including technology, resource endowments and institutions (particularly institutions for environmental protection), the shape and location of the EKC vary across countries. Three paths in Figure 3 illustrate the consequences of different EKC shapes. Path c illustrates the case in which the economy hits the $\dot{Y} = 0$ curve before reaching the top of the EKC. At this point, the economy is trapped by the mutual causality between environmental degradation and poverty. The environmental quality is low because the economy is poor. Such an economy cannot afford to employ better and cleaner technology because everyday consumption is their first priority and they cannot finance the costs of required investments that would improve their life and environment in the future. Similarly, it would be difficult for people in such an economy to agree to set stricter environmental regulations because such regulations would seem to (at least temporarily) further reduce their low incomes. At the same time, the economy is poor because environmental degradation lowers the productivity of workers, reduces their life expectancy, gives less incentives for parents to provide good education for their children, and so forth. We call such a situation the “poverty-environment trap”.

In Figure 3, path d shows that an economy that maintains low pollution intensity along the process of economic development can get over the top of the EKC and reaches a steady state in which both the environment and income are better than
Figure 4: Income and Air Pollution in Asian countries. Vertical axis: Annual mean PM10 concentrations in the capital city, where PM10 means particulate matter with a diameter of 10 µm or less. Source: Urban outdoor air pollution database, Department of Public Health and Environment, World Health Organization, September 2011. Per capita income is from World Development Indicators (WDI), Worldbank.

those of economies trapped by the poverty-environment trap. Path e shows that it is theoretically possible that an economy can grow indefinitely without facing limits to growth. These considerations suggest that in the long run we will observe large differences across countries in terms of the intensity of pollution and income level and will also find a negative relationship between these two variables. Figure 4 confirms this expectation, which shows that there is a negative relationship between air pollution (PM10 concentrations) and the per capita income level among Asian countries. In Section 3, we present a formal model with a microeconomic foundation that explains the existence of multiple steady states—the poverty-environment trap and a better steady state—and we discuss how the environment is related to international income differences.

Pollution is a serious problem not only at the level of individual countries but also at the global scale, particularly regarding the issue of global warming. In this case, we should view the whole global economy as one entity because the emission of global warming gases depends on the economic activities in all countries. Can we then observe an inverted-U relationship between the average income in the world and the emission of greenhouse gases? Thus far, the answer is negative. In many papers (e.g., Dinda,
2004; Kijima et al., 2010), the existence of the EKC has not been supported for the global warming gases such as carbon dioxide (CO\textsubscript{2}). If global pollution continues to increase as the world economy grows, it will pose a threat to the sustainability of future growth.

In fact, NASA suggests that an increase in global temperatures results in an increased intensity of storms, including tropical cyclones with higher wind speeds, a wetter Asian monsoon, and, possibly, more intense mid-latitude storms. Figure 5 shows that in the last 50 years total economic damages in the world have increased more rapidly than the world’s GDP, and most of the increase was due to weather related disasters. For example, Hurricane Katrina in August 2005 caused total economic damages of $125 billion in the United States. More recently, typhoon Haiyan (or Yolanda) in the Philippines in November 2013 generated $12 billion in economic damages, an enormous sum for a small country. CRED reports that floods appeared to be most frequent during the last two decades, and the highest number of floods occurred in Asia. The total damage and losses from the 2011 floods in Thailand amounted to $40 billion, more than 1/10th of the country’s GDP. Given that the economic damages

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6CRED Crunch, Issue No.32, August 2013, Center for Research on the Epidemiology of Disasters.
from natural disasters come primarily in the form of capital destruction, a higher risk of natural disasters inhibits the process of capital accumulation, not only by direct destruction of the stock but also by reducing the expected return from investing in new production facilities. If global warming continues with economic growth, and if these weather related disasters are intensified accordingly, it is clear that at some point further economic growth will become unsustainable.

We can again illustrate such a consequence in a phase diagram. Two panels in Figure 6 show the hypothetical evolution of the income of the world $Y$ and the intensity of greenhouse gases $P$ in a phase diagram. We again have a downward sloping $\dot{Y} = 0$ curve. A higher intensity of greenhouse gases will cause a higher risk of natural disasters. Given that the risk of natural disasters lowers new investments for production, it will lead to a smaller steady-state stock of capital and, hence, a lower steady-state level of world income. One difference from Figure 2 is that we now consider the possibility of endogenous growth. In the literature of endogenous growth, it is considered that physical and human capital can be accumulated without reaching a steady state, and that the rate of accumulation is determined endogenously by underlying economic conditions such as technology and preference (we will present a formal model in Section 4). In the current setting, a key factor in the economic conditions is the risk of natural disasters, and it would be legitimate to suppose that the long-term rate of economic growth becomes positive only when the greenhouse gas intensity $P$ is lower than some threshold value $\hat{P}$. This means that the $\dot{Y} = 0$ locus asymptotes to the $P = \hat{P}$ line as $Y$ becomes larger.

Path f in figure 6 (i) shows the evolution of the economy when there is no effort
to reduce emissions per output. In this case, $P/Y$ is constant. As the world’s income grows, the pollution increases proportionally, as does the risk of natural disasters. The magnitude of the risk eventually reaches the point at which firms do not want to invest in additional stock of capital, and this is the limit of growth for the economy in the case of a constant $P/Y$ ratio. To sustain economic growth, the economy needs to stay $P$ below the threshold value of $\hat{P}$, and this requires continued reductions in the $P/Y$ ratio as $Y$ increases. The $P/Y$ ratio could be reduced by a number of factors, including the introduction of more advanced production technologies, the substitution of polluting inputs for cleaner ones, and abatement activities. However, because it is often costly for private firms to reduce pollution, it is necessary for the authorities to encourage them to do so by appropriate policies, for example, by raising the rate of environmental tax on the emissions of pollutants.

Path $g$ in figure 6 (i) shows one such possibility, where the amount of pollution is kept barely below threshold $\hat{P}$. On this path, the $P/Y$ ratio is continually reduced, for example, by increasing the environmental tax rate, but the amount of pollution $P$ itself increases gradually toward the threshold level of $\hat{P}$. In this case, economic growth can be sustained in the meaning that the amount of output increases without bound, but the long run rate of growth will become lower because the risk of natural disasters gradually rises, which gives disincentives for further investments. Path $h$ illustrates a growth path under a stricter environmental policy, for example, where the tax rate is raised at a quicker rate than in path $g$. When such a policy induces the amount of pollution to become lower than the current level, we will eventually observe the EKC for global pollution. In this case, the adverse effects of global warming on growth (including the risk of natural disasters) will become milder in the long run. If the positive effect of the lower disaster risk exceeds the negative effect of higher taxation, such a policy will enable a higher long-term rate of economic growth than in path $g$.

The preceding discussion implicitly assumed that the current level of global pollution has not yet exceeded the threshold level, but this is actually far from obvious. Figure 6 (ii) depicts the possibility that the current level of pollution is already too high to maintain long-term growth. If this is the case, it is necessary to adopt a stricter environmental policy that reduces not only the $P/Y$ ratio (e.g., path $i$) but also the global level of pollution $P$ (e.g., path $j$). This means that economic growth is sustainable only when the amount of global pollution follows the EKC; in other words, the EKC for global pollution is a requirement for sustained growth. Although it might be considered that the EKC is a result of a successful process of economic growth, the above discussion suggests a possibility of reverse causality in that the sustained economic growth can be a result of appropriate environmental policies that achieve the
Note that even when a strict environmental policy is required for maintaining economic growth, it does not necessarily mean that this is always desirable in terms of welfare because, in the short run, consumers might need to reduce consumption because of increased production costs (and, hence, higher prices). Even in the long run, a stricter environmental policy does not always imply a higher long-term rate of growth because increased production costs mean lower profits, which might reduce the incentives to invest even under favorable natural environments. Therefore, we need to develop an economic model to explicitly investigate the mutual causality between the environment and growth and, by using it, examine the desirable policy. Also, in the case of local pollution it is necessary to develop a formal model to see the precise cause of the poverty-environmental trap, which will be indispensable in understanding the root of the international income inequalities and helping those trapped countries. The next two sections are devoted to these tasks.

3 The poverty-environment trap and international inequalities

In this section, we develop a model of local pollution and economic development, and explain the mechanism of the poverty-environment trap. The following model is based on a simplified version of Ikefuji and Horii (2007).

3.1 A model of local pollution and technological choice

Consider an overlapping generations model where each individual lives for two periods. Individuals in their first and second periods are called young and adult agents, respectively. In youth, agents invest in human capital through education, which is necessary if they want to adopt both more productive and cleaner technology later in their life. In adulthood, each agent works and bears a single child (a young agent). The efficiency of both education and production depends on their health, which in turn depends on the amount of pollution in the environment.

Let us call an agent who is born in period $t$ a generation-$t$ agent. We normalize the number of agents of each generation to one. The lifetime utility of a generation-$t$ agent is given by

$$U_t = \log c_{t}^{y} + (1 - \beta) \log c_{t+1}^{a} + \beta \log x_{t+1}, \quad 0 < \beta < 1,$$

(1)

where $c_{t}^{y}$, $c_{t+1}^{a}$, and $x_{t+1}$ represent the amount of consumption in youth, in adulthood, and the amount of transfer that is given to their children, respectively.
Suppose that the health status of a generation-$t$ agent is negatively affected by the amount of pollution in her youth. Specifically, we assume that the ability of an agent is given by

$$\ell_t = L - P_t,$$

where $L$ is a constant representing the ability of an agent under the pristine environment, while $P_t$ denotes the actual amount of pollution in period $t$.\(^7\) Let $x_t$ be the amount of transfer that each young generation-$t$ agent receives from her parents. We consider a situation of a developing country where the credit market is imperfect, and we therefore assume that agents can neither borrow nor lend. For simplicity, we also assume that goods are not storable, so they must be used within a given period. A part of the transfer is used for consumption $c_t^y$. The remaining $e_t$ is used as an input to human capital investment, which is combined with her ability to learn $\ell_t$ and yields $h_{t+1} = \phi e_t \ell_t$ units of human capital for her adulthood, where $\phi > 0$ is a parameter. The budget constraint in her youth can be written as:

$$c_t^y + e_t = x_t, \quad 0 \leq e_t \leq x_t.$$

(2)

In adulthood (period $t + 1$), each agent produces goods by employing two types of technologies. One is sustainable technology, which produces goods from labor and human capital according to

$$y_{s_{t+1}} = A^s(h_{t+1})^\theta(s_{t+1} \ell_t)^{1-\theta}, \quad A^s > 0, \quad 0 < \theta < 1,$$

where $s_{t+1}$ denotes the fraction of generation-$t$ agents’ time devoted to sustainable technology. This production technology does not cause pollution and, in that sense, is clean. The other technology is called primitive technology, which uses only labor to produce goods, according to

$$y_{p_{t+1}} = A^p(1-s_{t+1})\ell_t, \quad A^p > 0,$$

(4)

but it emits pollution. We assume that the emission is proportional to the amount of output from the primitive technology and that the amount of pollution in the environment evolves according to

$$P_{t+1} = (1-\delta)P_t + \hat{\eta}y_{p_{t+1}}, \quad 0 < \delta < 1, \quad \hat{\eta} > 0.$$

(5)

An adult agent uses her total output $y_{t+1} = y_{s_{t+1}} + y_{p_{t+1}}$ for consumption and transfer for her child:

$$c_{t+1}^a + x_{t+1} = y_{t+1} = y_{s_{t+1}} + y_{p_{t+1}}.$$

(6)

\(^7\)Here, we simplify the model of Ikefuji and Horii (2007), and ignore the variations in the abilities among individuals in a generation, abstracting from the issue of income distribution within an economy.
3.2 Choice between dirty and clean technologies

The problem of a generation-\(t\) individual can be described as follows: given the amount of transfer from her parent \(x_t\) and the pollution \(P_t\), she chooses education \(e_t\), the fraction of time devoted to sustainable technology \(s_{t+1}\), consumption \(c_y\) and \(c_a\), and transfer to her child \(x_{t+1}\). Her objective is to maximize lifetime utility (1), subject to budget constraints (2) and (6), and production technology (3) and (4). Because condition (2) includes inequality constraints due to credit market imperfection, this problem can be solved by the Kuhn-Tucker method. We find that the solution to the above problem critically depends on the amount of transfer from her parent \(x_t\). Note that under the utility function (1), \(x_t = \beta y_t\) holds because adult agents always leave the fraction \(\beta\) of their income for their children as a transfer. Because it is easier to interpret the result in terms of income level (rather than amount of transfer), we describe the solution using \(y_t\).

If the parent generation was poor and their income \(y_t\) was smaller than a threshold level of \(\overline{y} \equiv (1 - \theta)/2\sigma\theta\), agents cannot receive education \((e_t = 0)\) and have to rely completely on the primitive technology \((s_{t+1} = 0)\), which worsens the quality of the environment.\(^8\) Conversely, if the income of previous generation \(y_t\) was higher than \(\underline{y} \equiv (1 + \theta)/2\sigma\theta\), i.e., if their parents are sufficiently rich, agents can receive sufficient education \((e_t = \theta \beta y_t / (1 + \theta))\) such that they rely only on the sustainable technology \((s_{t+1} = 1)\), which improves the environmental quality. Finally, if \(y_t\) was between \(\underline{y}\) and \(\overline{y}\), agents receive some education \((e_t = (y_t - \underline{y})/2)\) but have to rely partly on the primitive technology \((s_{t+1} = \sigma(y_t - \underline{y}) < 1)\). Still, it can be seen that the dependence on the primitive technology decreases \((s_{t+1}\) increases) as the parents become richer. To summarize, we can write \(s_{t+1}\) in terms of \(y_t\) as:

\[
\begin{aligned}
s_t+1 &= s(y_t) \\
&= \begin{cases}
0 & \text{if } y_t \leq \underline{y} \equiv (1 - \theta)/2\sigma\theta, \\
\sigma(y_t - \underline{y}) & \text{if } y_t \in (\underline{y}, \overline{y}), \\
1 & \text{if } y_t \geq \overline{y} \equiv (1 + \theta)/2\sigma\theta,
\end{cases}
\end{aligned}
\tag{7}
\]

which is consistent with the observation that richer countries tend to use cleaner technologies in a larger fraction of their production.

The amount of production \(y_{t+1}^s + y_{t+1}^P\) is determined by the relative dependence on the two types of technologies \(s_{t+1} = s(y_t)\) in (7) and the ability of agents \(\ell_t = L - P_t\) as well as human capital \(h_{t+1} = \phi e_t \ell_t = \phi e_t (L - P_t)\). We thus obtain the evolution of

\[\text{As can be seen from equation (8), } \sigma \equiv (1/2)\beta \phi (A^s(1 - \theta)/A^p)^{1/\theta} \text{ represents the response of technology choice } s_{t+1} \text{ to a change in the parent’s income } y_t \text{ when } y_t \text{ is in the intermediate range.} \]

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Figure 7: Evolution of income (left) and pollution (right) over generations

income $y_t$ over the generations:

$$y_{t+1} = \bar{y}(y_t)(L - P_t), \text{ where } \bar{y}(y_t) \equiv \begin{cases} A^p & \text{if } y_t \leq \bar{y}, \\ A^p \left( \frac{y + y_t}{2y} \right) & \text{if } y_t \in (\bar{y}, y), \\ A^p \left[ \phi \beta \gamma_t / (1 + \theta) \right]^\theta & \text{if } y_t \geq y. \end{cases}$$  \hspace{1cm} (8)

Let us examine the “limits to growth” of this economy. The $\dot{y} = 0$ locus can be derived by setting $y_{t+1} = y_t$ in equation (8). The result is

$$y^*(P_t) \equiv \begin{cases} \left[ A^p \left( \frac{\phi \beta \gamma_t}{1 + \theta} \right)^\theta \right]^{1/(1-\theta)} \geq \bar{y} & \text{if } P_t \leq \bar{P}, \\ A^p \left[ \frac{2}{(L - P_t) - 1/(L - \bar{P})} \right]^{-1} \in (\bar{y}, y) & \text{if } P_t \in (\bar{P}, \bar{P}), \\ A^p (L - P_t) \leq \bar{y} & \text{if } P_t \geq \bar{P}, \end{cases}$$  \hspace{1cm} (9)

where $\bar{P} \equiv L - y/A^p$ and $\bar{P} \equiv (1 + \theta)\bar{P} - \theta L$.

Figure 7 (left) depicts the $\dot{y} = 0$ locus (i.e., function $y = y^*(P_t)$ in equation 9) in $(y, P)$ space. The level of income increases over generations if and only if the $(y, P)$ pair is to the left of this locus. Similarly to Figure 2, the $\dot{y} = 0$ locus is downward sloping. This means that the economy can grow up to a higher level of income when pollution is lower and, hence, the environment is better. In this economy, this occurs for two reasons. First, when the environment is better ($P_t$ is lower), the agents have greater ability to work ($\ell_t = L - P_t$), such that they can produce more output. This is a direct effect of the environment on income. There is also an indirect effect that

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9Because the model in this section is formulated in discrete time, it is more precise to call it the $y_{t+1} = y_t$ locus. However, for comparison between the results of this model and the discussion in section 2 (e.g., Figure 2), we intentionally do not make a clear distinction here between continuous time models and discrete time models.
is manifested over the generations: when the environment is better, parents can leave a larger amount of income to their children, and children themselves also have high ability to learn ($\ell_{t+1} = L - P_{t+1}$), both of which enable agents in the next generation to adopt a better technology. In Figure 7 (left), the effect of the environment on the technological shift appears when the amount of pollution is between $P_0$ and $P$. When $P_t$ is within this range, a marginal change $P_t$ has a larger effect on long-term income through inducing agents to employ the productive (and sustainable) technology in a larger portion of total production (i.e., the long-run level of $s_{t+1}$ increases with $P_t$). This explains why the $\dot{y} = 0$ locus is flatter in this segment than in other segments.

### 3.3 Dynamic interaction between income and environment

We have shown that, given the amount of pollution $P_t$, the evolution of income is determined by equation (8). How, then, is $P_t$ determined? Will it follow the environmental Kuznets curve? From (4), (5), (7) and $\ell_t = L - P_t$, the evolution of the amount of pollution in equilibrium can be written as

$$P_{t+1} = (1 - \delta)P_t + \eta(1 - s(y_t))(L - P_t),$$

where $\eta \equiv \hat{\eta}_AP$. Equation (10) shows that the evolution of $P_t$ is also determined by the $(y,P)$ pair. By applying $P_{t+1} = P_t$ for (10), we obtain the stationary level of pollution for each given income level $y_t$:

$$P^*(y_t) = \begin{cases} 
\frac{\eta L}{(\eta + \delta)} & \text{if } y_t \leq \bar{y}, \\
\frac{\eta L}{(\eta + \delta/\sigma(\bar{y} - y_t))} & \text{if } y_t \in (\bar{y}, \bar{y}), \\
0 & \text{if } y_t \geq \bar{y}.
\end{cases}$$

Let us call the curve given by (11) the $\dot{P} = 0$ locus, as depicted in Figure 7 (right). The amount of pollution in this economy increases toward the $\dot{P} = 0$ locus whenever the $(y,P)$ pair is below this locus. Observe that the $\dot{P} = 0$ locus is (weakly) downward sloping because a richer economy can afford to invest more in human capital and, hence, can employ cleaner technologies (recall equation 7), which implies lower pollution in the long run. Note, however, that the amount of income $y_t$ itself changes depending on $P_t$, and hence we need to examine the dynamic interaction between $y_t$ and $P_t$ over the process of economic development. This can be done by combining the $\dot{y} = 0$ locus and the $\dot{P} = 0$ locus in one figure.

Figure 8 depicts the phase diagram of the dynamic system in $(y,P)$ space. We can observe that there are two stable steady states, $T$ and $B$, and one saddle point $U$.}

\footnote{Here, we assume that the parameters satisfy $\eta L/(\eta + \delta) > \bar{P}$ and $\bar{P} > 0$, so that the two loci have intersections.}
It depends on the initial conditions which steady-state the economy converges to in the long run. In this system, both $y_t$ and $P_t$ are state variables, and therefore, the initial condition is given by a pair of the income of the initial adult generation $y_0$ (i.e., the parents of generation-0 agents) and the initial amount of pollution $P_0$. Because we are interested in the process of economic growth, we suppose $y_0$ to be small so that we can examine the process from the initial stage of development. It will also make sense to assume the initial amount of pollution $P_0$ to be small if we consider that the economy starts from a pre-industrial society, but the precise values of $P_0$, as well as the parameters of the model, will vary across economies.

Figure 8 shows three representative equilibrium paths that start from slightly different initial combinations of $(y_0, P_0)$. Path $k$ illustrates an equilibrium path when the economy starts from a low $P/Y$ ratio. On the first half of this path, pollution gradually accumulates while the output increases. However, once the income level sufficiently rises and the path moves past the $\dot{P} = 0$ locus, the accumulated amount of pollution begins to decrease. This is because the economy now has enough income to invest in human capital and, therefore, no longer needs to rely as much on dirty primitive technologies. Thereafter, as the environment improves, the ability (or health status) of the workers also improves, which enables the income level to increase further toward the better steady state $B$. This path explains that the interaction between the

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$11$ Strictly speaking, the predetermined variables in this system are $x_t$ and $P_t$. However, because $x_t = \beta y_t$ always holds, specifying $y_t$ is equivalent to specifying $x_t$. 

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income level and pollution can endogenously generate the EKC. However, this path is not the only possibility.

Path 1 in Figure 8 illustrates another equilibrium path when the economic development begins with a higher $P/Y$ ratio. In this case, the economy hits the $\dot{y} = 0$ locus (the limits to growth) before encountering the $\dot{P} = 0$ locus. This means that the economic growth has come to its limit due to the environmental degradation, before the income level reaches the top of the EKC. This is not the end of the story. From this point, environmental degradation further continues because the economy is still below the $\dot{P} = 0$ curve. After passing the $\dot{y} = 0$ locus, the income actually decreases due to the deteriorated ability of workers who are seriously affected by a poor environment. The economy eventually converges to steady state $T$, which we call the poverty-environment trap. In this trap, workers cannot escape from poverty because the deteriorated environment lowers their ability and productivity. At the same time, the economy cannot escape from the deteriorated environment because workers are too poor to obtain human capital and, hence, cannot employ cleaner technologies. This mutual causality creates a stagnating economy that suffers both from poverty and a deteriorated environment. Path 2 in Figure 8 shows the case where the initial $P/Y$ ratio is even higher. In this case, the economy converges directly to the poverty-environment trap $T$, which is locally stable and can be approached from any direction in the phase diagram.

These two long-term possibilities, the better steady state and the poverty-environment trap, are grossly different both in terms of environmental quality and income. What, then, separates the successful economies that get past the peak of the EKC from the economies that stagnate in the mutual trap of poverty and environmental degradation? Observe from Figure 8 that there exists a saddle path that converges to the saddle point $U$. Because both $y_0$ and $P_0$ are predetermined state variables, there is virtually zero possibility that the economy happens to be on this path. However, the location of this saddle path is important because it separates the two long-term outcomes: the economy converges to the poverty-environment trap if and only if the initial $(y_0, P_0)$ pair is above the saddle path. Therefore, even when all parameters are identical, a slight difference in the initial conditions (which depends on many factors, e.g., whether a country has been colonized or not and, if so, by what country) may explain persistent international inequality in income and environmental quality. In addition, if the parameters of economies are not identical (e.g., because of regional characteristics), the location of the saddle path as well as the locations of the steady states would differ across countries. This explains another possible reason why some economies have successfully developed along with a cleaner environment, while others are still suffering...
from low income and poor environmental conditions, as we have observed in Figure 4.

3.4  Environmental policies for trapped economies

Now let us discuss how environmental policies can or cannot save economies that are currently trapped in the poverty-environment trap and whether such policies can mitigate the international inequality both in terms of income and the environment. We have explained that, in a trapped economy, the environmental quality and thus the productivity was low because people rely on the primitive technologies that emit pollution. A direct approach to solve this problem is to limit the use of such technologies, i.e., to force them to reduce pollution even if it is costly for individuals. Alternatively, the authorities can tax the use of dirty technologies (or, equivalently, tax emissions) and use the tax revenue for pollution abatement activities. In either case, the net income from using the primitive technology $A^p$ will fall,\(^{12}\) but the amount of pollution per unit output from the primitive technology, given by parameter $\eta$, will also fall. To examine the equilibrium outcome of such policies in a convenient way, we suppose that both $A^p$ and $\eta$ are functions of the strictness of the environmental policy, denoted by $\alpha \in [0, 1]$, and that both are decreasing in $\alpha$.\(^{13}\)

Figure 9 illustrates how such environmental policies affect the trapped economy. Recall that in the poverty-environment trap, both the environmental quality and the income are low so that $P_t > \overline{P}$ and $y_t < \overline{y}$ hold. In this region, from (9) and (11), we can confirm that the $\dot{y} = 0$ locus shifts leftward, while the $\dot{P} = 0$ locus shifts downward. This implies that there are two opposing effects of environmental policies on the income of a trapped economy. First, the leftward shift of the $\dot{y} = 0$ locus means that, given the quality of the environment, the household income declines. This result comes directly from our assumption that environmental policies that aim to reduce emissions are costly for individuals. If it takes time for the environmental quality to change, as assumed in equation (10), then the short-term effect of environmental policy on income is necessarily negative.

In the long-run, however, the environment improves, as reflected in the downward shift in the $\dot{P} = 0$ locus. With a better environment, the productivity of workers will improve, increasing their incomes. The long-term net effect of environmental policy

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\(^{12}\)Originally we assumed $A^p$ to be a technology parameter. Here, we reinterpret $A^p$ as the productivity after deducting the cost of abatement activity or after deducting the environmental tax.

\(^{13}\)Specifically, in Figure 9 we assume that a stricter $\alpha$ reduces the two parameters proportionally: $A^p(\alpha) = (1-\alpha)A^p_0$ and $\eta(\alpha) = (1-\alpha)^2\eta_0$. Equation (10) implies that for a given amount of labor input the increments of pollution are reduced by a factor of $(1-\alpha)^2$. 

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Figure 9: Effect of a small environmental tax or the enforcement of mild pollution reduction in a trapped economy (a magnified view around steady state $T$). The dashed loci show the phase diagram without the environmental tax. The solid loci show the case of $\alpha = 0$.

15 (i.e., when both $A^p$ and $\eta$ are 85% of their original values).

on income depends on the relative magnitude of these two effects. (In other words, it depends on the relative significance of the reductions in $A^p$ and $\eta$, or more generally, it depends on the elasticity of the productivity loss to the reduction of emissions). As illustrated in Figure 9, the net effect can be negative even in the long-run. The steady state level of income in steady state $T'$ under an environmental policy of $\alpha = 0.15$ is lower than the income in steady state $T$ with $\alpha = 0$.

The above results suggest that it is not easy to form a consensus on the environmental policy to reduce emissions in a poverty-environmentally trapped economy because it will further undermine the already low income in the short run, and it is not certain whether it will raise income even in the long-run. However, a sufficiently strong environmental policy can have quite different implications. As illustrated by Figure 10, the $\dot{P} = 0$ and $\dot{y} = 0$ loci become detached from each other when $\alpha$ is large enough. This means that the poverty-environmental trap no longer exists in the new phase diagram, and because of this structural change the economy necessarily converges to the now unique steady state $B$. In this transition, the environment improves simultaneously with the rising income, as in the latter half of the EKC.

Why does a strong environmental policy give rise to such a drastic change? One possible reason is that a better environmental quality improves the productivity and, hence, the incomes of workers and enables their children to invest in human capital and
cleaner technologies. However, as we have explained above, environmental policies to reduce emissions do not necessarily improve income as long as individuals are relying on dirty primitive technologies (i.e., when $P_t > P$ and $y_t < y$), and therefore, this first cause does not always work. The second and more definite cause of the structural change is that because a strong environmental policy reduces the private returns from adopting primitive technologies, workers are induced to invest in human capital and adopt cleaner technologies even at lower income levels. This can be confirmed by the fact that the threshold levels of income for education, $y$ and $\eta$ in equation (7), become smaller when $A^p$ falls.\footnote{Recall that $\sigma \equiv (1/2)\beta \phi (A^*(1-\theta)/A^p)^{1/\theta}$, $\underline{y} \equiv (1 - \theta)/2\sigma \theta$ and $\bar{y} \equiv (1 + \theta)/2\sigma \theta$.} Obviously, such a policy temporarily reduces the income of poor households (given that the environmental quality does not change instantly). As time passes, however, the environment improves gradually, which increases productivity and income. Once the income level passes a threshold level (specifically, when $y_t$ and $P_t$ pair falls below the saddle path in Figure 8), workers are now willing to invest in human capital and cleaner technologies even without further policy interventions, and the economy autonomously improves toward the better steady state.

To summarize, once an economy falls into the poverty-environment trap, it is difficult to build a consensus on environmental policies because such policies are likely to worsen income and welfare, at least temporarily. In addition, when the policy intervention is insufficient, poverty can be aggravated even in the long run. However,

\[ T(\alpha = 0) \]

\[ \eta L \]

\[ \eta + \delta \]

\[ y_t \]

\[ P_t \]

\[ y \]

\[ y \]

\[ 0 \]

\[ \dot{y} = 0 \]

\[ \dot{P} = 0 \]

Figure 10: Effect of a large environmental tax or the enforcement of large pollution reduction in a trapped economy. The dashed loci show the phase diagram with $\alpha = 0$. The solid loci show the case of $\alpha = 0.27$ (i.e., when both $A^p$ and $\eta$ are 73% of their original values).
the situation can be solved permanently if a sufficiently strong environmental policy is continued for a certain period until the economy reaches the autonomous process of economic growth and environmental improvements. Given the short-term cost of such a drastic intervention, assistance from developed economies (e.g., by providing funds for the abatement cost, subsidies for education, or income assistance for households whose earnings are adversely affected by environmental policies) will certainly be a key in helping economies escape from the poverty-environment trap, thereby reducing the wide international inequality both in incomes and environmental quality.15

4 Sustainability of long-term growth under global warming

In the previous section, we examined the problems of large international differences in income and environmental quality. We showed the possibility that the interaction between the environment and growth within each country creates a poverty-environment trap and that this mechanism can explain the long-lasting international inequality in terms of both environmental quality and income. Now, let us turn to the growth of the world economy as a whole and examine how it can be harmonized with the global environment, particularly regarding the problem of global warming. As we have discussed in the latter half of Section 2, the growing world economy emits increasing amounts of global warming gases, which is suspected to intensify the risk of natural disasters. The aggregate economic damage from natural disasters, which mostly takes the form of the destruction of capital stocks, has been increasing at a speed faster than the growth of the world GDP (see Figure 5). If this trend continues, the risk of losing the capital stock will sooner or later exceed a threshold at which the economy does not want to invest further in capital stock. This is the “limits to growth” for the world economy (see path f in Figure 6). Can any environmental policy prevent such a situation from occurring and sustain long-term growth? Based on a simplified version of the endogenous growth model by Ikefuji and Horii (2012), this section presents a formal model of emissions, natural disasters, and the “limits to growth.”

15Ikefuji and Horii (2007) also examined the effect of income redistributive policies within the economy. It is found that at the initial stage of development, a smaller redistribution might help economies escape the poverty-environment trap, while in developed countries a larger redistribution will contribute toward a better environment.
4.1 A model with global pollution and capital destruction

While the previous section considered only human capital, in this section, we consider explicitly the accumulation of physical and human capital. Note that the damage from natural disasters occurs most strikingly in the form of capital destruction. In addition, natural disasters entail many human casualties and therefore destroy a substantial amount of skills and knowledge (i.e., human capital). A knowledge-based economy with a large stock of human capital can be vulnerable to natural disasters once its telecommunication network is damaged. Therefore, it is appropriate to develop a model where the economy accumulates both physical and human capital and where both are subject to the risk of natural disasters. This specification also allows us to examine the properties of long-term economic growth, rather than one-time transitional dynamics from a poor to a rich economy.

However, considering two types of capital simultaneously makes the analysis substantially complex. Therefore, we make several innocuous simplifications. First, rather than explicitly considering different generations (youth and adults, as in the previous section), let us consider a representative household and assume that they divide their human capital (or their disposable time) between production (fraction $u_t$) and education $(1 - u_t)$. In addition, rather than considering the choice between the two technologies, we suppose that production always uses fossil fuels $P_t$ that cause the emission of greenhouse gases of the same amount $P_t$, but that it is possible to adjust the amount of such an input in the production process. The output of the world economy is then given by a constant-returns-to-scale production function:

\[ Y_t = AK_t^\alpha (u_t H_t)^{1-\alpha-\beta} P_t^\beta, \]  

(12)

where $K_t$ and $H_t$ are the aggregate amounts of physical and human capital, respectively. It is possible to interpret that the primitive technology in the previous section corresponds to the situation where the economy uses a large amount of $P_t$ while making little use of $H_t$. Sustainable technology corresponds to the opposite combination. Because this section’s objective to examine how the economy’s reliance on fossil fuels $P_t$ limits the possibility of sustained growth through global warming and increased risk of natural disasters, we do not explicitly consider the finiteness of such resources.

16 While classical growth models require exogenous technological change to explain long-term growth, the literature on endogenous growth (with a seminal study by Lucas 1988) has shown that long-term growth can be explained within a model if we consider both physical and human capital accumulations simultaneously.

17 See the previous footnote 1 for the relationship between the finiteness of resources and the problem of pollution.
We also simplify the process of education. When the representative household uses amount \((1 - u_t)H_t\) of their human capital for education, it produces additional \(B(1 - u_t)H_t\) units of new human capital, where \(B\) is a constant parameter for the efficiency of education (see equation 14 below).

In the model of the previous section, we assumed that local pollution reduces the productivity (ability) of agents. Instead, we now assume that global pollution (i.e., the emission of greenhouse gases) raises the risk of capital stock destruction. Suppose that when the world economy emits amount \(P_t\) of greenhouse gases, then, on average, a fraction \(\phi P_t\) of physical capital is destroyed by natural disasters within a year.\(^{18}\) Natural disasters will also erode a fraction \(\psi P_t\) of human capital, where we assume \(0 < \psi < \phi\).\(^{19}\) Then, the aggregate amounts of physical and human capital evolve according to

\[
\dot{K}_t = Y_t - C_t - (\delta_K + \phi P_t)K_t, \tag{13}
\]
\[
\dot{H}_t = B(1 - u_t)H_t - (\delta_H + \psi P_t)H_t, \tag{14}
\]

where \(\delta_K\) and \(\delta_H\) are depreciation rates of physical and human capital, respectively, excluding the effect of global warming. Because we are concerned about long-term growth rather than short term fluctuations caused by individual events of natural disasters, we simply assume that the whole economy consists of many regions, and, by the law of large numbers, there is no aggregate uncertainty on the aggregate damage in each year. In this setting, the use of fossil fuels \(P_t\) in effect accelerates the depreciation of capital through the larger damages caused by natural disasters.

The representative household has the standard CRRA utility function:

\[
\int_0^\infty \frac{C^{1-\theta} - 1}{1 - \theta} e^{-\rho t} dt. \tag{15}
\]

Let us consider a market economy where the authorities levy a per-unit tax of \(\tau > 0\) on the use of fossil fuels \(P_t\) (or, equivalently, the amount of greenhouse gas emissions). Here, we abstract from the international politics and assume that the whole economy (i.e., the world’s economy) can set a common rate of environmental tax.\(^{20}\) Suppose the markets are perfectly competitive and there is a representative firm that produces

\(^{18}\)For simplicity, we do not explicitly consider the accumulation of greenhouse gases in the atmosphere. Ikefuji and Horii (2012) show that the basic property of the result is the same when we explicitly consider the accumulation process.

\(^{19}\)While human and physical capital are vulnerable to natural disasters, it is reasonable to think that physical capital suffers more directly from natural disasters and, therefore, that the damage fraction of physical capital \(\phi P_t\) would be larger than that for human capital \(\psi P_t\).

\(^{20}\)Of course, this is a mere abstraction. Among countries that differ in many aspects, such as income
output $Y_t$ according to (12). Let the price of the output be normalized to one, and assume that there are no other costs of using $P_t$ other than the environmental tax $\tau$. Then, the first order condition for the profit maximization implies

$$P_t = \beta Y_t / \tau. \quad (16)$$

Equation (16) clearly shows that the emission of greenhouse gases $P_t$ increases proportionally with the world’s GDP, $Y_t$, if there is no strengthening of environmental policy $\tau$. At the same time, this equation shows that emission $P_t$ can be reduced if the environmental tax rate $\tau$ is raised, confirming the discussion in section 2. However, this does not come without a cost. By substituting (16) into production function (12), we see that output can be expressed as

$$Y_t = \left( \tilde{A} \tau^{-\frac{1-\beta}{\beta}} \right) K_t^{\tilde{\alpha}} (u_t H_t)^{1-\tilde{\alpha}}, \quad (17)$$

where $\tilde{A} \equiv \beta^\beta / (1-\beta) A^{1/(1-\beta)}$ and $\tilde{\alpha} \equiv \alpha / (1-\beta)$. Equation (17) can be interpreted as the aggregate production function given the level of environmental tax $\tau$. It has the form of a standard Cobb-Douglas function with two inputs, $K_t$ and $u_t H_t$, and its total factor productivity (TFP) is given by $\tilde{A} \tau^{-\beta/(1-\beta)}$. This expression clearly shows that, given the current amounts of physical and human capital, the environmental tax lowers the productivity.

### 4.2 “Limits to growth” under constant tax rate

We first show that if the environmental tax rate $\tau$ is kept constant, the interacting processes of economic growth and environmental degradation eventually lead to the “Limits to growth.”

In this economy, the only source of externality is $P_t$, which represents the use of fossil fuels and, hence, the accompanying emission of greenhouse gases. Other than this aspect, the conditions for the market equilibrium with the representative household and the representative firm coincide with the conditions for the welfare maximization problem.\(^{22}\) The problem is to maximize the utility (15) subject to the production levels and geography, it is not easy to agree on a common standard for greenhouse gas emissions, and it will be even more difficult to strengthen it over time. As we will see, such conflicts in international politics will create a threat to the sustainability of the world’s economic growth.

\(^{21}\)In the present setting, we assume that the tax revenue is returned to the household in a lump sum fashion. With a minor modification of the model, it is possible to interpret that $\tau$ includes other costs of using fossil fuels, such as extraction costs. In this case, only the remaining fraction of $\tau$ will be returned to households.

\(^{22}\)Aside from the use of fossil fuels and the resulting global warming, our model framework is similar to
function (12) and the resource constraints (13) and (14), where we take the evolution of $P_t$ in (16) as given. By setting up a Hamiltonian, we obtain the following first order conditions for this problem, which should also hold in the market equilibrium:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta} \left[ \left( \frac{\alpha Y_t}{K_t} - \delta K - \phi P_t \right) - \rho \right],$$

(18)

$$\frac{\dot{H}_t}{H_t} + \frac{\dot{u}_t}{u_t} - \frac{\dot{Y}_t}{Y_t} = (B - \delta H - \psi P_t) - \left( \frac{\alpha Y_t}{K_t} - \delta K - \phi P_t \right).$$

(19)

Equation (18) is the Euler equation for the intertemporal consumption. Note that the term $(\alpha Y_t/K_t - \delta K - \phi P_t)$ in the right-hand side (RHS) represents the (expected) rate of return from physical capital investment, i.e., the marginal product of capital $\alpha Y_t/K_t$ minus the depreciation rate $\delta K$ minus the expected loss from natural disasters $\phi P_t$. Recall from (16) that if the environmental tax rate $\tau$ is constant and the economic growth continues ($\dot{Y}_t > 0$), the representative firm uses an ever increasing amount of fossil fuels $P_t$. The resulting increase in the risk of natural disasters lowers the rate of return from physical capital investment $(\alpha Y_t/K_t - \delta K - \phi P_t)$. Equation (18) shows that economic growth can be sustained $(\dot{C}_t/C_t > 0)$ only when the rate of return from physical capital investment is kept above the rate of time preference, $\rho$. For this condition to hold along economic growth, the continual rise in the expected damage $\phi P_t$ must be offset by an increase in the marginal product of physical capital $\alpha Y_t/K_t$.

Will such an increase in $\alpha Y_t/K_t$ actually occur? Note that under production function (17) the marginal product of physical capital $\alpha Y_t/K_t$ rises when the economy uses more human capital $u_t H_t$ relative to the amount of output $Y_t$. The rate of change in this ratio, $u_t H_t/Y_t$, is represented by the left-hand side (LHS) of equation (19). The first term in the RHS, $(B - \delta H - \psi P_t)$, represents the rate of return from investing in human capital. The second term is the rate of return from physical capital investment, as explained above. Therefore, equation (19) shows that the household uses more human capital in production if the rate of return from human capital investment is higher than that from physical capital investment.

As the amount of emission $P_t$ increases in the RHS of (19), the rate of return for physical capital falls more rapidly than that for human capital (recall that $\phi > \psi$), and therefore, the economy increases its reliance on human capital. This is consistent with the empirical findings of Skidmore and Toya (2002), who suggested that a higher frequency of climatic disasters leads to a substitution from physical capital investment toward human capital. This substitution process actually increases the

Lucas (1988), where it is well known that the market equilibrium coincides with the welfare maximization problem because there is no externality. For details regarding deriving the market equilibrium and the welfare maximization solution, see the online appendix of Ikefuji and Horii (2012).
marginal product of physical capital $\alpha Y_t/K_t$, which encourages growth. However, this process cannot perpetually sustain economic growth under a constant tax rate. As we discussed, Euler equation (18) implies that nonnegative growth in consumption requires the rate of return from physical capital $(\alpha Y_t/K_t - \delta K - \phi P_t)$ to be at least $\rho$. However, as $P_t = \beta Y_t/\tau$ rises with economic growth, the rate of return from human capital investment $(B - \delta H - \psi P_t)$ will eventually fall to $\rho$. This means that the RHS of (19) becomes zero (or negative), and substitution from physical capital to human capital stops. This means that $\alpha Y_t/K_t$ cannot rise further, and therefore, the rate of return from physical capital investment $(\alpha Y_t/K_t - \delta K - \phi P_t)$ must eventually fall until it reaches the rate of time preference $\rho$. At this point, people are unwilling to invest further in physical capital to increase their consumption, and economic growth comes to an end.

By setting time derivatives to zero in equations (18) and (19), we find that this steady state, or the “limits to growth,” is reached when the amount of emissions increases up to

$$\bar{P} = \frac{B - \delta H - \rho}{\psi}. \quad (20)$$

Figure 11 illustrates the processes of economic growth in $(Y,P)$ space for three different environmental tax rates. These three paths start from the same level of capital accumulation. However, as we have shown in equation (17), the initial amount of production (income) $Y_t$ varies negatively with the environmental tax rate because
the higher tax rate reduces the effective TFP. When the tax rate is higher, the initial level of emission \( P_t = \beta Y_t / \tau \) is also lower due to both higher \( \tau \) and lower initial \( Y_t \).

As the economy grows, the amount of emission \( P_t \) increases proportionally with output \( Y_t \). Observe from the figure that the slope of the path, \( P_t / Y_t = \beta / \tau \), is less steep when the tax rate is higher. This means that the level of income at which the economy reaches the "limits to growth" (i.e., when \( P_t \) reaches \( \hat{P} \)) is proportionally related to the environmental tax rate:

\[
\hat{Y} = \frac{\tau}{\beta \psi} (B - \delta_H - \rho).
\]

Therefore, although a higher environmental tax initially corresponds to a lower output, it also implies there remains larger room for economic growth.

While Figure 11 illustrates the existence of the limits to growth of the world economy under a constant environmental tax rate, it also suggests that economic growth can be maintained if the environmental tax rate is continually raised. Whenever the world economy reaches the limit of growth, it is possible to bring the economy to a less steep path of growth that leads to a higher long-term level of output, although output temporarily falls (e.g., from point A to point B, or from point C to point D in Figure 11). It might be even better to raise the environmental tax before the emission \( P_t \) reaches \( \hat{P} \) because doing so will allow the possibility of keeping the rates of return from physical and human capital investment higher, thereby encouraging faster capital accumulation. However, it is not always better to raise the environmental tax faster because it lowers the effective productivity of production. In the next subsection, we examine the desirable environmental tax policy both in terms of the long-term economic growth rate and in terms of welfare.

Before leaving this subsection, let us explain the difference between Figure 6 and Figure 11. While we discussed in Section 2 that the \( \dot{Y} = 0 \) locus would appear to be downward sloping (see Figure 6), the \( \dot{Y} = 0 \) locus we derived so far (Figure 11) is actually horizontal because \( \hat{P} = (B - \delta_H - \rho) / \psi \) does not depend on the income level \( Y_t \). This result is due to several simplifications. In particular, we simply assumed that \( \delta_H \) and \( \psi \) do not change with the income level or the level of human capital. At the initial stage of economic growth, however, when production relied more on basic labor than advanced human capital, we may interpret that \( \delta_H \) and \( \psi \) must have been smaller. As the economy develops and the aggregate human capital accumulates (i.e., the development of complex systems of skill, knowledge and information in the world economy), the depreciation or the obsolescence of existing human capital will accelerate (i.e., \( \delta_H \) increases). In addition, as the system of human capital becomes more complex, it might become more vulnerable to natural disasters (i.e., \( \psi \) rises). If
we incorporate these changes into the model, the $\dot{Y} = 0$ locus ($\dot{P} = (B - \delta_H - \rho)/\psi$) will become downward sloping, as depicted in Figure 6.

Note, however, that if we do not expect $\delta_H$ and $\psi$ to rise indefinitely, they will converge to certain constants in the very long run. Because this section examines the long-run consequences of the interaction between the environment and economic growth, we consider the situation where economic development has already sufficiently advanced so that $\delta_H$ and $\psi$ can be seen as constants. In Figure 6, this corresponds to the region where $Y_t$ is sufficiently large that the $\dot{Y} = 0$ comes close to the dashed horizontal line at $P_t = \hat{P}$.

### 4.3 Environmental policy and sustained growth

In the previous subsection, we confirmed that economic growth can be sustained only when the environmental tax rate is continually raised. Now let us define the rate of tax increase $g_r \equiv \dot{\tau_t}/\tau_t$ and examine how it determines the long-term rate of economic growth $g^* \equiv \dot{Y_t}/Y_t$.\(^{23}\)

The long-term rate of economic growth $g^*$ cannot be higher than the rate of tax increase, $g_r$, because otherwise $P_t = \beta Y_t/\tau_t$ would continue to increase and eventually face the “limits to growth,” $\hat{P}$. Therefore, sustained growth ($g^* > 0$) requires a positive rate of environmental tax increase. Under such a policy, there are two possible outcomes in the state of the environment. If the long-term economic growth occurs at the same rate as the tax increase (i.e., $g^* = g_r$), then the growth rate of $P_t = \beta Y_t/\tau_t$ will become zero, and therefore, the amount of emissions will converge to a constant, which we denote by $P^*$. Another possibility is that output grows slower than the tax rate ($g^* < g_r$). In such a scenario, the amount of emission $P_t = \beta Y_t/\tau_t$ falls at the rate of $g_P = g^* - g_r < 0$ and will converge toward zero. That is, the emissions are asymptotically eliminated in the long run; $P^* = 0$.

In either case, the amount of emissions and, hence, the risk of natural disasters converge to a constant level. This by itself should be good for economic growth. However, when the tax rate $\tau$ is continually increased, is it possible to maintain output growth, given that a higher $\tau$ reduces the effective productivity? Calculating the rates of change on both sides of equation (17), we find that the growth rate of human capital $g_H \equiv \dot{H_t}/H_t$ should be higher than the output growth so that it offsets the effective

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\(^{23}\)Strictly speaking, because we are interested in the long-term rate of economic growth, not the growth rate in the transition, the precise definition of $g^*$ should be $= \lim_{t \to \infty} \dot{Y_t}/Y_t$, and it should be called the asymptotic rate of economic growth. Similar remarks can be applied to growth rates of other variables, such as $g_r$. 
productivity loss caused by $g_r > 0$.\textsuperscript{24}

\[ g_H = g^* + \frac{\beta}{1 - \alpha - \beta} g_r. \]

Note that (22) means $H_t/Y_t$ increases over time, whereas the constancy of $P^*$ implies $P_t/Y_t$ will fall. Equation (22) can also be understood in such a way that the environmental tax policy induces the substitution from fossil fuels to human capital in the production process. In other words, the policy encourages the shift toward a knowledge-oriented production process rather than heavily relying on natural resources. In a broader sense, this can be interpreted as a process of technological shift, as in the model of Section 3.\textsuperscript{25}

Using the above properties in equations (13), (14), (16), (18) and (19), we obtain the asymptotic value of emission $P^*$ and the long-term growth rate $g^*$ as a function of environmental policy $g_r$. The result is summarized in Figure 12, which also depicts $g_H$ from (22) and $g_P = g^* - g_r$. Let us first explain the case in which the rate of tax increase $g_r$ does not exceed a threshold of

\[ g_{\text{max}} = \left( \theta + \frac{\beta}{1 - \alpha - \beta} \right)^{-1} (B - \delta_H - \rho). \]

When the tax policy is within the range of $g_r \leq g_{\text{max}}$, we find that the output of the economy can grow at the same speed as the tax increase, i.e., $g^* = g_r$. In this case, $P_t = \beta Y_t/\tau_t$ converges to a positive constant

\[ P^* = \frac{1}{\psi} \left[ B - \delta_H - \rho - \left( \theta + \frac{\beta}{1 - \alpha - \beta} \right) g_r \right], \]

which is decreasing in $g_r$. This means that by raising the tax rate faster it is possible to reduce the long-term level of emissions and, hence, the risk of natural disasters. The lower risk of natural disasters encourages the economy to accumulate more capital, thereby enabling faster economic growth. Thus, $g^*$ increases with $g_r$.

\textsuperscript{24}To derive (22), we use $\dot{K}_t/K_t = \dot{Y}_t/Y_t \equiv g^*$ and $\dot{u}_t/u_t = 0$, which is justified as follows. Observe from Euler equation (18) that steady-state growth (constant growth of $C_t$) occurs only when $K_t/Y_t$ is asymptotically constant. Therefore, the growth rate of $K_t$ should be the same as that of the output, $g^*$. Note also that $\dot{u}_t/u_t > 0$ is not possible because it has an upper bound of 1, whereas $\dot{u}_t/u_t < 0$ violates the transversality condition.

\textsuperscript{25}While we simplify the process of the technological change by focusing only on the relative use of human capital and the polluting input, a number of studies explicitly examine the determinant of technological change and how it is affected by environmental policies: see, for example, Gradus and Smulders (1993), Bovenberg and Smulders (1995), Groth and Schou (2002), Grimaud and Rougé (2003), Smulders and de Nooij (2003), Hart (2004), and Ricci (2007). Recent studies, such as Di Maria and Valente (2008), Pittel and Bretschger (2010), and Acemoglu et al. (2012), also examine the direction of technological change.
Figure 12: Rates of changes in output, human capital, and emissions (above) and the asymptotic value of emissions (below) as a function of the rate of environmental tax increase.

Figure 13 depicts the growth paths of the economy in \((P, Y)\) space under different tax policies. Suppose that thus far the tax rate on fossil fuels (or, more generally, the user cost of pollution-generating inputs) has been constant and that \(P_t\) and \(Y_t\) have increased proportionally to the current status of \((P_0, Y_0)\). As we have seen in the previous subsection, the economy will face the “limits to growth” if the tax rate is kept constant (path a). However, if the tax rate is continually raised at an appropriate speed, the economy can overcome this limit. Path b depicts the evolution of the economy when the tax rate is raised gradually, but the rate of tax increase \(\dot{\tau}_t/\tau_t\) is small. Under such an environmental tax policy, the amount of emission \(P_t\) can be kept below the threshold \(\hat{P}\) but will come close to it. Then, the long-term rate of economic growth \(g^*\) is positive, but not large (see Figure 12). Note that when the speed of increase in \(\tau_t\) is positive but less than an exponential increase, \(\dot{\tau}_t/\tau_t\) will fall to zero as the denominator \(\tau_t\) increases. In such a case, the economy will grow similarly to path b, but in the limit the long-term rate of growth \(g^* \equiv \dot{Y}_t/Y_t\) will fall to zero. This is a natural result because \(Y_t\) cannot grow more than proportionally with \(\tau_t\), and hence, if \(\tau_t\) is not raised at an exponential rate, an exponential growth of \(Y_t\) (i.e., \(g^* > 0\)) is not possible either.

If the environmental tax rate is raised at a faster rate, the long-term level of emissions can be reduced to a lower level, as shown by path c. (See also equation 24 and
Figure 13: Environmental tax policy and the future evolutions of income and environmental quality

Figure 12). The long-term rate of growth, then, is higher than that in Path a because the environmental tax rate is raised at a faster rate and the output grows at the same rate. The fastest rate of long-term growth can be obtained when \( \tau_t \) is raised at exactly the rate of \( g^{\text{max}} \) in (23). This environmental tax policy achieves asymptotically zero emissions in the long-run. This does not mean \( P_t \) becomes 0 at some date \( t \), which is not possible because the production function (12) always requires a positive input of \( P_t \). However, it is possible to reduce \( P_t \) toward zero while using increasingly more human capital \( H_t \) over time. With the risk of natural disasters falling to the minimum level, the economy is encouraged to accumulate physical and human capital at the fastest speed. In this sense, such an environmental policy perfectly harmonizes the maximization of economic growth with environmental improvements in the long run.

Note also that under a policy such that the long-term level of emission \( P^* \) is lower than the current level of emission, \( P_0 \), we will observe the EKC in the future. Therefore, in the case of greenhouse gas emissions, the EKC does not emerge automatically but will only be realized when the world economy agrees on setting sufficiently strict environmental policies that enable faster long-term economic growth. In other words, following the EKC is a requirement for achieving a high rate of economic growth in the long run.

What will happen if the world economy decides to adopt a stricter environmental policy? If \( g_\tau > g^{\text{max}} \), the negative effect of tax on the productivity dominates, and the...
economy cannot grow at the same speed as the tax increase \((g^* < g_\tau)\). Specifically, the long-term rate of growth becomes a decreasing function of \(g_\tau\),

\[
g^* = \frac{1}{\theta} \left( B - \delta_H - \rho - \frac{\beta}{1 - \alpha - \beta g_\tau} \right).
\]

(25)

The emission \(P_t\) falls to zero \((P^* = 0)\) at the rate of \(g_P = g^* - g_\tau\) (see Figure 12), which becomes more negative when \(g_\tau\) is larger. Therefore, as depicted by path \(e\) in Figure 13, when the environmental tax rate is raised too quickly \((g_\tau > g^{\text{max}})\), the emissions can be reduced at a faster rate but at the cost of a slower long-term rate of growth. Path \(f\) shows that the extremely strict environmental policy actually chokes economic growth, which occurs when the rate of tax increase is raised to \(g^{\text{lim}} \equiv (1 - \alpha - \beta)^{-1} (B - \delta_H - \rho)\).

4.4 Welfare-maximizing environmental policy for global warming

We have seen that an appropriate environmental policy can both maximize the long-term rate growth and reduce emissions toward zero. This particular policy \((g_\tau = g^{\text{max}})\) can be characterized as the mildest environmental tax policy among those that asymptotically achieve zero emissions.\(^{26}\) Note that this is still a strict environmental policy in the sense of reducing emissions toward zero, and such a policy incurs a substantial cost in terms of productivity loss in the short run, as we have seen in Subsection 4.2. While this policy can minimize the risk of natural disasters, is it desirable in terms of welfare?

Thus far, we have considered the equilibrium in the market economy. As we mentioned, in this economy the only difference between the welfare maximization problem and the market economy is the determination of the amount of emission (or, equivalently, the use of fossil fuels) \(P_t\). In the market economy, the amount of emissions is determined as \(P_t = \beta Y_t / \tau_t\) by (16). In the welfare maximization problem, the social cost of using \(P_t\), including the negative externality from a higher risk of natural disasters, should be determined such that it is equalized to the social benefit of using fossil fuels, i.e., the marginal product of using fossil fuels. This condition can be written as

\[
\phi K_t + \psi H_t \frac{(1 - \alpha - \beta) Y_t}{B u_t H_t} = \frac{\beta Y_t}{P_t},
\]

(26)

Note that \(\phi K_t\) and \(\psi H_t\) represent the marginal increases in the damages to physical and human capital due to a marginal increase in \(P_t\), while \(\beta Y_t / P_t\) is the marginal

\(^{26}\) We can confirm that when \(g_\tau = g^{\text{max}}\) while \(P_t\) falls to \(P^* = 0\), the asymptotic rate of reduction \(\dot{P}_t / P_t\) is \(g^* - g_\tau = 0\). This means that \(P_t\) converges to zero at a less than exponential speed. If \(g_\tau < g^{\text{max}}\), \(P_t\) converges to a positive constant \(P^* > 0\), whereas if \(g_\tau > g^{\text{max}}\), \(P_t\) falls to 0 at an exponential rate of \(g^* - g_\tau < 0\).
Figure 14: Welfare-maximizing environmental tax policy

Condition (26) can be solved for the amount of emissions as:

$$P_t = \beta \left( \phi \frac{K_t}{Y_t} + \psi \frac{(1 - \alpha - \beta)}{Bu_t} \right)^{-1}.$$  

Figure 14 illustrates the determination of the welfare-maximizing tax policy. Once $g_\tau$ is given, the capital output ratio $K_t/Y_t$ and the fraction of human capital used for production $u_t$ becomes constant in the long run. Therefore, equation (27) implies that it is optimal to keep emission $P_t$ at a positive constant value in the long run. On the other hand, the actual amount of emissions under environmental tax policy $g_\tau$ is determined as in Figure 12, where its long-term value is positive only when the rate of tax increase $g_\tau$ is slower than $g^{\text{max}}$. The welfare-maximizing rate of the tax increase, $g^{\text{opt}}_\tau$, is determined so that the above two coincide with each other. Observe that in our model framework it is neither necessary nor desirable to pursue zero emissions even in the long-run. As a result, if welfare is the main concern, we should adopt a milder environmental policy than when long-term economic growth is the first priority. In other words, growth maximization is achieved when the world economy adopts a stricter policy to improve the environment than is actually desired by the general public.

This result might be contrary to the common perception that there is a tradeoff between growth and environmental conservation. Because the welfare-maximizing environmental tax policy falls within the range of $g_\tau < g^{\text{max}}$, it is quite unlikely that international politicians will agree on a tax policy that is stricter than $g^{\text{max}}$, which is neither desirable in terms of growth or of welfare. Therefore, for the purpose of policy comparison, it will be sufficient to consider the tax policy within the range of
Then, the presented model of the global environment clearly shows that although there is a short term trade-off between the environment and income, in the long run the environment and growth are positively linked. An acceleration in the rate of tax increase simultaneously improves the long-term level of emissions and economic growth. Of course, adopting too fast a tax increase \( g^* > g^{\text{max}} \) will be harmful both for growth and welfare and will not improve the long-term quality of the environment \( (P^* = 0) \). However, it will be safe to rule out such a possibility given that the global environmental policy must be agreed upon by the majority of countries.

5 Conclusion

In this paper, we have discussed the implications of the mutual causality between environmental quality and growth. If the economy simply expands the scale of production, it will cause increasing amounts of pollution, which will deteriorate the environment. The environmental degradation in turn harms economic growth in various ways. Local pollution (such as air pollution), for example, adversely affects the health of workers and, hence, the productivity of the economy. Global pollution, particularly in the case of global warming, will destabilize the climate and raise the risk of natural disasters. When such negative effects become too large, the economy is no longer able to accumulate physical and human capital, which we call “limits to growth.”

This mutual link creates serious problems, both at the level of individual countries and at the level of the global economy. We have shown the possibility that in the least developed countries (LDCs), poverty and environmental degradation reinforce each other, creating the “poverty-environment trap,” in which people cannot afford to obtain adequate education and therefore have to rely on dirty technologies that cause further pollution. The existence of such a trap can be a cause of long-lasting international inequality both in terms of income level and environmental quality. The growth potential of the global economy is also limited by this mutual link if greenhouse gas emissions increase proportionally with the world’s output.

The key to overcoming the “limits to growth” is technological change, or the transition from “dirty” to “cleaner” inputs, which enables the production of outputs with less pollution. In fact, the Environmental Kuznets Curve (EKC) is observed for some air pollutants, which means that while the intensity of pollution increases with income for some range, once the income level exceeds a threshold value the pollution declines with income. It seems that developed countries have mostly already exceeded the threshold level of income and, thus, have achieved better environmental quality along with higher income. However, this situation does not hold for every economy. It
appears that the least developed countries and some developing countries are trapped before they exceed the threshold income. In such a case, the authorities need to adopt appropriate policies to encourage technological change, such as taxing dirty technologies. Such policies will help them escape the poverty-environment trap and achieve better environmental quality in the long run.

In the case of global pollution, the emission of greenhouse gases is still increasing along with the world's increasing output, and thus far there is no sign that it follows the EKC. Our theory suggests that emissions will increase proportionally with output if the environmental policy is unchanged. Therefore, to make the world's economic growth sustainable, it is necessary to strengthen environmental policies over time so producers will rely less on pollution generating output (e.g., fossil fuels) and more on new technologies, skills and knowledge (i.e., human capital). If such a policy is successful at achieving a high rate of long-term growth, we will observe the EKC for greenhouse gases in the future.

For both local and global environmental issues, we have shown that the appropriate environmental policy improves both the environmental quality and income level by altering the mutual causality between the environment and growth that leads to the "limits to growth" if there is no such policy. Therefore, although it may appear counterintuitive, economic growth and the environment are not subject to trade-offs in the long run. However, there are short term costs of environmental conservation that must be incurred by the economy. Banning dirty primitive technologies in LDCs will certainly lower their income, which is already quite low, in the short run. This creates a significant hurdle for such countries to adopt stronger environmental policies that are necessary to escape from the poverty-environment trap. The same can be said of the global economy. Raising the environmental tax rate will reduce the effective productivity of aggregate production, which will lower the growth rate for some time. However, such a cost is necessary to sustain economic growth in the long run. Still, this temporary adverse effect on the world economy makes it difficult for international authorities to agree on strengthening environmental policy at a sufficiently fast rate. Overcoming this political situation is not easy, and is clearly beyond the scope of this paper, but a correct understanding of the long-term positive relationship between environmental quality and growth, as examined in this paper, will certainly facilitate international cooperation for environmental conservation.
References


