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Real-Time Nowcasting Nominal GDP Under Structural Break

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Abstract

This paper provides early assessments of current U.S. Nominal GDP growth, which has been considered as a potential new monetary policy target. The nowcasts are computed using the exact amount of information that policy makers have available at the time predictions are made. However, real time information arrives at different frequencies and asynchronously, which poses the challenge of mixed frequencies, missing data, and ragged edges. This paper proposes a multivariate state space model that not only takes into account asynchronous information inflow it also allows for potential parameter instability. We use small scale confirmatory factor analysis in which the candidate variables are selected based on their ability to forecast GDP nominal. The model is fully estimated in one step using a nonlinear Kalman filter, which is applied to obtain simultaneously both optimal inferences on the dynamic factor and parameters. Differently from principal component analysis, the proposed factor model captures the comovement rather than the variance underlying the variables. We compare the predictive ability of the model with other univariate and multivariate specifications. The results indicate that the proposed model containing information on real economic activity, inflation, interest rates, and Divisia monetary aggregates produces the most accurate real time nowcasts of nominal GDP growth.

Keywords: Mixed Frequency, Ragged Edges, Real-Time, Nowcasting, Missing Data, Nonlinear, Structural Breaks, Dynamic Factor, Monetary Policy.

JEL Classification: C32, E27, E31, E32

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1 Introduction

In recent years interest rates reached a technical lower bound level, but unemployment rate still remained at high levels and capacity utilization has yet to rebound to levels previous to the Great Recession in 2007-2009. In view of this situation, the Federal Reserve has additionally been using complementary tools to carry out monetary policy. One of them, which is the motivation of our analysis, is ‘forward guidance.’ As discussed by Bernanke (2012) and Woodford (2012) at the Annual Jackson Hole Economic Symposium, this tool consists of explicit statements of a central bank about its future medium and long run actions to developments in the economy, in addition to its announcements about immediate short-run policy. The idea is that, depending on the target and rule that central banks are committed to follow, pursuing ‘forward guidance’ could lead to changes in expectations by economic agents, which could hasten achievement of the Fed’s target.

During the last recession, the trend of nominal GDP showed a substantial contraction associated with several large negative shocks, and the gap between the current and pre-crisis trend level is still large (Figure 1). Many economists have suggested that the Fed should start targeting the path of nominal GDP (Hall and Mankiw 1994, Romer 2011, and Woodford 2012, among others), as they consider this would constitute a powerful communication tool. Under this proposal, the funds rate would remain around the lower bound until nominal GDP reaches the pre-crisis level and, once this is achieved, the funds rate would increase as necessary to ensure normal level growth in the long run. Since nominal GDP is the output of the economy times the price level, setting the objective of returning nominal GDP to its pre-crisis trajectory could improve expectations about future economic conditions. The conjecture is that such expectations would increase households’ incentives to consume more in the present, and firms would be more optimistic regarding their future demand and, therefore, their present investment decisions.

Under nominal GDP targeting scenario, monitoring output path plays a fundamental role in assessing policy effectiveness and its future direction. The goal of this paper is to provide early real time nowcasts of nominal GDP growth that can be useful to inform monetary policy and economic agents. The work of Croushore and Stark (2001) was the starting point of a large forecasting literature that emphasizes the use of unrevised real-time data, which allows evaluation of how models performed at the time events were taking place. Accordingly, nowcasts of nominal GDP are computed using only the exact information available at the time predictions are made in order to reproduce the real time forecasting problem of policy makers and economic agents, based on a real-time data set for each vintage constructed for this paper.

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1 For an extensive discussion on forward guidance and targeting nominal GDP, see Woodford (2012), Belongia and Ireland (2012) and Del Negro et al. (2012).

2 Given lags of at least one month in the release of many macroeconomic variables, forecasting the present and even the near past is required to assess current economic situation. The literature has named this ‘nowcast’, which is a widespread term used by the U.S. National Weather Service for current weather.
However, at any point in time data arrive asynchronously, at different frequencies and, at first, based on preliminary and incomplete information. This poses the challenge of handling mixed frequencies, missing observations, and lags in the availability of primary data (ragged edges). Some advances in forecast methods have been proposed to address these problems. This is particularly the case in the growing literature on short term forecasting and nowcasting using multivariate state space models, which rely on the methods by Trehan (1989), Mariano and Murasawa (2003), Evans (2005), Proietti and Moauro (2006), or Giannone, Reichlin and Small (2008). Other mixed frequency methods have been proposed and applied to univariate and multivariate autoregressive (VAR) processes such as the mixed data sampling MIDAS proposed by Ghysels, Santa-Clara, and Valkanov (2004) or the mixed frequency VAR in Banbura, Giannone, and Reichlin (2010), Kuzin, Marcellino, and Schumacher (2011), Gotz and Hecq (2014), and the mixed frequency Bayesian VAR in Schorfheide and Song (2011).

Our paper combines the multivariate state space system with mixed frequency approach of Mariano and Murasawa (2003), and the small scale dynamic factor model of Stock and Watson (1989), which is extended in a nonlinear version to allow for potential structural breaks. Stock and Watson (1989) proposed a widely popular low-dimensional linear dynamic factor model to construct coincident indicators of the U.S. economy. Linear and nonlinear extensions of this small-scale dynamic factor model have been successful used in real time forecasting. For the U.S., see, for example, Chauvet (1998), Chauvet and Hamilton (2006), Chauvet and Piger (2008), Aruoba and Diebold (2009), Aruoba, Diebold and Scotti (2010); for Europe, see Camacho and Perez-Quiros 2010; and for Brazil see Chauvet (2001).3

Several recent papers such as Bai and Ng (2008a, 2008b), Jungbacker and Koopman (2008), and Doz, Giannone and Reichlin (2012), among several others, find that small scale factor models estimated through maximum likelihood display desirable properties such as efficiency gains when fewer but more informative predictors are carefully selected. Boivin and Ng (2006) and Bai and Ng (2008a, 2008b) contend that exploratory large factor models with uninformative data can result in large idiosyncratic error variances and cross-section correlated errors, reducing estimates accuracy, and the model predictive content. They argue for the benefits of supervised (confirmatory) factor models - even in a data rich environment - with pre-screened series based on economic reasoning and their predictive ability for the target variable. More recently, Alvarez, Camacho, and Perez-Quiros (2013) show through Monte Carlo analysis that small scale factor models outperform large scale models in factor estimation and forecasting.

Following this literature, we use small scale confirmatory factor analysis in which the candidate variables are carefully selected based on their marginal predictive ability to the target variable nominal GDP growth. In addition, we extend existing frameworks by proposing a multivariate state space system that considers the possibility of parameter instability in addition to asynchronous information inflow.

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3The indicators for the U.S. based on these papers are updated on a regular basis and posted on the website of the Saint Louis Fed: http://research.stlouisfed.org/fred2/series/RECPROUSM156N, Atlanta Fed: http://www.frbatlanta.org/cqer/researchcq/chauvet_real_time_analysis.cfm, and Philadelphia Fed: http://www.philadelphiafed.org/research-and-data/real-time-center/business-conditions-index/. The one for Brazil is updated by the Center for Research on Economic and Financial Cycles: https://sites.google.com/site/crefcus/brazil; and the one for the Euro area is updated regularly by the Bank of Spain but not posted on their website.
We propose a single fully specified dynamic factor model with mixed frequency and potential structural break (MFDFB model). Our paper differs from large factor models not only in the scale, but also in the estimation procedure, which yields very different factors. Most large factor models rely on two-step estimations, in which the factors are extracted as principal components. Within this methodology, the resulting factors represent the maximum variance underlying variables.\footnote{See, e.g. Giannone, Reichlin and Small (2008), Doz, Giannone and Reichlin (2012), Banbura, Giannone, Modugno, and Reichlin (2013), Banbura and Modugno (2010), etc.} In contrast, in our paper the factor and model parameters are estimated simultaneously in one step through maximum likelihood. The method yields optimal inferences on the dynamic factor, which captures the common correlation underlying the observable variables. The main difference between these two approaches is that in the proposed model the factor does not extract all variance from the variables, but only that proportion that is due to the commonality shared by all observable variables (i.e. their common variance). In addition, the set of hypotheses that form the conceptual basis of the fully estimated confirmatory factor analysis enables interpretation of the factor and specification testing.

We compare the predictive ability of the model with alternative univariate and multivariate specifications, which are combined with the best leading indicators of nominal GDP growth. The results indicate that the linear mixed frequency dynamic factor models containing information on real economic activity, inflation, monetary indicators, and interest rates outperform univariate specifications, linear and nonlinear. However, the proposed small scale mixed frequency dynamic factor model under structural break outperforms all other specifications considered. The results provide evidence of substantial gains in real time nowcasting accuracy when allowing for parameter instability.

The structure of the paper is as follows. Section 2 introduces Mariano and Murasawa mixed frequency method in a simple sum (naïve) model. Section 3 presents the linear and the proposed nonlinear mixed frequency dynamic factor model with structural break. Section 4 presents alternative univariate frameworks, Section 5 discusses the timing of forecasts, real time data, variable selection and the empirical results. Section 6 reports the real time nowcasting findings, and Section 7 concludes.

### 2 Simple Sum Mixed Frequency (Naïve) Model

Nominal GDP (NGDP) is the market value at current prices of all final goods and services produced within a country in a given period of time. It can also be viewed as the real GDP times the price level of the economy. Therefore, letting $Z_t$ be nominal GDP, $X_t$ real GDP, and $P_t$ the price level, there is a conceptual link between these three variables:

\[
Z_t = X_t P_t
\]

\[
\ln(Z_t) - \ln(Z_{t-1}) = \ln(X_t) - \ln(X_{t-1}) + \ln(P_t) - \ln(P_{t-1})
\]

\[
z_t = x_t + p_t
\]

We can take advantage of the fact that the target variable contains a real activity component and an inflation component and proxy $x_t$ and $p_t$, which are on quarterly frequency, with indicators available.
at monthly frequency, such as Industrial Production (IP) and Consumer Price Index (CPI), respectively. Charts A and B of Figure 2 show real GDP growth and GDP deflator growth in quarterly frequency, while charts C and D plot IP and CPI growth rates in monthly frequency, respectively. The NBER recessions are represented by the shaded areas. The monthly series display similar dynamics to the quarterly ones, but are available in a more timely manner.

We obtain a "naïve" monthly index of our target variable NGDP growth by adding IP and CPI growth rates and standardizing them with respect to NGDP. Since the naïve index is in monthly frequency and NGDP is in quarterly frequency, we use the transformation in Mariano and Murasawa (2003) to compare both variables in quarterly terms. Quarterly time series $Z_t$ can be expressed into monthly time series $W_t$ as:

$$Z_t = 3 \left( \frac{W_t + W_{t-1} + W_{t-2}}{3} \right),$$

which can be approximated using the geometric mean instead of the arithmetic mean, since when variations are small the difference between the two tends to be negligible:

$$Z_t = 3 (W_t W_{t-1} W_{t-2})^{1/3}$$

Taking logs of both sides, taking three period differences, and after some algebra, we obtain:

$$z_t = \frac{1}{3} w_t + \frac{2}{3} w_{t-1} + w_{t-2} + \frac{2}{3} w_{t-3} + \frac{1}{3} w_{t-4} \quad (2)$$

where the quarter-on-quarter growth rates, $z_t$, are expressed in month-on-month growth rates, $w_t$.

Chart A of Figure 3 plots both series in quarterly frequency, and the NBER recessions. The naïve index yields a relatively good in sample fit. However, as it will be discussed later, the performance of the index is not accurate in real time nowcasting of NGDP (Figure 3 Chart B, Table 4). In order to obtain more accurate real time forecasts, we explore the information contained in real and nominal indicators by extracting their underlying comovement using factor models, rather than relying on simple sum.

### 3 Mixed Frequency Dynamic Factor Model

#### 3.1 Linear Framework (MFDF)

In this section we specify the linear nowcasting dynamic factor model that allows for the inclusion of both mixed frequency data and missing observations. We use the approach proposed by Mariano and Murasawa (2003) in equation (2) to express quarterly data in terms of monthly data. The dynamic factor model extracts the comovement among the target variable NGDP, denoted $y_{1,t}$, an indicator of real economic activity, $y_{2,t}$, an indicator of inflation dynamics, $y_{3,t}$, and other candidate variables, $y_{h,t}, h = 4, \ldots, N$. The model separates out common cyclical fluctuations underlying these variables in the the unobservable factor, $f_t$, and idiosyncratic movements not representing their intercorrelations captured by the associated
idiosyncratic terms, \( v_{n,t} \) for \( n = 1, 2, \ldots, N \). The model is:

\[
\begin{bmatrix}
y_{1,t} \\
y_{2,t} \\
y_{3,t} \\
\vdots \\
y_{N,t}
\end{bmatrix}
= \begin{bmatrix}
\gamma_1 \left( \frac{1}{3} f_{t} + \frac{2}{3} f_{t-1} + f_{t-2} + \frac{2}{3} f_{t-3} + \frac{1}{3} f_{t-4} \right) \\
\gamma_2 f_{t} \\
\gamma_3 f_{t} \\
\vdots \\
\gamma_N f_{t}
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{3} v_{1,t} + \frac{2}{3} v_{1,t-1} + v_{1,t-2} + \frac{2}{3} v_{1,t-3} + \frac{1}{3} v_{1,t-4} \\
v_{2,t} \\
v_{3,t} \\
\vdots \\
v_{N,t}
\end{bmatrix},
\tag{3}
\]

where \( \gamma_n \) are the factor loadings, which measure the sensitivity of the common factor to the observable variables. The dynamics of the unobserved factor and error terms are modeled as autoregressive processes:

\[
f_t = \phi_1 f_{t-1} + \ldots + \phi_P f_{t-P} + e_t, \quad e_t \sim i.i.d. N(0, 1) \tag{4}
\]

\[
v_{n,t} = \varphi_{n1} v_{n,t-1} + \ldots + \varphi_{nQ} v_{n,t-Q} + \epsilon_{n,t}, \quad \epsilon_{n,t} \sim i.i.d. N(0, \sigma_{\epsilon_n}^2), \quad \text{for } n = 1, 2, \ldots, N \tag{5}
\]

The model assumes additionally that \( f_t \) and \( v_{nt} \) are mutually independent at all leads and lags for all \( N \) variables. This assumption together with \( \epsilon_{n,t} \sim i.i.d. N(0, \sigma_{\epsilon_n}^2) \) is at the core of the definition of the small scale dynamic factor model as in Stock and Watson (1989), since it implies that the model separates out common correlation underlying the observed variables from individual variations in each series.

In order to obtain optimal inferences on the unobserved variables \( f_t \) and \( v_{nt} \), the system of equations (3) - (5) is cast into a state space representation, which is estimated using the Kalman filter:

\[
y_t = H F_t + \xi_t, \quad \xi_t \sim i.i.d. N(0, R) \tag{6}
\]

\[
F_t = T F_{t-1} + \zeta_t, \quad \zeta_t \sim i.i.d. N(0, \mathbf{Q}) \tag{7}
\]

Equation (6) corresponds to the measurement equation that relates observed variables with the unobserved common component and idiosyncratic terms from equation (3). Equation (7) is the transition equation, which specifies the dynamics of the unobserved variables in equations (4) and (5).

Using Mariano and Murasawa (2003) and the adaptation of Camacho and Perez-Quiros (2010) we modify the state space model (6) - (7) to incorporate potential missing observations into the system. The strategy consists of substituting each missing observation with a random draw \( v_t \) from a \( N(0, \sigma_v^2) \). This substitution keeps the matrices conformable without affecting the estimation of the model parameters. The components of the model (6) - (7) are updated depending on whether \( y_{n,t} \) is observed or not, in the
following way:

\[ y_{n,t} = \begin{cases} y_{n,t} \text{ if } y_{n,t} \text{ observed} \\ v_t \text{ otherwise} \end{cases}, \quad H_t^* = \begin{cases} H_n \text{ if } y_{n,t} \text{ observed} \\ 0_{1\kappa} \text{ otherwise} \end{cases} \]

\[ \xi_{n,t}^* = \begin{cases} 0 \text{ if } y_{n,t} \text{ observed} \\ v_t \text{ otherwise} \end{cases}, \quad R_{n,t}^* = \begin{cases} 0 \text{ if } y_{n,t} \text{ observed} \\ \sigma_v^2 \text{ otherwise} \end{cases} \]

where \( H_t^* \) is the \( n \)-th row of matrix \( H \) which has \( \kappa \) columns, and \( 0_{1\kappa} \) is a \( \kappa \) row vector of zeros. Therefore, in the model robust to missing observations, the measurement equation (6) is replaced by

\[ y_t = H_t^* F_t + \xi_t^*, \quad \xi_t^* \sim i.i.d.N(0, R_t^*) \]

The Kalman filter is applied to the time-varying state space model (7)-(8) to obtain in one step optimal linear prediction of the model parameters and the latent state vector \( F_t \), which contains information on the comovement among the economic indicators, \( y_{n,t} \) for \( n = 1, 2, ..., N \), collected in the dynamic factor \( f_t \). The filter tracks the course of the dynamic factor, which is calculated using only observations on \( y_{n,t} \). It computes recursively one-step-ahead predictions and updating equations of the conditional expectation of the dynamic factor and the associated mean squared error matrices. The output, \( f_{t|t} \), is an optimal estimator of the dynamic factor constructed as a linear combination of the variables \( y_{i,t} \), using information available through time \( t \). As new information becomes available, the filter is applied to update the state vector on a real time basis. A by-product of the filter is the conditional likelihood of the observable variables. The filter simultaneously evaluates this likelihood function, which is maximized with respect to the model parameters using an optimization algorithm. These parameters and the observations on \( y_{n,t} \) are then used in a final pass of the filter to yield the optimal latent dynamic factor based on maximum likelihood estimates.

### 3.2 Mixed Frequency Dynamic Factor Model under Structural Break (MFDFB)

Over the years, the U.S. economy has experienced different regimes that could have strongly impacted the dynamics of nominal GDP, such as the Great Moderation or the Great Recession. In order to account for this possibility, we propose a nonlinear dynamic factor model that allows nowcasting with mixed frequency and structural breaks. In particular, this paper extends the model (3)-(5) to allow for potential endogenous breaks in the common factor, which are modeled by considering two independent absorbing Markov processes. Specifically, equation (4) is replaced by:

\[ f_t = \mu S_t^m + \phi_1 f_{t-1} + \ldots + \phi_p f_{t-p} + e_t, \quad e_t \sim i.i.d.N(0, \sigma_{S_t^v}) \]

\[ \mu_S^m = \mu_0 (1 - S_t^m) + \mu_1 S_t \]

\[ \sigma_{S_t^v} = \sigma_0 (1 - S_t^v) + \sigma_1 S_t^v \]

\(^5\)Note that in this model, identification of the factor is achieved by setting one of the factor loadings to unity. The choice of normalization does not affect the parameter estimation.
where $S^m_t$ and $S^v_t$ are distinct unobserved two-state Markov variables that capture permanent changes in the factor mean or variance, respectively:

\[ S^m_t = 0 \text{ for } 1 \leq t \leq \tau^m \text{ and } S^m_t = 1 \text{ for } \tau^m < t \leq T - 1 \]
\[ S^v_t = 0 \text{ for } 1 \leq t \leq \tau^v \text{ and } S^v_t = 1 \text{ for } \tau^v < t \leq T - 1 \]

We model the one-time break as an unknown change point, $k^k$, for $k = m, v$, which follows constrained unobservable Markov state variables, as in Chib (1998)\(^6\):

\[
\begin{align*}
\Pr(S^k_t = 1 | S^k_{t-1} = 1) &= p^{k}_{11} = 1 \\
\Pr(S^k_t = 0 | S^k_{t-1} = 1) &= 1 - p^{k}_{11} = 0 \\
\Pr(S^k_t = 1 | S^k_{t-1} = 0) &= 1 - p^{k}_{00} \\
\Pr(S^k_t = 0 | S^k_{t-1} = 0) &= p^{k}_{00}, 0 < p^{k}_{11} < 1
\end{align*}
\]

That is, in order to capture structural break, the transition probabilities $p^{k}_{ij} = \Pr(S^k_t = j | S^k_{t-1} = i)$ are restricted so that the probability that $S^k_t$ will switch from state 0 at the unknown change point $\tau^k$ to state 1, at $\tau^k + 1$ is greater than zero. On the other hand, once the economy switches to state 1, it will stay at this state permanently. The corresponding transition probability matrices, for $p^{k}_{ij}$ with row $j$th, column $i$th are given by:

\[
P^k = \begin{bmatrix}
p^{k}_{00} & 0 \\
1 - p^{k}_{00} & 1
\end{bmatrix}.
\]

The proposed mixed frequency dynamic factor model with structural break (MFDFB) can be represented in the following state space form:

\[
\begin{align*}
y_t &= H_t^* F_t + \xi_t^*; \xi_t^* \sim \text{i.i.d.} N(0, R_t^*) \\
F_t &= \lambda^m_{S_t} + T \xi_{t-1}^* + \zeta_t^*; \zeta_t^* \sim \text{i.i.d.} N(0, Q_{S_t}^*)
\end{align*}
\]

In this case, the model is estimated in one step via maximum likelihood through a combination of the Kalman filter and Hamilton’s (1989) algorithm. The nonlinear filter forms forecasts of the unobserved state vector. As in the linear Kalman filter, the algorithm calculates recursively one-step-ahead predictions and updating equations of the dynamic factor and the mean squared error matrices, given the parameters of the model and starting values for the state vector, the mean squared error and, additionally, the probabilities of the Markov states. The updating equations are computed as averages weighted by the probabilities of the Markov states. The conditional likelihood of the observable variables is obtained as a by-product of the algorithm at each $t$, which is used to estimate the unknown model parameters. The filter evaluates this likelihood function, which is then maximized with respect to the model parameters using a nonlinear optimization algorithm. The maximum likelihood estimators and the sample data are then used in a final application of the filter to draw inferences about the dynamic factor and probabilities, based on information available at time $t$. The outputs are the conditional expectation of the state vector at $t$ given $I_t$, and the filtered probabilities of the Markov states $\Pr(S^k_t = j | I_t)$, where $I_t$ is the information set at $t$, based on the observable variables. For details see Kim (1994).

\(^6\)See also Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Chauvet and Su (2013).
4 Univariate Autoregressive Models

4.1 Linear Autoregressive Model

We compare the real-time nowcasts obtained from the multivariate mixed frequency model of nominal GDP growth and monthly indicators with those obtained from univariate models based solely on quarterly NGDP growth. Consider the following autoregression model:

\[
y_{t|v} = \phi_{0|v} + \sum_{p=1}^{P} \phi_{p|v} y_{t-p|v} + u_{t|v}, \quad u_{t|v} \sim i.i.d. N(0, \sigma^2_{u|v}) \tag{16}
\]

where \(y_{t|v}\) denotes NGDP growth of quarter \(t\) that is observed at monthly vintage \(v\), and \(\phi_{p|v}\) are the autoregressive parameters computed with all the available information up to \(v\). At the end of the sample \(T\) a forecast for the next period is computed as:

\[
\hat{y}_{T+1|V} = \hat{\phi}_{0|V} + \sum_{p=1}^{P} \hat{\phi}_{p|V} y_{T-p+1|V} \tag{17}
\]

where \(V\) denotes the last available vintage.

4.2 Autoregressive Model under Structural Break

In order to account for potential parameter instability in the autoregressive specifications, we follow the same method proposed for the mixed frequency dynamic factor model with breaks. That is, the coefficients in Equation (16) as subject to potential one-time breaks at unknown date \(\tau\), which follow unobserved two-state Markov variables, \(S^m_t\) and \(S^v_t\):

\[
y_{t|v} = \phi_{0|v,S^m_t} + \sum_{p=1}^{P} \phi_{p|v,S^m_t} y_{t-p|v} + u_{t|v}, \quad u_{t|v} \sim i.i.d. N(0, \sigma^2_{u|v,S^v_t}). \tag{18}
\]

The dynamics of \(S^k_t\), \(k = m, v\) are subject to the same restrictions as in the multivariate approach under structural break in Section 3.2. The estimation of model in Equation (18) is performed by maximum likelihood.\(^7\) Out-of-sample nowcasts with real-time data, \(\hat{y}_{T+1|V}\), are obtained from:

\[
\hat{y}_{T+1|V} = E(y_{T+1|V}) = \sum_{j=0}^{1} \Pr(S^k_{T+1} = j|V) y_{T+1|S^k_{T+1}=j,V}, \tag{19}
\]

where \(\Pr(S^k_{T+1} = j|V)\) can be computed by using the transition probability matrix \(P^k\) and \(y_{T+1|S^k_{T+1}=j,V}\) can be obtained from Equation (16) conditioned on the Markov state variables.

\(^7\)Since the probabilities to initialize the filter are unknown, and the ergodic ones are not suitable due to the truncation in the transition probabilities, we treat the initial probabilities as additional parameters to be estimated in the maximization.
5 Empirical Results

5.1 Timing of Forecasts

The U.S. nominal GDP (NGDP) series is first released by the Bureau of Economic Analysis (BEA) based on timely but incomplete information. Subsequent releases may involve large revisions to mend inconsistencies caused by lags in the data availability. There are three main releases of NGDP for a quarter, which occur in the three subsequent months following that quarter. For example, the first release of NGDP (‘advance’ estimate) for the last quarter of a year occurs in the end of January of the following year. The ‘second estimate’ is released in the end of February, and the ‘third estimate’ is released in the end of March. This allows us to compute three monthly inferences of NGDP for each quarter.

Our interest is in an assessment based solely on information that was available at each date, reproducing the real time forecasting problem for monetary policy monitoring at the time events were unfolding. We have collected vintages of NGDP and many other macroeconomic and financial time series as they would have appeared at the end of each month. For each vintage the sample collected begins in January 1967. The models are estimated with data from 1967:M1 to 2000:M12, and then recursively estimated for the period starting in 2001:M1 and ending in 2012:12 using only collected real time vintage as released at each period to generate nowcasts of NGDP growth.\textsuperscript{8} For example, the first prediction is for nominal GDP growth in the first quarter of 2001, \( y_{01Q1} \), which uses monthly indicators and NGDP growth up to its ‘advance release’ of 2000:Q4, \( y_{00Q4}^{1} \), based on information up to January 2001. The first nowcast is \( y_{01Q1|01Jan}^{1} \). The second nowcast of \( y_{01Q1} \) is \( y_{01Q1|01Feb}^{2} \) obtained in the end of February 2001, using monthly information up to 2001M02 and NGDP growth up to its second release of 2000:Q4, \( y_{00Q4}^{2} \). The third nowcast of \( y_{01Q1} \) is \( y_{01Q1|01Mar}^{3} \) obtained in the end of March 2001, using monthly information up to 2001M03 and NGDP growth up to its third release of 2000:Q4, \( y_{00Q4}^{3} \). Notice that this is the last prediction of \( y_{01Q1} \) since in the end of April its ‘advance release’ is published by the BEA. The timing of the nowcasts is:

\[
\begin{align*}
01/31/01 & \quad \text{Advance release } y_{00Q4}^{1} \\
& \quad + \text{ monthly series} \\
& \quad \downarrow \\
& \quad \text{First nowcast } y_{01Q1|01Jan}^{1} \\
02/28/01 & \quad \text{Second release } y_{00Q4}^{2} \\
& \quad + \text{ monthly series} \\
& \quad \downarrow \\
& \quad \text{Second nowcast } y_{01Q1|01Feb}^{2} \\
03/31/01 & \quad \text{Third release } y_{00Q4}^{3} \\
& \quad + \text{ monthly series} \\
& \quad \downarrow \\
& \quad \text{Third nowcast } y_{01Q1|01Mar}^{3} \\
04/30/01 & \quad \text{Advance release of } y_{01Q1} \\
& \quad \downarrow \\
& \quad \text{RMSE}
\end{align*}
\]

Given our interest in reproducing real time forecasts for monetary policy monitoring, we use the ‘advance release’ real time of GDP for each quarter as our target variable, as its publication dates closely match the Federal Open Market Committee (FOMC) meetings of January, April, July/August and October/November. By the time the FOMC meets on those months, most of the information on the

\textsuperscript{8}The real sample data is determined by the availability of data.
‘advance release’ of NGDP is available and used in their conjectures. The nowcasting accuracy of the models is then assessed with the root mean square error (RMSE) associated with this release.

5.2 Confirmatory Factor Analysis

Several recent papers find that small scale dynamic factor models can produce more accurate nowcasts than large scale models, such as Chauvet (2001), Boivin and Ng (2006), Bai and Ng (2008a, 2008b), Alvarez, Camacho, and Perez-Quiros (2013), etc. One reason is the potential misspecification of the factor and idiosyncratic error autoregressive dynamics. Another reason is that most economic series can be classified into a small number of categories. Thus, large models that include all available variables without pre-screening can lead to large cross-correlation in the idiosyncratic errors of the series. However, Bai and Ng (2008a) find that even when there is weak cross-correlation, models that include carefully selected variables display higher signal to noise ratio and outperform large-scale models.

The approach in this paper is ‘confirmatory’ dynamic factor analysis, in which models are specified based on prior knowledge of the economic variables’ dynamics and relationships. The proposed single fully estimated framework allows diagnostic tests that enable assessment of the reliability of the nowcasts. The variables included in the models are selected based on whether they represent similar economic or financial sectors, on their marginal predictive contribution to nowcast NGDP growth, and on model specification tests. The nowcasts are then compared using Diebold and Mariano’s test (DM 1995) for non-nested models, and Clark and McCracken’s (CM 2001) test for nested models.

5.2.1 Data

The series were obtained from the Federal Reserve Bank of Philadelphia and from the Federal Reserve Bank of Saint Louis real time data archives, and from data collected by the authors for the papers Chauvet (1998), Chauvet and Hamilton (2006), and Chauvet and Piger (2008).

We note that, although there is a large database of series available, only a smaller subset of monthly real time vintage series have a sample long enough to allow reasonable estimation inferences. We collected all available NIPA series at the month frequency, nominal variables from the product side, industrial production and capacity utilization, consumption expenditures, labor market variables, all price indices from production and consumption sides, and monetary and financial series. All variables were transformed to rate of growth, with the exception of those already expressed in rates.

5.2.2 Variable Selection

Several selection criteria were implemented to find series that display simultaneous movements with NGDP growth. The underlying guidelines were the economic significance of the variables, their statistical adequacy, and their overall conformity to the U.S. business cycle and inflation fluctuations. First, the series were ranked according to their marginal predictive content for NGDP growth similarly to Chauvet (2001), Camacho and Perez Quiros (2010), and Bai and Ng (2008a), and their ability to Granger-cause NGDP growth. Second, we evaluate their contemporaneous and cross correlation with NGDP growth.
The confirmatory dynamic factor model captures the common cyclical comovements underlying the observable variables. Thus, it is important that the series selected display a strong contemporaneous correlation with the target variable. If the series have off-set cycles, the upturn in NGDP growth may be offset by the downturn in the other variables, which will generate a latent dynamic factor with a lower signal to noise representation of common cyclical movements. Another important criterion used is the availability of real time vintages of the series and their availability at a reasonable sample length, which allows for testing the reliability of the NGDP nowcasts in real time.

From these procedures we classified and ranked the top variables. These series represent different measurements of real economic activity, inflation, and monetary and financial activities.

5.3 Multivariate Mixed-Frequency Dynamic Factor (MFDF) Model

5.3.1 Benchmark Model

Chauvet (2001), Boivin and Ng (2006), Bai and Ng (2008a) and Alvarez, Camacho, and Perez-Quiros (2013) find that small scale dynamic factor models that use one representative indicator of each classification outperform large scale dynamic factor models that includes all economic indicators as this minimizes cross-correlation in the idiosyncratic errors of series from the same classification.

We also find that fewer, pre-selected variables lead to more accurate nowcasts. We, thus, start with the construction of a three-variable "benchmark" model, based on the definition of nominal GDP, which incorporates information of our target variable NGDP growth, one real activity indicator, and one inflation indicator. This benchmark is then enlarged with additional variables that were highly ranked in the procedure described above, and based on diagnostic tests.

Among all variables, the top three representative indicators of real economic activity concur with the traditional coincident indicators used by the NBER business cycle dating committee: Industrial Production (IP), Real Personal Income Less Transfer Payments (PILT), and an employment measurement, which in our case, is Nonfarm Labor (NFL). The three leading representative indicators of U.S. inflation dynamics are Consumer Price Index (CPI), Producer Price Index (PPI), and Personal Consumption Expenditures Price Index (PCEP). These best three real activity indicators and best three inflation indicators yield nine possible pairwise (one real, one inflation) combinations, which will constitute the new set of potential benchmark models to nowcast NGDP growth. We estimate these nine models as in equations (7) - (8), always using NGDP growth and one of the pairwise in the benchmark set to obtain an index based on the common component among the variables. We then compute the RMSE with respect to the ‘advance release’ of NGDP growth for the corresponding quarter.

The results are reported in Table 2. The combination that displays the best predictive performance is model A: {NGDP, IP, CPI}, with a $RMSE = 0.297$, which is significantly lower than the RMSE for all other combinations, based on DM test (1995). Figure 4 plots the best three-variable MFDF benchmark model A and the target variable, and NBER recessions. Nowcasts from Model A closely match NGDP growth, and show a substantial improvement with respect to the naïve simple sum model (Figure 3).
5.3.2 Augmented Multivariate Mixed-Frequency Factor Models

We augment the basic three-variable benchmark dynamic factor model that include NGDP growth, an indicator of real economic activity, and an indicator of inflation by including additional highly ranked series. We assess the contribution of these additional indicators in several ways. First, since the small-scale dynamic factor structure captures cyclical comovements underlying the observable variables, we test whether the resulting augmented dynamic factor is highly correlated with the series used in its construction. This indicates whether the structure was or not simply imposed on the data by assuming large idiosyncratic errors. Second, since the model assumes that the factor summarizes the common dynamic correlation underlying the observable variables, this implies that the idiosyncratic errors should be uncorrelated with the observed variables. In order to test this assumption, the disturbances are regressed on lags of the observable variables. The additional series are kept if the parameters of the equations are found to be insignificantly different from zero. Third, we adjust the number of lags based on maximum likelihood tests, Bayesian Information criteria, and on whether the one-step-ahead conditional forecast errors, obtained from the filter described in section 3, are not predictable by lags of the observable variables, as implied by the model. Finally, the i.i.d. assumption of the residuals from equations (4) and (5) or (9) is tested using Ljung-Box statistics on their sample autocorrelation.

Additional Indicators  We next consider four-variable mixed frequency dynamic factor models. Table 2 reports the RMSE of the best four-variable models. Some interesting findings emerge from the results. In particular, the RMSE increases substantially if the additional fourth series is another measure of real activity or inflation. That is, once one real and one inflation indicator have been already incorporated into the model, any additional indicator in the same category (real or inflation) yields substantial decreases in the accuracy of the enlarged model. This corroborates the results of Chauvet (2001), Alvarez, Camacho, and Perez-Quiros (2013), and substantiates the arguments of Boivin and Ng (2006) and Bai and Ng (2008a, 2008b).

The next step is to assess the marginal predictive ability of additional indicators from category others than inflation and real activity, which could improve the fitting between our index and NGDP growth. We consider a large number of series. However, we find that most of the larger models display an inferior performance in terms of RMSE compared to the best three-variable benchmark (Model A). The exceptions are when some monetary and financial variables are considered. From those, the best performing indicators are the 3-Month Treasury Bill (TBILL), the S&P500 index, and Divisia measures of M3 and M4 computed by the Center for Financial Stability (CFS), which rely on the methodology proposed by Barnett (1980). The results are shown on Table 2. The lowest RMSE occurs when this series are included in the best three-variable benchmark model. That is, the best four-variable combinations correspond to Model B: {NGDP, IP, CPI, M3}, Model C: {NGDP, IP, CPI, M4}, and Model D: {NGDP, M3}, and Model D: {NGDP, M4}.

9The Divisia monetary aggregates for the U.S., including the broad measures M3 and M4 (both quantity and dual user cost-aggregates), are made available to the public by a program directed by William A. Barnett at the Center for Financial Stability at http://www.centerforfinancialstability.org/amfm.php. For an explanation of the methods underlying the data, see Barnett, Liu, Mattson, and Noort (2013).
IP, CPI, TBILL}. The difference between the nowcasts of these models and the ones from the others (all non-nested) is significantly different at the 1% or 5% level based on DM test.

We next estimate models with five and six variables. The results are reported on Table 3. Once again, we find that including more than one series from the same category (e.g. interest rates, monetary aggregates, stock market indices, etc.) substantially reduces the models’ predictive performance. The best five-variable models are Model E: \{NGDP, IP, CPI, M3, TBILL\}, and Model F: \{NGDP, IP, CPI, M4, TBILL\}. Notice, however, that the RMSE of these larger models are not substantially different from the benchmark Model A based on DM (1995) test of CM (2001) test.

The combined results indicate that the top ranked variables and specifications that have the best predictive performance to the target variable NGDP growth are different combinations of real activity, inflation, Divisia monetary aggregates, and interest rates.

6 Real-Time Nowcasting

6.1 Nowcasting with Linear Models

In this section we discuss the results of the mixed frequency dynamic factor (MFDF) models estimated over real time recursive samples from 2001:M1 to 2012:M12, as described in subsection 5.1. We use the six MFDF models that yield the best predictive performance so far to assess their ability to predict current growth of NGDP, using the exact amount of data available at the time of the prediction, and by taking into account all possible revisions in previous releases of variables. For comparison, we also estimate the naive simple sum model (section 2) and the autoregressive models (section 3) over real time recursive samples.

The RMSEs for these models are reported on Table 4. The MFDF models show relatively similar performance compared to each other over the full real time sample. However, there are substantial differences between the nowcasts from the MFDF models and the ones from the alternative models. The best performing specification for this period is the MFDF Model B ($RMSE = 0.512$), followed closely by Models C and F. The RMSE of Model B is approximately 24% and 45% lower than the ones from the autoregressive model and the naive simple sum model, respectively. These differences are statistically significant at the 5% level based on the DM test. The worst performing model is the naive simple sum with $RMSE = 0.740$.

\footnote{We have also estimated larger models. However, since this incurs in including series that represent similar economic and financial sectors, we find that these models are not top ranked as they 1) fail in diagnostic and specification tests; 2) display lower predictive performance to NGDP growth than the smaller scale factors considered. We do not report their results due to space consideration, but they are available upon request.}

\footnote{There are two recessions in the real time period studied. The NBER dated the 2001 recession as starting in 2001:M3 and ending in 2001:M11. The ‘Great Recession’ in 2007-2009 started in 2007:M12 and ended in 2009:M6.}

\footnote{As in Leiva-Leon (2014), we have also estimated a model with two dynamic factors, one that uses information on real activity indicators and the other based on inflation indicators. We find that the model with two separate factors performed similarly to the naïve model. The results are available from the authors upon request.}
The real-time nowcasts of the MFDF models and NBER recessions are plotted in Figure 5 and the nowcasts for the autoregressive model are shown in Figure 6 for \( p = 1, 2, 3 \) in equation (16), from the left to right chart, respectively. The nowcasts from the naive model are shown in Chart B of Figure 3. NBER recessions are represented as shaded areas. As can be seen, the performance of the autoregressive models is generally not accurate, with actual NGDP growth overestimated most of the period. This is also the case for nowcasts from the naive simple sum model, which also overestimate NGDP growth but to a much lesser extent. Chauvet and Potter (2013) study several univariate and multivariate models and find, in contrast, that the univariate autoregressive model has a good performance for real GDP growth compared to other more sophisticated models. However, we find that this is not the case here when considering nominal GDP growth as the target. The nowcasts from the linear mixed frequency dynamic factor model show a substantially better fit compared to the other models, although they also overestimate NGDP growth after the Great Recession.

In effect, we notice that the performance of all models seems to change over sub-periods. Chauvet and Potter (2013) find that it is more difficult to predict real output growth during recessions than during expansions. We also find that this is the case for nominal GDP growth. The autoregressive models miss the two recessions in the sample: the 2001 and the Great Recession, as they predict only a small decrease in growth. Although the 2001 recession was a mild downturn, the Great Recession was characterized by large negative NGDP growth. The nowcasts from the simple sum naive model and from the MFDF model display much better performance in predicting the timing and intensity of the fall in NGDP growth during these recessions, although the simple sum model overestimates the severity of the downturns.\(^{13}\)

We investigate the potential changes in predictive performance across different samples, by computing the RMSE also for the period before, during, and after the Great Recession that took place between 2007:M12 and 2009:M6. The results are also reported on Table 4. The MFDF models exhibit lower RMSE in the period before the Great Recession compared to the autoregressive models and the naive simple sum model. The differences in accuracy are even more substantial during the Great Recession, with the values of the RMSE from the MFDF models generally around half of the ones from the alternative models. In contrast, all models show a more similar performance since the end of this last recession, in the period from 2009:M7 and 2012:M12. The autoregressive models show a slightly better performance but the difference with the best MFDF model for this period (Model C) is not statistically significant at the 5% level using DM test. Interestingly, Figures 3, 5, and 6 show that the nowcasts of all models tend to overestimate NGDP growth since the Great Recession.

### 6.2 Real-Time Nowcasting under Parameter Instability

Although linear MFDF models have shown markedly improvements in nowcasting performance in comparison to univariate and naive models, there are differences in the predictive performance over sub-samples across all models. These results could be due to instability in the models’ parameters. Over the years,\(^{13}\)

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\(^{13}\)This is related to the fact that the simple sum model uses monthly Industrial Production, which displays a much larger fall (Figure 2 Chart C) than NGDP during recessions (Figure 1 Chart B).
the U.S. economy has experienced some abrupt changes that could have strongly impacted nominal GDP dynamics, such as the Great Moderation or the Great Recession. Structural changes generate breakdown in real-time forecasts. In this section we account for this possibility. We report the results of the proposed a nonlinear mixed frequency dynamic factor model that produces nowcasts in the presence of structural breaks (MFDFB).

We first estimate the MFDFB model using the same variables from the linear MFDF model B \{NGDP, IP, CPI, M3\} in the previous section, which presented the overall best nowcasting performance for the full real time sample compared to all other linear models (Table 4). The estimation is based on equations 8 and 9-12, as explained in section 3.2.

Figure 7 Chart A shows the estimated factor along with the probabilities of regime change. The probabilities of a break show some increases during periods that have been discussed in previous literature as potential permanent breaks such as in 1970-1971 (productivity slowdown), 1975 (oil crisis), and 1982 (end of Great Inflation period). However, the probabilities of a permanent break reached one in 1990M12, after which they remained at this regime until the end of the sample. This period is related to a change in volatility of NGDP growth, as shown in Figure 1 Chart B, which is associated with the Great Moderation. It is interesting to notice that an extensive literature has found that real GDP displays a break in volatility around 1984 (e.g. Kim and Nelson 1999, McConnell and G. Perez-Quiros 2000, Chauvet and Potter 2001, etc.) while other authors find that inflation volatility shows a break in the late 1980s (Chauvet and Popli 2003). The breaks in NGDP growth seem to reflect abrupt changes in its components, specifically, inflation and real GDP growth. Figure 7 Chart B plots the estimated factors and the probabilities of mean break, which clearly reflects the abrupt change in the economy associated with the Great Recession. The probabilities of a break switched to regime one and reached the value of 1 in 2008M10, associated with the impact of the Lehman Brothers’ crisis, and remained at this level until the end of the sample.

We also examine whether univariate models of NGDP growth also display structural breaks. We first obtain the recursive autoregressive parameters from the real time estimation of the AR models using equation (16), which are plotted in Figure 8 for \(p = 1, 2, 3\), from the left to right chart. The parameter of the AR(1) model shows evidence of parameter instability during the Great Recession. This is the case for the AR(2) and AR(3) models but to a lesser extent. This could be the origin of the overestimation of the real-time AR models nowcasts during this period (Figure 6).

We next estimate the univariate autoregressive model under structural break, based on equation 18. We focus on the AR(1) specification as it shows the largest parameter instability. Figure 9 Chart A plots nominal GDP growth and the probabilities of a break from the autoregressive model, which are similar to the ones obtained for the MFDFB model. The probabilities also indicate a structural change at around the same time as the MFDFB model, switching to state one in 1989Q3. Figure 9 Chart B plots the corresponding probabilities of mean break regime, which reach the value of 1 in 2008Q4, coinciding with the date of the break found with the multivariate model in 2008M10. Thus, both univariate and multivariate approaches unveil structural break in nominal output growth around 1989-1990 and in the midst of the Great Recession, in 2008.
Based on these findings, we estimate the proposed MFDFB model presented in section 3.2 over real time recursive samples to obtain NGDP growth nowcasts. Notice that we only use information available at the time of the predictions. Thus, assessments of potential breaks in the sample are endogenously recursively estimated for every vintage. The predictions associated with this model along with the NGDP growth data are plotted in Figure 10. The results show marked improvements compared with the performance of the linear version of this model (Figure 7). In particular, the previous finding of overestimated nowcasts is substantially reduced with this framework. This can also be seen in Table 5, which reports the RMSE associated with the MFDFB models and the AR(1) specification subject to break across different subsamples. The MFDFB specification that generally displays the best performance across periods is Model D, which contains information on NGDP, IP, CPI, and TBILL. However, for the period since the Great Recession, the model with the best performance is the smaller benchmark, which contains only the series NGDP, IP and CPI. The reason might be that the TBILL became uninformative at very low values and almost no volatility, as it has been since the last recession.

Overall, the nowcasting performance of the MFDFB models subject to breaks is more accurate compared to the nowcasting from the linear MFDF models, with the exception of during the Great Recession. Specifically, during the full sample period and before the Great Recession, all MFDFB nonlinear models present considerably lower RMSE than the linear MFDF models. This is also the case for most nonlinear model the Great Recession. The RMSE for the best MFDFB for this period (Model A) has a RMSE significantly lower than the best linear MFDF (Model C) at the 5% level using CM’s test (2001). Compared to the autoregressive model (Figure 11 and Table 5), the nowcasts from the MFDFB models under break show clear improvements. This is more accentuated for the period since the Great Recession, for which all linear models generate overestimated nowcasts. The RMSE of the MFDFB is less than half of the linear AR(1) model and around half of the RMSE for the AR(1) with break.

In summary, the results provide evidence of substantial gains in the nowcasting ability of the proposed mixed frequency multivariate models when allowing for potential structural break in parameters.

7 Conclusions

Given the non-conventional situation that the Fed faces regarding the lower bound level of the interest rate, many economists have suggested that alternative strategies should be adopted to decrease unemployment rate, one of the proposals is forward guidance through targeting nominal GDP. This paper proposes a nonlinear nowcasting dynamic factor model that includes mixed-frequency and parameter instability that can be helpful in the assessment of current economic conditions.

We evaluate the performance of univariate and multivariate, linear and nonlinear econometric models that can be useful to earlier assessments of current nominal GDP growth, under real conditions that policy makers face at the time the predictions are made. The univariate analysis shows that classical autoregressive models provide poor performance regarding real-time nowcasts of the target variable. However, when allowing for parameter instability, the performance of the univariate model substantially
increases.

We then estimate the proposed small scale nonlinear mixed frequency dynamic factor models. We find the presence of two breaks in NGDP growth dynamics: the first in the late 1980s, associated with the Great Moderation, and the second in the midst of the Great Recession, in 2008. The multivariate models that allow parameter instability outperform linear multivariate and linear and nonlinear univariate specifications, yielding the highest nowcasting performance. The best specifications are parsimonious and include economic activity, inflation, monetary indicators, and/or interest rates.
References


### Table 1. RMSE for 3-Variable MFDF Model Benchmark: NGDP, Real Activity, and Inflation

<table>
<thead>
<tr>
<th>Inflation Indicators</th>
<th>Real Activity Indicators</th>
<th>IP</th>
<th>PILT</th>
<th>NFL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td></td>
<td>0.297**</td>
<td>0.822</td>
<td>0.810</td>
</tr>
<tr>
<td>PPI</td>
<td></td>
<td>0.370</td>
<td>0.449</td>
<td>0.810</td>
</tr>
<tr>
<td>PCEP</td>
<td></td>
<td>0.341</td>
<td>0.862</td>
<td>0.810</td>
</tr>
</tbody>
</table>

### Table 2. RMSE for 4-Variable MFDF Models: NGDP, Real Activity, Inflation, and Others

<table>
<thead>
<tr>
<th>Variables</th>
<th>NFL</th>
<th>PILT</th>
<th>PPI</th>
<th>PCEP</th>
<th>PI</th>
<th>PCE</th>
<th>M3</th>
<th>M4</th>
<th>SP500</th>
<th>TBILL</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP, CPI</td>
<td>0.814</td>
<td>0.805</td>
<td>0.986</td>
<td>0.958</td>
<td>0.435</td>
<td>0.327</td>
<td><strong>0.298</strong></td>
<td><strong>0.295</strong></td>
<td>0.330</td>
<td><strong>0.297</strong></td>
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<tr>
<td>NFL, CPI</td>
<td>—</td>
<td>0.812</td>
<td>0.968</td>
<td>0.971</td>
<td>0.534</td>
<td>0.423</td>
<td>0.809</td>
<td>0.809</td>
<td>0.809</td>
<td>0.809</td>
</tr>
<tr>
<td>PILT, CPI</td>
<td>—</td>
<td>—</td>
<td>0.995</td>
<td>0.959</td>
<td>0.762</td>
<td>0.420</td>
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<td>0.805</td>
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<tr>
<td>IP, PPI</td>
<td>0.814</td>
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<td>—</td>
<td>0.960</td>
<td>0.421</td>
<td>0.318</td>
<td>0.375</td>
<td>0.364</td>
<td>0.380</td>
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<tr>
<td>NFL, PPI</td>
<td>—</td>
<td>0.812</td>
<td>—</td>
<td>—</td>
<td>0.960</td>
<td>0.784</td>
<td>0.418</td>
<td>0.810</td>
<td>0.810</td>
<td>0.811</td>
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<tr>
<td>PILT, PPI</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.960</td>
<td>0.760</td>
<td>0.385</td>
<td>0.476</td>
<td>0.477</td>
<td>0.443</td>
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<tr>
<td>IP, PCEP</td>
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<td>0.809</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.463</td>
<td>0.368</td>
<td>0.869</td>
<td>0.865</td>
<td>0.338</td>
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<tr>
<td>NFL, PCEP</td>
<td>—</td>
<td>0.812</td>
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<td>0.529</td>
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<td>0.809</td>
<td>0.809</td>
</tr>
<tr>
<td>PILT, PCEP</td>
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<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.762</td>
<td>0.452</td>
<td>0.858</td>
<td>0.854</td>
<td>0.865</td>
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### Table 3. RMSE for 5- and 6-Variable MFDF Models: NGDP, Real Activity, Inflation, and Others

<table>
<thead>
<tr>
<th>Variables</th>
<th>RMSE</th>
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<tr>
<td>IP, CPI, M3, M4</td>
<td>1.261</td>
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<tr>
<td>IP, CPI, M3, TBILL</td>
<td><strong>0.298</strong></td>
</tr>
<tr>
<td>IP, CPI, M4, TBILL</td>
<td><strong>0.294</strong></td>
</tr>
<tr>
<td>IP, CPI, M3, M4, TBILL</td>
<td>1.494</td>
</tr>
</tbody>
</table>

Note: RMSE stands for root mean squared errors, and MFDF is the dynamic factor mixed frequency model

(*) and (**) stand for statistically significant at the 5% and 1% level based on DM test used to compare non-nested models.

(·) and (··) stand for statistically significant at the 5% and 1% level based on CM test used to compare nested models.
Table 4. RMSE for Real-Time Nowcasts from Best MFDF Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Variables</th>
<th>Full Sample</th>
<th>Great Recession</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>RMSE</td>
<td>RMSE</td>
<td>RMSE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Before</td>
<td>During</td>
<td>After</td>
</tr>
<tr>
<td>A</td>
<td>IP, CPI</td>
<td>0.524</td>
<td>0.532</td>
<td>0.584</td>
</tr>
<tr>
<td>B</td>
<td>IP, CPI, M3</td>
<td>0.512**</td>
<td>0.530**</td>
<td>0.589</td>
</tr>
<tr>
<td>C</td>
<td>IP, CPI, M4</td>
<td>0.514</td>
<td>0.531</td>
<td>0.603</td>
</tr>
<tr>
<td>D</td>
<td>IP, CPI, TBILL</td>
<td>0.560</td>
<td>0.532</td>
<td>0.777</td>
</tr>
<tr>
<td>E</td>
<td>IP, CPI, M3, TBILL</td>
<td>0.521</td>
<td>0.546</td>
<td>0.576**</td>
</tr>
<tr>
<td>F</td>
<td>IP, CPI, M4, TBILL</td>
<td>0.514</td>
<td>0.532</td>
<td>0.608</td>
</tr>
<tr>
<td>AR(1)</td>
<td></td>
<td>0.700</td>
<td>0.628</td>
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<tr>
<td>AR(2)</td>
<td></td>
<td>0.671</td>
<td>0.570</td>
<td>1.235</td>
</tr>
<tr>
<td>AR(3)</td>
<td></td>
<td>0.719</td>
<td>0.568</td>
<td>1.336</td>
</tr>
<tr>
<td>Naive</td>
<td>Model</td>
<td>0.740</td>
<td>0.647</td>
<td>1.035</td>
</tr>
</tbody>
</table>

Table 5. RMSE for Real-Time Nowcasts with Break

<table>
<thead>
<tr>
<th>Model</th>
<th>Variables</th>
<th>Full Sample</th>
<th>Great Recession</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>RMSE</td>
<td>RMSE</td>
<td>RMSE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Before</td>
<td>During</td>
<td>After</td>
</tr>
<tr>
<td>A</td>
<td>IP, CPI</td>
<td>0.546</td>
<td>0.447</td>
<td>1.031</td>
</tr>
<tr>
<td>B</td>
<td>IP, CPI, M3</td>
<td>0.476</td>
<td>0.400</td>
<td>0.812</td>
</tr>
<tr>
<td>C</td>
<td>IP, CPI, M4</td>
<td>0.509</td>
<td>0.410</td>
<td>0.944</td>
</tr>
<tr>
<td>D</td>
<td>IP, CPI, TBILL</td>
<td>0.449**</td>
<td>0.377**</td>
<td>0.737**</td>
</tr>
<tr>
<td>E</td>
<td>IP, CPI, M3, TBILL</td>
<td>0.505</td>
<td>0.393</td>
<td>0.792</td>
</tr>
<tr>
<td>F</td>
<td>IP, CPI, M4, TBILL</td>
<td>0.545</td>
<td>0.503</td>
<td>0.755</td>
</tr>
<tr>
<td>AR(1)</td>
<td>with Break</td>
<td>0.630</td>
<td>0.536</td>
<td>0.916</td>
</tr>
</tbody>
</table>

Note: RMSE stands for root mean squared errors, MFDF is the dynamic factor mixed frequency model and MFDFB is the dynamic factor mixed frequency model with break. Full sample in the third column refers to the real time period from 2001:M1 to 2012:M12. The fourth column refers to the real time sample: 2001:M1-2007:M11, fifth column to 2007:M12 to 2009:M6, and sixth column to 2009:M7-2012:M12. (*) and (**) stand for statistically significant at the 5% and 1% level based on DM test used to compare non-nested models. (·) and (··) stand for statistically significant at the 5% and 1% level based on CM test used to compare nested-models.
Figure 1. Nominal GDP (NGDP)

Chart A. Level

Chart B. Growth

Figure 2. Real Activity and Inflation

Chart A. Real GDP growth

Chart B. GDP Deflator growth

Chart C. IP growth

Chart D. CPI growth

Figure 3. Naïve Simple Sum Model

Chart A. In-Sample

Chart B. Real Time Nowcast
Figure 4. Nowcasts from Best Benchmark Linear MFDF Model

Figure 5. Real-Time Nowcasts from Best Augmented Linear MFDF Models

Figure 6. Real-Time Nowcasts from Univariate Autoregressive Models
Figure 7. Probability of a Break - MFDF with Structural Break Model

Chart A. Chart B.

Figure 8. Recursive Autoregressive Parameters from Univariate Models

Figure 9. Break Probability: Univariate Autoregressive Model

Chart A. Chart B.
Figure 10. Real-Time Nowcasts from MFDFB under Structural Break Model

Figure 11. Real-Time Nowcasts from Univariate Autoregressive Models with Structural Break