Public Procurement in Times of Crisis: The Bundling Decision Reconsidered

Patrick W. Schmitz

University of Cologne

2013

Online at http://mpra.ub.uni-muenchen.de/53712/
MPRA Paper No. 53712, posted 16. February 2014 16:01 UTC
Public Procurement in Times of Crisis: The Bundling Decision Reconsidered

Patrick W. Schmitz*

University of Cologne, Germany, and CEPR, London, UK

**Abstract.** The government wants two tasks to be performed. In each task, unobservable effort can be exerted by a wealth-constrained private contractor. If the government faces no binding budget constraints, it is optimal to bundle the tasks. The contractor in charge of both tasks then gets a bonus payment if and only if both tasks are successful. Yet, if the government has only a limited budget, it may be optimal to separate the tasks, so that there are two contractors each in charge of one task. In this case, high efforts in both tasks can be implemented with smaller bonus payments.

**Keywords:** moral hazard; limited liability; procurement contracts; bundling; public goods provision

**JEL Classification:** D86; L24; L33; H57; H12

---

* Department of Economics, University of Cologne, Albertus-Magnus-Platz, 50923 Köln, Germany. Tel.: +49 221 470 5609; fax: +49 221 470 5077. E-mail: <patrick.schmitz@uni-koeln.de>.
1 Introduction

An important question in public procurement is whether the government should bundle different tasks together and let one private contractor be responsible for these tasks, or whether the government should contract with different private parties each in charge of only one task.¹ The present paper reconsiders the bundling decision in a model where the principal (i.e., the government) has only a limited budget. In an influential paper, Hart (2003) has argued that financing issues may be secondary in the context of public-private partnerships, since the government has “enormous powers of taxation” (Hart, 2003, p. C75). Yet, in times of financial crises, governments may well face binding budget constraints. The purpose of the present study is to explore the implications of such constraints on the optimal bundling decision.²

In the principal-agent literature, many authors have studied moral hazard problems in which the agent is risk-neutral but wealth-constrained, such that a “limited liability rent” must be paid to motivate an agent to exert high effort (see Laffont and Martimort, 2002). In most papers, only the agent is wealth-constrained, while the principal faces no (binding) wealth constraints.³ When in such a framework two technologically independent tasks have to be

---

¹For discussions of bundling in public procurement, see e.g. Hart (2003), Bennett and Iossa (2006), Chen and Chiu (2010), Iossa and Martimort (2012), and De Brux and Desrieux (2013).

²In a model encompassing agency problems and property rights, Martimort and Pouyet (2008) find that the question whether tasks are bundled may be more important than the ownership structure. For discussions of public versus private ownership in incomplete contracting frameworks, see Hart, Shleifer, and Vishny (1997) and Hoppe and Schmitz (2010).

performed, bundling these tasks may reduce the limited liability rent that the principal has to pay in order to induce high efforts. If one agent is in charge of both tasks, the principal must pay a bonus only if both tasks are successful. In contrast, if two different agents are each in charge of one task, then the principal must also pay a bonus if only one agent is successful.4

Yet, if the principal has only a limited budget which she can use for making payments to the agent(s), then she may be better off when she does not bundle the tasks. The intuition for this novel finding is as follows. If one agent is in charge of both tasks, the payment that the principal has to make if both tasks are successful may be so large that the principal cannot afford to induce the agent to exert high efforts in both tasks. In contrast, if there are two agents each in charge of one task, the principal can induce both agents to exert high efforts, since now the payments can be smaller, as they are also paid when only one task is successful.

2 The model

Consider a principal (a government agency) who wants two tasks to be performed in order to improve the provision of public goods. In each task $i \in \{1, 2\}$, unobservable effort $e_i \in \{e_l, e_h\}$ can be exerted, where $0 < e_l < \frac{1}{2} < e_h < 1$. Effort in task $i$ leads to a success ($y_i = 1$) with probability $e_i$ and to a failure ($y_i = 0$) otherwise. A success in task $i$ yields a non-monetary benefit $b$ to the principal, capturing the improved quality of public good provision.

4See e.g. Bolton and Dewatripont (2005, section 6.2.2), when their parameter $\gamma$ is zero. For experimental evidence, see the “no conflict” treatments in Hoppe and Kusterer (2011).
A failure yields no benefit to the principal. The outcome $y_i$ of each task is verifiable.

The principal can either decide to have one agent in charge of both tasks (*bundling*), or to have two different agents in charge of the two different tasks (*separation*). An agent’s effort costs in a task $i$ are given by $\psi > 0$ if high effort is chosen and by 0 if low effort is chosen. Note that the two tasks are technologically unrelated. All parties are risk-neutral and the reservation utilities are given by zero. Moreover, we assume that the agents have no wealth and are protected by limited liability; i.e., payments to the agents must be non-negative.

If the principal decides to bundle the tasks, then a contract is given by $(w_{11}, w_{10}, w_{01}, w_{00})$, where $w_{y_1y_2}$ denotes the payment from the principal to the agent given the outcomes $y_1$ of task 1 and $y_2$ of task 2. Analogously, if the principal hires agent $A$ to perform task 1 and agent $B$ to perform task 2, the contracts are given by $(w_{11}^A, w_{10}^A, w_{01}^A, w_{00}^A)$ for agent $A$ and $(w_{11}^B, w_{10}^B, w_{01}^B, w_{00}^B)$ for agent $B$.

Finally, our key assumption is that also the principal has limited resources. This assumption distinguishes the present paper from previous studies on public procurement contracting with limited liability.\textsuperscript{5} Specifically, the payment to the agent in case of bundling and the sum of the payments that the principal makes to the agents in case of separation must not be larger than the principal’s budget $W$.

Throughout, we suppose that the principal’s benefit $b$ is sufficiently large to make high effort attractive for the principal to implement even in a second-best

\textsuperscript{5}See e.g. Martimort and Straub (2012) and Hoppe and Schmitz (2013).
Assumption 1. $b > e_h\psi/(e_h - e_l)^2$.

3 Bundling

Let us first suppose the principal contracts with only one agent to perform both tasks. The agent’s expected payoff when he exerts high effort in both tasks is given by

$$u_{hh} = e_h^2 w_{11} + e_h(1 - e_h)(w_{10} + w_{01}) + (1 - e_h)^2 w_{00} - 2\psi.$$  

When the agent exerts high effort in task 1 and low effort in task 2, his expected payoff is

$$u_{hl} = e_h e_l w_{11} + e_h(1 - e_l)w_{10} + (1 - e_h)e_l w_{01} + (1 - e_h)(1 - e_l)w_{00} - \psi.$$  

Analogously, when the agent chooses low effort in task 1 and high effort in task 2, his expected payoff is

$$u_{lh} = e_l e_h w_{11} + (1 - e_l)w_{10} + e_l(1 - e_h)w_{01} + (1 - e_l)(1 - e_h)w_{00} - \psi.$$  

Finally, when the agent chooses low effort in both tasks, his expected payoff is given by

$$u_{ll} = e_l^2 w_{11} + e_l(1 - e_l)(w_{10} + w_{01}) + (1 - e_l)^2 w_{00}.$$  

The expected total surplus is $2(e_h b - \psi)$ if high effort is exerted in both tasks, $(e_h + e_l) b - \psi$ if high effort is exerted in only one task, and $2e_l b$ if low effort is exerted in both tasks. Thus, in a first-best world, high effort in both tasks would be chosen whenever $b \geq \psi/(e_h - e_l)$, while low effort in both tasks would be chosen otherwise. Yet, it will become clear in Section 4 that in the second-best world under separation the principal would never implement high effort if $b < e_h\psi/(e_h - e_l)^2$. Hence, if Assumption 1 is violated, separation cannot be strictly better than bundling.
High effort in both tasks can be implemented if it is possible to simultaneously satisfy the incentive compatibility constraints $u_{hh} \geq u_{hl}$, $u_{hh} \geq u_{lh}$, and the constraints that the payments must be non-negative and smaller than the budget $W$.

The principal’s expected payoff when high effort in both tasks is implemented is given by

$$2e_h b - \frac{e_h^2}{e_l} w_{11} - e_h (1 - e_h)(w_{10} + w_{01}) - (1 - e_h)^2 w_{00}.$$

It is straightforward to see that it is optimal for the principal to set $w_{00} = 0$ and $w_{10} = w_{01} =: w_1$. The incentive compatibility constraints $u_{hh} \geq u_{hl}$ and $u_{hh} \geq u_{lh}$ can thus be written as

$$w_{11} \geq C_I(w_1) := \frac{\psi}{e_h(e_h - e_l)} + \frac{2e_h - 1}{e_h} w_1$$

and the incentive compatibility constraint $u_{hh} \geq u_{ll}$ can be written as

$$w_{11} \geq C_{II}(w_1) := \frac{2\psi}{e_h^2 - e_l^2} - 2 \frac{1 - e_h - e_l}{e_h + e_l} w_1.$$

**Lemma 1** Suppose that the two tasks are bundled and the principal wants to implement high effort in both tasks.

(i) Suppose that $e_h + e_l \geq 1$. The principal can implement high effort in both tasks whenever $W \geq 2\psi/(e_h^2 - e_l^2)$. In this case, she sets $w_{11} = 2\psi/(e_h^2 - e_l^2)$ and $w_1 = 0$, yielding the expected payoff $2e_h b - 2\psi e_h^2/(e_h^2 - e_l^2)$.

(ii) Suppose that $e_h + e_l < 1$. The principal can implement high effort in both tasks whenever $W \geq 2\psi/(e_h - e_l)$. If $W \geq 2\psi/(e_h^2 - e_l^2)$, the principal sets $w_{11} = 2\psi/(e_h^2 - e_l^2)$ and $w_1 = 0$, yielding the expected payoff $2e_h b - 2\psi e_h^2/(e_h^2 - e_l^2)$. If $2\psi/(e_h - e_l) \leq W < 2\psi/(e_h^2 - e_l^2)$, the principal sets

7Throughout, participation constraints are redundant, as they are implied by incentive compatibility and non-negativity of the payments.
$w_{11} = W$ and $w_1 = \frac{2\psi - (e_h^2 - e_i^2)w}{2(1-e_h-e_i)(e_h-e_i)}$, yielding the expected payoff $2e_hb - e_h^2W - e_h(1-e_h)\frac{2\psi - (e_h^2 - e_i^2)w}{2(1-e_h-e_i)(e_h-e_i)}$.

**Proof.** The principal’s problem is to maximize $2e_hb - e_h^2w_{11} - 2e_h(1-e_h)w_1$ subject to the constraints $w_{11} \geq C_I(w_1)$, $w_{11} \geq C_{II}(w_1)$, $0 \leq w_{11} \leq W$, and $0 \leq w_1 \leq W$.

Consider case (i). Note that $C_{II}(w_1)$ is increasing. Hence, the constraints $w_{11} \geq C_{II}(w_1)$ and $w_{11} \leq W$ cannot be simultaneously satisfied if $W < 2\psi/(e_h^2 - e_i^2)$, so under this condition it is impossible to implement high effort in both tasks. Suppose $W \geq 2\psi/(e_h^2 - e_i^2)$. Ignore for a moment the constraint $w_{11} \geq C_I(w_1)$. Then the constraints $w_{11} \geq C_{II}(w_1)$ and $w_1 \geq 0$ must be binding, since otherwise the principal’s expected payoff could be increased by reducing $w_{11}$ (resp., $w_1$), which would not violate the remaining constraints. Thus, the solution to the principal’s relaxed problem is $w_{11} = 2\psi/(e_h^2 - e_i^2)$ and $w_1 = 0$. It is easy to see that this solution also satisfies the omitted constraint $w_{11} \geq C_I(w_1)$, so we have found the solution to the principal’s original maximization problem.

Consider now case (ii). Note that $C_{II}(w_1)$ is decreasing, $C_I(w_1)$ is increasing, and $C_{II}(0) > C_I(0)$. Moreover, if $w_1 = \psi/(e_h - e_i)$, then $C_I(w_1) = C_{II}(w_1) = 2\psi/(e_h - e_i)$. Hence, it is impossible to simultaneously satisfy the constraints $w_{11} \geq C_I(w_1)$, $w_{11} \geq C_{II}(w_1)$, and $w_{11} \leq W$ if $W < 2\psi/(e_h - e_i)$. Suppose that $W \geq C_{II}(0) = 2\psi/(e_h^2 - e_i^2)$. Ignore for a moment the constraints $w_{11} \geq C_I(w_1)$, $0 \leq w_{11} \leq W$, and $w_1 \leq W$. The constraint $w_{11} \geq C_{II}(w_1)$ must be binding, since otherwise the principal’s expected payoff could be increased by reducing $w_{11}$. The principal’s problem is then to maximize $2e_hb - e_h^2w_1 - 2\frac{\psi}{e_h-e_i} - 2\frac{1-e_h-e_i}{e_h+e_i}w_1 - 2e_h(1-e_h)w_1$ subject to $w_1 \geq 0,$
which is solved by \( w_1 = 0 \). It is straightforward to check that the solution \( w_1 = 0, w_{11} = 2\psi/(e_h^2 - e_i^2) \) satisfies the omitted constraints. Next, suppose that \( 2\psi/(e_h - e_i) \leq W < 2\psi/(e_h^2 - e_i^2) \). Ignore for a moment the constraints \( w_{11} \geq C_I(w_1), 0 \leq w_{11}, 0 \leq w_1 \leq W \). The constraint \( w_{11} \geq C_{II}(w_1) \) must again be binding. The principal’s problem is to maximize \( 2e_h b - e_h^2(\frac{2\psi}{e_h^2 - e_i^2} - 2\frac{1-e_h-e_i}{e_h+e_i}w_1) - 2e_h(1 - e_h)w_1 \) subject to \( w_{11} \leq W \). The principal thus makes \( w_1 \) as small as possible given that \( w_{11} = C_{II}(w_1) \leq W \) must hold. The solution is given by \( w_{11} = W, w_1 = \frac{2\psi-e_h^2W}{2(1-e_h-e_i)(e_h-e_i)}, \) which also satisfies the omitted constraints.

Of course, the principal can always implement low effort in both tasks by setting all wages equal to zero, so her expected payoff is \( 2e_l b \). Moreover, the principal may want to implement high effort in only one task, say task 1. Then she maximizes

\[
(e_h + e_i)b - e_h e_l w_{11} - e_h (1 - e_i) w_{10} - (1 - e_h) e_i w_{01} - (1 - e_h)(1 - e_i) w_{00}
\]

subject to the incentive compatibility constraints \( u_{hl} \geq u_{hh}, u_{hl} \geq u_{lh}, u_{hl} \geq u_{ll} \) as well as the constraints that the payments must be non-negative and smaller than the budget \( W \). Ignore for a moment the constraints \( u_{hl} \geq u_{hh} \) and \( u_{hl} \geq u_{lh} \). The principal then maximizes her expected payoff subject to \( e_i(w_{11} - w_{01}) + (1 - e_i)(w_{10} - w_{00}) \geq \psi/(e_h - e_i) \) and the constraints on the payments. Hence, she sets \( w_{00} = w_{01} = 0 \). The principal can implement high effort in one task and low effort in the other task whenever \( W \geq \psi/(e_h - e_i) \). In this case, it is optimal for her to set \( w_{11} = w_{10} = \psi/(e_h - e_i) \). It is easy to check that the omitted constraints are also satisfied. The principal’s expected payoff then is \( (e_h + e_i)b - e_h \psi/(e_h - e_i) \).

Note that the principal’s expected payoff when she implements high effort
in only one task is strictly larger than her expected payoff when she implements low effort in both tasks if and only if $b > e_h \psi / (e_h - e_l)^2$, which holds according to Assumption 1. Thus, the principal implements low effort in both tasks only if $W < \psi / (e_h - e_l)$. Moreover, under Assumption 1 the principal’s expected payoff when she implements high effort in both tasks is larger than when she implements high effort in only one task. Hence, the principal always implements high effort in as many tasks as possible, so that the maximum expected payoffs that the principal can attain under bundling are as follows.

**Proposition 1** Suppose the two tasks are bundled.

(i) Suppose that $e_h + e_l \geq 1$. The principal’s maximum expected payoff is

$$
\Pi^\text{bundle} = \begin{cases} 
2e_h b - \frac{2\psi e_h^2}{e_h - e_l} & \text{if } \frac{2\psi}{e_h - e_l} \leq W; \\
(e_h + e_l)b - \frac{e_h \psi}{e_h - e_l} & \text{if } \frac{\psi}{e_h - e_l} \leq W < \frac{2\psi}{e_h - e_l}; \\
2e_l b & \text{if } W < \frac{\psi}{e_h - e_l}.
\end{cases}
$$

(ii) Suppose that $e_h + e_l < 1$. The principal’s maximum expected payoff is

$$
\Pi^\text{bundle} = \begin{cases} 
2e_h b - \frac{2\psi e_h^2}{e_h - e_l} & \text{if } \frac{2\psi}{e_h - e_l} \leq W; \\
2e_h b - e_h^2 W - e_h (1 - e_h) \frac{2\psi - (e_h - e_l)^2 W}{(1 - e_h - e_l)(e_h - e_l)} & \text{if } \frac{2\psi}{e_h - e_l} \leq W < \frac{2\psi}{e_h - e_l}; \\
(e_h + e_l)b - \frac{e_h \psi}{e_h - e_l} & \text{if } \frac{\psi}{e_h - e_l} \leq W < \frac{2\psi}{e_h - e_l}; \\
2e_l b & \text{if } W < \frac{\psi}{e_h - e_l}.
\end{cases}
$$

4 Separation

Let us now consider the case of separation, such that agent $A$ is in charge of task 1 while another agent $B$ is in charge of task 2.
Suppose first the principal wants to implement high effort in both tasks. She maximizes her expected payoff

\[ 2e_h b - e_h^2 (w_{11}^A + w_{11}^B) - e_h (1 - e_h) (w_{10}^A + w_{10}^B + w_{01}^A + w_{01}^B) - (1 - e_h)^2 (w_{00}^A + w_{00}^B) \]

subject to the incentive compatibility constraints

\[ e_h^2 w_{11}^A + e_h (1 - e_h) (w_{10}^A + w_{01}^A) + (1 - e_h)^2 w_{00}^A - \psi \geq e_l e_h w_{11}^A + e_l (1 - e_h) w_{10}^A + (1 - e_l) e_h w_{01}^A + (1 - e_l)(1 - e_h)w_{00}^A, \]

\[ e_h^2 w_{11}^B + e_h (1 - e_h) (w_{10}^B + w_{01}^B) + (1 - e_h)^2 w_{00}^B - \psi \geq e_l e_h w_{11}^B + e_l (1 - e_h) w_{10}^B + (1 - e_l) e_h w_{01}^B + (1 - e_l)(1 - e_h)w_{00}^B, \]

and the constraints on the payments, \( w_{y1y2}^A \geq 0, w_{y1y2}^B \geq 0, \) and \( w_{y1y2}^A + w_{y1y2}^B \leq W, \) for \( y_1 \in \{0, 1\}, y_2 \in \{0, 1\}. \) It is easy to verify that in a solution \( w_{00}^A = w_{00}^B = w_{10}^A = w_{10}^B = 0 \) and \( e_h w_{11}^A + (1 - e_h) w_{10}^A = e_h w_{11}^B + (1 - e_h) w_{10}^B = \psi / (e_h - e_l) \) must hold. Hence, the principal’s expected payoff from implementing high effort in both tasks is \( 2e_h (b - \psi / (e_h - e_l)). \) Note that high effort in both tasks is implementable whenever \( W \geq 2\psi / [(2 - e_h)(e_h - e_l)], \) since the principal can set \( w_{11}^A = w_{11}^B = \psi / [(2 - e_h)(e_h - e_l)] \) and \( w_{10}^A = w_{10}^B = 2\psi / [(2 - e_h)(e_h - e_l)]. \)

Suppose now the principal implements high effort in only one task, say task 1. It is straightforward to see that she will make no payments to agent B. The incentive compatibility constraint of agent A reads

\[ e_h e_l w_{11}^A + e_h (1 - e_l) w_{10}^A + (1 - e_h) e_l w_{01}^A + (1 - e_l)(1 - e_h)w_{00}^A - \psi \geq e_l^2 w_{11}^A + e_l (1 - e_l) (w_{10}^A + w_{01}^A) + (1 - e_l)^2 w_{00}^A. \]

Thus, it is optimal for the principal to set \( w_{00}^A = w_{01}^A = 0 \) and \( w_{11}^A = w_{10}^A = \psi / (e_h - e_l). \) The principal’s expected payoff then is \( (e_h + e_l)b - e_h \psi / (e_h - e_l). \)
Note that high effort in only one task is implementable whenever $W \geq \psi/(e_h - e_l)$.

Finally, the principal can always implement low effort in both tasks by setting all payments to zero. Her expected payoff then is $2e_lb$. The principal’s expected payoff when she implements high effort in only one task is strictly larger than when she implements low effort in both tasks if and only if $b > e_h\psi/(e_h - e_l)^2$, which is satisfied according to Assumption 1. Moreover, under Assumption 1 the principal’s expected payoff when she implements high effort in both tasks is larger than when she implements high effort in only one task. Hence, the following result holds.

**Proposition 2** Suppose the two tasks are separated. The principal’s maximum expected payoff is

$$\Pi^{\text{sep}} = \begin{cases} 
2e_h(b - \frac{\psi}{e_h - e_l}) & \text{if } \frac{2\psi}{(2 - e_h)(e_h - e_l)} \leq W, \\
(e_h + e_l)b - \frac{e_h\psi}{e_h - e_l} & \text{if } \frac{\psi}{e_h - e_l} \leq W < \frac{2\psi}{(2 - e_h)(e_h - e_l)}, \\
2e_l b & \text{if } W < \frac{\psi}{e_h - e_l}.
\end{cases}$$

5 Bundling versus separation

We can now state our main result, which follows immediately from a comparison of the principal’s expected payoffs as characterized in Propositions 1 and 2.

**Proposition 3** (i) Suppose that $e_h + e_l \geq 1$. If $W > 2\psi/(e_h^2 - e_l^2)$, then the principal strictly prefers bundling. If $2\psi/[(2 - e_h)(e_h - e_l)] < W < 2\psi/(e_h^2 - e_l^2)$,
e_l^2), then the principal strictly prefers separation. Otherwise, the principal is indifferent between the two modes.

(ii) Suppose that \( e_h + e_l < 1 \). If \( W > 2\psi/(e_h - e_l) \), then the principal strictly prefers bundling. If \( 2\psi/[(2 - e_l)(e_h - e_l)] < W < 2\psi/(e_h - e_l) \), then the principal strictly prefers separation. Otherwise, the principal is indifferent between the two modes.

Hence, if the principal does not face a relevant budget constraint (i.e., if \( W \) is sufficiently large), it is optimal for her to bundle the two tasks. This result is in line with the existing literature on multi-task moral hazard problems with wealth-constrained agents. However, when the principal has only a limited budget, a new effect arises. Separation can now be optimal, because it may allow the principal to implement high efforts in both tasks even when in the case of bundling she could implement high effort in one task only.\(^8\)

Intuitively, in the case of bundling, the principal must make a very large payment to the agent when both tasks are successful. In contrast, under separation the payments can be smaller, since each agent can also be incentivized with a payment that is made if the other agent is not successful, so that the payments can be spread more evenly over the different states of the world.

6 Conclusion

We have considered a government agency that wants two tasks to be performed.

In each task, unobservable effort can be exerted by a wealth-constrained private

\(^8\)Note that budget levels \( W \) such that separation is strictly optimal always exist in case (ii), while they exist in case (i) whenever \( e_l < 2(1 - e_h) \).

12
party. In line with the principal-agent literature, if the government faces no binding budget constraints, it is optimal to bundle the tasks. In this case, the contractor responsible for both tasks gets a large payment whenever both tasks are successful. However, we have shown that if the government has only a limited budget, then it may be optimal to separate the two tasks, so that there are two different contractors, each responsible for one task only. High efforts in both tasks can then be implemented with smaller payments, since each contractor can also be incentivized with a payment that is made when the other contractor is not successful.

Acknowledgement

I have benefitted from helpful discussions with Eva Hoppe.
References


