



Munich Personal RePEc Archive

## **Public goods and the hold-up problem under asymmetric information**

Schmitz, Patrick W.

University of Cologne

2013

Online at <https://mpra.ub.uni-muenchen.de/53717/>

MPRA Paper No. 53717, posted 16 Feb 2014 16:03 UTC

# Public goods and the hold-up problem under asymmetric information\*

Patrick W. Schmitz

*University of Cologne, Germany, and CEPR, London, UK*

## **Abstract**

An agent can make an observable but non-contractible investment. A principal then offers to collaborate with the agent to provide a public good. Private information of the agent about his valuation may either decrease or increase his investment incentives, depending on whether he learns his type before or after the investment stage.

*Keywords:* asymmetric information, incomplete contracts, investment incentives, public goods

*JEL Classification:* D82; D86; H41

---

\* Department of Economics, University of Cologne, Albertus-Magnus-Platz, 50923 Köln, Germany. E-mail: <patrick.schmitz@uni-koeln.de>.

# 1 Introduction

In the incomplete contracting literature, the hold-up problem plays an important role (see Hart, 1995). In a standard hold-up problem under symmetric information, an agent has insufficient incentives to invest today, because tomorrow a part of the returns of his investment will go to the agent's trading partner. It has been shown in the literature that hold-up problems can be mitigated if the agent privately learns his type (before or after the investment stage). Intuitively, due to his private information the agent will get an information rent tomorrow, which today increases his incentives to invest.<sup>1</sup>

Most papers in the literature on hold-up problems consider private goods only. In a notable exception, Besley and Ghatak (2001) have studied an incomplete contracting model with public goods.<sup>2</sup> Yet, they consider the case of symmetric information only. In contrast, in the present paper we analyze a variant of Besley and Ghatak's (2001) public goods framework under asymmetric information.

It turns out that in a hold-up problem with public goods, the effects of asymmetric information crucially depend on the sequence of events. If the agent privately learns his type *after* his investment decision, the presence of asymmetric information can only *improve* investment incentives (as in the case of private goods). However, if the agent privately learns his type *before* the investment stage, asymmetric information can only *decrease* the agent's incentives to invest.

---

<sup>1</sup>See e.g. the early contribution by Tirole (1986), the papers by Gul (2001) and González (2004) who study unobservable investments, and the papers by Schmitz (2006, 2008) and Goldlücke and Schmitz (2011) who analyze observable investments in settings with incomplete information.

<sup>2</sup>See also the subsequent work by Halonen-Akatwijuka and Pafilis (2009), Francesconi and Muthoo (2011), and Halonen-Akatwijuka (2012).

## 2 The model

There are two risk-neutral parties, a principal (the government) and an agent (a non-governmental organization). At some initial date 1, the agent can make an observable but non-contractible investment  $i \geq 0$ . Following the incomplete contracting approach, it is assumed that ex ante the public good which can be produced with the help of the agent's investment cannot yet be described, so that no contract can be written before the investment is made.<sup>3</sup> However, at date 2 the principal can offer a contract to the agent. If the agent accepts the contract, the parties collaborate so that they together produce the quantity  $y(i)$  of the public good (where  $y(0) = 0$  and  $y'(i) > 0$ ,  $y''(i) < 0$  for all  $i > 0$ ). In contrast, if the agent rejects the offered contract, there is no collaboration, but the agent can still use his investment to produce the quantity  $\lambda y(i)$  of the public good good, where  $\lambda \in (0, 1)$ .

The principal's valuation of the good is commonly known and normalized to 1. As in Hart, Shleifer, and Vishny (1997), the principal's valuation can be interpreted as the benefit that the rest of the society (i.e., everyone except the agent) derives from the public good. The agent's valuation is denoted by  $\theta \in \{\theta_l, \theta_h\}$ , where  $0 < \theta_l < \theta_h < 1$  and  $\Pr\{\theta = \theta_l\} = p \in (0, 1)$ . Thus, we assume that society's benefit from the public good is larger than the agent's benefit.<sup>4</sup> In line with Besley and Ghatak (2001), we make the following

---

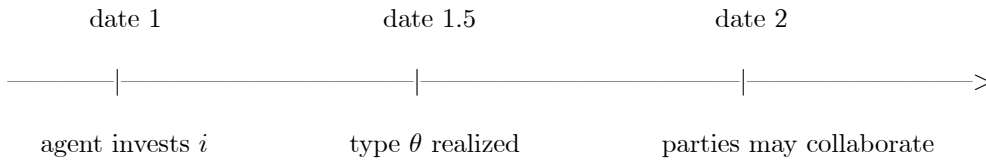
<sup>3</sup>See Hart and Moore (1999) and Maskin and Tirole (1999) for discussions of the incomplete contracting paradigm.

<sup>4</sup>Note that in the case of a private good, if the parties agree to collaborate, then the principal uses the good (since her valuation is larger). Hence, the principal's payoff is  $u_p = y(i) - t$  and the agent's payoff is  $u_A = t - i$ , where  $t$  is a transfer payment. If the agent rejects, then the principal's payoff is  $u_P = 0$  and the agent's payoff is  $u_A = \theta \lambda y(i) - i$ . In such a private good model, compared to the symmetric information benchmark, the agent's investment incentives can only increase if he privately learns his type (regardless of whether he learns his type before or after the investment stage). For details, see e.g. the more general models in Schmitz (2006) and Goldlücke and Schmitz (2011).

assumption. If the parties collaborate at date 2, then both parties benefit from the produced quantity  $y(i)$ . Thus, the principal's payoff is  $u_P = y(i) - t$  and the agent's payoff is  $u_A = \theta y(i) + t - i$ , where  $t$  is a transfer payment on which the parties agree. In contrast, if the parties do not reach an agreement to collaborate, so that only the quantity  $\lambda y(i)$  of the public good is produced, then the principal's payoff is  $u_P = \lambda y(i)$  and the agent's payoff is  $u_A = \theta \lambda y(i) - i$ .

### 3 Scenario I

In scenario I, the agent's type is realized after the investment stage.



**Figure 1.** The sequence of events in Scenario I.

#### 3.1 The first-best benchmark

In a first-best world, ex post efficiency would always be achieved (i.e., the parties would collaborate at date 2). The first-best investment level is given by  $i_I^{FB} = \arg \max E[(1 + \theta)y(i)] - i$ .

For any  $\xi \geq 0$ , let  $I(\xi) := \arg \max \xi y(i) - i$ . Hence, the first-best investment level in scenario I is given by  $i_I^{FB} = I(1 + E[\theta])$ .

#### 3.2 Symmetric information

Now consider an incomplete contracting world in which contracts can only be written at date 2. There is symmetric information about the agent's type.

At date 2, the principal offers to collaborate with the agent when the agent accepts the transfer payment  $t$ . The agent will accept if  $t \geq T(\theta, i) := (\theta\lambda - \theta)y(i)$ , because then the agent's date-2 payoff is larger in case of acceptance  $(\theta y(i) + t)$  than in case of rejection  $(\theta\lambda y(i))$ . Anticipating the agent's behavior, the principal will make the offer  $T(\theta, i)$ , so that her payoff is  $(1 + \theta - \theta\lambda)y(i)$ , which is larger than the payoff she gets when no agreement is reached  $(\lambda y(i))$ . At date 1, the agent thus chooses the investment level  $i_I^{SI} = I(\lambda E[\theta])$ .

Note that  $I(\xi)$  is an increasing function. Thus, there is underinvestment compared to the first-best benchmark. The social marginal return of the investment is  $1 + E[\theta]$ , but since ex post the principal will push the agent to his disagreement payoff  $\theta\lambda y(i)$ , the agent's marginal return is only  $\lambda E[\theta]$ .

### 3.3 Asymmetric information

Now consider the case in which only the agent privately learns the realization of his type  $\theta$  at date 1.5. If the principal makes the offer  $T(\theta_l, i)$ , the agent will always accept the offer regardless of his type, so that the principal's payoff is  $(1 + \theta_l - \theta_l\lambda)y(i)$ . If instead the principal offers  $T(\theta_h, i)$ , the agent will accept whenever  $\theta = \theta_h$ , so that the principal's expected payoff is  $[p\lambda + (1-p)(1 + \theta_h - \theta_h\lambda)]y(i)$ . Therefore,<sup>5</sup> if  $p \geq (\theta_h - \theta_l)/(1 + \theta_h)$  the principal offers  $t = T(\theta_l, i)$  (so that the agent's expected date-2 payoff is  $[p\theta_l\lambda + (1-p)(\theta_h - \theta_l + \theta_l\lambda)]y(i)$ ), while otherwise the principal offers  $t = T(\theta_h, i)$  (so that the agent's expected date-2 payoff is  $E[\theta]\lambda y(i)$ ). Thus, the following proposition holds.

**Proposition 1** *Consider scenario I.*

(a) *Under asymmetric information, the agent invests  $i_I^{AI} = I(\lambda E[\theta] +$*

---

<sup>5</sup>Note that offers strictly smaller than  $T(\theta_h, i)$  would always be rejected, offers strictly between  $T(\theta_h, i)$  and  $T(\theta_l, i)$  would be accepted by the type  $\theta_h$  only (so that the offer  $T(\theta_h, i)$  is more profitable for the principal), and offers strictly larger than  $T(\theta_l, i)$  would always be accepted (so that the offer  $T(\theta_l, i)$  is more profitable).

$(1 - p)(1 - \lambda)(\theta_h - \theta_l)$  if  $p \geq (\theta_h - \theta_l)/(1 + \theta_h)$ , and  $i_I^{AI} = I(\lambda E[\theta])$  otherwise.

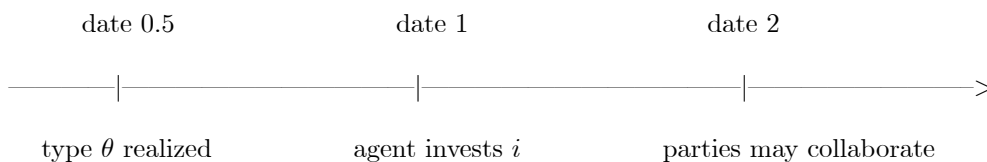
(b) Hence, the presence of asymmetric information can only increase the agent's incentives to invest.

If the prior probability  $p$  of the type  $\theta_l$  is sufficiently large, the principal offers  $T(\theta_l, i)$ . Type  $\theta_l$  then gets his disagreement payoff, while type  $\theta_h$  enjoys an information rent. As a consequence, while there is still underinvestment compared to the first-best benchmark, at date 1 the agent's investment incentives are *larger* than in the case of symmetric information.<sup>6</sup>

In contrast, if  $p$  is small, then the principal offers  $T(\theta_h, i)$ , which will be accepted by the type  $\theta_h$  and rejected by the type  $\theta_l$ , so that both types are pushed to their reservation utilities and the investment incentives are thus as in the case of symmetric information.

## 4 Scenario II

In scenario II, the agent's type is realized before the investment stage.



**Figure 2.** The sequence of events in Scenario II.

---

<sup>6</sup>Note that in the private good setting of footnote 4, an agent of type  $\theta_l$  would get an information rent if  $p$  were sufficiently small, which however would similarly increase the incentives to invest at date 1.

## 4.1 The first-best benchmark

In a first-best world, ex post efficiency would always be achieved and the first-best investment levels depending on the agent's type  $\theta$  are given by  $i_{II}^{FB}(\theta) = I(1 + \theta)$ .

## 4.2 Symmetric information

Consider an incomplete contracting world in which contracts can only be written at date 2 and there is symmetric information. In analogy to the analysis of Section 3.2, the principal will make the offer  $T(\theta, i)$ . At date 1, the agent thus chooses the investment level  $i_{II}^{SI}(\theta) = I(\lambda\theta)$ . Thus, there is again underinvestment compared to the first-best benchmark.

## 4.3 Asymmetric information

Now consider the case in which only the agent privately learns the realization of his type  $\theta$  at date 0.5. The principal may now update his belief about the agent's type when she observes the chosen investment level  $i$ . If the principal believes that  $\theta = \theta_l$  with probability  $\hat{p}$ , then in analogy to Section 3.3, the principal offers  $T(\theta_l, i)$  if  $\hat{p} \geq (\theta_h - \theta_l)/(1 + \theta_h)$ , and  $T(\theta_h, i)$  otherwise.

Consider an agent of type  $\theta_l$ . He knows that either he will get the offer  $T(\theta_l, i)$  which he will accept, or he will get the offer  $T(\theta_h, i)$  which he will reject. In both cases, his date-2 payoff is  $\theta_l \lambda y(i)$ . Hence, an agent of type  $\theta_l$  will always choose the investment level  $I(\lambda\theta_l)$ .

An agent of type  $\theta_h$  reveals his type if he chooses an investment level different from  $I(\lambda\theta_l)$ . In a separating equilibrium, an agent of type  $\theta_h$  invests  $I(\lambda\theta_h)$  and gets the offer  $T(\theta_h, I(\lambda\theta_h))$  (since the principal then believes  $\hat{p} = 0$ ), while an agent of type  $\theta_l$  invests  $I(\lambda\theta_l)$  and gets the offer  $T(\theta_l, I(\lambda\theta_l))$  (since the principal believes  $\hat{p} = 1$ ). The separating equilibrium exists if the agent of type  $\theta_h$  has no incentive to mimic an agent of type  $\theta_l$ , i.e. whenever



$$\theta_h \lambda y(I(\lambda \theta_h)) - I(\lambda \theta_h) \geq (\theta_h - \theta_l + \theta_l \lambda) y(I(\lambda \theta_l)) - I(\lambda \theta_l).$$

If the separating equilibrium does not exist and if  $p \geq (\theta_h - \theta_l) / (1 + \theta_h)$ , there is pooling equilibrium, so that at date 1 both types of the agent invest  $I(\lambda \theta_l)$  and at date 2 the principal offers  $T(\theta_l, I(\lambda \theta_l))$  (since the principal's belief  $\hat{p}$  is always equal to the prior probability  $p$ ).

Finally, if the separating equilibrium does not exist and  $p < (\theta_h - \theta_l) / (1 + \theta_h)$ , there is a semi-separating equilibrium in which an agent of type  $\theta_h$  chooses the investment level  $I(\lambda \theta_h)$  with probability  $\alpha$  and he chooses  $I(\lambda \theta_l)$  with probability  $1 - \alpha$ . Recall that an agent of type  $\theta_l$  always invests  $I(\lambda \theta_l)$ . Hence, when the principal observes the investment level  $I(\lambda \theta_h)$ , her belief  $\hat{p}$  is equal to 0 and she offers  $T(\theta_h, I(\lambda \theta_h))$ . When the principal observes the investment level  $I(\lambda \theta_l)$ , her belief  $\hat{p}$  is equal to  $p / [1 - \alpha(1 - p)]$ . Suppose she then makes the offer  $T(\theta_h, I(\lambda \theta_l))$  with probability  $\beta$  and the offer  $T(\theta_l, I(\lambda \theta_l))$  with probability  $1 - \beta$ .

Specifically,  $\beta = [(\theta_h - \theta_l + \theta_l \lambda) y(I(\lambda \theta_l)) - I(\lambda \theta_l) - (\theta_h \lambda y(I(\lambda \theta_h)) - I(\lambda \theta_h))] / [(1 - \lambda)(\theta_h - \theta_l) y(I(\lambda \theta_l))]$ , so that an agent of type  $\theta_h$  is indifferent between the investment levels  $I(\lambda \theta_h)$  (yielding the payoff  $\theta_h \lambda y(I(\lambda \theta_h)) - I(\lambda \theta_h)$ ) and  $I(\lambda \theta_l)$  (yielding the payoff  $\beta \theta_h \lambda y(I(\lambda \theta_l)) + (1 - \beta)(\theta_h - \theta_l + \theta_l \lambda) y(I(\lambda \theta_l)) - I(\lambda \theta_l)$ ). Note that  $\beta$  is positive whenever the separating equilibrium does not exist, and it is straightforward to check that  $\beta$  is smaller than 1. Moreover,  $\alpha = [(\theta_h - \theta_l) - p(1 + \theta_h)] / [(1 - p)(\theta_h - \theta_l)]$ , so that the principal is indifferent between the offers  $T(\theta_l, I(\lambda \theta_l))$  and  $T(\theta_h, I(\lambda \theta_l))$  when she observes the investment level  $I(\lambda \theta_l)$ , because  $\hat{p} = p / [1 - \alpha(1 - p)] = (\theta_h - \theta_l) / (1 + \theta_h)$ . Note that  $\alpha$  is smaller than 1 and that  $\alpha$  is positive whenever  $p < (\theta_h - \theta_l) / (1 + \theta_h)$ .

**Proposition 2** *Consider scenario II.*

(a) *Under asymmetric information, there exists a separating equilibrium whenever  $\theta_h \lambda y(I(\lambda \theta_h)) - I(\lambda \theta_h) \geq (\theta_h - \theta_l + \theta_l \lambda) y(I(\lambda \theta_l)) - I(\lambda \theta_l)$ , so that an agent of type  $\theta$  invests  $i_{II}^A(\theta) = I(\lambda \theta)$ . When the separating equilibrium does not exist, if  $p \geq (\theta_h - \theta_l) / (1 + \theta_h)$  there is a pooling equilibrium in which*

$i_{II}^{AI}(\theta_l) = i_{II}^{AI}(\theta_h) = I(\lambda\theta_l)$ , and otherwise there is a semi-separating equilibrium in which an agent of type  $\theta_l$  invests  $i_{II}^{AI}(\theta_l) = I(\lambda\theta_l)$  and an agent of type  $\theta_h$  mixes between  $i_{II}^{AI}(\theta_h) = I(\lambda\theta_l)$  and  $i_{II}^{AI}(\theta_h) = I(\lambda\theta_h)$ .

(b) Hence, in scenario II the presence of asymmetric information has no influence on the investment incentives of an agent of type  $\theta_l$ , while it can only decrease the investment incentives of an agent of type  $\theta_h$ .

Note that in the separating equilibrium, the investment levels are identical to the investment levels in the case of symmetric information, since both types are pushed to their disagreement payoffs. In contrast, in the pooling equilibrium and in the semi-separating equilibrium, an agent of type  $\theta_h$  gets an information rent. Nevertheless, the investment incentives of the agent are now *smaller* than in the case of symmetric information, since he mimics the behavior of an agent of type  $\theta_l$ .<sup>7</sup> Finally, note that the condition under which the separating equilibrium does not exist and hence the investment incentives are strictly smaller may well be satisfied.<sup>8</sup>

---

<sup>7</sup>This is in contrast to the case of a private good (cf. footnote 4), in which a low type has an incentive to mimic a high type.

<sup>8</sup>For example, let  $y(i) = \sqrt{i}$ ,  $\lambda = 0.2$ ,  $p = 0.3$ , and  $\theta_l = 0.1$ . Then a separating equilibrium exists only if  $\theta_h$  is larger than 0.9.

## References

- Besley, T., Ghatak, M., 2001. Government versus private ownership of public goods. *Quarterly Journal of Economics* 116, 1343–1372.
- Francesconi, M., Muthoo, A., 2011. Control rights in complex partnerships. *Journal of the European Economic Association* 9, 551–589.
- Goldlücke, S., Schmitz, P.W., 2011. Investments as signals of outside options. CEPR Discussion Paper 8366.
- González, P., 2004. Investment and screening under asymmetric endogenous information. *Rand Journal of Economics* 35, 502–519.
- Gul, F., 2001. Unobservable investment and the hold-up problem. *Econometrica* 69, 343–376.
- Halonen-Akatwijuka, M., 2012. Nature of human capital, technology and ownership of public goods. *Journal of Public Economics*, forthcoming.
- Halonen-Akatwijuka, M., Pafilis, E., 2009. Reputation and ownership of public goods. University of Bristol, CMPO Working Paper 09/211.
- Hart, O.D., 1995. *Firms, Contracts and Financial Structure*. Oxford University Press.
- Hart, O., Moore, J., 1999. Foundations of incomplete contracts. *Review of Economic Studies* 66, 115–138.
- Hart, O., Shleifer, A., Vishny, R.W., 1997. The proper scope of government: theory and application to prisons. *Quarterly Journal of Economics* 112, 1127–1161.
- Maskin, E., Tirole, J., 1999. Unforeseen contingencies, property rights, and incomplete contracts. *Review of Economic Studies* 66, 83–114.

- Schmitz, P.W., 2006. Information gathering, transaction costs, and the property rights approach. *American Economic Review* 96, 422–434.
- Schmitz, P.W., 2008. Incomplete contracts, the hold-up problem, and asymmetric information. *Economics Letters* 99, 119–122.
- Tirole, J., 1986. Procurement and renegotiation. *Journal of Political Economy* 94, 235–259.