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Abstract

This paper investigates the rates of technological progress, total output growth, and per capita output growth when population growth is negative by using a semi-endogenous R&D growth model. The analysis shows that within finite time, the employment share of the final goods sector reaches unity, the employment share of the R&D sector reaches zero, and accordingly, the rate of technological progress leads to zero. In this case, the growth rate of per capita output asymptotically approaches a positive value.

Keywords: technological progress; semi-endogenous growth; negative population growth

JEL Classification: O11; O41; O31

1 Introduction

This paper investigates the rates of technological progress, total output growth, and per capita output growth when population growth is negative by using a semi-endogenous R&D growth model.

Japan first experienced a fall in population in 2005 since the war, and then also experienced negative population growth in 2009 and 2011. According to the data from the Ministry of Internal Affairs and Communications, the rates of decrease in population in Japan are −0.1% in 2005, −0.4% in 2009, and −2.0% in 2010. Moreover, in Italy and Germany too, concern about population decline has been increasing (World Bank, 2013). Therefore, population decline is an urgent problem in developed economies.

Given that population growth can be negative in reality, we need to consider this case as well. However, considerations of negative population growth in the field of economic

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growth theory have just started (Ferrara, 2011). Among them, Christiaans (2011) is a very interesting study.[1] He shows the importance of negative population growth by using a simple growth model. Consider a neo-classical growth model with a production function that exhibits increasing, but relatively small, returns to scale. When the population growth rate is negative, contrary to expectations, per capita output growth is positive. To obtain increasing returns to scale, he uses externality arising from capital accumulation. However, he does not explicitly consider endogenous technological progress.

Based on the above observation, we use Jones' (1995) R&D growth model in which technological progress is endogenously determined to investigate how growth rates of key variables are determined when population growth is negative. He points out that scale effects specific to endogenous growth models à la Romer (1990), that is, the larger the level of population becomes, the faster per capita output grows, are not realistic. Then, he removes scale effects by modifying the specification of an R&D function, and obtains that the faster the growth rate of population becomes, the faster per capita output grows.[2] In this model, labor allocation between the final goods producing sector and the R&D sector are endogenously determined, which determines the rates of technological progress and economic growth.[3]

We apply Jones' (1995) model to a case in which population growth is negative. Our analysis shows that when population growth is negative, the long-run rate of technological progress is zero, that of total output growth is negative, and that of per capita output growth is positive.

The remainder of the paper is organized as follows. Section 2 presents the framework of our model and derives the system of differential equations. Sections 3 and 4 investigate the dynamics of the model when population growth is positive and negative, respectively. Section 5 concludes the paper.

[1] Sasaki (2014) builds a small-open-economy, semi-endogenous-growth model with negative population growth and investigates the relationship between trade patterns and per capita consumption growth.

[2] In Jones' (1995) model, scale effects in terms of growth are removed whereas scale effects in terms of level, that is, the larger the level of population becomes, the higher the level of per capita income becomes, are not removed. For a systematic exposition of scale effects and semi-endogenous growth, see Jones (1999), Jones (2005), Aghion and Howitt (2005), and Dinopoulos and Sener (2007).

[3] Some studies criticize the empirical validity of Jones' (1995) semi-endogenous growth model. For example, Abdih and Joutz (2006) and Madsen (2008) conduct empirical analysis and conclude that Romer's (1990) endogenous growth model is more realistic than Jones' (1995) semi-endogenous growth model. Strulik et al. (2013) empirically show that there is a negative correlation between population growth and total factor productivity growth. Moreover, Sasaki (2011) states that the relationship between population growth and per capita real income growth differs for developed and developing countries.

2 The model

We present a slightly simplified Jones' (1995) model for ease of exposition. However, the essence is the same, and hence, we briefly explain the model. A closed economy with no government consists of three sectors: the final goods producing, capital goods producing, and R&D sectors. The production function of the final goods sector is given by

$$Y = L_Y^{1-\alpha} \int_0^A x_i^\alpha \, di, \quad 0 < \alpha < 1, \tag{1}$$

where Y denotes the output of final goods; L_Y, the employment of the final goods sector; x_i, the input of capital goods; A, the number of capital goods; and α, a positive parameter. Final goods are used as numéraire.

The market for final goods is perfectly competitive. Profits of final goods producing firms are given by

$$\Pi_Y = L_Y^{1-\alpha} \int_0^A x_i^\alpha \, di - w_Y L_Y - \int_0^A p_i x_i \, di, \tag{2}$$

where w_Y denotes the final goods sector's wage rate; and p_i, the rental price of capital good that the i-th capital good firm produces. From the profit maximization condition and equation (1), we obtain

$$w_Y = (1-\alpha)\frac{Y}{L_Y}, \tag{3}$$

$$p_i = \alpha L_Y^{1-\alpha} x_i^{-(1-\alpha)}. \tag{4}$$

The market for capital goods is monopolistically competitive. The i-th capital good is produced only by the i-th capital good firm. The i-th capital good firm buys a blueprint from the R&D sector, and produces finished capital goods by borrowing unfinished capital goods at an interest rate r. Unfinished capital goods can be converted into finished capital goods at zero cost. Accordingly, profits of capital goods producing firms are given by

$$\Pi_i = p_i(x_i)x_i - r x_i, \tag{5}$$

where $p_i(x)_i$ is the inverse demand function for the i-th capital good and given by equation (4). Considering symmetric equilibrium, from the profit maximization condition, we obtain

$$p_i = p = \frac{r}{\alpha}, \tag{6}$$

$$x_i = x = \left(\frac{\alpha L_Y^{1-\alpha}}{p}\right)^{\frac{1}{1-\alpha}}. \tag{7}$$

Substituting equations (6) and (7) into equation (5), we obtain

$$\Pi_i = \Pi = \alpha(1-\alpha)\frac{Y}{A}. \tag{8}$$

Total capital stock K is the sum of capital goods.

$$K = \int_0^A x\,di = Ax. \tag{9}$$

Substituting equation (9) into equation (1), we obtain the aggregate production function as follows:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}. \tag{10}$$

Thus, with equations (4), (6), (9), and (10), the interest rate is given by

$$r = \alpha^2\frac{Y}{K}. \tag{11}$$

The market for blueprints is perfectly competitive. In equilibrium, the price of blueprints P_A is equal to the discounted present value of profits that new capital goods produce. Accordingly, the following non-arbitrage condition holds.

$$\frac{\Pi}{r} = P_A. \tag{12}$$

Here, for simplicity, we consider a situation in which there is neither capital gain nor capital loss, that is $\dot{P}_A = 0$ ($\dot{x} = dx/dt$, hereafter).[4]

Let L_A be the employment of R&D sector. Then, the full employment condition leads to

$$L_Y + L_A = L, \tag{13}$$

where L denotes total population. We assume that the growth rate of total population n is constant and can be positive ($n > 0$) or negative ($n < 0$).

[4] If we assume that only a market for newly invented blueprints exists and that old blueprints are not traded, neither capital gain nor capital loss accrues (Adachi, 2000, ch. 9).

The production function of the R&D sector is given by

$$\dot{A} = \delta L_A, \quad \text{where } \delta = A^{\gamma}, \quad 0 < \gamma < 1, \tag{14}$$

where δ denotes externality specific to knowledge production. An individual firm takes δ as given to maximize profits. The aggregate production function of the R&D sector is given by

$$\dot{A} = L_A A^{\gamma}, \tag{15}$$

where γ is the degree of externality.

Profits of the R&D sector are given by

$$\Pi_A = P_A \delta L_A - w_A L_A. \tag{16}$$

From the profit maximization and free entry conditions, we obtain

$$w_A = P_A \delta. \tag{17}$$

Using equations (14) and (17), we obtain

$$w_A = P_A A^{\gamma}. \tag{18}$$

Equalizing the wage rate of the final goods sector with that of the R&D sector from equations (3) and (18), that is, $w_Y = w_A$, we obtain

$$\alpha \frac{Y}{L_Y} = P_A A^{\gamma}. \tag{19}$$

From equations (8), (11), and (12), we can eliminate the interest rate r:

$$\frac{K}{P_A A} = \frac{\alpha}{1-\alpha}. \tag{20}$$

We now turn to consumers' behavior. According to Zhang (2007), consumers solve the following utility maximization problem.

$$\max_{C,S} U = C^{1-s} S^{s}, \quad 0 < s < 1, \tag{21}$$

$$\text{s.t. } C + S = Y, \tag{22}$$

where C denotes consumption of final goods; and S, savings. From this, we obtain

$$C = (1 - s)Y, \tag{23}$$

$$S = sY. \tag{24}$$

Therefore, the assumption of a constant saving rate adopted by Solow (1956) has some micro-foundations. The original version of Jones' (1995) model uses dynamic optimization for consumer behavior. However, for simplicity, we assume the constant saving rate.

From the final goods market clearing condition, total saving S is equal to investment I.

$$\dot{K} = I = sY, \quad 0 < s < 1, \tag{25}$$

where we assume that the rate of depreciation is zero for simplicity.

Eliminating P_A from equations (19) and (20), and substituting equation (10) into the resultant expression, we obtain

$$\sigma = \left(\frac{\alpha^2}{1-\alpha}\right)^{\frac{1}{\alpha}} A^{\frac{2-\gamma-\alpha}{\alpha}} K^{-\frac{1-\alpha}{\alpha}} L^{-1} = \sigma(\underset{+}{A}, \underset{-}{K}, \underset{-}{L}), \tag{26}$$

where $\sigma = L_Y/L$ denotes the employment share of the final goods sector. Accordingly, the employment share of the R&D sector is given by $1 - \sigma = L_A/L$. Equation (26) states that if A, K, and L are given, the value of σ is determined.

Summarizing the above equations, we obtain the following two differential equations.

$$\frac{\dot{K}}{K} = sK^{-(1-\alpha)}(A\sigma L)^{1-\alpha}, \tag{27}$$

$$\frac{\dot{A}}{A} = (1 - \sigma)LA^{-(1-\gamma)}. \tag{28}$$

3 Analysis when population growth is positive

When $n > 0$, there exists a balanced growth path (BGP, hereafter) along which A and K grow at constant rates and σ stays constant. In the following analysis, g_x denotes $\dot{x}/x$. Calculating $\dot{g}_K/g_K$ and $\dot{g}_A/g_A$ from equations (27) and (28), and letting the resultant expressions be zero, we obtain the BGP growth rates of A and K as follows:

$$g_A^* = \phi n > 0, \quad \phi \equiv \frac{1}{1-\gamma} > 1, \tag{29}$$

$$g_K^* = (1 + \phi)n > 0, \tag{30}$$

where an asterisk "$*$" denotes the BGP value of a variable. Accordingly, A and K continue to increase at constant rates. In this case, from equation (26), we can know that σ stays constant. Based on equations (29) and (30), we introduce the following scale-adjusted variables.

$$a \equiv \frac{A}{L^{\phi}}, \tag{31}$$

$$k \equiv \frac{K}{L^{1+\phi}}. \tag{32}$$

In addition, from equation (26), σ is rewritten as follows:

$$\sigma = \left(\frac{\alpha^2}{1-\alpha}\right)^{\frac{1}{\alpha}} a^{\frac{2-\gamma-\alpha}{\alpha}} k^{-\frac{1-\alpha}{\alpha}} = \sigma(\underset{+}{a}, \underset{-}{k}). \tag{33}$$

Accordingly, when a and k are given, σ is determined. The growth rate of σ is given by

$$\frac{\dot{\sigma}}{\sigma} = \frac{2-\gamma-\alpha}{\alpha}\frac{\dot{a}}{a} - \frac{1-\alpha}{\alpha}\frac{\dot{k}}{k}. \tag{34}$$

Summarizing the above discussions, we obtain the following system of differential equations.

$$\dot{k} = k\left[sk^{-(1-\alpha)}a^{1-\alpha}\sigma(a,k)^{1-\alpha} - (1+\phi)n\right], \tag{35}$$

$$\dot{a} = a\left\{[1-\sigma(a,k)]a^{-(1-\gamma)} - \phi n\right\}. \tag{36}$$

When $n > 0$, we can show that there exists the steady state values such that $k^* > 0$ and $a^* > 0$. From $\dot{k} = \dot{a} = 0$, we obtain

$$k^* = \frac{[s(1-\alpha)]^{\frac{2-\gamma-\alpha}{(1-\alpha)(1-\gamma)}}}{[\alpha^2(1+\phi)n]^{\frac{\alpha}{1-\alpha}}\left\{[s\phi(1-\alpha)+\alpha^2(1+\phi)]n\right\}^{\frac{2-\gamma}{1-\gamma}}} > 0, \tag{37}$$

$$a^* = \left\{\frac{s(1-\alpha)}{[s(1-\alpha)\phi + \alpha^2(1+\phi)]\, n}\right\}^{\frac{1}{1-\gamma}} > 0. \tag{38}$$

We now turn to the stability analysis. The elements of the Jacobian matrix $\mathbf{J}$ that corresponds to equations (35) and (36) are given by

$$J_{11} = \frac{\partial \dot{k}}{\partial k} = -\frac{(1-\alpha)(1+\phi)n}{\alpha} < 0, \tag{39}$$

$$J_{12} = \frac{\partial \dot{k}}{\partial a} = \frac{(1-\alpha)(2-\gamma)(1+\phi)n}{\alpha}\frac{k^*}{a^*} > 0, \tag{40}$$

$$J_{21} = \frac{\partial \dot{a}}{\partial k} = \frac{\alpha(1+\phi)n}{s}\frac{a^*}{k^*} > 0, \tag{41}$$

$$J_{22} = \frac{\partial \dot{a}}{\partial a} = -\frac{(1-\gamma)\phi n}{1-\sigma^*} - \frac{\alpha(2-\gamma)(1+\phi)n}{s} < 0. \tag{42}$$

All elements are evaluated by the steady state values. In this simplified Jones' (1995) model, both k and a are state-variables. Accordingly, the necessary and sufficient conditions for the local stability of the steady state are that the trace of $\mathbf{J}$ is negative and the determinant of $\mathbf{J}$ is positive. From equations (39) and (42), we can easily find that $\mathrm{tr}\,\mathbf{J} = J_{11} + J_{22} < 0$. The determinant of $\mathbf{J}$ is given by

$$\det \mathbf{J} = J_{11}J_{22} - J_{12}J_{21} = \frac{(1-\alpha)(1-\gamma)\phi(1+\phi)n^2}{\alpha(1-\sigma^*)} > 0. \tag{43}$$

Accordingly, the sign of $\det \mathbf{J}$ is positive. Therefore, the local stability condition is satisfied: k and a converge to their respective steady state values from arbitrary initial values k_0 and a_0.[5]

With equations (10), (31), and (32), the production function for final goods is rewritten as

$$Y = \sigma^{1-\alpha} a^{1-\sigma} k^{\alpha} L^{1+\phi}. \tag{44}$$

Hence, the growth rate of per capita output $y = Y/L$ is given by

$$g_y = (1-\alpha)\frac{\dot{\sigma}}{\sigma} + (1-\alpha)\frac{\dot{a}}{a} + \alpha\frac{\dot{k}}{k} + \phi n. \tag{45}$$

At the steady state, $\dot{k} = \dot{a} = \dot{\sigma} = 0$. Accordingly, the BGP growth of per capita output leads to

$$g_y^* = \phi n > 0. \tag{46}$$

Therefore, the BGP growth rate of per capita output is proportional to population growth.

[5]If we consider dynamic optimization of consumers, the Euler equation for consumption appears. In addition, if we consider capital gain and capital loss, the differential equation for P_A also appears. Hence, Jones's model consists of four differential equations. The dynamic stability of Jones' model in this case is fully analyzed by Arnold (2006).

4 Analysis when population growth is negative

When $n < 0$, the right-hand sides of equations (35) and (36) are always positive. Hence, we find that there never exists a situation in which $\dot{k} = \dot{a} = 0$. Since $\sigma(a,k)$ is restricted to the range of $\sigma \in [0,1]$, the growth rate of a is always positive even if σ takes any value. Thus, a continues to increase through time, and consequently, the growth rate of a asymptotically approaches $-\phi n > 0$ because $a^{-(1-\gamma)}$ in equation (36) approaches zero with $\gamma < 1$.

At this stage, the dynamics of $\sigma(a,k)$ are uncertain: σ may increase, decrease, or converge to a constant value. First, if σ continues to decrease and reaches $\sigma = 0$, then the growth rate of k becomes $-(1+\phi)n > 0$. Next, we consider a case in which σ continues to increase and reaches $\sigma = 1$ and a case in which σ converges to a constant value. In both cases, we can prove that the term $k^{-(1-\alpha)}a^{1-\alpha}$ of equation (35) converges to zero in the long run. Define $z = k^{-(1-\alpha)}a^{1-\alpha}$. Differentiating both sides with respect to time, we obtain

$$\dot{z} = (1-\alpha)[(1-\sigma)a^{-(1-\gamma)} - s\sigma^{1-\alpha}z + n]z. \tag{47}$$

Note that σ in equation (47) is unity or a constant. For $t \to \infty$, we have $a^{-(1-\gamma)} \to 0$ because g_a is always positive. Accordingly, we rewrite equation (47) as follows:

$$\dot{z} = -(1-\alpha)(s\sigma^{1-\alpha}z - n)z. \tag{48}$$

Since $n < 0$, we have $s\sigma^{1-\alpha}z - n > 0$, and hence, the steady state value is $z^* = 0$. Moreover, since $d\dot{z}/dz|_{z^*} = (1-\alpha)n < 0$, the steady state is locally stable. Therefore, the term $z = k^{-(1-\alpha)}a^{1-\alpha}$ approaches zero in the long run. Therefore, in these cases too, the growth rate of k becomes $-(1+\phi)n > 0$.

Given the growth rates of a and k, from equation (34), the growth rate of σ becomes $-n > 0$ in the long run: σ continues to increase in the long run. Note that σ is restricted to the range of $\sigma \in [0,1]$. Accordingly, within finite time, σ becomes $\sigma = 1$. This means that within finite time, the employment share of the R&D sector becomes zero and that of the final goods sector becomes unity.

We confirm the above discussions by using numerical simulations. We set the baseline parameters and initial values as follows:

$$n = -0.01,\ \alpha = 0.3,\ \gamma = 0.8,\ s = 0.5,\ k(0) = 0.01,\ a(0) = 0.01. \tag{49}$$

Figure 1 shows the time paths of σ when $n = -0.01$, -0.02, and -0.03. Figure 2 shows the time paths of σ when $\gamma = 0.6$, 0.7, and 0.8. Figure 3 shows the time paths of σ when $s = 0.5$, 0.6, and 0.7. All figures show that the employment share of the final goods sector continues to increase through time and reaches unity within finite time.[6]

[Figures 1, 2, and 3 around here]

Therefore, within finite time, the employment share of the R&D sector becomes zero, and hence, the growth rate of A becomes zero within finite time. Accordingly, we can say that there exists a $t_1 \in (0, \infty)$ such that we have $\sigma \in (0, 1)$ during $t \in [0, t_1)$ and we have $\sigma = 1$ during $t \in [t_1, \infty)$. Hence, the system of differential equations is decomposed into two sub-systems:

$$\text{S1 : for } t \in [0, t_1) \begin{cases} \dot{k} = k\left[sk^{-(1-\alpha)}a^{1-\alpha}\sigma(a,k)^{1-\alpha} - (1+\phi)n\right], \\ \dot{a} = a\left\{[1-\sigma(a,k)]a^{-(1-\gamma)} - \phi n\right\}. \end{cases} \tag{50}$$

$$\text{S2 : for } t \in [t_1, \infty) \begin{cases} \dot{k} = k\left[sk^{-(1-\alpha)}a^{1-\alpha} - (1+\phi)n\right], \\ \dot{a} = -\phi n a. \end{cases} \tag{51}$$

System S1 corresponds to $t \in [0, t_1)$ while system S2 corresponds to $t \in [t_1, \infty)$. Then, at time t_1, system S1 switches to system S2. By investigating system S2, we find that in the long run, the growth rates of k and a asymptotically approaches $-(1+\phi)n > 0$ and $-\phi n > 0$, respectively. In this case, from equation (45), the growth rate of per capita output $y = Y/L$ is given by

$$g_y = -(1-\alpha)(\phi n) - \alpha[(1+\phi)n] + \phi n = -\alpha n > 0. \tag{52}$$

That is, even if population growth is negative, the growth rate of per capita output is positive in the long run.

Suppose that some economic policy could keep the employment share of the final goods sector, σ, constant through time. In this case too, in the long run, the growth rate of per capita output asymptotically approaches $g_y = -\alpha n > 0$.

Therefore, when population growth is negative, per capita output continues to increase at the rate of $-\alpha n > 0$ in the long run. In this case, the rates of economic growth, capital accumulation, and technological progress are respectively given by $g_Y = (1-\alpha)n < 0$, $g_K = 0$, and $g_A = 0$.

Proposition 1. *Suppose that the growth rate of population is constant and negative. Then, in a semi-endogenous R&D growth economy, the growth rates of total output, technological progress, and per capita output are negative, zero, and positive, respectively.*

[6] Figure 4 shows that if we take a more longer span, σ increases exponentially.

Figure 5 shows the long-run relationship between population growth and per capita output growth. The larger the absolute value of population growth, the faster per capita output growth becomes.

[Figure 5 around here]

Why do we obtain these results? From equation (10), per capita output growth is rewritten as follows:

$$g_y = \underbrace{(1-\alpha)\left(\frac{\dot{\sigma}}{\sigma} + \frac{\dot{A}}{A}\right)}_{\text{RD effect}} + \underbrace{\alpha \frac{\dot{\tilde{k}}}{\tilde{k}}}_{\text{CD effect}}, \tag{53}$$

where $\tilde{k} = K/L$ denotes per capita capital stock. Thus, the per capita output growth is decomposed into the two effects: the R&D effect (RD effect, hereafter) and the capital deepening effect (CD effect, hereafter). After $t = t_1$, we have $\sigma = 1$ and $g_A = 0$, and thus, the RD effect vanishes and only the CD effect lasts. The CD effect is given by

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = \frac{\dot{K}}{K} - n = s\bar{A}^{1-\alpha}\tilde{k}^{-(1-\alpha)} - n > 0, \tag{54}$$

where $\bar{A}$ denotes the constant value of A after t_1. The CD effect is always positive, and thus, $\tilde{k}$ increases indefinitely. Hence, $g_{\tilde{k}}$ converges to

$$\lim_{\tilde{k}\to\infty} g_{\tilde{k}} = -n > 0. \tag{55}$$

From equation (53), we obtain $g_y = -\alpha n > 0$.

5 Conclusions

By using Jones' (1995) semi-endogenous growth model, we investigate the long-run growth rates of per capita output when population growth is negative. Our results show that when population growth is negative, in the long run, the growth rate of technological progress is zero, that of total output is negative, and that of per capita output is positive. Therefore, incorporating negative population growth in growth models is more complicated than simply replacing a positive population growth rate with a negative population growth rate.

Our analysis focuses only on the long-run relationship between negative population growth, economic growth, and technological progress. In particular, we only investigate growth rates after sufficiently long time has passed. Accordingly, analysis of transitional dynamics along which growth rates approaches constant values is inadequate. Hence, detailed analysis of transitional dynamics will be left for future research. In addition, our analysis neglects effects of negative population growth on population composition, social security system, and so forth. These effects should be included in future research.

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References

Abdih, Y., Joutz, F., 2006. Relating the knowledge production function to total factor productivity: an endogenous growth puzzle. IMF Staff Papers 53 (2), 242–271.

Adachi, H., 2000. Endogenous growth and scale effects. In: Imperfect Competition and Theories of Macroeconomic Dynamics, Tokyo: Yuhikaku (in Japanese).

Aghion, H., Howitt, P., 2005. Growth with quality-improving innovations: an integrated framework. In: Aghion, H., Durlauf, S. N. (Eds.). Handbook of Economic Growth, ch. 2, Amsterdam: North-Holland, 67–110.

Arnold, L. G., 2006. The dynamics of the Jones R&D growth model. Review of Economic Dynamics 9 (1), 143–152.

Christiaans, T., 2011. Semi-endogenous growth when population is decreasing. Economics Bulletin 31 (3), 2667–2673.

Dinopoulos, E., Sener, F., 2007. New directions in Schumpeterian growth theory. In: Hanush, H., Pyca, A. (Eds.). Edgar Companion to Neo-Schumpeterian Economics, ch. 42, New York: Edward Elgar.

Ferrara, M., 2011. An AK Solow model with a non-positive rate of population growth Applied Mathematical Sciences 5 (25), 1241–1244.

Jones, C. I., 1995. R&D-based models of economic growth. Journal of Political Economy 103 (4), 759–784.

Jones, C. I., 1999. Growth: with or without scale effects? American Economic Review 89 (2), 139–44.

Jones, C. I., 2005. Growth and ideas. In: Aghion, P., Durlauf, S. N. (Eds.). Handbook of Economic Growth, ch. 16, Amsterdam: North Holland, 1063–1111.

Madsen, J. B., 2008. Semi-endogenous versus Schumpeterian growth models: testing the knowledge production function using international data. Journal of Economic Growth 13 (1), 1–26.

Romer, P. M., 1990. Endogenous technological change. Journal of Political Economy 98 (5), 71–102.

Sasaki, H., 2011. Population growth and north-south uneven development. Oxford Economic Papers 63 (2), 307–330.

Sasaki, H., 2014. International trade and industrialization with negative population growth. Macroeconomic Dynamics, forthcoming.

Solow, R., 1956. A contribution to the theory of economic growth. Quarterly Journal of Economics 70 (1), 65–94.

Strulik, H., Frnkranz-Prskawetz, A., Prettner, K., 2013. The past and future of knowledge-based growth. Journal of Economic Growth 18 (4), 411–437.

World Bank 2013. *World Development Indicators*.

Zhang, W.-B., 2007. A discrete two-sector economic growth model. Discrete Dynamics in Nature and Society, doi:10.1155/2007/89464

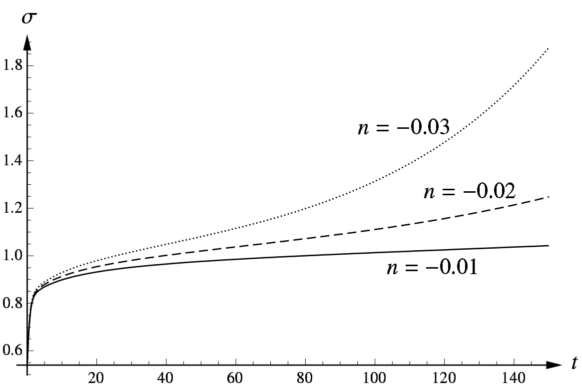

Figure 1: Employment shares of the final goods sector through time for different values of n

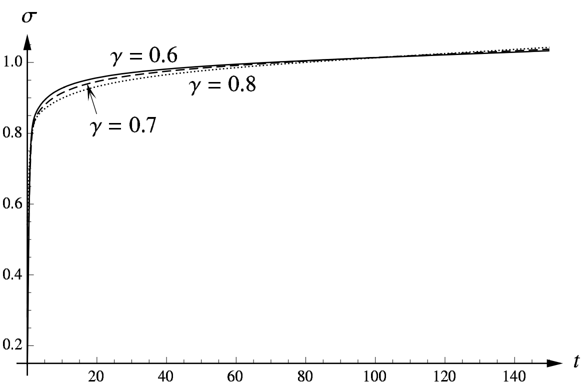

Figure 2: Employment shares of the final goods sector through time for different values of γ

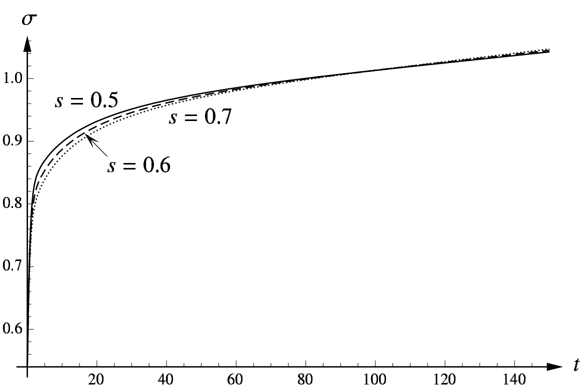

Figure 3: Employment shares of the final goods sector through time for different values of s

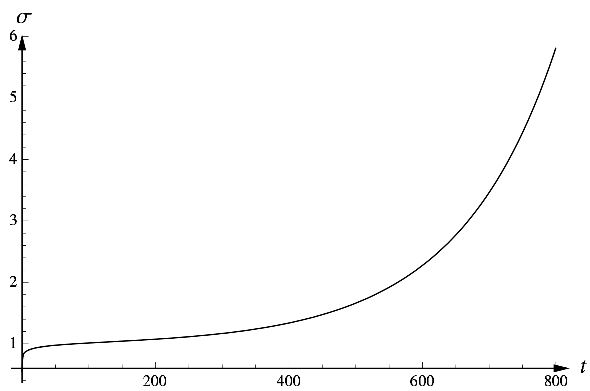

Figure 4: Employment share of the final goods sector during the period $t \in [0, 500]$

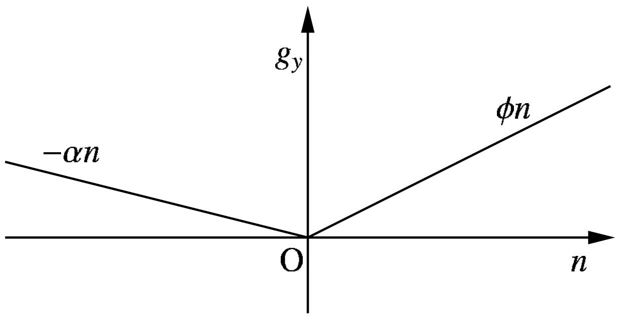

Figure 5: Relationship between population growth and per capita output growth