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Abstract

Firms frequently offer refunds, both when physical products are returned and when service contracts are terminated prematurely. We show how refunds act as a "metering device" when consumers learn about their personal valuation while experimenting with the product or service. Our theory predicts that low-quality firms offer inefficiently strict terms for refunds, while high-quality firms offer inefficiently generous terms. This may help to explain the observed variety in contractual terms. As in our model strict cancellation terms and low refunds are used to price discriminate, rather than to trap consumers into purchasing inferior products, the imposition of a statutory minimum refund policy would not, in general, improve consumer surplus or welfare.

Keywords: Refunds; Cancellation terms; Metering.

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1 Introduction

The annual turnover of product return in the U.S. retail industry exceeds 100 Billion U.S. Dollars, of which 70% are due to reasons of taste and fit[1] (see Posselt et al. 2008 and Anderson et al. 2009). Likewise, contracts for services, including insurances, utilities, or subscriptions to health clubs, frequently involve cancellation clauses allowing customers to terminate prematurely. Refunds and termination clauses are thus ubiquitous. As we document below, however, there is wide variety in the use that firms make of such contracts. Our paper presents a simple theory of refunds that generates such heterogeneity. We present a model where firms price discriminate between consumers by using refunds as a device for "metering" (cf. Schmalensee 1981). In our model, high-quality firms offer excessively generous terms, while low-quality firms offer excessively strict terms.

Several studies find that higher-quality retailers, such as up-market stores or internet retailers with a higher customer rating, offer more generous terms (e.g., Heiman et al. 2002; Bonifield et al. 2010). When firm characteristics and product quality are unobservable to customers at the time of purchase, this relationship could be explained through signaling, similar to the theory of warranties put forward by Grossman (1981).[2] However, when the reported measures of quality, such as customer ratings, are readily observable by customers, signaling alone cannot explain the observed heterogeneity. In our model, high-quality firms extract more surplus from consumers by offering an excessively high refund, while the opposite holds for competing low-quality firms. These distortions "at both ends" also distinguish our theory from models that explain contract heterogeneity by an efficiency rationale, e.g., as goods differ in their salvage value to firms after they are returned.[3]

Our setting borrows heavily from the literature on "sequential screening" (Courty and Li 2000).[4] Consumers hold only privately observed prior beliefs about their valuation and learn from experimenting with the product or service. In our baseline model of a monopolistic seller, the departure from the set-up in Courty and Li (2000) is that we do not allow for a menu of contracts, as we restrict the firm to offer a single contract. Such a restriction may be particularly realistic with physical products. Otherwise, a firm would have to ascertain that a customer who bought under a less generous refund policy does not

[1] Such returns are also classified as "returns of not damaged goods".

[2] Cf. also theories of moral hazard such as Mann and Wissink (1990).

[3] Davis et al. (1995) document a positive correlation between the salvage value and the refund.

[4] Matthews and Persico (2007) allow, in addition, for the acquisition of information before purchasing.

claim a higher refund by returning a product that was bought by another customer under a more generous refund policy. That is, the firm must ensure that product-customer matches remain uniquely identified as, otherwise, a "grey market" for second-hand products could allow returning customers to always use the most generous refund policy available. While it is common in retailing to make product return contingent on holding a valid receipt, such a "plain receipt policy" would not be sufficient for this purpose. Letting customers choose between different contracts, specifying different prices and refunds, may also be unprofitable when it consumes too much valuable assistance time at the point of sale. Finally, customers may shy away from the additional complexity that this decision involves. These arguments suggest that sometimes it may be realistic to restrict consideration to a single contract policy, instead of a menu. This generates a role for refunds and cancellation terms as a "metering device".

In our model of competition, two firms with a known high or low quality are in the market. Following the approach in Shaked and Sutton (1982), consumers who ultimately derive a higher utility from the product have also a higher marginal valuation for high quality. Prior to experimenting with a product or service, consumers have only imprecise knowledge of their utility (their "true type"). We characterize an equilibrium where the market is segmented as follows. Customers who, ex-ante, have a lower expected utility turn to the low-quality firm, and customers who, ex-ante, have a higher expected utility turn to the high-quality firm. Consequently, for the low-quality firm its "marginal" customer has the highest ex-ante valuation among all its customers and, therefore, the lowest marginal valuation for a higher refund, given that he is less likely to return the product (or to terminate a contract prematurely). By reducing the refund below the efficient level, which is equal to the salvage value or to the cost of continuing service, the low-quality firm can extract more of the consumer surplus of "inframarginal" customers, who have low ex-ante valuation.

For the high-quality firm the opposite picture emerges. There, the "marginal" customer has the lowest ex-ante valuation and, therefore, the highest marginal valuation for a higher refund. To extract relatively more consumer surplus from its "inframarginal" customers, who have a higher ex-ante valuation, the high-quality firm optimally offers a refund that is excessively generous.

As noted above, the setting borrows heavily from Courty and Li (2000). There, a monopolistic seller offers an optimal menu, which prescribes an efficient refund for the highest-value customer and a higher refund for all other customer types. A high refund is also the outcome in Che (1996), where customers are risk averse. As we noted above, theories of signaling would predict an inefficiently high refund for a firm with an unobservable high quality, while the refund policy of the low-quality firm would be efficient. The novelty of our model is thus to generate a large heterogeneity in contractual terms as low-quality firms offer inefficiently low refunds and high-quality firms offer inefficiently high refunds.

Several contributions (cf. Loewenstein et al. 2003 or Inderst and Ottaviani 2009) have shown that when consumers are naive with respect to sellers' incentives or their own future valuation (projection bias), the imposition of a statutory minimum right of product return may be beneficial as it protects consumers. In our model, the low-quality firm offers an inefficiently low refund or, likewise, inefficiently strict cancellation terms in order to better practice price discrimination. As is well known, restricting such a practice has, in general, ambiguous effects on welfare and expected consumer surplus (cf. also the discussion in Section 6). It is also important that according to our model, consumers who purchase under such a strict refund policy are not unknowingly trapped, but they purchase the respective product or service under perfect information about its quality, though they are still uncertain about their personal valuation. From this perspective, our model thus also provides a warning to consumer protection policy: A negative correlation between cancellation terms and product quality is not necessarily a sign that consumers are systematically fooled by some firms. Instead, it may be the outcome of price discrimination, in which case the impact of policy intervention should, in general, be ambiguous.

The rest of this paper is organized as follows. Section 2 introduces the baseline model. In Sections 3 and 4 we derive the results under monopoly and competition, respectively. Section 5 extends the model to the case where consumers vary in the quality of information they possess ex-ante. Section 6 offers some concluding remarks.

2 The Model

Utility. We model a market for differentiated goods. For a given consumer in this market, utility depends both on a common quality measure y that differs between firms and an intrinsic fit t that differs between consumers:

$$u(t, y) = yt.$$

Importantly, we postulate that t, which is not observed by the firms, is also only partially observed by consumers *before* they purchase. Specifically, we stipulate that

$$t = \theta + s, \tag{1}$$

where θ is observed before purchase and s only after purchase. Here, $\theta \in [\underline{\theta}, \overline{\theta}] \in \Theta$ is distributed according to $G(\theta)$ with density $g(\theta) > 0$, while $s \in [\underline{s}, \overline{s}] = S$ is distributed according to $F(s)$ with density $f(s) > 0$. For clarity of exposition we assume that these are drawn independently, both for a given consumer and across consumers. In Section 5 we generalize this assumption.[5] There is a mass one of consumers in the market.

Firms and Contracts. At most two firms operate in the market, which we denote by $i = l, h$. Firm h's intrinsic quality is given by $y_h > 0$ and firm l's is given by y_l where $0 < y_l < y_h$. That is, firm h's intrinsic quality is better than that of firm l.

Firms have constant production costs $c > 0$ and offer the following contracts. A contract specifies a sales price p together with a refund q. When a good is returned, it has the salvage value k with $0 < k < c$. Note that the salvage value is independent of the firm's "quality" y. For instance, we may suppose that after early return the good is no longer suitable for its primary usage. Further, the salvage value's independence of quality allows us to focus on the metering effects.

As noted in the Introduction, our model applies to service contracts. Then, $c - k$ is the cost of initiating a contract, while k is the cost of continuing to service a customer who has not terminated earlier. Likewise, $p - q$ is the price for initiating the contract and q is the extra pay for continuing the service.

Firms offer a single contract (p, q). As we discussed in the Introduction, a menu may not be feasible when a returned product cannot be matched surely to a particular purchase contract, so that there is scope for "arbitrage" between returning and non-returning customers. Also, administering different contracts, instead of a single contract, may be too costly at the point of sale, while also customers may prefer a simple, transparent choice.

[5] It should be noted that the additive structure of t is by itself not restrictive. In fact, the part of t that is unknown to a consumer ex-ante (the "error term") can always be defined as $s = t - \theta$.

Timing. We consider the following market game. At $\tau = 0$, each firm chooses a contract (p, q). At $\tau = 1$, each consumer learns his ex-ante "type" θ and chooses whether to purchase and, if so, from which firm. At $\tau = 2$, provided a consumer purchased, he learns the full fit $t = \theta + s$ and thus the respective utility $u(t, y)$. He can then choose whether to return the product, in which case he realizes the refund q. Otherwise, he keeps the good and realizes the utility $u(t, y)$ in $\tau = 3$. All parties are risk neutral, and we abstract from discounting. Note that, according to this specification, the customer only derives utility when consuming the service over the time interval between $\tau = 2$ and $\tau = 3$, but not over the "experimentation" phase between $\tau = 1$ and $\tau = 2$. Our qualitative results all hold, however, when a fraction of $u(t, y)$ is enjoyed over the first interval and the residual fraction over the second interval.

Finally, we make the following parameter restrictions that ensure that we can safely rule out some corner solutions. From

$$y_h(\overline{\theta} + \underline{s}) < k, \tag{2}$$

returning the product with positive probability is efficient also for the highest *ex-ante* customer type. From

$$y_l(\underline{\theta} + \overline{s}) > c, \tag{3}$$

purchasing and consuming the product is efficient for the lowest *ex-ante* type, at least for the highest realization $\overline{s}$.

3 Monopoly Benchmark

Suppose for now that only a single firm is in the market. Since only a single firm operates, presently we drop the subscript $i \in \{l, h\}$ for clarity.

A customer returns a product when $y(\theta + s) < q$, while he continues to consume the good when $y(\theta + s) \geq q$.[6] His expected utility from purchasing is given by

$$U(q, p; \theta) := V(q; \theta) - p, \tag{4}$$

[6] As this is a zero-probability event, we stipulate without loss of generality that the consumer does not return the product when he is indifferent.

where[7]

$$\begin{aligned} V(q;\theta) &= \int_S \max[y(\theta+s), q] dF(s) \\ &= qF(q/y-\theta) + \int_{q/y-\theta}^{\overline{s}} y(\theta+s)\, dF(s). \end{aligned}$$

Note further that $dV/d\theta \geq 0$, which holds strictly whenever the good is consumed with positive probability. Note also that the consumer's decision to ask for a refund is interim efficient only when $s < s^{FB}$, where

$$y(\theta + s^{FB}) = k.$$

(For ease of exposition we suppress the dependency of s^{FB} on the parameters, in particular on the type θ.) When $q > k$ holds, the refund is too generous, and the product will be returned too frequently. Instead, when $q < k$ holds, the refund is inefficiently low, and the product will be consumed too frequently.

The firm's expected profit with a consumer of type θ consists of two parts: The up-front margin, $p - c$, and the additional profit or loss when the product is returned:

$$\pi(p, q; \theta) := p - c + (k - q)F(q/y - \theta). \tag{5}$$

We have

$$\frac{d\pi(p,q;\theta)}{d\theta} = -(k-q)f(q/y-\theta) \gtreqless 0 \Leftrightarrow q \gtreqless k.$$

Thus, when the refund is set efficiently with $q = k$, the firm realizes the same expected profits with all customers. Instead, since the return probability declines with the ex-ante type θ, for $q > k$ the firm would prefer to contract with consumers who initially have a higher expected value for the good (high θ). Likewise, when the refund is inefficiently low, as $q < k$, the firm would prefer to contract with low-θ consumers. It is finally useful to define the total surplus with a type-θ customer:

$$\omega(q;\theta) = \int_{q/y-\theta}^{\overline{s}} [y(\theta+s) - k]\, dF(s) + k - c.$$

[7] Note that when $y(q-\theta) < \underline{s}$ or $y(q-\theta) > \overline{s}$, we can evaluate F simply by extending the boundary of the support beyond S.

Optimal Refund. Recall that $V(q,\theta)$ is increasing in θ. Consequently, as long as there is a sale with positive probability, at $\tau = 1$ there is a critical type $\theta^* \in [\underline{\theta}, \overline{\theta})$ so that all consumers $\theta \geq \theta^*$ purchase, while consumers $\theta < \theta^*$ do not purchase.[8] From inspection of (4) we can immediately conclude that θ^* increases with p and decreases with q. Hence, the firm chooses (p,q) so as to maximize expected profits

$$\Pi(p,q;\theta^*) = \int_{\theta^*}^{\overline{\theta}} \pi(p,q;\theta) dG(\theta) \tag{6}$$

subject to the participation constraint of the lowest type: $U(q,p;\theta^*) = 0$.[9]

Proposition 1 *Suppose there is a single firm in the market. The monopolist chooses a refund that is inefficiently generous, $q^* > k$.*

Proof. For the proof we can take some θ^* as given, which for a given q pins down p from the requirement that $U(q,p;\theta^*) = 0$. After substitution we have

$$\Pi(q,p;\theta^*) = \int_{\theta^*}^{\overline{\theta}} \left[V(q;\theta^*) - c + (k-q)\, F(q/y - \theta)\right] dG(\theta). \tag{7}$$

We now differentiate with respect to q to obtain the first-order condition

$$\frac{d\Pi(q,p;\theta^*)}{dq} = 0,$$

which yields

$$\int_{\theta^*}^{\overline{\theta}} \left[F(q/y - \theta^*) - F(q/y - \theta)\right] dG(\theta) - (q-k)\frac{1}{y}\int_{\theta^*}^{\overline{\theta}} f(q/y - \theta) dG(\theta) = 0. \tag{8}$$

The first term in (8) is strictly positive whenever at least the lowest participating type θ^* returns with positive probability,

$$q/y - \theta^* > \underline{s}, \tag{9}$$

and when at least the highest participating type does not return with positive probability,

$$q/y - \overline{\theta} < \overline{s}. \tag{10}$$

[8] Again, resolving the indifference of the ex-ante type θ^* in this way is without loss of generality, as the realization $\theta = \theta^*$ is a zero-probability event.

[9] Strictly speaking, when $\theta^* = \underline{\theta}$ holds, this already uses that, by optimality for the firm, the price p is set so as to make $\underline{\theta}$ just indifferent between purchasing and not purchasing.

In this case, i.e., when (9) and (10) hold, we have immediately that $q - k > 0$.

In what follows, we show that (9) and (10) must hold. Take first (10). If this did not hold, profits would be negative from $k < c$. Take next (9). We argue to the contrary and suppose that there are no returns in equilibrium. Note that this requires that $q < k$ (cf. condition (2)). The firm then makes profits of $(p - c)[1 - G(\theta^*)]$. Profits clearly remain unchanged as we increase q until $q/y - \theta^* = \underline{s}$ holds with equality. As we then increase $q < k$ further, however, we have from inspection of (8) that $d\Pi(p, q; \theta^*)/dq > 0$, which contradicts the optimality of the initial choice q. Thus, also (9) must hold at an optimal value q. **Q.E.D.**

This result is best understood in terms of "metering". Recall that high-θ consumers $\theta > \theta^*$ receive a strictly positive consumer rent: $U(p, q; \theta) > 0$. Recall also that consumers with a higher ex-ante type θ are less likely to make use of the refund option. In fact, the type that is most likely to return the product is the marginal type θ^*. When the firm increases the refund q and the price p simultaneously, so as to keep the marginal consumer just indifferent, this makes $U(p, q; \theta)$, as a function of the type θ, flatter. It allows to extract more consumer surplus from higher types, though at the cost of inefficiently reducing available surplus with all consumer types.

This trade-off between rent extraction for high-θ consumers and maximization of total surplus is most evident when we rewrite the first-order condition for q (cf. (8)) as follows:

$$(q - k)\frac{1}{y}\int_{\theta^*}^{\overline{\theta}} f(q/y - \theta)dG(\theta) = \int_{\theta^*}^{\overline{\theta}} \left[F(q/y - \theta^*) - F(q/y - \theta)\right] dG(\theta). \tag{11}$$

There, the rent-extraction rationale is captured by the term on the right-hand side and the rationale to maximize ex-ante surplus by the term on the left-hand side.

Incidentally, note that as consumers make different use of the refund option, the cost that this imposes on the seller, given that $q - k > 0$, is also type-dependent. This is different, for instance, in the seminal contribution by Spence (1975), where a change in quality may affect the valuation of the marginal consumer differently from that of "inframarginal" consumers, but where the cost of servicing a customer is the same.

Comparative Statics. To conduct a comparative analysis, we stipulate for brevity's sake that the seller's problem is strictly quasiconcave. Further, we invoke the standard hazard rate assumption that for $G(\theta)$

$$\frac{d}{d\theta}\left[\frac{g(\theta)}{1-G(\theta)}\right] > 0. \tag{12}$$

The following Corollary then provides the intuitive result that the "metering distortion" decreases, so that the refund becomes closer to the efficient (salvage) value, as the difference between the "marginal" type θ^* and the "inframarginal" types $\theta > \theta^*$ decreases.

Corollary 1 *When (12) holds, the monopolist's optimal choice of $q^* > k$ becomes less distorted (lower q^* and thus lower difference $q^* - k > 0$) when his coverage of the market decreases (higher θ^*).*

Proof. Note first that as $\theta^* \to \overline{\theta}$, we have from rewriting the first-order condition (11)

$$q^* - k = y\frac{\int_{\theta^*}^{\overline{\theta}} \left[F(q^*/y-\theta^*) - F(q^*/y-\theta)\right] dG(\theta)}{\int_{\theta^*}^{\overline{\theta}} f(q^*/y-\theta)dG(\theta)}$$

and by applying l'Hopital's rule that $q^* - k$ converges to

$$y\frac{[1-G(\theta^*)]}{g(\theta^*)}\bigg|_{\theta^*=\overline{\theta}} = 0.$$

Next, from implicit differentiation of (8), together with the stipulated strict quasiconcavity of the seller's program, we have that the sign of the continuous function $dq^*/d\theta^*$ is determined by the expression

$$\frac{d^2\Pi}{dqd\theta^*} = f(q^*/y-\theta^*)\left[1-G(\theta^*)\right]\left[(q^*-k)\frac{1}{y}\frac{g(\theta^*)}{1-G(\theta^*)} - 1\right]. \tag{13}$$

To sign (13), note first that $q^* > k$ holds whenever $\theta^* < \overline{\theta}$ (cf. Proposition 1) and that $q^* \to k$ as $\theta^* \to \overline{\theta}$, which implies that $dq^*/d\theta^*$ must be decreasing somewhere. Arguing to a contraction, suppose thus that $dq^*/d\theta^* > 0$ were to hold for some values θ^*. Then, by continuity of $dq^*/d\theta^*$, there must be some value(s) $\theta^* < \overline{\theta}$ where the term (13) is zero and where, in addition, it cuts zero from above. To see that this cannot be the case, note that the derivative of (13), when evaluated at such a point θ^*, equals:

$$\begin{aligned}
& \frac{d\left(f(q^*/y-\theta^*)\left[1-G(\theta^*)\right]\right)}{d\theta^*}\left[(q^*-k)\frac{1}{y}\frac{g(\theta^*)}{1-G(\theta^*)} - 1\right] \\
& + f(q^*/y-\theta^*)\left[1-G(\theta^*)\right]\left[\frac{dq^*}{d\theta^*}\frac{1}{y}\frac{g(\theta^*)}{1-G(\theta^*)} + (q^*-k)\frac{1}{y}\frac{d}{d\theta^*}\left(\frac{g(\theta^*)}{1-G(\theta^*)}\right)\right] \\
= \; & f(q^*/y-\theta^*)\left[1-G(\theta^*)\right](q^*-k)\frac{1}{y}\frac{d}{d\theta^*}\left(\frac{g(\theta^*)}{1-G(\theta^*)}\right) \\
> \; & 0.
\end{aligned}$$

Here, the equality follows as at the considered value θ^*, both $dq^*/d\theta^* = 0$ and $\left[(q^* - k)\frac{1}{y}\frac{g(\theta^*)}{1-G(\theta^*)} - 1\right] = 0$. The inequality, in turn, follows from $q^* > k$ and from the hazard rate assumption (12). Thus, the term (13) is indeed strictly positive in the immediate right-side neighborhood of the considered value θ^*, and we obtain a contradiction. **Q.E.D.**

Equilibrium Market Coverage. How is the marginal ex-ante type θ^* pinned down? To keep our exposition brief, suppose that the firm's program to choose q is strictly quasi-concave for a given θ^*, so that there is always a unique value q^* that solves the respective first-order condition (8). Then, differentiating the firm's profits in (7) and making use of the envelope theorem with respect to q^*, we have that

$$\frac{d\Pi(q^*, p; \theta^*)}{d\theta^*} = y \int_{\theta^*}^{\overline{\theta}} \left[1 - F(q^*/y - \theta^*)\right] dG(\theta) - g(\theta^*) \left[U(q^*, p; \theta^*) + \pi(q^*, p; \theta^*)\right]. \quad (14)$$

Observe that the last term in (14) captures the total surplus that is realized with the marginal type θ^*. When θ^* is interior, then the first-order condition implies that this must be strictly positive. In this case, we have

$$\omega(q^*; \theta^*) = U(q^*, p; \theta^*) + \pi(q^*, p; \theta^*) = \frac{1 - G(\theta^*)}{g(\theta^*)} y \left[1 - F(q^*/y - \theta^*)\right].$$

In standard terminology, in this case the virtual surplus of θ^* is zero, but the true surplus is strictly positive. Note finally that the optimal price p^* is then, for given θ^* and q^*, derived immediately from the requirement that

$$\begin{aligned} p^* &= V(q^*; \theta^*) \\ &= q^* F(q^*/y - \theta^*) + \int_{q^*/y - \theta^*}^{\overline{s}} y\,(\theta^* + s)\, dF(s). \end{aligned}$$

4 Competition

Initially, we proceed under the joint assumption that, first, both firms have a positive share of the market and that, second, there is full market coverage. When both firms are active and when customers do not randomize between where to purchase, we can segment the type space Θ into two subsets: All types $\theta \in \Theta_l$ purchase at firm l and all types $\theta \in \Theta_h$ purchase at firm h, where full market coverage implies $\Theta_l \cup \Theta_h = \Theta$. Denote now the two pairs of contracts by the respective subscripts l and h, i.e., by writing (q_l, p_l) and (q_h, p_h).

We further abbreviate the respective expected utility for a customer of type θ by writing $U_l(\theta)$ and $U_h(\theta)$. The profits of each firm are given by[10]

$$\Pi_i = \int_{\theta \in \Theta_i} \pi(p_i, q_i; \theta) dG(\theta).$$

From consumer optimality, it must hold that $U_h(\theta) \geq U_l(\theta)$ for all $\theta \in \Theta_h$ and $U_l(\theta) \geq U_h(\theta)$ for all $\theta \in \Theta_l$. Note that the offer made by the other firm j generates for each consumer who purchases at firm $i \neq j$ a type-dependent reservation value. As is standard in the theory of contracting with ex-ante private information, we now apply the following procedure. We first solve a *relaxed* program for each firm, where we assume that the "participation constraint", i.e., that $U_h(\theta) \geq U_l(\theta)$ or, respectively, $U_l(\theta) \geq U_h(\theta)$, binds only at a single customer type: There is a critical type $\underline{\theta} < \theta^* < \overline{\theta}$, so that from $U_l(\theta^*) = U_h(\theta^*)$, together with $U_h(\theta) > U_l(\theta)$ for all $\theta > \theta^*$ and $U_h(\theta) < U_l(\theta)$ for all $\theta < \theta^*$, the participation constraint binds only at θ^*.

Optimal Refund. When the relaxed program applies, the contract design problems for the two firms simplify as follows. As in our preceding analysis with a monopoly, we first take θ^* as given, now together with the respective reservation value $U^* \geq 0$, so that $U_l(\theta^*) = U_h(\theta^*) = U^*$. For the two firms, the respective profit functions from the two relaxed programs are then given by

$$\begin{aligned} \Pi_h &= \Pi_h(p_h, q_h; \theta^*) = \int_{\theta^*}^{\overline{\theta}} \pi(p_h, q_h; \theta) dG(\theta), \\ \Pi_l &= \Pi_l(p_l, q_l; \theta^*) = \int_{\underline{\theta}}^{\theta^*} \pi(p_l, q_l; \theta) dG(\theta). \end{aligned}$$

After substitution for the respective prices p_l and p_h from the participation constraints, in analogy to the procedure in the proof of Proposition 1, this yields

$$\begin{aligned} \Pi_h &= \int_{\theta^*}^{\overline{\theta}} \left[V(q_h, y_h; \theta^*) - c - (q_h - k) F(q_h/y_h - \theta) - U^* \right] dG(\theta), \\ \Pi_l &= \int_{\underline{\theta}}^{\theta^*} \left[V(q_l, y_l; \theta^*) - c - (q_l - k) F(q_l/y_l - \theta) - U^* \right] dG(\theta). \end{aligned}$$

Note that, to avoid confusion, we have made explicit the dependency of $V(\cdot)$ on the quality of the consumed product, y_l or y_h.

[10] This presumes that both sets of types are measurable.

Proposition 2 *Suppose that with competition the market is fully covered and that it is sufficient to consider, for each firm, only the participation constraint of some marginal type θ^* that segments the market. Then, the high-quality firm h chooses a refund that is inefficiently generous, $q_h^* > k$, while the low-quality firm l chooses a refund that is inefficiently strict, $q_l^* < k$.*

Proof. When the refund for either firm is characterized by the respective first-order condition, we have in analogy to the proof of Proposition 1 (cf. condition (8)) that

$$(q_h - k)\frac{1}{y_h}\int_{\theta^*}^{\overline{\theta}} f(q_h/y_h - \theta)dG(\theta) = \int_{\theta^*}^{\overline{\theta}} \left[F(q_h/y_h - \theta^*) - F(q_h/y_h - \theta)\right] dG(\theta) \qquad (15)$$

and

$$(q_l - k)\frac{1}{y_l}\int_{\underline{\theta}}^{\theta^*} f(q_l/y_l - \theta)dG(\theta) = \int_{\underline{\theta}}^{\theta^*} \left[F(q_l/y_l - \theta^*) - F(q_l/y_l - \theta)\right] dG(\theta). \qquad (16)$$

When these first-order conditions apply, the assertions that $q_h^* > k$ and $q_l^* < k$ follow immediately. Finally, by the same arguments as in the proof of Proposition 1, we can rule out for either firm the cases where the good is either returned with probability one or never returned with probability one. **Q.E.D.**

The intuition for the competitive result under market segmentation is immediate from our previous discussion of how a monopolistic firm can use the refund for metering, thereby extracting a higher surplus from "inframarginal" consumer types. For the low-quality firm, the marginal type θ^* is now, however, the highest type that it serves. Under the assumption that the relaxed program applies for both firms, we have for all lower types $\theta < \theta^*$ that $U_l(\theta) > U_h(\theta)$: Their utility from purchasing the low-quality product is strictly higher than their respective (type-dependent) reservation value, which they would realize when purchasing, instead, the high-quality product. To extract more of the low types' consumer surplus, the low-quality firm finds it optimal to reduce the refund below the efficient level. By reducing q_l and adjusting the price p_l accordingly, $U_l(\theta)$ becomes steeper, which reduces the expected utility for all lower types. Recall, instead, that for the (high-quality) firm, which serves the upper segment of the market, it is optimal to make $U_h(\theta)$ flatter, which is achieved by setting $q_h^* > k$.

Market Segmentation. To complete the characterization, we have from $U_h(\theta^*) = U_l(\theta^*)$ that $p_h^* = V(q_h^*, y_h; \theta^*) - U_l(\theta^*)$ and $p_l^* = V(q_l^*, y_l; \theta^*) - U_h(\theta^*)$. Further, to determine the marginal type θ^*, we obtain from differentiation of the two profit functions, similar to the procedure in the monopoly case, that

$$\begin{aligned} \omega_h^*(\theta^*) &= \omega_h(q_h^*, p_h^*, y_h, q_l^*, p_l^* y_l; \theta^*) \\ &= V(q_h^*.y_h; \theta^*) - p_h^* + \pi(q_h^*, p_h^*; \theta^*) \\ &= \frac{1 - G(\theta^*)}{g(\theta^*)} \left[\frac{dU_h(\theta^*)}{d\theta^*} - \frac{dU_l(\theta^*)}{d\theta^*} \right] \\ &= \frac{1 - G(\theta^*)}{g(\theta^*)} \left[y_h \left[1 - F(q_h^*/y_h - \theta^*) \right] - y_l \left[1 - F(q_l^*/y_l - \theta^*) \right] \right] \end{aligned} \tag{17}$$

and, likewise, that

$$\begin{aligned} \omega_l^*(\theta^*) &= \omega_l(q_h^*, p_h^*, y_h, q_l^*, p_l^* y_l; \theta^*) \\ &= \frac{G(\theta^*)}{g(\theta^*)} \left[y_h \left[1 - F(q_h^*/y_h - \theta^*) \right] - y_l \left[1 - F(q_l^*/y_l - \theta^*) \right] \right]. \end{aligned} \tag{18}$$

Note that

$$\frac{dU_h(\theta^*)}{d\theta^*} > \frac{dU_l(\theta^*)}{d\theta^*} \tag{19}$$

is necessary to ensure that the participation constraint for each firm is indeed slack around the marginal type. Generally, for $y_h > y_l$, we have from

$$\frac{dU_i(\theta)}{d\theta} = y_i \left[1 - F(q_i^*/y_i - \theta) \right] \tag{20}$$

that a sufficient condition for the assumed segmentation is given by

$$\frac{q_h^*}{y_h} \leq \frac{q_l^*}{y_l}. \tag{21}$$

Though condition (21) is not in terms of the primitives alone, it is easily checked when results can be explicitly computed. For instance, when $F(s)$ is uniformly distributed with support $[0, \overline{s}]$ and $G(\theta)$ is uniformly distributed with support $[\underline{\theta}, \overline{\theta}]$, we obtain

$$q_h^* = k + \frac{1}{2} y_h \left(\overline{\theta} - \theta^* \right)$$

and

$$q_l^* = k - \frac{1}{2} y_l \left(\theta^* - \underline{\theta} \right).$$

After substituting also for the optimal θ^*, condition (21) becomes

$$\frac{1}{2}\left(\overline{\theta}-\underline{\theta}\right) < k\left(\frac{1}{y_l}-\frac{1}{y_h}\right). \tag{22}$$

Note also that condition (21), respectively condition (22), is only a sufficient, but not a necessary condition.

When there is no such segmentation, each firm possibly serves a set of disjunct intervals. A simple characterization is then no longer obtained. However, our key observation on how refunds change in quality across firms would then still be preserved.

Proposition 3 *Under full coverage, when both firms are active in the market and when $y_h > y_l$, then $q_l^* < q_h^*$.*

Proof. See Appendix.

Comparative Statics. We now consider the partition of the market, as given by θ^*, as exogenous and conduct a comparative analysis. As in the monopoly case, this helps to bring out the mechanism driving the metering distortion. For brevity's sake, we stipulate again that the firms' problems are strictly quasiconcave. We continue to maintain the hazard rate condition (12) while adding its standard complementary condition, namely, that for $G(\theta)$

$$\frac{d}{d\theta}\left[\frac{g(\theta)}{G(\theta)}\right] < 0. \tag{23}$$

With this we can extend the previous monotonicity result from the monopoly case, albeit now the distortions in the two firms' contracts change inversely: While one contract becomes more distorted, the other contract becomes less distorted.

Corollary 2 *When (12) and (23) hold, both $q_h^* > k$ and $q_l^* < k$ decrease as θ^* increases.*

Proof. See Appendix.

Equilibrium Market Partition. Finally, we turn to the question how the marginal type θ^* is pinned down with competition. From the first-order conditions for the two firms, with respect to θ^*, we have

$$p_h^* = c - (k - q_h^*)\,F(q_h^*/y_h - \theta^*) + \frac{1-G(\theta^*)}{g(\theta^*)}\left[\frac{dU_h(\theta^*)}{d\theta^*} - \frac{dU_l(\theta^*)}{d\theta^*}\right] \tag{24}$$

and

$$p_l^* = c - (k - q_l^*)\,F(q_l^*/y_l - \theta^*) + \frac{G(\theta^*)}{g(\theta^*)}\left[\frac{dU_h(\theta^*)}{d\theta^*} - \frac{dU_l(\theta^*)}{d\theta^*}\right], \tag{25}$$

where we can use $dU_i(\theta^*)/d\theta^*$ from (20). When both firms have a positive market share, these conditions must hold together with the indifference condition

$$V(q_h^*, y_h; \theta^*) - p_h^* = V(q_l^*, y_l; \theta^*) - p_l^*.$$

To further pin down market shares, we would have to make specific functional choices.

5 Heterogeneity in the Value of Information

In the following Section, we extend the analysis to the case where, ex-ante, consumers do not differ in their expected valuation, but rather in the expected value from experimenting with the product, i.e., in the quality of information that they expect to thereby obtain (cf. Courty and Li 2000). We thus postulate that t, the fit value, is now given by

$$t = \theta s. \tag{26}$$

Also, it holds that $\int_S s dF(s) = 0$ and for all types $\theta \geq \underline{\theta} > 0$. Thus, θ orders the conditional distribution of t in the sense of a Mean-Preserving Spread.

To rule out corner solutions, we assume in analogy to the parameter restriction (3) that $y_l \underline{\theta} \overline{s} > c$. Note that from $\underline{s} < 0$, the restriction corresponding to (2), namely that $y_h \overline{\theta} \underline{s} < k$, is now always satisfied, given $\overline{\theta} > 0$.

A consumer returns firm i's product whenever $y_i \theta s < q_i$, while keeping it otherwise. His gross expected utility is thus given by

$$\begin{aligned} V(q_i, y_i; \theta) &= \int_S \max[y_i \theta s, q_i] dF(s) \\ &= q_i F(q_i/(y_i \theta)) + \int_{q_i/(y_i\theta)}^{\overline{s}} y_i \theta s dF(s). \end{aligned} \tag{27}$$

This is strictly increasing in the quality, y_i, provided that the product is not always returned. Expected utility is also increasing in the customer's type

$$\frac{dV(q_i, y_i; \theta)}{d\theta} = \int_{q_i/(y_i\theta)}^{\overline{s}} y_i s dF(s) \geq 0,$$

which holds strictly when the product is not returned for sure. From this we have also that

$$\frac{d^2V(q_i, y_i; \theta)}{d\theta dq_i} = -f(q_i/(y_i\theta))\frac{1}{\theta} \leq 0,$$

which holds strictly when the threshold $q_i/(y_i\theta)$ is in the interior of the support of s. Thus, as long as a customer both returns and consumes the product with positive probability, the marginal valuation for a higher refund is strictly decreasing in θ.

For generality, we also want to account for the case where the fit value turns out to be negative. To avoid "forcing" the buyer to then consume the product, we stipulate that there is free disposal. Note that, in principle, this also imposes a restriction on the lowest feasible refund, $q \geq 0$. However, this restriction will not bind in equilibrium.

Similar to the procedure above, we first assume again that the respective participation constraints $U_i(\theta) \geq U_j(\theta)$ with $i \neq j$ bind only at some interior type $\underline{\theta} < \theta^* < \overline{\theta}$, so that we can focus on the thereby relaxed program. A firm's profit with consumer θ is now given by

$$\pi(p_i, q_i; \theta) = p_i - c + (k - q_i) F(q_i/(\theta y_i)).$$

In analogy to Proposition 1, substitution for the respective prices p_l and p_h yields expected firm profits

$$\begin{aligned}
\Pi_h &= \int_{\theta^*}^{\overline{\theta}} [V(q_h, y_h; \theta^*) - c - (q_h - k) F(q_h/(\theta y_h)) - U^*] \, dG(\theta), \\
\Pi_l &= \int_{\underline{\theta}}^{\theta^*} [V(q_l, y_l; \theta^*) - c - (q_l - k) F(q_l/(\theta y_l)) - U^*] \, dG(\theta),
\end{aligned}$$

where U^* is the utility that is obtained by the marginal type.

Proposition 4 *Take now the utility function $u = ty$, with type $t = \theta s$ (cf. (26)). Suppose the market is fully covered and that it is sufficient to consider only the participation constraint of some marginal type θ^* that segments the market. Then, as in Proposition 2, firm h sets an inefficiently generous refund, $q_h^* > k$, and firm l sets an inefficiently strict refund, $q_l^* < k$.*

Proof. When the refund for either firm is characterized by the respective first-order condition, we have in analogy to the proof of Proposition 2 (respectively, Proposition 1; cf. condition (8)) that

$$(q_h - k)\frac{1}{y_h}\int_{\theta^*}^{\overline{\theta}} \frac{1}{\theta} f(q_h/(\theta y_h)) dG(\theta) = \int_{\theta^*}^{\overline{\theta}} [F(q_h/(\theta^* y_h)) - F(q_h/(\theta y_h))] \, dG(\theta) \tag{28}$$

and

$$(q_l - k)\frac{1}{y_l}\int_{\underline{\theta}}^{\theta^*} \frac{1}{\theta} f(q_l/(\theta y_l))dG(\theta) = \int_{\underline{\theta}}^{\theta^*} \left[F(q_l/(\theta^* y_l)) - F(q_l/(\theta y_l))\right] dG(\theta). \tag{29}$$

When these hold, the assertions that $q_h^* > k$ and $q_l^* < k$ follow immediately from the fact that the probability of return decreases with the consumer's type, as noted above. Note also that $q_l > 0$.

Finally, by the same arguments as in the proof of Proposition 1, we can rule out for either firm the cases where the good is either returned with probability one or never returned with probability one (i.e., the cases where q_l and q_n would be determined by the respective corner solutions, rather than the respective first-order conditions). **Q.E.D.**

Proposition 4 mirrors the characterization from Proposition 2. We have thus established another channel through which customer heterogeneity together with vertical differentiation of products can lead to a wide variety of refund policies, in line with the empirical observations (cf. the Introduction). Note finally that, as previously, we have focused on the case where the market is segmented in two type sets. Again, condition (19) is necessary for this to hold, which now transforms to

$$\frac{dU_h(\theta^*)}{d\theta^*} - \frac{dU_l(\theta^*)}{d\theta^*} = \int_{q_h^*/(y_h\theta^*)}^{q_l^*/(y_l\theta^*)} y_h s dF(s) + \int_{q_l^*/(y_l\theta^*)}^{\overline{s}} (y_h - y_l)\, s dF(s)$$

and which holds for any θ^* whenever $q_h^*/y_h \leq q_l^*/y_l$ (cf. condition (21)). Again, while this is not in terms of the model's primitives, it can be readily verified once a solution has been calculated. For instance, when both θ and s are distributed uniformly, in analogy to condition (22), we obtain the condition

$$y_l\left(2 - \frac{1}{\Delta}\frac{\overline{\theta} - \underline{\theta}}{\overline{\theta}}\right) \leq y_h\left(2 - \frac{1}{\Delta}\frac{\overline{\theta} - \underline{\theta}}{\underline{\theta}}\right),$$

where $\Delta = \ln\left(\overline{\theta}\right) - \ln\left(\underline{\theta}\right)$.

General Signal Structure. So far we considered two different ways to model the interaction of customer types and information. In the first case, we could use immediately from the specification $t = \theta + s$ that the type θ orders the distribution of "fit values" in the sense of First-Order Stochastic Dominance. This implied, in turn, that for higher types θ it was less likely that they return the product or cancel a contract prematurely. Recall that this feature was essential to derive the "metering result", which then lead to the two distortions in the refund offered by low-quality and high-quality firms.

Clearly, in the second case, where $t = \theta s$, the type θ no longer implies such a general ordering of the distribution of "fit values". In fact, now θ orders the distributions in the sense of a Mean-Preserving Spread. However, given that, in equilibrium, the refund is (strictly) positive, $q_i > 0$, we still have that $d\Pr(t < q_i/y_i|\theta)/d\theta < 0$. In fact, our results thereby apply whenever the conditional distribution of $\Pr(t|\theta)$ exhibits this property over the relevant range.

Finally, it should be noted that the obtained results as well as the simple characterization are also more general in the following way. Recently, Esö and Szentes (2007) use an orthogonalizing procedure to separate between the "old information", already contained in θ, and the "new information" that is provided by s, and which we may term z. Then, θ and z are, by construction, independently distributed. They then consider joint distributions where (i) $E(t|\theta, s)$ increases with s and where (ii) $d\Pr(t|\theta)/d\theta < 0$, showing that there always exists an equivalent representation of the "fit value", as a direct function of θ and z, that still satisfies (ii). As noted above, for our results we only need this ordering for all values where the product is certainly not "disposed off", $t \geq 0$.

6 Concluding Remarks

Firms frequently offer refunds, both when physical products are returned and when service contracts are terminated prematurely. This paper shows how refunds can be used by firms as a "metering device" in case consumers, while having different expectations about their utility, still learn when experimenting with the product or service. In our benchmark monopoly case, which borrows from Courty and Li (2000), the refund is inefficiently high, as this allows the firm to extract more of the consumer rent of consumers with a high ex-ante valuation and thus a lower marginal valuation for the refund. As we argued above, the case where firms are restricted to offer a single refund policy for all consumers, so that they cannot achieve perfect sorting through a menu, seems reasonable in many instances, as matching consumers to particular contracts may be time-consuming at the point-of-sale and, in addition, may require additional after-sales monitoring to prevent consumer arbitrage.

Our main contribution lies in the analysis of the competitive model. There, two vertically differentiated firms, as in Shaked and Sutton (1982), compete through offering prices and refund terms. Our main positive prediction is that the low-quality firm offers a lower refund, though the salvage value is the same across products and even though product quality is observable to consumers ("no signaling"). We showed that this result is robust to a different source of consumer heterogeneity, where consumers differ in the precision of information that they obtain from experimenting with the product or service. Our results may thus offer an explanation for the wide variety of contractual terms that have been observed in empirical work (cf. the Introduction).

From a normative perspective, refunds are inefficient at both firms. In particular, the low-quality firm offers an inefficiently strict refund policy. In non-reported calculations we have explored the impact of stipulating a binding, mandatory minimum refund, similar to a consumer protection policy. This restricts the low-quality firm's scope for price discrimination. The impact that this has on welfare and expected consumer surplus is generally ambiguous. This observation is in line with the standard notion that restricting price discrimination has ambiguous effects. In terms of consumer protection, this suggests that imposing a minimum mandatory refund or minimum cancellation terms could backfire by reducing, instead of increasing, consumer surplus and welfare, provided that strict terms of refund are used as a means to price discriminate, as in our model.

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7 Appendix: Omitted Proofs

Proof of Proposition 3. We argue to a contradiction. Suppose that in equilibrium $q_l^* \geq q_h^*$. Given $y_h > y_l$, condition (21) holds and the market is separated with firm h

operating on $[\theta^*, \overline{\theta}]$ and firm l on $[\underline{\theta}, \theta^*)$. Fix a cutoff point θ^*. When both firms are active in the market, θ^* is interior. We need to analyze three cases: (i) $q_l^* \geq q_h^* \geq k$, (ii) $k \geq q_l^* \geq q_h^*$, (iii) $q_l^* \geq k \geq q_h^*$.

Take first case (i). From inspection of $d\Pi_l(\theta^*)/dq_l$, using the first-order condition in (16), note first that independent of q_l, it holds for $q_l \geq 0$ that

$$\int_{\underline{\theta}}^{\theta^*} [F(q_l/y_l - \theta^*) - F(q_l/y_l - \theta)]\, dG(\theta) \leq 0.$$

Thus, since $f(s) > 0$, $d\Pi_l(\theta^*)/dq_l$ is strictly negative for all $q_l \in (k, q_l^*]$. This implies that neither $q_l^* \geq q_h^* > k$ nor $q_l^* > q_h^* \geq k$ can hold in equilibrium for firm l. Still staying with case (i), suppose next that $q_l^* = q_h^* = k$. Since $y_h\left(\overline{\theta} + \underline{s}\right) < k < y_l\left(\underline{\theta} + \overline{s}\right)$, cf. (2) and (3), it must hold that

$$\int_{\underline{\theta}}^{\theta^*} [F(q_l/y_l - \theta^*) - F(q_l/y_l - \theta)]\, dG(\theta) < 0$$

and hence $d\Pi_l(\theta^*)\backslash dq_l|_{q_l=k} < 0$, so that with $q_l^* = q_h^* = k$ firm l would still want to deviate.

The argument proving that $k \geq q_l^* \geq q_h^*$ (case (ii)) cannot arise in equilibrium is analogous, with the only difference that this time one must consider firm h's unilateral deviation. Finally, with regards to case (iii), observe that we have already ruled out that $q_l^* = q_h^* = k$ can arise in equilibrium. When $q_l^* > k \geq q_h^*$, we can argue, as in case (i), that firm l has an incentive to deviate, as $d\Pi_l(\theta^*)\backslash dq_l$ is strictly negative for all $q_l \in (k, q_l^*]$; while when $q_l^* \leq k < q_h^*$ we can argue likewise for firm h. Taken together, we can thus conclude that $q_l^* \geq q_h^*$ cannot arise in equilibrium. **Q.E.D.**

Proof of Corollary 2. The proof that $dq_h^*/d\theta^* < 0$ follows directly from Corollary 1. The proof for $dq_l^*/d\theta^* < 0$ proceeds now analogously, making use of (23). As $\theta^* \to \underline{\theta}$, we have from

$$q_l^* - k = y_l \frac{\int_{\underline{\theta}}^{\theta^*} [F(q_l^*/y_l - \theta^*) - F(q_l^*/y_l - \theta)]\, dG(\theta)}{\int_{\underline{\theta}}^{\theta^*} f(q_l^*/y_l - \theta) dG(\theta)}$$

and by applying l'Hopital's rule that $q_l^* - k$ converges to

$$-y_l \left.\frac{G(\theta^*)}{g(\theta^*)}\right|_{\theta^*=\underline{\theta}} = 0.$$

Next, from implicit differentiation of (16), together with the stipulated strict quasiconcavity of the seller's program, the sign of the continuous function $dq_l^*/d\theta^*$ is determined by the expression

$$\frac{d^2\Pi_l}{dq_l d\theta^*} = f(q_l^*/y_l - \theta^*)G(\theta^*)\left[(k - q_l^*)\frac{1}{y_l}\frac{g(\theta^*)}{G(\theta^*)} - 1\right]. \tag{30}$$

We know that $q_l^* < k$ for all $\theta^* > \underline{\theta}$ and that $q_l^* \to k$ as $\theta^* \to \underline{\theta}$, implying that $dq_l^*/d\theta^*$ must be decreasing somewhere. Arguing to a contraction, suppose thus that $dq_l^*/d\theta^* > 0$ were to hold for some values θ^*. Then, by continuity of $dq_l^*/d\theta^*$, there must be some value(s) $\theta^* > \underline{\theta}$ where the term (30) is zero and where, in addition, it cuts zero from below. To see that this can, however, not be the case, note that the derivative of (30) evaluated at such a point θ^* equals:

$$\begin{aligned}
& \frac{d\left(f(q_l^*/y_l - \theta^*)G(\theta^*)\right)}{d\theta^*}\left[(k - q_l^*)\frac{1}{y_l}\frac{g(\theta^*)}{G(\theta^*)} - 1\right] \\
& + f(q_l^*/y_l - \theta^*)G(\theta^*)\left[-\frac{dq_l^*}{d\theta^*}\frac{1}{y_l}\frac{g(\theta^*)}{G(\theta^*)} + (k - q_l^*)\frac{1}{y_l}\frac{d}{d\theta^*}\left(\frac{g(\theta^*)}{G(\theta^*)}\right)\right] \\
= \; & f(q_l^*/y_l - \theta^*)G(\theta^*)(k - q_l^*)\frac{1}{y_l}\frac{d}{d\theta^*}\left(\frac{g(\theta^*)}{G(\theta^*)}\right) \\
< \; & 0,
\end{aligned}$$

where the equality is due to the fact that at the considered value θ^*, both $dq_l^*/d\theta^* = 0$ and $\left[(k - q_l^*)\frac{1}{y_l}\frac{g(\theta^*)}{G(\theta^*)} - 1\right] = 0$. The inequality, in turn, follows from $q_l^* < k$ and from the hazard rate assumption (23). This completes the contradiction. **Q.E.D.**