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A MODEL FOR ECONOMIC PLANNING AND ANALYSIS
FOR THE BRAZILIAN ECONOMY

BY

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THESIS

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Urbana, Illinois

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ABSTRACT

In this work a general purpose multisectoral economy-wide model, solved for growth rates, is constructed for the Brazilian economy.

In constructing the Brazilian model, the ORANI model for the Australian Economy was chosen as the starting point and was modified in a way that it can reflect and can be used to study the Brazilian reality. The main differences between both models are that in the Brazilian model:

- a) A special treatment is giving to the government sector;
- b) The demand for household consumption is broken down by different income groups, and an equation linking the workers income with their expenditure is introduced; allowing in this way for the study of income distribution problems;
- c) An industry by industry framework is used, opposing to an industry by commodity framework used in the ORANI model;
- d) Prices are assumed to be formed through a mark-up price theory, while the ORANI model assumes that prices are formed by maximizing profits.

The Brazilian model is constructed for: a) 21 industries; b) 3 types of primary factors (3 categories of labor, fixed capital, and agricultural land); c) one type of other costs; d) 2 sources of products (domestic and imported); e) 6 types of product use (inputs

to current production, inputs to capital formation, commodity flows to household consumption, exports, government demands, and other demands); and f) 3 income groups. The model also presents a detailed specification for trade margins and taxes.

The basic input-output data used in the model refers to the 1975 input-output matrices for the Brazilian economy.

To My Parents
(Aos Meus Pais)

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CHAPTER 1

INTRODUCTION

Long-term plans setting out sectoral priorities will be necessary to provide some degree of consistency in the extended economic control system of Brazil. Werner Baer, The Brazilian Economy

1.1. Introduction

The task of planning in any economy is a hard one. First of all one has to face the ideological barrier, should planning be made only in socialist and communist economies or can it also be made in capitalist economies.

The answer to that question does not seem to be harder, as a look at most of developed and developing capitalist economies shows that planning, in some degree, is always made.

It is interesting to note that any kind of planning that is made in the capitalist economies is always directed to allow a smoothness in the growth process, i. e., make the economy stable. And a stable economy is one of the things that the private sector

more desire, so it can make its how plans about growth. As one can see, planning and capitalism are not antagonists, but complements.

In the particular case of the Brazilian economy, one sees a widely use of planning in the economy. And, given the strong participation of the government in the economy (see Chapter 3), planning becomes a must.

The work being developed here, the construction of a economy-wide multisectoral model for the Brazilian economy, is an effort in the direction of allowing a better process of planning in the Brazilian economy.

While no one expects that a model gives the right answers about how the economy will behave, one expects that a model gives results that show in which direction the economy will move. This is the objective of the model being constructed here; for a given set of economic policies the model is expected to give results that help in the planning process.

The choice of constructing a multisectoral model to be used in the planning process is in part due to the extremely favorable characteristic of these kind of models, which allow the study of the effects of sectoral policies over the economy, and vice-versa, i. e., the study of the effects of aggregated economic policies over the different economic sectors.

The model being constructed here is a economy-wide multisectoral model of the Johansen-type (see Chapter 2), i. e., it is solved for growth rates. The Brazilian model is based in the ORANI model (see Dixon, Parmenter, Sutton, and Vincent (DPSV), 1982) that is currently in use in the Australian economy; the basic

differences between both models is that the Brazilian model allows for the study of the income distribution problem and the final demands of the government have a special treatment, those are features that the ORANI model does not have.

Those features were incorporated due to the characteristics of the Brazilian economy (see Chapter 3): the strong influence of the government over the economy; and the problem of the uneven distribution of income that must be solved.

The basic input-output data used in the model refers to the 1975 input-output matrices for the Brazilian economy (see IBGE, 1984b). The 1975 matrices present more detailed input-output relationships than the 1970 input-output matrices (see IBGE, 1979), i. e., the 1975 matrices present a special treatment for trade margins and taxes. This made possible the construction of a more complex and detailed multisectoral model for the Brazilian economy.

In the other sections of this chapter, one will first go over a discussion of the desirable characteristics in a model as well as some of the problems that the model being constructed here presents. Then, in the last section, an outline of the dissertation chapters will be presented.

1.2. Building a Model

When constructing a model, a series of things needed to be considered, e. g., what is the purpose of the model, what kind of approach should be used in its construction, who should build the

model, etc.. The answers to those questions are not so easy, but the following recommendations, presented in Kornai (1975, pp. 18-22), should shed some light in the discussion:

- a. The entire task, each individual phase of it - from data compilation, to design of the model, to appraisal of results - should be performed as a team project;
- b. Constant, lively, working relationship between model builders, local practical economic leaders, economic politicians, and official planners;
- c. The structure of the model should be simple;
- d. A model should be designed to be able to furnish valuable information about the real-world problems of the planners and practical decision makers;
- e. It is highly desirable that both the structure of the model and the series of computations to be performed with it are fully designed at an early stage;
- f. The estimates, forecasts, and plans of the nonmathematical planners should be utilized to a considerable degree;
- g. The model's results should be summarized in a report clearly understandable to practical people;
- h. The practical planners and economic leaders should consider the results meaningful and interpretable;
- i. The proponents of mathematical planning should try to instill a familiarity with their methods and assure their inclusion into the systematic institutional planning framework.

Concerning the above recommendations, the comments about their

relation with the model being constructed here are discussed below.

The discussion about the construction of the model for the Brazilian economy was basically restricted to the author, the dissertation committee, and some author's friends; it is expected that when the model will be implemented, a wider discussion of the model will be made.

The structure of the model, despite its complex economic relationships, was kept in a simple way. The model was constructed in a way that its users will find it simple and flexible, in the sense that there is no hard time in shifting between exogenous and endogenous variables, and in getting solutions for the model.

One can say that most of the desirable characteristics of a model are presented in the model being constructed here, and the ones that are not discussed here are supposed to be fulfilled when the model will be implemented.

But the model being constructed here is not free of problems. Due to the characteristics of a economy-wide multisectoral model, the model does not take in consideration, e. g., the monetary sector; the influence of monopolies and oligopolies over the economy; and the problem of non-utilized resources.

The fact that the model does not take in consideration the monetary sector, can in part, be solved by incorporating a macroeconometric model to this model (see Cooper and McLaren (1980 and 1981), and Powell (1981)).

The problem of monopolies and oligopolies, and the fact that non-utilized resources are not taken in consideration are yet to be solved.

A more detailed discussion of the weakness of the model constructed in this dissertation is presented in the concluding chapter.

The next section will go over the chapter organization in this dissertation.

1.3. Chapter Organization

Of the remaining 7 chapters of the dissertation: 3 are directed to placing the model being constructed here among the other models and the Brazilian reality; 3 chapters deal with the specification, estimation, and implementation of the model; and the last one is a conclusion about the work being done here. Besides these chapters, 3 appendixes with data, and explanation of data calculation are presented.

Chapter 2 presents an overview of input-output analysis and the different types of multisectoral models that can be constructed from this kind of approach. This chapter also places the model being constructed here among the different kinds of input-output models.

In Chapter 3: a historical perspective of the role of the state in intervening and planning the Brazilian economy; and the problem of the uneven distribution of income in Brazil, are presented. Those aspects are also related to the process of building the model presented in Chapter 5.

Chapter 4 will go over the economy-wide models constructed for the Brazilian economy, these models are reviewed under a critical

point of view as they are compared with the model being constructed here.

The objective of Chapter 5 is to present the specification of the equations, and the theory behind them, that form the multisectoral model.

A discussion of the data required to estimate, and the estimation of, the coefficients and parameters in the model is made in Chapter 6.

Chapter 7 deals with the problem of how to estimate the model, i. e., make simulations with it. A discussion of solution methods, and closure problems is made; also, the original model, as presented in Chapter 5, is reduced to a workable size, and the equations of this reduced system are presented.

In Chapter 8 a conclusion about the work being done here is presented.

Appendix A goes over the methodology used in the construction, from the original 1975 input-output matrices for Brazil, of the input-output data used in the model. Appendix B presents the data used in the econometric estimation and the algebraic calculation of some of the coefficients and parameters in the model. And, Appendix C presents the values of the coefficients and parameters that are different from zero and are not presented in Chapter 6.

CHAPTER 2

THEORETICAL ELEMENTS

2.1. Introduction

In constructing a planning model for the Brazilian economy one is faced with different kinds of approaches. The objective of this chapter is to place the model being built here among the vast literature on multisectoral planning models. As these kinds of models, usually use input-output analysis as one of the main ingredients in their construction, a brief presentation of input-output theory will be made.

The way that the multisectoral models use input-output analysis can be either in the form of manipulation of the basic input-output relations, as in the case of: a) static input-output analysis; b) dynamic input-output analysis; c) static linear programming models; and d) dynamic optimizing models. Or they can use the data and the theory behind input-output analysis to derive coefficients and parameters needed for the simulations of economy-wide models that in their construction make use of elements other than the input-output relations.

The organization of this chapter will be as follows: first a brief presentation of input-output theory will be made, then different kinds of multisectoral models will be discussed. In the

discussion of different kinds of multisectoral models, some examples of those models will be presented. Also, the model being constructed here will be related to the different models. The literature review presented here does not plan to be exhaustive, as the only objective in this chapter is to give an overview of planning models.

2.2. Basic Input-Output Theory

Input-output theory dates to the pioneering work of Wassily Leontief, about fifty years ago, and his classical contribution is certainly the construction of the first input-output matrices for the United States (see Leontief, 1951). Most of the presentation in this section draws on elements from Leontief (1951).

Consider the following identity, in which the economy is divided into n sectors:¹

$$\sum_{j=1}^n x_{ij} + c_i + I_i + x_i - m_i \equiv Y_i$$

$$i = 1, 2, \dots, n \quad (2.1)$$

where:

x_{ij} is the quantity of product produced by sector i and used as an intermediary input by sector j

c_i is the domestic consumption from sector i

I_i is the domestic investment from sector i

x_i are the exports from sector i

m_i are the imports of sector i

y_i is the total domestic product of sector i

Making the assumption that the intermediary flows per unit of final product are fixed, one can derive the open Leontief system, which is expressed in the following way:

$$\sum_{j=1}^n a_{ij} Y_j + f_i - m_i = Y_i$$

$$i = 1, 2, \dots, n \quad (2.2)$$

where:

a_{ij} is the quantity of product from sector i needed for the production of one unit of total product in sector j

f_i is the final demand for products in sector i , i.e.,

$$C_i + I_i + X_i$$

All the other variables are defined as before

One can write (2.2) in matrix form as:

$$AY + f - m = y \quad (2.3)$$

where:

A is a matrix of direct inputs of order n by n

f , m and y are column vectors with n elements

Solving for (2.3) it is possible to get the total product that is needed to satisfy the final demand less imports, i. e.,

$$y = (I - A)^{-1} (f - m) \quad (2.4)$$

where:

$(I - A)^{-1}$ is the matrix of direct and indirect requirements,
or the Leontief matrix

In $Z = (I - A)^{-1}$, the element z_{ij} should be interpreted as the total product of sector i needed to produce one unit of final demand in sector j .

In theory, matrices A and Z are expressed in physical relations between inputs and outputs, and their elements are called technical

coefficients. However, in practical terms, these matrices are estimated from flows measured in monetary values; this may cause some problems when those matrices are used.

Even if one can estimate matrices A and Z from physical relations, problems will also arise: such as the stability in the technical coefficients over time; definition of how the sectors should be aggregated; etc. For a good review of these issues see Miller and Blair (1985).

Besides the above problems, when matrices A and Z are estimated from monetary flows, the problem of change in relative prices also affects the value of the technical coefficients. What usually is done to solve this problem, in analytical terms, is to assume that prices are constant.

Despite all of those problems, input-output analysis is still a powerful tool, if not the best one available, when one needs to develop a multisectoral study of the economy. In essence, it serves as the foundation for such a study.

2.3. Multisectoral Models

As was noted earlier, multisectoral models can use input-output analysis by manipulation of its basic relations or they can use the data and the theory behind input-output analysis to derive coefficients and parameters needed for the simulations of economy-wide models.

Concerning the manipulation of basic input-output relations, surveys and discussions of these models can be found in Manne (1974), Taylor (1975), Bulmer-Thomas (1982), Dervis, Melo, and Robinson (1984), and Miller and Blair (1985). These models are classified as either static or dynamic according to the existence of a theory of investment that sets the system in motion; they are classified as optimizing or non-optimizing if they use any form of linear or non-linear programming to get results for the system.

Surveys and discussion of economy-wide models can be found in Blitzler, Clark, and Taylor (1975), Bulmer-Thomas (1982), Dervis, Melo, and Robinson (1984), Scarf and Shoven (1984), Shoven and Whalley (1984), and Stone (1984). These kinds of models can basically be divided into two: the ones that are solved for levels and the ones that are solved for growth rates, the Johansen-type models (see Johansen, 1974).

The models that are solved for levels usually require a non-linear solution, and as a consequence of that, each kind of model has to have its own solution algorithm. The Johansen model can be solved only with matrix algebra, which make it easier to solve these kinds of models. Besides the advantage in the solution of the model, the Johansen models provide for an easy assignment of variables between exogenous and endogenous, which is not the case in the models solved for levels. The model being constructed here is an economy-wide model of the Johansen-type. In the following subsections each type of multisectoral model will be discussed in more details.

2.3.1. Static Input-Output Models

Static input-output models are usually based on the coefficients of $(I - A)^{-1}$ and are used to predict factor uses, i.e., given a structure of final demand what would be total production, labor absorption, import requirements, etc.. As an example, suppose that the government decides to increase its demand for public services, what would be the effect on total production? The answer is obtained by changing the value of the public services sector in the final demand vector in equation (2.4), and then calculating the new levels of total production in the economy (vector y). Naturally this is only a simple example, and more elaborate models, using static input-output models, are available, see, for more references, Manne (1974), Taylor (1975), Dervis, Melo, and Robinson (1984), and Miller and Blair (1985).

2.3.2. Dynamic Input-Output Models

A good definition of dynamic input-output models can be taken from Taylor (1975) who states: "this widely used models incorporate [in the static input-output models] an accelerator-type investment theory in which current demands for investment goods depend on future expected growth of output" (pp. 50-51). Due to its nature, these kind of models can only be applied, forgetting the data problems for the moment, "in countries where there is a relatively

advanced capital goods industry ..., because where capital goods are imported one can ignore the interaction between output increases and the capital goods industries" (Bulmer-Thomas, 1982, p. 222).

The following brief description of the equations leading to the dynamic input-output models is based on Bulmer-Thomas (1982). Consider the following balance equation (based on equation 2.3):

$$y(t) = A(t)y(t) + I(t) + (c+x-m)(t) \quad (2.5)$$

where all variables were given a time dimension and I is the vector of investment by origin explained by the following accelerator-type relationship:

$$I(t) = B[y(t+1) - y(t)] \quad (2.6)$$

where B is the capital matrix in which the ij th element shows the requirement of the i th capital good per unit of output in the j th sector. Assuming that the two technology matrices (A , and B) are invariant with respect to time, one gets:

$$y(t) = Ay(t) + By(t+1) - By(t) + (c+x-m)(t) \quad (2.7)$$

The general solution of equation (2.7) is given by:

$$y(t) = [I + B^{-1}(I - A)]^t y(0) + y^*(t) \quad (2.8)$$

where the first term on the RHS is the 'homogeneous' equation and the second is the 'particular' solution.

The resulting equation (2.8) presents two basic problems: a) matrix B is not always invertible; b) the results for the model when extrapolated too far in the future does not give consistent results. For a discussion of those problems see, e. g., Taylor (1975). Examples of applications of dynamic input-output models can be found, e.g., in Manne (1974) and Taylor (1975).

2.3.3. Static Linear Programming Models

The following quote from Taylor (1975) very well describes the relationship between input-output analysis and linear programming:

"A natural complement to the input-output production specification is optimization of some welfare function to select the 'best' pattern of final demand and resource allocation from the many which are possible. Since input-output technological assumptions are all of a constant (linear) type, linear programming is the appropriate computational means for doing this"

(Taylor, 1975, p. 59).

In linear programming terms, the primal would make use of input-output quantities and the dual of input-output prices. Due to the costs involved in using linear programming models linked with input-output analysis, and the fact that the results from this kind of approach do not differ from those using input-output alone (see Taylor, 1975), these integrated models are used less and less in the literature (see Bulmer-Thomas, 1982).

For a detailed discussion of linear programming models see Bruno (1975), and Taylor (1975).

2.3.4. Dynamic Optimizing Models

One can see dynamic optimizing models as static programming models that repeat themselves overtime, the successive periods are

linked through capital accumulation equations. Thus, they can be related to the dynamic input-output models.

The advantages of dynamic optimizing models over the dynamic input-output models are (see Taylor, 1975, pp. 94-5): a) they can be used to set discount rates for investment project analysis; b) they can be used to better know the results of economic processes which require a number of years to work themselves out; c) they can be useful in building up the exogenous projections needed for more detailed short- and medium-term planning.

For a detailed discussion of dynamic optimizing models see Taylor (1975).

2.3.5. Economy-Wide Models

As discussed earlier, the economy-wide models are the ones that use the data and the theory behind input-output analysis to derive coefficients and parameters needed for their simulations. These kind of models are of Walrasian type, i. e., of general equilibrium, and as they are computable they are also known in the literature as Computable General Equilibrium (CGE) models. The CGE models are constructed by first stating the economic relations in equation forms. Then the next step is to search for the values of the coefficients and parameters of these equations. Most of these coefficients and parameters are derived from input-output relationships, e.g., technical coefficients, consumption structure, imports and exports composition, etc.. The other coefficients and

parameters are derived from: a) other sources, such as national accounts; b) econometric estimation; c) and some are even model builders 'guesses', based on economic theory, the particularities of the model, and the country to where it has been applied.

Another important source of data for the CGE models are the Social Accounting Matrices (SAMs). A SAM is basically a "matrix that provides an accounting record for the whole economy (not just transactions among producers), although not all entries will be considered in the same detail" (Bulmer-Thomas, 1982, p.1), i. e., input-output relations are part of a SAM.

For a discussion of the use of SAM in CGE models see Dervis, Melo, and Robinson (1982), who also present a good discussion of input-output models and their application in the planning process. Besides that, they present model applications for South Korea and Turkey. Another example of this approach (SAM in CGE models) is given in Taylor (1983), in which a model for India is constructed.

From the development of SAM and CGE models a new kind of approach was developed, the Transaction Value (TV) approach (see Drud, Grais, and Pyatt, 1983), that basically "is the extension of the simple formulation and implementation of input-output models that are not constrained to be linear and where prices are not necessarily independent of excess demand" (Drud, Grais, and Pyatt, 1983, p. 2).

The interesting point about TV models is that they start with a SAM and then constructed the equations that will explain each entry in the matrix. Contrary to the CGE models that first construct the equations and, only after that, a SAM is constructed in such way as

to supply the model with the data that it needs. One can say that the TV approach is the SAMs that are fighting back the CGE models, i. e., after being used by CGE models, now are the SAMs that use the CGE models (equations of general equilibrium). But what one would expect in the future is that CGE and TV models will be combined.

A good discussion of CGE models applied to developing planning is presented in Blitzler, Clark and Taylor (1975).

In Shoven and Whalley's (1984) survey of applied general-equilibrium models of taxation and international trade, some of the CGE models developed are also discussed. Taylor (1979) presents a series of models for developing countries. For a discussion of CGE models see also Scarf and Shoven (1984).

The CGE models can be divided, basically, into models that are solved for levels, or for growth rates. Examples of CGE models that are solved for levels are: a) the MODIS model for Norway (see Bjerkholt and Longva, 1980), that is a medium-term model, and is directed towards an equilibrium solution of quantity and relative prices; b) the model by Adelman and Robinson (1978a, and 1978b) to study income distribution, and that is applied to the specific case of South Korea; c) the model by Lysy and Taylor (1980) for the Brazilian economy is also directed to the study of income distribution.

The Johansen-type model, as the name says, started with the pioneering work of Leif Johansen who in the late 1950's, constructed a CGE model for the Norwegian economy (see Johansen, 1974). The model, in growth rates, is obtained by logarithmically differentiating the equations of the model with respect to time in

order to get a simultaneous system of equations that are linear in all growth rates. This is a medium- to long-term model and gives results for a period between 2 to 3 years.

The work being done here is based on the Johansen-type of model, and more than that it is based on the ORANI model being used in the Australian economy (see DPSV, 1982). Rijckeghem (1969) presents a Johansen-type model applied to the Brazilian economy.

An important point when constructing a CGE model is the problem of closure, i. e., as usually in a CGE model the number of variables is greater than the number of equations, with variables should be exogenous and with ones should be endogenous. Taylor and Lysy (1979) have shown, for a two sector model, that depending on the theory that one uses to close the model, the results can be completely different. A discussion about closure in the model being constructed here is presented in Chapter 7.

Contrary to macroeconometric models, when a CGE model is constructed it does not have a time period defined. To define a time period for a CGE model, one needs to compare the results of this model with projections from macroeconomic models, or, by making simulations for the past, with the observed value of economic variables.

Another drawback from CGE models is that they usually are static, giving results only for a specific period of time. However, this problem can be solved in part by running the model more than one time, i. e., defined the time interval for the model, one can plug the results from the first simulation in the model, and run the model to get results for the following period, and so on.

Usually one can transfer models that make use of basic input-output relations from one country to another, is the same true for CGE models? The answer for that question is not an easy one, it will depend on the characteristics of the model and the countries from where the model was originated and to where it is going to be transferred, and how it is going to be transferred, i. e., with or without alterations.

In the case of the work being done here, the ORANI model, for the Australian economy, is transferred to the Brazilian economy. Due to its general characteristics, it is believed that this model can be applied to the Brazilian economy. However, some changes need to be made before it can be used. The main changes are: a) the role of the government sector is to be highlighted; and b) the model is to be altered such that income distribution problems could be studied. A discussion of why these changes need to be made is presented in Chapter 3, which presents the characteristics of the Brazilian economy. In Chapter 4, a discussion of previous economy-wide models constructed for the Brazilian economy is made, those models are also compared with the model being constructed here.

The reason of why a Johansen-type model was chosen for the Brazilian economy, instead of a CGE model solved for levels, can be explained by Taylor (1975):

"In view of its robustness and the range of problems with which it can deal, it is surprising that the Johansen technique has not been applied more widely" (p. 101).

Indeed it is!

NOTES

1. The notation used in this chapter is different from the one used in the remaining chapters of this work.

CHAPTER 3

CHARACTERISTICS OF THE BRAZILIAN ECONOMY

3.1. Introduction

In Brazil of the 1980's two main topics, among others equally important, need to be considered when building a model for its economy. The first one is the great power that the government has over the economy, either through its regulatory process, or through the various state companies; and the second one is the uneven distribution of income, among individuals and among regions (see Baer, 1983).

Those two topics are dealt with in the model constructed for the Brazilian economy. For the government sector, while no specifications are made for the state companies, the effects of government control over production, prices, and investment of its companies can be studied in the model by making those variables exogenous to the system; a vector of final demands for the government is also specified, allowing for the study of the direct demands of the government. Concerning the distribution of income, the model distinguishes between three classes of income. These three classes have different structures of final demand and the income of these classes is linked with different skills of labor; in this way it is possible to study the effects of different policies on the

demand for labor of the different skills and on the final demand, for consumption, by the different income groups. No specification for the problem of the uneven distribution of income between the different Brazilian regions is made; this is so because the present model does not have a regional dimension. However, in the future, the model could be expanded so that regional problems could be studied.

The other sections of this chapter are as follows: the next section will go over the history of the increasing participation of the state in the Brazilian economy, and will also present a discussion of the Brazilian government's experiences with planning models; the third and last section of this chapter will discuss the problem of the uneven distribution of income in the Brazilian economy. During the presentation of those two sections, some of the institutional and economic characteristics of Brazil will also be presented.

3.2. State Participation in the Economy

Until the 1930's, the participation of the state in the Brazilian economy was limited to the traditional role of the government according to the notion of the "laissez-faire", i.e., the interference of the government in the economy should be limited to the minimum necessary to maintain peace and property rights.

In the 1930's, when the world economy was in crisis, Brazil, whose coffee exports made up the dynamic sector of the economy, also

faced a crisis. The latter was not more serious because the government, for the first time, took a series of political and economic measures directed toward the stabilization of the level of aggregate demand and to the regulation of the external sector. In a certain sense one can say that the Brazilian government was Keynesian before there was a Keynesian theory. For a good presentation of the intervention of the Brazilian government in the economy in the 1930's see Furtado (1972).

Together with the stabilization program of the 1930's, the Brazilian government started a process of industrialization that led the industrial sector to grow at annual real rates above 10% in the 1933-39 period (see Villela, 1972). In the 1940's the intervention of the state in the economy was relatively high during the war period, but decreased drastically after the war until the beginning of the 1950's.

During the war the Brazilian government started to build the infrastructure necessary for the development of the industrial sector. The construction of the steel mill at Volta Redonda is a good example of this development. Also during this period, an American Mission visited Brazil (the Cooke Mission during 1942-43). Its objective was to study the possibilities of the Brazilian economy for the war effort. The basic recommendations of this mission can be summarized as follows:

"The task of industrialization, according to the mission's report, should be left to the private sector, while the government should concentrate on general industrial planning, developing industrial credit facilities and

providing technical education" (Baer, 1983, p. 53).

The decrease of the role of the state in the economy during the second half of the 1950's is linked to the fall of the Vargas regime in 1945 and the idea of a free political system linked with the idea of no intervention of the government in the economy. Despite this decrease in its role, the state presented a five-year expenditure program, the SALTE plan, that was directed to the areas of health, food, transport, and energy. This plan was in effect only during one year, due to its overoptimistic estimation about the possibilities of revenues to allow its execution.

In the 1950's the participation of the state in the economy come back strongly, first with the Vargas government in the first half of the 1950's, and then with the Kubitschek government in the second half of the 1950's. However strong the participation of both governments in the economy, the focus of both programs were different. The Vargas policies were directed to the creation of a national capitalist system, while Kubitschek policies were direct to the integration of the Brazilian into the international economy.

In the period from 1951 to 1953 a Joint Brazil-United States Economic Commission was formed. Its main contribution to the Brazilian economy was the creation of BNDE (National Bank for Economic Development), an institution linked with the financing of long-term economic projects. This institution was to be of crucial importance in the development of the industrial sector in Brazil. And, at the same time, this institution contributed for the increasing participation of the state in the economy; this happen as some of the loans made by BNDE were not paid, and as a form of

receiving them, BNDE took over the debtor enterprises; making, indirectly, the government as the owner of those enterprises. BNDE also helped in the implementation of the first economic plans.

During the Vargas government a series of other state enterprises were created, besides BNDE, such as PETROBRAS (the state company linked with oil production, refinery, distribution, etc.), Banco do Nordeste do Brasil (Northeast Bank of Brazil), etc..

The first economic plan made for the Brazilian economy was the "Programa de Metas" (Target Plan) of the Kubitschek government; it predominated in the period 1956 to 1960. The basic idea of the plan was the development of an industrial complex for the Brazilian economy, through the implementation of a policy of Import Substitution Industrialization (ISI). It had the automotive industry as its leading sector. Besides the industrialization aspect, the plan also had as an objective the construction of a new capital (Brasília), that had, among other things, the objective of promoting national integration.

One can say that, in general, the plan was successful, as the most of the targets linked to the industrial production were attained, and as Brasília was built in a record time, 4 years. However, the implementation of the plan caused distortions in the economy, as imbalances in trade appeared and the inflation level increased.

At the end of the plan, one can say that the ISI process in the consumer goods industry was finished, i. e., there were no more possibilities for internal growth through the substitution of imported consumer goods for domestic consumer goods. This in

conjunction with the problems mentioned above, and to a certain extent, the inability of the next government to solve this problems, led to a crisis in the Brazilian economy at the beginning of the 1960's that culminate in a military coup in 1964.

The next government, of João Goulart, introduced the "Plano Trienal" (Three-Year Plan), prepared by his Planning Ministry (Celso Furtado), in 1963 "that was to control inflation drastically and systematically deal with the economy's principal imbalances. This plan was soon shelved when it became obvious that the government had neither the means nor the will to impose its stabilization and reform measures" (Baer, 1983, p.94).

After the military coup of 1964, the next president, Castello Branco, stated his basic economic measures in the "Programa de Ação Econômica do Governo" - PAEG (Program of Economic Action of the Government) that was used from 1964 to 1966. The basic economic policies in the plan were directed to stabilization programs whose main goals were to decrease inflation and to increase economic growth, as the economy was at an inflation level of 80% and had had very low or negative real growth rates (as measured by the GNP) at the beginning of the 1960's.

The results attained with the plan where not as good as the plan projected, in the sense that inflation decreased to 37.4% in 1966 while the plan forecasted 10%, and real GNP grew 4.4% while the plan forecasted 6.0% (see Martone, 1975). The economic measures stated in the plan included: the reorganization of capital markets; tax reform; changes in the exchange system; creation of a housing bank (BNH); changes in the labor relations; and a freeze in wages.

If the plan was not a success, it helped to set the basic institutional measures that: a) led to the Brazilian "miracle" at the end of the 1960's and beginning of the 1970's; and b) led to a process of increase in the concentration of income that lasted from the late 1960's to the early 1980's.

The Castello Branco administration also made a Ten-Year Plan for Economic and Social Development, but this one was never used, as the next government of Costa e Silva used its own plan from 1968 to 1970: "Plano Estratégico de Desenvolvimento" - PED (Strategic Development Plan).

The PED in a certain sense continued with the main goals of the PAEG, i. e., decrease in the inflation rate and increase in the growth rate of the GNP. In this sense, the PED was more successful as high real growth rates of GNP were attained in this period (average of 9.7% during the three year period), and the inflation level decreased to about 20% in 1970 (see Baer, 1983). This favorable results for the economy were attained, in part, due to the utilization of idle capacity in the industrial sectors.

The second half of the 1960's saw an increase in the participation of the government in the economy, as new state enterprises were created and the military government took over the control of the economy. It is interesting to note that the increased control of the government over the economy was linked with the basic objective of making the private sector stronger, and, at the same time, to continue the process of integration of Brazil with the international economy, a process begun in the Kubitschek government.

Following the PED, the government of President Médici presented the "I Plano Nacional de Desenvolvimento" - I PND (I National Development Plan) for the period 1972-1974, as the plan to guide the economic policies of his government. Due to the favorable aspects of the economy when the plan was presented, one can see an overoptimistic approach in it. The plan contained gigantic projects of development, such as the Tranzamazônica highway, the nuclear project, etc., all of which were beyond the country's possibilities. In a certain sense, they started the problems of trade balances that Brazil faces today. These problems were aggravated by the oil shocks, but without question, the overoptimistic government policies have a great share in today's problems.

In the period from 1971 to 1974, the economy grew at a real average rate of 12.18% (GNP) and the inflation level, measured by the general price index, went down to 16.3% in 1973, but increased to 33.8% in 1974, showing the first signs of problems with the economy (see Baer, 1983).

The II PND presented by President Geisel for the period 1975-1979 was still optimistic in relation to the Brazilian economy, but not as much as the I PND. The plan recognized the necessity for certain adjustments to be made in the economy. During the II PND period the government continued with the following policies initiated in the I PND: a) ISI in the capital goods industries; and b) diversification and expansion of exports.

The growth rate of real GNP fell from 9.5% in 1974 to 5.6% in 1975 and to 4.8% in 1978. The inflation level increased from 30.1%

in 1975 to 40.5% in 1978 (see Baer, 1983). Those were the first signs of the crisis that Brazil would face in the 1980's.

The government of President Figueiredo presented the III PND for the period 1980-1985. The interesting characteristic of this plan was that, at most, it was merely a letter of intentions. To start with, the III PND had no numbers in it. The plan in itself is contradictory, in the sense that there are a lot of conflicting goals, e. g., the plan expects that the agriculture sector will solve the Brazilian problems of food for internal consumption, production of energy, and exports without taking into account the existence, in the short- and medium-run, of trade-offs among those three goals.

As a result, one can say that the Figueiredo government did not have a defined economic policy. As a consequence of that, and of the structural problems started in the beginning of the 1970's, Brazil went through a period of crisis in the 1980's when negative real growth rates were attained and the inflation level skyrocketed to the 200% level. A positive point in Figueiredo government was the liberalization of the political system that led to the implementation of a true democracy in 1985.

The new democratic government was called "The New Republic", and as such, the economic plan developed has the name of "I Plano Nacional de Desenvolvimento - Nova República" - I PND - NR (I Nacional Development Plan - New Republic).

The main goals of the plan are to try to solve the economic problems created by the past Brazilian government, i. e., reduction in the level of inflation; maintenance of a real GNP growth rate

high enough to absorb the new labor force and to reduce the level of unemployment and underemployment in the economy; reduction of the external pressures over the Brazilian economy; reduction in the uneven distribution of income; etc.

It is still too early to estimate the results of this plan, but, without question, it is aimed in the direction of facing and correcting the economic and social problems that Brazil is experiencing today.

As one can see from the above, since 1956, when the first economic plan was laid out, every government has had its own plan; establishing in this way a tradition of planning in the Brazilian economy. Hence, this process creates a need for the construction of a multisectoral model that can be used to help in formulating the goals of any economic plan. The effort being made in this dissertation can be interpreted as one more step in this direction.

As a consequence of the constant intervention of the state in the economy, either through its direct economic policies or through its companies, the state became the main economic agent in the system. The discussion of whether this is bad or not is beyond the scope of this work. Of one thing one can be sure, that if the state had not intervened in the economy as it entered in the past, Brazil would not have reached the level of development that it has today.¹

The next section of this chapter will present an overview of the evolution of the income distribution from the 1960's to the 1980's, as well as a discussion of the theories behind the trends towards increasing disparities.

3.3. Income Distribution

In the process of economic development, in a capitalist economy, there is always a relationship between the way that the economy will grow and the way that the national income is distributed among the country's population. This happens because the production structure of a capitalist economy is in great part a reflex of the demand structure, and the demand structure is a reflex, in the most of it, of the distribution of income. In this way, for a country with a low per-capita income, as the income is more concentrated, the demand structure will tend to concentrate towards the consumption of durable consumer goods; and as the income is less concentrated, the demand structure will tend to concentrate towards the consumption of non-durable consumer goods.

In the Brazilian case, the process of industrialization started in the 1950's was directed to the production of durable consumer goods for the domestic consumption. Due to the low per-capita income in Brazil, this meant that for this kind of industrialization to give results one needs to have some degree of concentration of income in the economy, and for these industries to grow you have either to export these industries products or to expand the internal demand. In the 1960's and 1970's the Brazilian economy did both things, expanded exports and internal consumption. The way that internal consumption was increased was basically by concentrating the effort of industrialization and growth on only a small part of the population, and in specific regions of the country (mainly the Southeast and South regions). This could be done due to the vast

size of the Brazil territory and the size of the Brazilian population. In this way one can say that the process of industrialization of the Brazilian economy was done at the expenses of the most part of its population and its territory.

Discussions that can, and need to, be made are: was there another possibility to establish a well developed industrial sector in the Brazilian economy without the concentration of income at the personal and regional level? and given the actual structure of the Brazilian economy, is it possible to grow with a more equalitarian distribution of income? As these questions are not in the scope of this work, they will not be addressed here. The only discussion that will be carried out in this dissertation is related to the concentration of income during the 1960's and the 1970's, as this will be enough to show that any model that is or is going to be constructed for the Brazilian economy must take in consideration the income distribution problem. To do so, some of the theories behind the process of income concentration in the 1960's and 1970's will be discussed, as well as some data will be presented.

At the regional level, in 1980, per capital income varied "to such an extent that in many states of the northeast Brazil it was less than half the national average, while in the more advanced regions it was more than three times the national average" (Baer, 1983, p.4).

At the personal level, one can see from Tables 3.1 and 3.2 that the share of the lowest 50% of Brazil's income group, in the national income, decreased from 17.71% in 1960 to 14.91% in 1970, and to only 13.87% in 1980; while the share of the highest 20%

Table 3.1Comparison of Income Distribution by Income DecilesShares (%)Brazil: 1960, 1970, and 1980

Decile		Shares			Percentage Change		
		1960*	1970	1980*	1960/70	1970/80	1960/80
Lowest	10%	1.17	1.11	1.06	-5.13	-4.50	-9.40
	10	2.32	2.05	1.91	-11.64	-6.83	-17.67
	10	3.42	2.97	2.94	-13.16	-1.01	-14.04
	10	4.65	3.88	3.49	-16.55	-10.05	-24.95
	10	6.15	4.90	4.47	-20.32	-8.78	-27.32
	10	7.66	5.91	5.54	-22.75	-6.26	-27.68
	10	9.41	7.37	7.19	-21.68	-2.44	-23.59
	10	10.85	9.57	9.61	-11.80	+0.42	-11.43
	10	14.69	14.45	15.33	-1.64	+6.09	+4.36
Highest	10%	39.66	47.79	48.36	+20.50	+1.19	+21.94
Total		99.98	100.00	99.90	-	-	-

Note: * Total do not add to 100.00% due to rounding problems.

Sources: Langoni (1973), and Benevides (1985).

Table 3.2Comparison of Income Distribution by Income DecilesAccumulated Shares (%)Brazil: 1960, 1970, and 1980

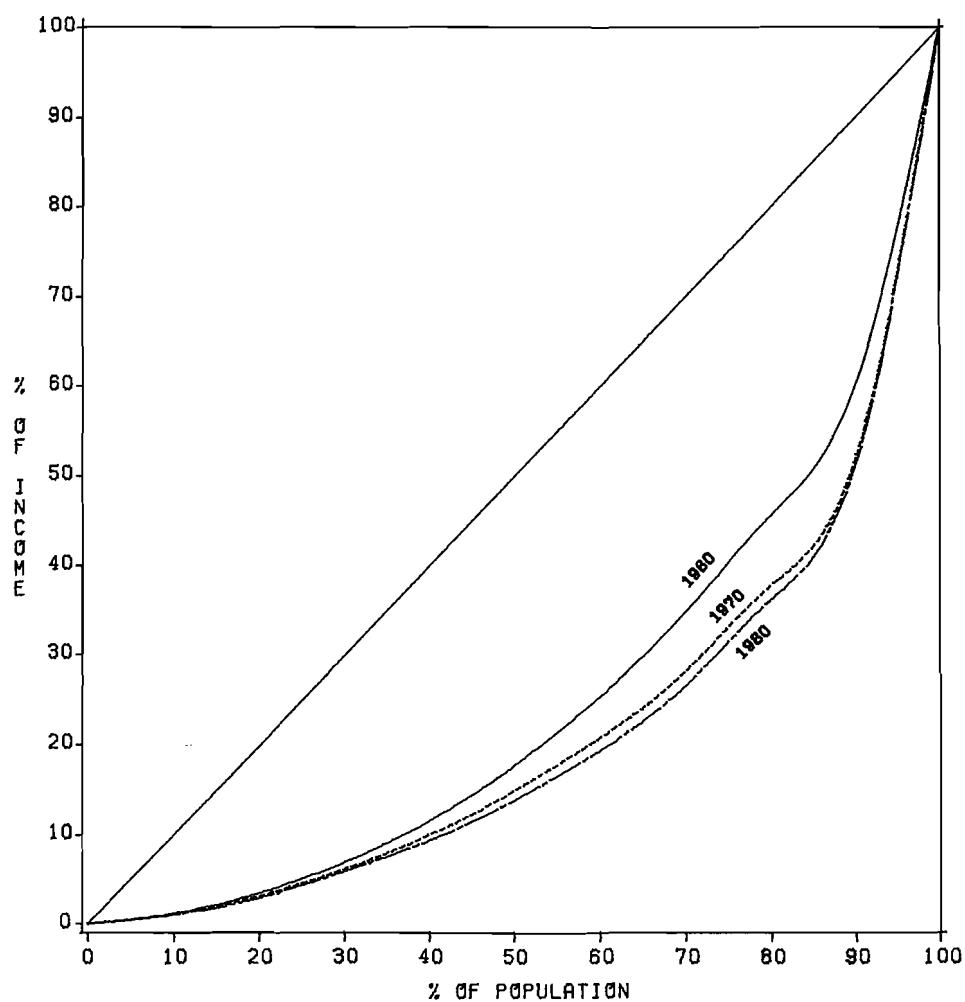
Decile		1960*		1970		1980*	
		From the		From the		From the	
		Lowest	Highest	Lowest	Highest	Lowest	Highest
Lowest	10%	1.17	99.98	1.11	100.00	1.06	99.90
	10	3.49	98.81	3.16	98.89	2.97	98.84
	10	6.91	96.49	6.13	96.84	5.91	96.93
	10	11.56	93.07	10.01	93.87	9.40	93.99
	10	17.71	88.42	14.91	89.99	13.87	90.50
	10	25.37	82.27	20.82	85.09	19.41	86.03
	10	34.78	74.61	28.19	79.18	26.60	80.49
	10	45.63	65.20	37.76	71.81	36.21	73.30
	10	60.32	54.35	52.21	62.24	51.54	63.69
Highest 10%		99.98	39.66	100.00	47.79	99.90	48.36

Note: * Total do not add to 100.00% due to rounding problems.

Source: Table 3.1.

Figure 3.1

Lorenz Curve for the Brazilian Economy:
1960, 1970, and 1980



Source: Table 3.2.

increased from 54.35% in 1960 to 62.24% in 1970, and to 63.69% in 1980. One can also see that the shares of each income group, with the exception of the highest 20%, decreased in the 1960/80 period. From those tables and from Figure 3.1, that shows the Lorenz curve for 1960, 1970, and 1980, one can also see that the concentration of income in the 1960's was greater than the one in the 1970's. This can be explained, in part by the fact that the most severe control of wages by the government occurred during the second half of the 1960's, soon after the military coup in 1964. During the 1970's the controls on wages decreased.

A look at Table 3.3 shows that the average income of each of the income groups increased from 1960 to 1970, and from 1970 to 1980. However, the groups from the 4th to the 7th decile (unskilled and semiskilled urban works) had their income growing less than the other groups (the lowest ones usually refer to agricultural workers, while the highest ones usually refer to urban skilled workers). This effect also can be seen in Figure 3.1. One can also see that the average income of the highest 20% was the one that increased the most. The results in Tables 3.1 and 3.2, and in Figure 3.1, are reflex of the way the average income increased in the 1960/80 period.

More support for the fact that there was a concentration of income in the 1960/80 period can be get from the inequality indices presented in Table 3.4, however measure of inequality one looks at, one can see a continuous increase in the concentration of income. The Gini index increased from 0.5 in 1960 to 0.568 in 1970, and 0.584 in 1980. The Theil index increased from 0.47 in 1960 to 0.644

Table 3.3Comparison of Income Distribution by Income DecilesAverage Income (1980 Cr\$ per month)Brazil: 1960, 1970, and 1980

Decile		Average Income			Percentage Change		
		1960	1970	1980	1960/70	1970/80	1960/80
Lowest	10%	618	791	1262	+27.99	+59.54	+104.21
	10	1187	1434	2503	+20.81	+74.55	+110.87
	10	1756	2077	3546	+18.28	+70.73	+101.94
	10	2374	2720	4341	+14.57	+59.60	+82.86
	10	3141	3437	5306	+9.42	+54.38	+68.93
	10	3907	4155	6562	+6.35	+57.93	+67.95
	10	4822	5193	8609	+7.69	+65.78	+78.54
	10	5564	6727	11690	+20.90	+73.78	+110.10
	10	7543	10164	18306	+34.75	+80.11	+142.69
Highest 10%		20155	33633	57612	+66.87	+71.30	+185.85
Total		5094	6974	11964	+36.91	+71.55	+134.86

Sources: Langoni (1973), and Benevides (1985).

Table 3.4Inequality IndicesBrazil: 1960, 1970, and 1980

Index	Value			Percentage Change		
	1960	1970	1980	1960/70	1970/80	1960/80
Gini	0.500	0.568	0.584	+13.60	+2.82	+16.80
Theil	0.470	0.644	0.703	+37.02	+9.16	+49.57
Variance of Logs	0.897	0.976	1.100	+8.81	+12.70	+22.63

Sources: Langoni (1973), and Benevides (1985).

in 1970, and 0.703 in 1980. And the Variance of Logs increased from 0.897 in 1960 to 0.976 in 1970, and 1.1 in 1980.

As it was show above, the income really concentrate in the Brazilian economy during the 1960/80 period. The question that should be asked now is: what factor(s) led to the income concentration during this period?

The answer to that question can be found by looking at the

various theories which one can draw to explain the process of concentration of income in Brazil during the last two decades. They can be summarized as:² a) Kuznets effects; b) skill differentials; c) wage squeeze; d) wage spread; and e) income concentration with growth. Each one of these theories will be discussed below.

The Kuznets effects theory states that changes in the employment structure due to the technological progress can lead to deterioration in the income distribution. This deterioration, in terms of increasing concentration, would be only in the intermediary phases of economic growth, after which there would be a decrease in the inequality after further economic growth.

This theory can be associated with the fact that when the workers migrate from the rural to the urban areas, there would be an increase in the concentration of income due to the fact that the distribution of income in the rural areas is more equalitarian than the one in urban areas.

Bacha and Taylor (1980) have shown that for the Brazilian case the migration rural-urban areas had an equalize effect, refuting the Kuznets effects theory. Refutes of this theory are also presented in Malan and Wells (1973), and Fishlow (1973).

The skill differentials theory is based on the idea that in the initial stages of economic growth there is a shortage of skilled labor and an excess of unskilled labor. Through the interaction of supply and demand, there would be an increase in the wages of skilled workers, generating in this way a process of concentration of income. Through the presentation of different models, Bacha and Taylor (1980) have presented arguments to refute this theory.

The wage squeeze theory is associated with the repressive wage policy applied after the 1964 military coup. This theory is linked with the fact that the wages that suffered with this policy were the ones of the unskilled and semiskilled urban workers (4th to 7th decile of income distribution), while the wages of skilled urban workers did not suffer with this policy. This contributed to the concentration of income, as showed above in the discussion of Table 3.3. This theory is also defended by Bacha and Taylor (1980).

The wage spread theory is linked with the idea that technological progress brings about increase in profits, and this increase in profits is distributed only among the administrative workers, leading in this was to a process of concentration of income.

This theory is supported by Bacha and Taylor (1980) who also put the public sector as the main agent of wage spread, and link this theory with the wage squeeze theory:

"These pieces and bits of evidence suggest that the public sector may have been the leading agent of wage spread since 1964. At least in part the higher salaries at the top were made possible by increases in taxes, public enterprise prices, and public utility rates. These could take place without reducing private profit rates because, at the same time, the wage policy was squeezing the remuneration of lower-paid occupations both in the public and private sector" (Bacha and Taylor, 1980, p. 335).

The income concentration with growth theory basically states that the concentration of income is needed because this is the only

way to generate the amount of savings necessary to invest and to continue the economic growth, without the need for the government to rely in the inflationary financing of the economic growth. This was achieved in the Brazilian case through tax incentives to invest, those incentives were basically used by high income groups. This theory is supported by Bacha and Taylor (1980).

From the above, what one can say, for the Brazilian case, is that the theories of Kuznets effects and skill differentials are dismissed, while the theories of wage squeeze, wage spread and income concentration with growth can be accepted as explanation for the worsening in the income distribution during the 1960's and 1970's.

NOTES

1. For a study of the evolution in the structure of Brazil's industrial sector from 1960 to 1980 see Baer, Guilhoto, and Fonseca (1985).
2. The following discussion about the theories of income distribution is based on Bacha and Taylor (1980).

CHAPTER 4

ECONOMY-WIDE MODELS APPLIED TO BRAZIL: A CRITICAL REVIEW

4.1. Introduction

As discussed in Chapter 2, the model being constructed here for the Brazilian economy is a economy-wide multisectoral model of the Johansen-type, i. e., it is solved for growth rates. This chapter will go over the literature of economy-wide models constructed for the Brazilian economy. It will not discuss the other types of input-output models constructed for Brazil,¹ as those models are not in the same class as the one to be presented in Chapter 5.

The author could only find in the literature three multisectoral economy-wide models constructed for the Brazilian economy. In chronological order, the first one is presented in Rijckeghem (1969) and it is of the Johansen-type. The model is used to make simulations about expected sectoral growth rates, given certain assumptions about the growth rates of other economic variables. The input-output data used in the model are for the year of 1959. The second model is presented in Lysy and Taylor (1980) and is a Computable General Equilibrium (CGE) model solved for levels and directed to the study of the income distribution problems in Brazil during the 1960's. The input-output source of data is again the 1959 matrix. The third and last model, as discussed in Werneck

(1984), is a economy-wide consistency model solved for growth rates;² the objective of the model is to examine the consequences of economic policies directed to programs of import substitution and expansion of exports, in the 1980's, on the structure of production, investment and growth of the economic sectors. The input-output source of data is the 1970 matrix for Brazil.

The above models can be said to be directed to the study of specific problems of the Brazilian economy. As such, they differ from the model being constructed here, in the sense that the model presented in Chapter 5 can be used to study different economic problems depending on the user's choice of exogenous and endogenous variables, and the value of the exogenous variables.

The Rijckeghem model is discussed in the next section; the Lysy and Taylor, and the Werneck models are discussed respectively in sections 4.3 and 4.4. A conclusion is presented in section 4.5.

4.2. The Rijckeghem Model

The Rijckeghem model (see Rijckeghem, 1969) is a simple economy-wide model solved for growth rates, in which the final result are sectoral growth rates for 32 sectors. Besides the input-output data (for 1959) the author makes use of other data, e. g., industrial censuses, family budgets, etc., to calculate the coefficients and parameters needed for the solution of the model. The economic relations presented in the model lead to a solution in which the growth rates in total production, of the various sectors,

are derived from the growth rates in final demands, i. e., individuals' consumption, government consumption, investment, exports, and the program of import substitution.

The model was used to simulate the possibility of the Brazilian economy growing at an real average rate of GNP of 7% in the period from 1968 to 1970. The model finds that this rate is feasible, and gives the resulting growth rates of the different sectors. In reality, the Brazilian economy grew at an average rate of 9.7% in the 1968-1970 period (see Baer, 1983).

The model does not present any specification for the problems of: income distribution, labor absorption, prices indices, trade margins, etc.. In summary, it is a simple model directed to relatively simple problems. It was also the first effort to build an economy-wide multisectoral model for the Brazilian economy, and given the state-of-the-art of computers and data at that time, one, probably, could not expect more than that from this model.

4.3. The Lysy and Taylor Model

The Lysy and Taylor model (see Lysy and Taylor, 1980) is a CGE model solved for levels. This model is a complete one, and it is directed to the study of the concentration of income in Brazil during the 1960's.

The general characteristics of the model can be summarized as follows: a) the model is solved for a 3 year time interval; b) there are 25 sectors, and a special sector to represent noncompetitive

imports of various types; c) there are 6 types of employed labor, 25 types of self-employed proprietors, and 4 types of employers; and d) there are 4 classes of expenditures.

The model constructed by Lysy and Taylor is highly data dependent; the input-output data for the model are derived from the 1959 input-output matrix and the remaining data needed for the estimation of the coefficients and parameters in the model are derived from a wide range of works. (The reader is referred to chapter 8 in Taylor, et. al. (1980) for a presentation of the data used in the model.)

As the Lysy and Taylor model is direct to the study of income distribution, one can see a very detailed specification in the types of employees and employers. Those are aggregated in four classes of expenditure, the individuals in each class have the same patterns of consumption. This is similar to the model being constructed here for the Brazilian economy, the difference being that the model presented in Chapter 5 has only 3 classes of labor, and 3 classes of expenditure.

The headings and sub-headings of the equations for the Lysy and Taylor model are summarized in Table 4.1.

As can be seen by the headings and sub-headings in Table 4.1, the Lysy and Taylor model is a very detailed one, but not as much as the one being constructed here. The greater detail in the model being constructed in this dissertation should not be interpreted to imply that it has a more complex solution. On the contrary, due to the fact that the Lysy and Taylor model is solved for levels and the model being constructed here is solved for growth rates, the

Table 4.1

Headings and Sub-Headings of the Equations in the
Lysy and Taylor Model

a. Cost Functions and Input Coefficients:

- | | |
|-------------------------|---------------------------------|
| a.1. Labor; | a.5. Technical progress; |
| a.2. Capital; | a.6. Interindustry factor price |
| a.3. Value added; | structure; |
| a.4. Producer's prices; | a.7. Price determination. |

b. Employment and Income Levels:

- | | |
|-------------------------|-------------------------------|
| b.1. Employment levels; | b.4. Incomes per participant; |
| b.2. Labor incomes; | b.5. Total consumption levels |
| b.3. Capital incomes; | by class. |

c. Sectoral Consumption Functions:

- | | |
|--|--------------------------------|
| c.1. Consumer's prices; | c.3. Total consumption demand. |
| c.2. Utility and expenditure
functions; | |

d. Investment Functions:

- | | |
|-----------------------------|------------------------------|
| d.1. Net capital formation; | d.2. Replacement investment. |
|-----------------------------|------------------------------|

e. Commodity Balances.

f. National Accounting:

- | | |
|---------------------------|------------------------------|
| f.1. Balance of payments; | f.3. Private saving; |
| f.2. Government; | f.4. Gross national product. |

Source: Lysy and Taylor (1980).

flexibility of the latter is greater than of the former.

If one takes a look in Chapter 5, or even in the headings of the equations presented in Table 5.1, one can see that the model being constructed here presents a more detailed approach to: trade margins; consumption of domestic and imported products; investment functions; government taxes, tariffs, and subsidies; demands for labor, capital, and land; change in the technical coefficients; etc.. In addition, this model presents price indices for consumer and capital-goods prices, these are not features present in the Lysy and Taylor model.

The Lysy and Taylor model is solved for levels, and the system of equations requires a non-linear solution. In the case of the model being constructed here, the equations are given in linear form, and the solution algorithm requires only matrix algebra.

Depending on the way that the Lysy and Taylor model is closed, the results can be completely different. This is shown by Taylor and Lysy (1979) who discuss the different results obtained by closing an one sector model in three different ways, one neo-classical and two Keynesian. The ways that the Lysy and Taylor model can be closed are presented in Chapter 7 of Taylor, et. al. (1980). A discussion about closure in the model constructed in this dissertation is presented in Chapter 7.

The method of solution for the Lysy and Taylor model is presented in Chapter 7 of Taylor, et. al. (1980). Simulations and results are presented and discussed in Chapters 8, 9, and 10 of Taylor, et. al. (1980). The reader is referred to these chapters for a discussion of these topics.

4.4. The Werneck Model

The Werneck model (see Werneck, 1984) is an economy-wide consistency model solved for growth rates, in which the final results are: sectoral growth rates of production; and the share of each sector in the value added, and in the total investment made in the economy. The model distinguishes 30 different sectors.

The input-output data used in the model are from the 1970 input-output matrix for Brazil (see IBGE (1979) for a presentation of this matrix). The remaining data needed for the simulation of the model is discussed in Werneck (1982), and the reader is referred to this work for further details.

The way that the model works can be summarized in Werneck's own words:

"Given the planning horizon and establishing, on one side, the sectoral targets of import substitution and export expansion, and on the other side, the expected average growth rate of the aggregated product over the plan horizon, it is expected that the model will generate the sectoral growth rates required, the investment program involved and the implicit modifications, also in sectoral terms, in the productive structure" (Werneck, 1984, p.314).

The planning horizon used in the model is the period from 1978 to 1990.

Due to the objectives of the model, its equations are highly detailed in relation to investment, and the external sector. The

model also highlights the role of the government in the economy. The model was not constructed for the purpose of allowing the study of any other topic than the one explained above. And as such, it is limited in its power of analysis.

The results for the model show that with low growth rates of the aggregated product the relative economic importance of the sectors linked with import substitution and export expansion increases. As the growth rates of the aggregated product increases, the importance of sectors linked with external trade decreases. This was a result already expected, because an increase in the growth rates of the aggregate product means that the internal demand for domestic products is increasing, and as such, the dynamic market becomes the internal rather than the external one.

Concluding, one can say that the Werneck model is constructed with the objective of studying a specific economic problem; in this way it fulfilled its target. The model being constructed here for the Brazilian economy, on the other hand, is a general purpose model; and as such, it is more flexible than the Werneck model.

4.5. Conclusion

The multisectoral economy-wide models discussed above were constructed with the objective of studying certain economic problems, while the model being constructed here is a general purpose model. When the previous models were constructed, they probably did not have the amount of data information that is

available today, e. g., the 1975 input-output matrix for Brazil only became available in 1984 (see IBGE, 1984b). This could also be a restrictive factor in the specification of the equations in the models. Another restrictive factor could also be in terms of computer capability.

The fact is that the model being constructed here is more detailed than the previous ones and, at the same time allows for a study of a diversity of economic policies and problems.

This can be viewed as a natural evolution in the construction of economy-wide models for the Brazilian economy. Evolution that is linked with: developments in the theory of economy-wide modelling; the existence of more and better data for the Brazilian economy; the progress in computers' technology; and the development of the Brazilian economy to more complex stages of economic development.

NOTES

1. For a presentation of the characteristics of the different types of input-output models see Chapter 2.
2. For a discussion of consistency models see Clark (1975).

CHAPTER 5

A MULTISECTORAL MODEL FOR THE BRAZILIAN ECONOMY

5.1. An Overview of the Model

The model to be presented in this chapter belongs to the Johansen class of multisectoral models (see Johansen, 1974), and is based on the ORANI model for the Australian economy, as presented in DPSV (1982).

The main differences from the ORANI model and the one presented here are: a) the government sector have a special treatment in the Brazilian model; b) the present model breaks down the demand for household consumption by different income groups, and introduces an equation linking the workers income with their expenditures; allowing, in this way, for the study of income distribution problems; c) the model presented here assumes that each industry produces only one type of commodity, contrary to the ORANI model that uses a more general industry by commodity framework; and d) in this model it is assumed that firms minimize costs, and form their prices through a mark-up theory; the ORANI model assumes that firms form their prices by maximizing profits.

The results of the model are given in percentage changes, which can be interpreted in the following way: for a given policy change

A, in the macroeconomic environment B, variable C will differ in the short run by x percent from the value it would have had in the absence of the policy change; in the long run it will differ by y percent. Thus the model involves a comparative static analysis.

The difference between the short- and the long-run is that in the long-run capital stocks are allowed to change.

The results for the model are not given for a specific period of time, but for the time necessary for the variables to adjust themselves to a new from an old equilibrium position that was disrupted by a given policy change A.

To derive the equations for the model, one starts from equations in level form and then derives the percentage-change form for this equation. In doing so, one will get a linear system of equations in which the number of variables will be greater than the number of equations. In using the model, some of the variables will have to be made exogenous to solve the system. The choice of what is exogenous or endogenous will vary according to the uses to which the model will be put; however, the choice does involve considerable judgement and is not without problems (for a discussion of closure in this model see Chapter 7).

Model equations are derived for industry demands, final demands, prices, investment allocation, market-clearing, and miscellaneous definitions.

The derivation of the equations in the Brazilian model is based on Chapter 3 in DPSV (1982). And the variables, coefficients and parameters in the model are presented first with a generic range.

The values of the ranges chosen to be used in the model are presented at the end of this Chapter, in section 5.13. Section 5.13 also presents Table 5.1 that contains the list of the equations that form the Brazilian model, and Table 5.2 with the list of variables in the model. For a list of the coefficients and parameter in the model, the reader is referred to Table 6.1 in Chapter 6.

The model distinguishes between: h types of industries; g types of products;¹ 3 types of primary factors: M categories of labor, fixed capital (building, plant and machinery), and agricultural land; one type of "other costs" (production taxes, costs of holding liquidities, cost of holding inventories, and other miscellaneous production costs); 2 sources of products (1. domestic, and 2. imported); 6 types of product use (1. inputs to current production, 2. inputs to capital formation, 3. commodity flows to household consumption, 4. exports, 5. government demands, and 6. "other" demands); and D income groups.

The type of notation used is as follows: upper case letters are used for variables levels and the corresponding lower case letters for their percentage changes (when this is not the case, it is indicated in the text); the superscript (0) refers to outputs, and superscripts (1) to (6) refer to the 6 types of products use indicated above. All the other notation will be explained as they appear in the model presentation.

One last observation is that the equations that belong to the final system are enclosed in boxes.

5.2. The Production Functions for Current Output

It is assumed that each industry produces only one type of commodity from the combination of the different inputs.

5.2.1. Inputs and Activity Level

For each industry j it is assumed that:

$$\text{Leontief}_{i=1, \dots, g+2} \{X_{ij}^{(1)} / A_{ij}^{(1)}\} = A_j^{(1)} Z_j$$

$$j = 1, \dots, h \quad (5.1)$$

where:

$$\text{Leontief}_{i=1, \dots, r} \{f_i\} = \text{minimum} \{f_1, f_2, \dots, f_r\} \quad (5.2)$$

and where $X_{ij}^{(1)}$ is the effective input (defined in eqs. 5.3 and 5.5) of good or factor i into current production,² Z_j is industry j 's activity level and the $A_{ij}^{(1)}$'s and $A_j^{(1)}$'s are technological coefficients.

Units of a given input are combined between domestic and imported sources in the following way:

$$X_{ij}^{(1)} = \text{CES}_{s=1,2} \{X_{(is)j}^{(1)} / A_{(is)j}^{(1)}; \rho_{ij}^{(1)}, b_{(is)j}^{(1)}\}$$

$$i = 1, \dots, g, \quad j = 1, \dots, h \quad (5.3)$$

where $X_{(is)j}^{(1)}$ is the input of i from source s to current production in industry j ,³ and $A_{(is)j}^{(1)}$ are positive coefficients. The notation $\text{CES}_{s=1,2} \{f_s; \rho, b_s\}$ means that the variables f_s , $s=1,2$, are to be aggregated according to a CES function with parameter ρ and b_s , i.e.

$$CES_S\{f_S; \rho, b_S\} \equiv (\sum_S f_S^{-\rho} b_S)^{-(1/\rho)} \quad (5.4)$$

where:

$$b_S \geq 0$$

$$-1 < \rho \quad \text{and} \quad \rho \neq 0$$

For simplicity, the LHS of eq. (5.4) can be written as $CES_S(f_S)$.

For primary factors, $X_{g+1,j}^{(1)}$, is given by:

$$X_{g+1,j}^{(1)} = CRESH_{s=1,2,3}\{X_{(g+1,s)j}^{(1)} / A_{(g+1,s)j}^{(1)}; \quad (5.5)$$

$$h_{(g+1,s)j}^{(1)}, Q_{(g+1,s)j}^{(1)}, \kappa_{g+1,j}^{(1)}\}$$

where $X_{(g+1,s)j}^{(1)}$ is the input of primary factor of type s to production in industry j and the $A_{(g+1,s)j}^{(1)}$'s are positive coefficients used in simulating the effects of technical change.⁴ The notation $CRESH_{s=1,2,3}\{f_S; h_S, Q_S, \kappa\}$ means that the variables f_S , $s = 1, 2, 3$, are to be aggregated according to a CRESH (constant ratios of elasticities of substitution, homothetic) function with parameters h_S , Q_S , $s = 1, 2, 3$, and κ ,⁵ i.e.

$$X = CRESH_{s=1,2,3}\{f_S; h_S, Q_S, \kappa\} \quad (5.6)$$

implies that:

$$\sum_{s=1}^3 (f_S / X)^{h_S} (Q_S / h_S) = \kappa \quad (5.7)$$

where:

$$h_S < 1 \text{ and } h_S \neq 0, \quad s = 1, 2, 3$$

$$Q_S \geq 0, \quad s = 1, 2, 3$$

$$\sum_{s=1}^3 Q_S = 1$$

For simplicity, the RHS of eq. (5.6) can be written as $CRESH_S(f_S)$.

CRESH functions have the following characteristics: 1. constant returns to scale; 2. positive marginal products and diminishing

marginal rates of substitution. To find the marginal product of input r , f_r is allowed to change, but all other input levels are held constant. Then the change in the LHS of (5.7) is given by

$$\frac{Q_r f_r^{h_r-1}}{x^{h_r}} df_r - \sum_s \frac{f_s^{h_s} Q_s}{x^{h_s+1}} dx = 0 \quad (5.8)$$

Hence, the marginal product of input r is

$$\frac{dx}{df_r} = Q_r \left(\frac{f_r}{x} \right)^{h_r-1} / \sum_s Q_s \left(\frac{f_s}{x} \right)^{h_s} \quad (5.9)$$

and the marginal rate of substitution of input r for input t is:

$$MRS_{tr} \equiv \frac{\partial X / \partial f_t}{\partial X / \partial f_r} = \frac{Q_t (f_t/x)^{h_t-1}}{Q_r (f_r/x)^{h_r-1}} \quad (5.10)$$

where (h_t-1) and (h_r-1) are negative.

CRESH is also a generalization of CES. If $h_s = h$, for all s , then (5.7) implies that:

$$X = CES_S \{f_s; -h, Q_s/h\}$$

The potential advantage of CRESH over CES is that it allows the elasticity of substitution between labor and capital to differ from that between capital and agricultural land. In turn, these elasticities can differ from the elasticity of substitution between labor and agricultural land. Under CES, on the other hand, all substitution elasticities are given by:

$$\sigma = 1 / (1 + \rho) \quad (5.11)$$

The primary factor, labor, is disaggregated into M skill categories. The effective input of labor into industry j is given by:

$$X_{(g+1,1)j}^{(1)} = CRESH_{m=1, \dots, M} \{X_{(g+1,1,m)j}^{(1)} / A_{(g+1,1,m)j}^{(1)}\} \\ j = 1, \dots, h \quad (5.12)$$

where $X_{(g+1,1,m)j}^{(1)}$ is the input of primary factor (g+1) from source 1 (i.e. labor) of skill group m used in current production in industry j. Again, the A's are positive coefficients that can be used in simulation of the effects of technical changes.

5.2.2. Demands for Directly Used Inputs for Current Production:

A Cost Minimizing Problem

It is assumed that producers treat all factors of production as variables. In particular they act as if they rent their fixed capital and agricultural land, whose rental prices are given. Also, capital and agricultural land are assumed to be nonshiftable between industries, i.e., industry-specific.

The cost minimizing problem for industry j, $j = 1, \dots, h$, is as follows: choose the input levels

$$X_{ij}^{(1)} \quad , \quad i = 1, \dots, g+2$$

(effective intermediate and primary inputs),

$$X_{(is)j}^{(1)} \quad , \quad i = 1, \dots, g \quad , \quad s = 1, 2$$

(intermediate inputs, domestic and imported),

$$X_{(g+1,s)j}^{(1)} \quad , \quad s = 1, 2, 3$$

(overall labor input, capital and agricultural land),

$$X_{(g+1,1,m)j}^{(1)} \quad , \quad m = 1, \dots, M$$

(labor by M different skill groups).

to minimize

$$\begin{aligned}
& \sum_{i=1}^g \sum_{s=1}^2 P_{(is)j}^{(1)} X_{(is)j}^{(1)} \\
& + \sum_{m=1}^M P_{(g+1,1,m)j}^{(1)} X_{(g+1,1,m)j}^{(1)} \\
& + \sum_{s=2}^3 P_{(g+1,s)j}^{(1)} X_{(g+1,s)j}^{(1)} \\
& + P_{g+2,j}^{(1)} X_{g+2,j}^{(1)}
\end{aligned} \tag{5.13}$$

subject to (5.1), (5.3), (5.5) and (5.12).

Where Z_j and the P 's are treated as exogenous variables. $P_{(is)j}^{(1)}$, for $i = 1, \dots, g$ and $s = 1, 2$, is the cost to industry j of one unit of intermediate input from source s . $P_{(g+1,1,m)j}^{(1)}$ is the price to industry j of a unit of labor of skill m . $P_{(g+1,s)j}^{(1)}$, $s = 2, 3$, are the rental costs to industry j of units of capital and agricultural land. $P_{g+2,j}^{(1)}$ is the price to industry j of "other costs".

The solution of the cost minimization problem will be carried out in stages, and will be presented in the following sub-sections.

5.2.2.1. Solution for the Demands for Domestic and Imported Intermediate Inputs

From (5.1) and (5.3) one can note that for each i , $i = 1, \dots, g$, $X_{(i1)j}^{(1)}$ and $X_{(i2)j}^{(1)}$ will be chosen to minimize

$$P_{(i1)j}^{(1)} X_{(i1)j}^{(1)} + P_{(i2)j}^{(1)} X_{(i2)j}^{(1)}$$

s.t.

(5.14)

$$A_j^{(1)} A_{ij}^{(1)} Z_j = CES_s(X_{(is)j}^{(1)} / A_{(is)j}^{(1)})$$

i.e., the effective input of good i , $A_j^{(1)} A_{ij}^{(1)} Z_j$, required to sustain the activity level Z_j will be provided by the cost minimizing combination of imported and domestic inputs of good i .

Problem (5.14) can be rewritten conveniently as: choose $\bar{X}_{(is)j}^{(1)}$, $s = 1, 2$ to minimize

$$\sum_{s=1}^2 \bar{P}_{(is)j}^{(1)} \bar{X}_{(is)j}^{(1)}$$

s.t.

(5.15)

$$\bar{Z}_j = CES_s (\bar{X}_{(is)j}^{(1)})$$

where:

$$\bar{X}_{(is)j}^{(1)} = X_{(is)j}^{(1)} / A_{(is)j}^{(1)} \quad (5.16)$$

$$\bar{P}_{(is)j}^{(1)} = A_{(is)j}^{(1)} P_{(is)j}^{(1)} \quad (5.17)$$

$$\bar{Z}_j = A_j^{(1)} A_{ij}^{(1)} Z_j \quad (5.18)$$

The first order condition for problem (5.14) are:

$$\bar{P}_{(is)j}^{(1)} - \Lambda \{ \partial CES_s (\bar{X}_{(is)j}^{(1)}) / \partial \bar{X}_{(is)j}^{(1)} \} = 0$$

$$s = 1, 2 \quad (5.19)$$

$$\bar{Z}_j - CES_s (\bar{X}_{(is)j}^{(1)}) = 0 \quad (5.20)$$

Where Λ is the Lagrangian multiplier. The first of these conditions may be rewritten as:

$$\bar{P}_{(is)j}^{(1)} - \Lambda b_{(is)j}^{(1)} (\bar{X}_{(is)j}^{(1)} / \bar{Z}_j)^{-\rho_{ij}^{(1)} - 1} = 0$$

$$s = 1, 2 \quad (5.21)$$

Then by using (5.20) and (5.21) one could derive functions of the form:

$$\bar{X}_{(is)j}^{(1)} = f_{(is)j}^{(1)} (\bar{Z}_j, \bar{P}_{(i1)j}^{(1)}, \bar{P}_{(i2)j}^{(1)})$$

$$s = 1, 2 \quad (5.22)$$

By substitution from (5.16)-(5.18) into (5.22) one would obtain demand functions for imported and domestic intermediate inputs to industry j . However, rather than finding the explicit form for $f_{(is)j}^{(1)}$, it will be sufficient to work in percentage changes. Next, the first-order conditions (5.20) and (5.21) are expressed in terms of percentage changes.

From (5.21), one gets:

$$\bar{p}_{(is)j}^{(1)} - \lambda + (\rho_{ij}^{(1)} + 1) (\bar{x}_{(is)j}^{(1)} - \bar{z}_j) = 0, \quad s = 1, 2 \quad (5.23)$$

where the lower case symbols denote percentage changes in the variables represented by the corresponding upper case symbols. In general, the following notation will be used:⁶

$$y = 100 (dY / Y)$$

By totally differentiating (5.20) and using (5.19), one finds that:

$$d\bar{z}_j = \sum_s (\partial CES_s / \partial \bar{x}_{(is)j}^{(1)}) d\bar{x}_{(is)j}^{(1)}$$

i.e.

$$d\bar{z}_j = \sum_s (\bar{p}_{(is)j}^{(1)} d\bar{x}_{(is)j}^{(1)} / \Lambda)$$

and

$$\bar{z}_j = \sum_s (\bar{p}_{(is)j}^{(1)} \bar{x}_{(is)j}^{(1)} / \Lambda \bar{z}_j) \bar{x}_{(is)j}^{(1)} \quad (5.24)$$

(5.24) can be written as:

$$\bar{z}_j = \sum_s S_{(is)j}^{(1)} \bar{x}_{(is)j}^{(1)} \quad (5.25)$$

where:

$$S_{(is)j}^{(1)} = (\bar{p}_{(is)j}^{(1)} \bar{x}_{(is)j}^{(1)}) / \Lambda \bar{z}_j \quad (5.26)$$

Equations (5.23) and (5.25) are linear in $\bar{x}_{(is)j}^{(1)}$, $\bar{p}_{(is)j}^{(1)}$, $s = 1, 2$, λ and \bar{z}_j . λ may be eliminated so that one can express the $\bar{x}_{(is)j}^{(1)}$'s as linear functions of the $\bar{p}_{(is)j}^{(1)}$'s and \bar{z}_j . However, before doing that, the $S_{(is)j}^{(1)}$ will be interpreted.

One can show that:

$$S_{(is)j}^{(1)} = P_{(is)j}^{(1)} X_{(is)j}^{(1)} / \sum_s P_{(is)j}^{(1)} X_{(is)j}^{(1)} \quad (5.27)$$

i.e., $S_{(is)j}^{(1)}$ is the share of good i from source s in the total cost of the inputs of i to industry j . To establish (5.27), one multiplies (5.19) through by $\bar{x}_{(is)j}^{(1)}$ and aggregate over s . This

gives:

$$\sum_s \bar{p}_{(is)j}^{(1)} \bar{x}_{(is)j}^{(1)} - \lambda \sum_s (\partial CES_s / \partial \bar{x}_{(is)j}^{(1)}) \bar{x}_{(is)j}^{(1)} = 0 \quad (5.28)$$

Applying Euler's theorem, (5.28) reduces to:

$$\sum_s \bar{p}_{(is)j}^{(1)} \bar{x}_{(is)j}^{(1)} - \lambda \bar{z}_j = 0 \quad (5.29)$$

(5.29) together with (5.26), (5.16) and (5.17) implies (5.27).

Now, back to equation (5.23); by multiplying (5.23) through by $S_{(is)j}^{(1)}$, aggregating over s , and using (5.25), one finds that:

$$\lambda = \sum_s \bar{p}_{(is)j}^{(1)} S_{(is)j}^{(1)} \quad (5.30)$$

Substituting (5.30) into (5.23) gives:

$$\bar{x}_{(is)j}^{(1)} = \bar{z}_j - \sigma_{ij}^{(1)} (\bar{p}_{(is)j}^{(1)} - \sum_{s=1,2} \bar{p}_{(is)j}^{(1)} S_{(is)j}^{(1)}) \quad (5.31)$$

where:

$$\sigma_{ij}^{(1)} = 1 / (1 + \rho_{ij}^{(1)})$$

$\sigma_{ij}^{(1)}$ is the elasticity of substitution in industry j between intermediate inputs of good i from domestic and foreign sources.

Finally, from (5.16)-(5.18) one has:

$$\bar{x}_{(is)j}^{(1)} = x_{(is)j}^{(1)} - a_{(is)j}^{(1)} \quad (5.32)$$

$$\bar{p}_{(is)j}^{(1)} = p_{(is)j}^{(1)} + a_{(is)j}^{(1)} \quad (5.33)$$

$$\bar{z}_j = a_j^{(1)} + a_{ij}^{(1)} + z_j \quad (5.34)$$

On substituting (5.32)-(5.34) into (5.31) one obtains the intermediate input demand functions:

$$x_{(is)j}^{(1)} = z_j - \sigma_{ij}^{(1)} (p_{(is)j}^{(1)} - \sum_s S_{(is)j}^{(1)} p_{(is)j}^{(1)}) + a_j^{(1)} + a_{ij}^{(1)} + a_{(is)j}^{(1)} - \sigma_{ij}^{(1)} (a_{(is)j}^{(1)} - \sum_s S_{(is)j}^{(1)} a_{(is)j}^{(1)})$$

$$i = 1, \dots, g, \quad s = 1, 2, \quad j = 1, \dots, h$$

(5.35)

5.2.2.2. Solution for the Demands for Primary Factors

Industry j will choose inputs of labor, fixed capital and agricultural land to minimize:

$$\sum_{m=1}^M \bar{P}_{(g+1,1,m)j}^{(1)} \bar{X}_{(g+1,1,m)j}^{(1)} + \bar{P}_{(g+1,2)j}^{(1)} \bar{X}_{(g+1,2)j}^{(1)} + \bar{P}_{(g+1,3)j}^{(1)} \bar{X}_{(g+1,3)j}^{(1)} \quad (5.36)$$

s.t.

$$\bar{Z}_j^{(1)} = \text{CRESH}_{s=1,2,3}(\bar{X}_{(g+1,s)j}^{(1)}) \quad (5.37)$$

$$\bar{X}_{(g+1,1)j}^{(1)} = \text{CRESH}_{m=1,\dots,M}(\bar{X}_{(g+1,1,m)j}^{(1)}) \quad (5.38)$$

where:

$$\bar{P}_{(g+1,1,m)j}^{(1)} = P_{(g+1,1,m)j}^{(1)} A_{(g+1,1)j}^{(1)} A_{(g+1,1,m)j}^{(1)} \quad m = 1, \dots, M \quad (5.39)$$

$$\bar{X}_{(g+1,1,m)j}^{(1)} = X_{(g+1,1,m)j}^{(1)} / (A_{(g+1,1)j}^{(1)} A_{(g+1,1,m)j}^{(1)}) \quad m = 1, \dots, M \quad (5.40)$$

$$\bar{P}_{(g+1,s)j}^{(1)} = P_{(g+1,s)j}^{(1)} A_{(g+1,s)j}^{(1)} \quad s = 1, 2, 3 \quad (5.41)$$

$$\bar{X}_{(g+1,s)j}^{(1)} = X_{(g+1,s)j}^{(1)} / A_{(g+1,s)j}^{(1)} \quad s = 1, 2, 3 \quad (5.42)$$

$$\bar{Z}_j^{(1)} = A_j^{(1)} A_{g+1,j}^{(1)} Z_j \quad (5.43)$$

Industry j chooses the combination of capital, agricultural land and labor of different skill categories to minimize the costs of providing the effective input, $A_j^{(1)} A_{g+1,j}^{(1)} Z_j$, of primary factors required to sustain the activity level Z_j .

The first order conditions for problem (5.36)-(5.38) are:

$$\bar{P}_{(g+1,1,q)j}^{(1)} - \bar{P}_{(g+1,1)j}^{(1)} \frac{\partial \text{CRESH}_m(\bar{X}_{(g+1,1,m)j}^{(1)})}{\partial \bar{X}_{(g+1,1,q)j}^{(1)}} = 0 \quad q = 1, \dots, M \quad (5.44)$$

$$\bar{X}_{(g+1,1)j}^{(1)} - \text{CRESH}_m(\bar{X}_{(g+1,1,m)j}^{(1)}) = 0 \quad (5.45)$$

$$\bar{P}_{(g+1,v)j}^{(1)} - \Lambda \frac{\partial \text{CRESH}_s(\bar{X}_{(g+1,s)j}^{(1)})}{\partial \bar{X}_{(g+1,v)j}^{(1)}} = 0$$

$$v = 1, 2, 3 \quad (5.46)$$

$$\bar{Z}_j^{(1)} - \text{CRESH}_s(\bar{X}_{(g+1,s)j}^{(1)}) = 0 \quad (5.47)$$

where Λ and $\bar{P}_{(g+1,1)j}^{(1)}$ are the Lagrangian multipliers.

The same process that one used to derive intermediate input demand equations, (5.35), from eqs. (5.19)-(5.20) is repeated here. That is, one expresses the first order conditions (5.44)-(5.47) in linear percentage form. First one works with eqs. (5.44) and (5.45) and focuses attention on industry j 's demand for labor by occupational group. Later on, one will derive industry j 's demand equations for labor in general, capital and agricultural land by looking at (5.46) and (5.47).

First, one rewrites eqs. (5.44) and (5.45) as:

$$\bar{P}_q - \bar{P} [\partial \text{CRESH}_m(\bar{X}_m) / \partial \bar{X}_q] = 0, \quad q = 1, \dots, M \quad (5.48)$$

$$\bar{X} - \text{CRESH}_m(\bar{X}_m) = 0 \quad (5.49)$$

What one has done is to drop those subscripts and superscripts which are inessential for the present purposes.

From (5.9) one knows that:

$$\frac{\partial \text{CRESH}_m(\bar{X}_m)}{\partial \bar{X}_q} = Q_q \left(\frac{\bar{X}_q}{\bar{X}} \right)^{h_q-1} / \sum_{u=1}^M Q_u \left(\frac{\bar{X}_u}{\bar{X}} \right)^{h_u} \quad (5.50)$$

Hence, in percentage change form (5.50) becomes:

$$\bar{P}_q = \bar{P} + (h_q-1)(\bar{X}_q/\bar{X}) - \sum_{u=1}^M h_u (\bar{X}_u/\bar{X}) S_u, \quad q = 1, \dots, M \quad (5.51)$$

where:

$$S_u = Q_u (\bar{X}_u/\bar{X})^{h_u} / \sum_{u=1}^M Q_u (\bar{X}_u/\bar{X})^{h_u} \quad (5.52)$$

It follows from (5.50) and (5.52) that:

$$S_q = [\partial \text{CRESH}_m(\bar{X}_m) / \partial \bar{X}_q] (\bar{X}_q / \bar{X}) \quad (5.53)$$

Thus, from (5.49) one sees that:

$$\bar{X} = \sum_{u=1}^M S_u \bar{X}_u \quad (5.54)$$

Notice, also, that substituting (5.53) into (5.48) yields:

$$S_q = \bar{P}_q \bar{X}_q / \bar{P} \bar{X}$$

i.e.:

$$S_q = P_q X_q / P X \quad (5.55)$$

Since from (5.52) one knows that:

$$\sum_u S_u = 1$$

it follows that:

$$P X = \sum_q P_q X_q$$

or using full notation:

$$P_{(g+1,1)j}^{(1)} X_{(g+1,1)j}^{(1)} = \sum_{q=1}^M P_{(g+1,1,q)j}^{(1)} X_{(g+1,1,q)j}^{(1)} \quad (5.56)$$

Consequently, in full notation one sees that (5.55) implies that:

$$S_{(g+1,q)j}^{(1)} = \frac{P_{(g+1,1,q)j}^{(1)} X_{(g+1,1,q)j}^{(1)}}{\sum_q P_{(g+1,1,q)j}^{(1)} X_{(g+1,1,q)j}^{(1)}} \quad (5.57)$$

$$q = 1, \dots, M$$

i.e., $S_{(g+1,1,q)j}^{(1)}$ is the share of skill q in the total labor cost of industry j .

Returning to the short-hand notation and rearranging (5.51) one has:

$$\bar{X}_q = [1 / (h_q - 1)] (\bar{P}_q - \bar{P}) + \bar{X} + [1 / (h_q - 1)] \sum_u h_u (\bar{X}_u - \bar{X}) S_u \quad (5.58)$$

$$q = 1, \dots, M$$

On multiplying (5.58) through by S_q , aggregating over all q and applying (5.54), one gets:

$$\sum_u h_u (\bar{X}_u - \bar{X}) S_u = - \sum_q S_q^* (\bar{P}_q - \bar{P}) \quad (5.59)$$

where S_q^* is the modified cost share defined by:

$$S_q^* = [S_q / (h_q - 1)] / \sum_{q=1}^M [S_q / (h_q - 1)] \quad (5.60)$$

Then substituting from (5.59) into (5.58), one sees that:

$$\bar{x}_q = \bar{x} + [1 / (h_q - 1)] (\bar{p}_q - \sum_{q=1}^M S_q^* \bar{p}_q) \quad , \quad q = 1, \dots, M \quad (5.61)$$

In short-hand notation, the percentage change forms for (5.39), (5.40) and (5.42) for $s=1$ are:

$$\bar{p}_q = p_q + a + a_q \quad , \quad q = 1, \dots, M \quad (5.62)$$

$$\bar{x}_q = x_q - a - a_q \quad , \quad q = 1, \dots, M \quad (5.63)$$

$$\bar{x} = x - a \quad (5.64)$$

Substituting (5.62)-(5.64) into (5.61) gives:

$$x_q = x - [1 / (1 - h_q)] (p_q - \sum_{q=1}^M S_q^* p_q) + a_q - [1 / (h_q)] (a_q - \sum_{q=1}^M S_q^* a_q) \quad (5.65)$$

Or in full notation:

$$\begin{aligned} x_{(g+1,1,q)j}^{(1)} &= x_{(g+1,1)j}^{(1)} + a_{(g+1,1,q)j}^{(1)} \\ &- \sigma_{(g+1,1,q)j}^{(1)} (p_{(g+1,1,q)j}^{(1)} - \sum_{q=1}^M S_{(g+1,1,q)j}^{*(1)} p_{(g+1,1,q)j}^{(1)}) \\ &- \sigma_{(g+1,1,q)j}^{(1)} (a_{(g+1,1,q)j}^{(1)} - \sum_{q=1}^M S_{(g+1,1,q)j}^{*(1)} a_{(g+1,1,q)j}^{(1)}) \\ &q = 1, \dots, M \quad , \quad j = 1, \dots, h \end{aligned} \quad (5.66)$$

where:

$$\sigma_{(g+1,1,q)j}^{(1)} = 1 / (1 - h_{(g+1,1,q)j}^{(1)}) \quad , \quad q = 1, \dots, M \quad (5.67)$$

$$S_{(g+1,1,q)j}^{*(1)} = \frac{\sigma_{(g+1,1,q)j}^{(1)} S_{(g+1,1,q)j}^{(1)}}{\sum_{q=1}^M \sigma_{(g+1,1,q)j}^{(1)} S_{(g+1,1,q)j}^{(1)}} \quad , \quad q = 1, \dots, M \quad (5.68)$$

Equation (5.66) relates each industry's demand for labor of particular types to the industry's demand for labor in general, to the costs of different types of labor and to various technical change variables. Equation (5.68) defines modified cost shares.

Next, one derives industry j 's demand functions for labor in general, capital and agricultural land from the first-order

conditions (5.46) and (5.47), which are analogous to (5.44) and (5.45). Hence, by reference to (5.61) one can write:

$$\begin{aligned}\bar{x}_{(g+1,v)j}^{(1)} &= \bar{z}_j^{(1)} - \sigma_{(g+1,v)j}^{(1)} (\bar{p}_{(g+1,v)j}^{(1)} \\ &\quad - \sum_{v=1}^3 S_{(g+1,v)j}^{*(1)} \bar{p}_{(g+1,v)j}^{(1)}) \\ v &= 1, 2, 3\end{aligned}\quad (5.69)$$

where:

$$\sigma_{(g+1,v)j}^{(1)} = 1 / (1 - h_{(g+1,v)j}^{(1)}) \quad , \quad v = 1, 2, 3 \quad (5.70)$$

$$S_{(g+1,v)j}^{*(1)} = \frac{\sigma_{(g+1,v)j}^{(1)} S_{(g+1,v)j}^{(1)}}{\sum_{v=1}^3 \sigma_{(g+1,v)j}^{(1)} S_{(g+1,v)j}^{(1)}} \quad , \quad v = 1, 2, 3 \quad (5.71)$$

$$S_{(g+1,v)j}^{(1)} = \frac{P_{(g+1,v)j}^{(1)} X_{(g+1,v)j}^{(1)}}{\sum_{v=1}^3 P_{(g+1,v)j}^{(1)} X_{(g+1,v)j}^{(1)}} \quad , \quad v = 1, 2, 3 \quad (5.72)$$

$P_{(g+1,v)j}^{(1)} X_{(g+1,v)j}^{(1)}$, $v = 2, 3$, are industry j 's rental payments for capital and agricultural land, while $P_{(g+1,1)j}^{(1)} X_{(g+1,1)j}^{(1)}$ is industry j 's expenditure on labor. So, the $S_{(g+1,v)j}^{(1)}$, $v = 1, 2, 3$ are the shares of labor, capital and agricultural land in industry j 's primary factor payments while the $S_{(g+1,v)j}^{*(1)}$'s are modified shares.

Applying the percentage change forms of (5.41), (5.42) and (5.43) to (5.69) yields the primary factor demand equations:

$$\begin{aligned}x_{(g+1,v)j}^{(1)} &= z_j + a_j^{(1)} + a_{g+1,j}^{(1)} + a_{(g+1,v)j}^{(1)} \\ &\quad - \sigma_{(g+1,v)j}^{(1)} (p_{(g+1,v)j}^{(1)} - \sum_{v=1}^3 S_{(g+1,v)j}^{*(1)} p_{(g+1,v)j}^{(1)}) \\ &\quad - \sigma_{(g+1,v)j}^{(1)} (a_{(g+1,v)j}^{(1)} - \sum_{v=1}^3 S_{(g+1,v)j}^{*(1)} a_{(g+1,v)j}^{(1)}) \\ v &= 1, 2, 3 \quad , \quad j = 1, \dots, h\end{aligned}\quad (5.73)$$

The variable $p_{(g+1,1)j}^{(1)}$ can be interpreted as the percentage change in the cost to industry j of a unit of labor.

The problem with equation (5.73) is that $P_{(g+1,1)j}^{(1)}$ is not exogenous to the cost minimization problem. To solve this problem,

this variable needs to be expressed in terms of other variables which are exogenous to the minimization problem. To do so, one writes (5.56) in linear percentage change form as:

$$p_{(g+1,1)j}^{(1)} = \sum_{q=1}^M p_{(g+1,1,q)j}^{(1)} s_{(g+1,1,q)j}^{(1)} + \sum_{q=1}^M x_{(g+1,1,q)j}^{(1)} s_{(g+1,1,q)j}^{(1)} - x_{(g+1,1)j}^{(1)} \quad (5.74)$$

Writing (5.54) in full notation and substituting for (5.63) and (5.64), one sees that:

$$x_{(g+1,1)j}^{(1)} - a_{(g+1,1)j}^{(1)} = \sum_{q=1}^M x_{(g+1,1,q)j}^{(1)} s_{(g+1,1,q)j}^{(1)} - a_{(g+1,1)j}^{(1)} - \sum_{q=1}^M a_{(g+1,1,q)j}^{(1)} s_{(g+1,1,q)j}^{(1)}$$

So, (5.74) reduces to:

$$\boxed{p_{(g+1,1)j}^{(1)} = \sum_{q=1}^M p_{(g+1,1,q)j}^{(1)} s_{(g+1,1,q)j}^{(1)} + \sum_{q=1}^M a_{(g+1,1,q)j}^{(1)} s_{(g+1,1,q)j}^{(1)} \quad j = 1, \dots, h} \quad (5.75)$$

If one assumes that:

$$\sum_{q=1}^M a_{(g+1,1,q)j}^{(1)} s_{(g+1,1,q)j}^{(1)} = 0$$

this implies that $p_{(g+1,1)j}^{(1)}$ is a weighted average of the percentage change in the costs to industry j of units of labor from the different skill groups, the weights being the shares of each skill group in j 's total labor costs.

5.2.2.3. Solution for the Demands for "Other Costs"

The demand for input $g+2$, "other costs", by industry j is given by:

$$x_{g+2,j}^{(1)} = A_j^{(1)} A_{g+2,j}^{(1)} z_j$$

and in percentage change form one has:

$$x_{g+2,j}^{(1)} = z_j + a_j^{(1)} + a_{g+2,j}^{(1)} \quad , \quad j = 1, \dots, h \quad (5.76)$$

5.3. Demand for Inputs for the Production of Fixed Capital

To determine the demand functions for inputs for the construction of fixed capital, one starts by defining the following production function:

$$A_j^{(2)} Y_j = \text{Leontief}_{i=1, \dots, g} \{X_{ij}^{(2)} / A_{ij}^{(2)}\} \quad , \quad j=1, \dots, h \quad (5.77)$$

where Y_j is the number of units of capital created for industry j ; $X_{ij}^{(2)}$, $i = 1, \dots, g$, is the direct effective input of good i to creating capital for industry j . $A_j^{(2)}$ and $A_{ij}^{(2)}$ are positive coefficients used to simulate changes in the technology of making units of capital for industry j .

The effective inputs, $X_{ij}^{(2)}$, $i = 1, \dots, g$, are defined by:

$$X_{ij}^{(2)} = \text{CES}_{s=1,2} (X_{(is)j}^{(2)} / A_{(is)j}^{(2)}) \quad (5.78)$$

where $X_{(i1)j}^{(2)}$ and $X_{(i2)j}^{(2)}$ are respectively inputs of domestic and imported sources of good i , and $A_{(is)j}^{(2)}$ are positive technological coefficients.

Capital creation requires no inputs of primary factors or "other costs". The use of labor, capital and land, the payment of production taxes and the costs of holding liquidity and inventories associated with the creation of capital are recognized via the inputs of construction, i.e., the construction industries use primary inputs and pay production taxes, etc. and the creation of fixed capital requires heavy inputs of construction.

Producers of capital for industry j treat input prices as

given, so they choose:

$$x_{(is)j}^{(2)} \quad , \quad i = 1, \dots, g \quad , \quad s = 1, 2$$

to minimize:

$$\sum_{i=1}^g \sum_{s=1}^2 p_{(is)j}^{(2)} x_{(is)j}^{(2)} \quad (5.79)$$

subject to (5.77) and (5.78).

Where $p_{(is)j}^{(2)}$ is the price of good i from source s when it is used as an input for creating capital for industry j .

By analogy with steps (5.14)-(5.35), the solution of the cost minimization problem (5.79) is given by:

$$\begin{aligned} x_{(is)j}^{(2)} = & y_j + a_j^{(2)} + a_{ij}^{(2)} + a_{(is)j}^{(2)} \\ & - \sigma_{ij}^{(2)} (p_{(is)j}^{(2)} - \sum_{s=1}^2 S_{(is)j}^{(2)} p_{(is)j}^{(2)}) \\ & - \sigma_{ij}^{(2)} (a_{(is)j}^{(2)} - \sum_{s=1}^2 S_{(is)j}^{(2)} a_{(is)j}^{(2)}) \\ & i = 1, \dots, g \quad , \quad s = 1, 2 \quad , \quad j = 1, \dots, h \end{aligned} \quad (5.80)$$

where $S_{(is)j}^{(2)}$ is the share of good i from source s in the total cost of good i used for creation of capital in industry j and $\sigma_{ij}^{(2)} = 1 / (1 + \rho_{ij}^{(2)})$ is the elasticity of substitution between imported and domestic good i as inputs for creation of capital of type j .

The way that the level of investment, Y_j , is determined is explained in section 5.9.

5.4 Household Demands

The household demands are divided by income groups, as each income group is supposed to have a different composition of final demand. There are D different income groups, and the household

demands in each group are explained by a utility maximizing model.

Letting Q_d , $d = 1, \dots, D$, be the number of households in each income group, the consumption bundle of effective inputs for the average household in the income group d is given by $X_{id}^{(3)} / Q_d$, $i = 1, \dots, g$, and it is chosen to maximize:

$$U(\bar{X}_{1d}^{(3)}, \dots, \bar{X}_{gd}^{(3)}) \quad (5.81)$$

subject to:

$$\bar{X}_{id}^{(3)} = CES_{s=1,2}(\bar{X}_{(is)d}^{(3)}) \quad , \quad i = 1, \dots, g \quad (5.82)$$

$$\sum_{s=1}^2 \sum_{i=1}^g \bar{P}_{(is)d}^{(3)} \bar{X}_{(is)d}^{(3)} = C_d \quad (5.83)$$

where:

$$\bar{X}_{id}^{(3)} = X_{id}^{(3)} / (A_{id}^{(3)} Q_d) \quad , \quad i = 1, \dots, g \quad (5.84)$$

$$\bar{X}_{(is)d}^{(3)} = X_{(is)d}^{(3)} / (A_{id}^{(3)} A_{(is)d}^{(3)} Q_d) \quad (5.85)$$

$$i = 1, \dots, g \quad , \quad s = 1, 2$$

$$\bar{P}_{(is)d}^{(3)} = P_{(is)d}^{(3)} A_{id}^{(3)} A_{(is)d}^{(3)} Q_d \quad (5.86)$$

$$i = 1, \dots, g \quad , \quad s = 1, 2$$

$X_{(is)d}^{(3)}$ and $P_{(is)d}^{(3)}$, $i = 1, \dots, g$, $s = 1, 2$, are the quantities consumed and prices paid by household in the income group d for units of good i from source s , $s = 1$ referring to domestic sources and $s = 2$ referring to imports. C_d is the aggregate consumer budget for income group d , and $A_{id}^{(3)}$ and $A_{(is)d}^{(3)}$ are positive coefficients that allow for differences in taste in income group d .

The first order conditions for the problem (5.81)-(5.83) can be written as:

$$(\partial U_d / \partial \bar{X}_{id}^{(3)}) - \Gamma_d \bar{P}_{id}^{(3)} = 0 \quad , \quad i = 1, \dots, g \quad (5.87)$$

$$\bar{P}_{id}^{(3)} [\partial CES_s(\bar{X}_{(is)d}^{(3)}) / \partial \bar{X}_{(is)d}^{(3)}] - \bar{P}_{(is)d}^{(3)} = 0 \quad (5.88)$$

$$i = 1, \dots, g \quad , \quad s = 1, 2$$

$$\bar{X}_{id}^{(3)} - CES_s(\bar{X}_{(is)d}^{(3)}) = 0 \quad , \quad i = 1, \dots, g \quad (5.89)$$

$$\sum_{s=1}^2 \sum_{i=1}^g \bar{p}_{(is)d}^{(3)} \bar{x}_{(is)d}^{(3)} = C_d \quad (5.90)$$

where Γ_d is the Lagrangian multiplier on constraint (5.83) and the $\bar{p}_{id}^{(3)}$'s are defined so that $\Gamma_d \bar{p}_{id}^{(3)}$ are the Lagrangian multipliers on the constraints (5.82).

From (5.88) and (5.89), by analogy with the argument by which one derived (5.31) and (5.30) from (5.19) and (5.20), one has that:

$$\bar{x}_{(is)d}^{(3)} = \bar{x}_{id}^{(3)} - \sigma_{id}^{(3)} (\bar{p}_{(is)d}^{(3)} - \sum_{s=1}^2 s_{(is)d}^{(3)} \bar{p}_{(is)d}^{(3)})$$

$$i = 1, \dots, g, \quad s = 1, 2 \quad (5.91)$$

$$\bar{p}_{id}^{(3)} = \sum_{s=1}^2 \bar{p}_{(is)d}^{(3)} s_{(is)d}^{(3)}, \quad i = 1, \dots, g \quad (5.92)$$

Where $\sigma_{id}^{(3)}$, $i = 1, \dots, g$, $d = 1, \dots, D$, is the elasticity of substitution between domestic and imported good i for income group d and $s_{(is)d}^{(3)}$ is the share of total consumer spending on good i which is devoted to good i from source s for income group d .

The next step is to derive equations for the $\bar{x}_{id}^{(3)}$. One can start by multiplying (5.88) through by $\bar{x}_{(is)d}^{(3)}$, aggregating over s and applying Euler's theorem, the result being:

$$\bar{p}_{id}^{(3)} \bar{x}_{id}^{(3)} = \sum_s \bar{p}_{(is)d}^{(3)} \bar{x}_{(is)d}^{(3)}, \quad i = 1, \dots, g \quad (5.93)$$

Hence, (5.90) may be rewritten as:

$$\sum_{i=1}^g \bar{p}_{id}^{(3)} \bar{x}_{id}^{(3)} = C_d \quad (5.94)$$

Equation (5.94) together with (5.87) give the set of first order conditions for the problem of choosing $\bar{x}_{id}^{(3)}$ to maximize:

$$U(\bar{x}_{1d}^{(3)}, \dots, \bar{x}_{gd}^{(3)}) \quad (5.95)$$

subject to:

$$\sum_{i=1}^g \bar{p}_{id}^{(3)} \bar{x}_{id}^{(3)} = C_d \quad (5.96)$$

This means that the choice of the $\bar{x}_{id}^{(3)}$'s can be obtained by the conventional unnested, utility maximizing model. So, one may conclude that:

$$\bar{x}_{id}^{(3)} = \epsilon_{id}c_d + \sum_{k=1}^g \eta_{ikd} \bar{p}_{kd}^{(3)} \quad , \quad i = 1, \dots, g \quad (5.97)$$

where for income group d , ϵ_{id} , η_{ikd} , may be interpreted as expenditure and own- and cross-price elasticities satisfying the usual restrictions: homogeneity, symmetry and Engel's aggregation (see Powell, 1974).

The percentage change forms of (5.84)-(5.86) are given by:

$$\bar{x}_{id}^{(3)} = x_{(id)}^{(3)} - a_{id}^{(3)} - q_d \quad , \quad i = 1, \dots, g \quad (5.98)$$

$$\begin{aligned} \bar{x}_{(is)d}^{(3)} &= x_{(is)d}^{(3)} - a_{id}^{(3)} - a_{(is)d}^{(3)} - q_d \\ i &= 1, \dots, g \quad , \quad s = 1, 2 \end{aligned} \quad (5.99)$$

$$\begin{aligned} \bar{p}_{(is)d}^{(3)} &= p_{(is)d}^{(3)} + a_{id}^{(3)} + a_{(is)d}^{(3)} + q_d \\ i &= 1, \dots, g \quad , \quad s = 1, 2 \end{aligned} \quad (5.100)$$

Substituting (5.98)-(5.100) into (5.91) yields:

$$\begin{aligned} x_{(is)d}^{(3)} &= x_{id}^{(3)} + a_{(is)d}^{(3)} \\ &\quad - \sigma_{id}^{(3)} (p_{(is)d}^{(3)} - \sum_{s=1}^2 S_{(is)d}^{(3)} p_{(is)d}^{(3)}) \\ &\quad - \sigma_{id}^{(3)} (a_{(is)d}^{(3)} - \sum_{s=1}^2 S_{(is)d}^{(3)} a_{(is)d}^{(3)}) \\ i &= 1, \dots, g \quad , \quad s = 1, 2 \quad , \quad d = 1, \dots, D \end{aligned} \quad (5.101)$$

Substituting (5.100) into (5.92) gives:

$$\begin{aligned} \bar{p}_{id}^{(3)} &= p_{id}^{(3)} + a_{id}^{(3)} + \sum_{s=1}^2 a_{(is)d}^{(3)} S_{(is)d}^{(3)} + q_d \\ i &= 1, \dots, g \end{aligned} \quad (5.102)$$

where:

$$\begin{aligned} p_{id}^{(3)} &= \sum_{s=1}^2 S_{(is)d}^{(3)} p_{(is)d}^{(3)} \\ i &= 1, \dots, g \quad , \quad d = 1, \dots, D \end{aligned} \quad (5.103)$$

Finally, one substitutes (5.98) and (5.102) into (5.97), giving:

$$\begin{aligned} x_{id}^{(3)} - q_d &= \epsilon_{id}(c_d - q_d) + \sum_{k=1}^g \eta_{ikd} p_{kd}^{(3)} + a_{id}^{(3)} \\ &\quad + \sum_{k=1}^g \eta_{ikd} [a_{kd}^{(3)} + \sum_{s=1}^2 S_{(ks)d}^{(3)} a_{(ks)d}^{(3)}] \\ i &= 1, \dots, g \quad , \quad d = 1, \dots, D \end{aligned} \quad (5.104)$$

In deriving (5.104) one made use of the homogeneity restriction (implied by the problem (5.95)-(5.96)), i.e.,

$$\sum_{k=1}^g \eta_{ikd} = -\epsilon_{id} \quad (5.105)$$

In assigning values for the elasticities ϵ_{id} and η_{ikd} it was assumed that the utility function (5.95) is of the Klein-Rubin form, i.e.,⁷

$$U(\bar{x}_{id}^{(3)}, \dots, \bar{x}_{id}^{(3)}) = \sum_{i=1}^g \delta_{id} \ln(\bar{x}_{id}^{(3)}) - \theta_{id} \quad (5.106)$$

where:

$$\delta_i > 0, \quad i = 1, \dots, g$$

$$\sum_i \delta_i = 1$$

Solving problem (5.95)-(5.96) under (5.106) one gets the linear expenditure system:

$$\bar{x}_{id}^{(3)} = \theta_{id} + \delta_{id}(C_d - \sum_{k=1}^g \bar{p}_{kd}^{(3)} \theta_{kd}) / \bar{p}_{id}^{(3)} \quad i = 1, \dots, g \quad (5.107)$$

On the basis of (5.107) one obtains:

$$\epsilon_{id} = \delta_{id} / S_{id}^{(3)}, \quad i = 1, \dots, g \quad (5.108)$$

$$\eta_{ikd} = -\delta_{id} S_{kd}^{*(3)} / S_{id}^{(3)}, \quad \text{for all } i \neq k \quad (5.109)$$

$$\eta_{iidd} = -\epsilon_{id} - \sum_{k \neq i} \eta_{ikd} \quad (5.110)$$

where:

$$S_{id}^{(3)} = \bar{p}_{id}^{(3)} \bar{x}_{id}^{(3)} / \sum_k \bar{p}_{kd}^{(3)} \bar{x}_{kd}^{(3)}, \quad i = 1, \dots, g \quad (5.111)$$

$$S_{id}^{*(3)} = \bar{p}_{id}^{(3)} \theta_{id} / \sum_k \bar{p}_{kd}^{(3)} \bar{x}_{kd}^{(3)}, \quad i = 1, \dots, g \quad (5.112)$$

Given (5.93), (5.85) and (5.86) one may interpret, for income group d, $S_{id}^{(3)}$ as being the share of household expenditure devoted to good i. The interpretation of $S_{id}^{*(3)}$ is less clear, it depends, first, on the interpretation of θ_{id} (minimum level for $x_{id}^{(3)}$). Unless:

$$(x_{id}^{(3)} / Q_d) > (\theta_{id} A_i^{(3)}) \quad \text{for all } i,$$

the utility function (5.106) is undefined. Thus, one interprets $\theta_{id}A_{id}^{(3)}$ as the current subsistence level for the consumption of good i per household in the income group d . Rewritten (5.112) as:

$$S_{id}^{*(3)} = S_{id}^{(3)} [\theta_{id}A_{id}^{(3)} / (X_{id}^{(3)} / Q_d)] \quad , \quad i = 1, \dots, g \quad (5.113)$$

one sees that $S_{id}^{*(3)}$ is the product of two shares: the share of good i in household expenditure of income group d , and the share that the subsistence level for the consumption of good i represents in the average household's consumption of good i for income group d .

5.4.1. The Aggregate Consumer Budgets

From the side of receipts, the aggregate consumer budget for income group d can be viewed as being equal to the sum of the income from the different labor skills, which belong to income group d , less a residual value (taxes, subsidies, savings, etc.), i.e.,

$$C_d = \sum_{m=1, \dots, M} (g+1, 1, m) j \in J_d (P_{(g+1, 1, m)j}^{(1)} X_{(g+1, 1, m)j}^{(1)}) - S_d \quad (5.114)$$

where S_d is a residual value and J_d is a subset of $\{(g+1, 1, m)j \mid m = 1, \dots, M, j = 1, \dots, h\}$, containing the elements that belong to income group d .

In percentage change form, equation (5.114) becomes:

$$c_d = \sum_{m=1, \dots, M} (g+1, 1, m) j \in J_d (P_{(g+1, 1, m)j}^{(1)} + x_{(g+1, 1, m)j}^{(1)}) H_{(g+1, 1, m)jd}^{(1)} - s_d H_d \quad (5.115)$$

where $H_{(g+1, 1, m)jd}^{(1)}$ is the share of individuals in the skill level m that work in industry j in the total aggregate income of income

group d. H_d is the share of the residual value in the total aggregate income of income group d.

5.5. Foreign Demand for Brazilian Exports

One assumes that:

$$P_{(il)}^e = g_i(X_{(il)}^{(4)})F_{(il)}^e, \quad i = 1, \dots, g \quad (5.116)$$

where $P_{(il)}^e$ is the f.o.b. price in foreign currency of good i domestically produced (it is assumed that imports are not exported before going through a domestic industry); g_i is a nonincreasing function of $X_{(il)}^{(4)}$ and $X_{(il)}^{(4)}$ is the volume of exports of good i ; $F_{(il)}^e$ is an exogenous shift variable which will increase if there is an increase in foreign demand for good i .

In percentage change form (5.116) becomes:

$$p_{(il)}^e = -\gamma_i x_{(il)}^{(4)} + f_{(il)}^e, \quad i = 1, \dots, g \quad (5.117)$$

where:

$$-\gamma_i \equiv (\partial g_i / \partial X_{(il)}^{(4)}) (X_{(il)}^{(4)} / g_i)$$

so, γ_i is non-negative and is the reciprocal of the foreign elasticity of foreign demand for good i .

5.6. Government and "Other" Demands

There is no specific theory suggesting how the government and "other" demands should behave, so, the following relationship in percentage change form is assumed:

$$\boxed{x_{(is)}^{(k)} = c_r h_{(is)}^{(k)} + f_{(is)}^{(k)} \quad i=1, \dots, g, \quad s=1, 2, \quad k=5, 6} \quad (5.118)$$

where:

$$\boxed{c_r = \sum_{d=1}^D c_d O_d - \xi^{(3)}} \quad (5.119)$$

In equations (5.118) and (5.119) one has that $x_{is}^{(k)}$ is the percentage change in the government ($k=5$) and "other" ($k=6$) demands for good i from source s ; c_r is the percentage change in real aggregate household expenditure; the $h_{(is)}^{(k)}$'s are parameters; the $f_{(is)}^{(k)}$'s are shift variables; O_d is the share of income group d in the aggregate private consumption; and $\xi^{(3)}$ is the percentage change in the model consumer price index (see section 5.12).

5.7. Demands for Margins

Until now one has considered direct demands for goods and services by producers, investors, households, foreigners and governments, but, the satisfaction of these direct demands creates demands for margins, i.e., transport, wholesale and retail services, etc..

The margins associated with the delivery of inputs for current production and capital construction are given by:

$$x_{(r1)}^{(is)jk} = A_{(r1)}^{(is)jk} x_{(is)j}^{(k)} \quad (5.120)$$

$$i, r = 1, \dots, g, \quad j = 1, \dots, h, \quad k, s = 1, 2$$

where $x_{(r1)}^{(is)jk}$ is the quantity of good $(r1)$ used as a margin to facilitate the flow of good i from source s to industry j for purpose k , $A_{(r1)}^{(is)jk}$ is a positive coefficient and $x_{(is)j}^{(k)}$ is

the quantity of good i from source s used in industry j for purpose k .

If there is no change in the $A_{(r1)}(is)jk$ in (5.120), margins flows are forced to be proportional to commodity flows. In the case of imports ($s=2$), (5.120) represents the demand for margins associated with deliveries from Brazilian ports to users within the country.

In percentage change form, (5.120) becomes:

$$\begin{aligned} x_{(r1)}(is)jk &= x_{(is)j}^{(k)} + a_{(r1)}(is)jk \\ i, r &= 1, \dots, g, \quad j = 1, \dots, h, \quad k, s = 1, 2 \end{aligned} \quad (5.121)$$

Changes in $A_{(r1)}(is)jk$ reflect changes in the amounts of margin services associated with various commodity flows.

Margin flows associated with the delivery of commodities to households, ports prior to export, government and "other" demands are giving respectively by:

$$X_{(r1)}(is)d3 = A_{(r1)}(is)d3 X_{(is)d}^{(3)} \quad (5.122)$$

$$i, r = 1, \dots, g, \quad d = 1, \dots, D, \quad s = 1, 2$$

$$X_{(r1)}(i1)4 = A_{(r1)}(i1)4 X_{(i1)}^{(4)}, \quad i, r = 1, \dots, g \quad (5.123)$$

$$X_{(r1)}(is)k = A_{(r1)}(is)k X_{(is)}^{(k)} \quad (5.124)$$

$$i, r = 1, \dots, g, \quad s = 1, 2, \quad k = 5, 6$$

where all variables are as defined before.

In percentage change form, (5.122)-(5.124) become:

$$x_{(r1)}(is)d3 = x_{(is)d}^{(3)} + a_{(r1)}(is)d3 \quad (5.125)$$

$$i, r = 1, \dots, g, \quad d = 1, \dots, D, \quad s = 1, 2$$

$$x_{(r1)}(i1)4 = x_{(i1)}^{(4)} + a_{(r1)}(i1)4, \quad i, r = 1, \dots, g \quad (5.126)$$

$$x_{(r1)}(is)k = x_{(is)}^{(k)} + a_{(r1)}(is)k \quad (5.127)$$

$$i, r = 1, \dots, g, \quad s = 1, 2, \quad k = 5, 6$$

5.8. The Price Systems

There are several sets of commodity prices in the model: purchasers' price, basic values, prices of capital units, f.o.b. foreign currency export prices and c.i.f. foreign currency import prices. This section will show the relationship between them.

In setting those relationships, three assumptions are going to be made: 1) there are no pure profits in any economic activity;⁸ 2) basic values are uniform across users and across producing industries in the case of domestic goods, and importers in the case of foreign goods;⁹ and 3) there are no margins on margins.

Under the first two assumptions, one has that:

$$\begin{aligned}
 P_{(j1)j}^{(0)} X_{(j1)j}^{(0)} &= \sum_{i=1}^g \sum_{s=1}^2 X_{(is)}^{(1)} P_{(is)j}^{(1)} \\
 &+ \sum_{m=1}^M P_{(g+1,1,m)j}^{(1)} X_{(g+1,1,m)j}^{(1)} \\
 &+ \sum_{s=2}^3 P_{(g+1,s)j}^{(1)} X_{(g+1,s)j}^{(1)} + P_{g+2,j}^{(1)} X_{g+2,j}^{(1)} \\
 j &= 1, \dots, h
 \end{aligned} \tag{5.128}$$

where all the variables are as defined before.

The LHS of (5.128) gives the basic value of output of industry j , while the RHS gives the total payment for inputs.

In percentage change form, (5.128) gives:

$$\begin{aligned}
 p_{(j1)j}^{(0)} + x_{(j1)j}^{(0)} &= \sum_{i=1}^g \sum_{s=1}^2 (x_{(is)j}^{(1)} + p_{(is)j}^{(1)}) H_{(is)j}^{(1)} \\
 &+ \sum_{m=1}^M (p_{(g+1,1,m)j}^{(1)} + x_{(g+1,1,m)j}^{(1)}) H_{(g+1,1,m)j}^{(1)} \\
 &+ \sum_{s=2}^3 (p_{(g+1,s)j}^{(1)} + x_{(g+1,s)j}^{(1)}) H_{(g+1,s)j}^{(1)} \\
 &+ (p_{g+2,j}^{(1)} + x_{g+2,j}^{(1)}) H_{g+2,j}^{(1)} \\
 j &= 1, \dots, h
 \end{aligned} \tag{5.129}$$

Where $H_{(is)j}^{(1)}$, $H_{(g+1,1,m)j}^{(1)}$, $H_{(g+1,s)j}^{(1)}$, and $H_{g+2,j}^{(1)}$

are the cost shares, of the different inputs, in the final product.

Each of the $\sum xH$ terms in (5.129) can be expressed in terms of z_j and of the various technical change variables (the a 's). For example, for $x_{(j1)j}^{(0)}$ one has that:

$$x_{(j1)j}^{(0)} = z_j - a_j^{(0)} \quad (5.130)$$

For $x_{g+2,j}^{(1)} H_{g+2,j}^{(1)}$, applying (5.76) one has that:

$$x_{g+2,j}^{(1)} = (z_j + a_j^{(1)} + a_{g+2,j}^{(1)}) H_{g+2,j}^{(1)} \quad (5.131)$$

Expressions similar to (5.130) and (5.131) can be generate for the others $\sum xH$. When the results are substitute into equation (5.129) one gets:

$$\begin{aligned} p_{(j1)}^{(0)} = & \sum_{i=1}^g \sum_{s=1}^2 p_{(is)j}^{(1)} H_{(is)j}^{(1)} \\ & + \sum_{m=1}^M p_{(g+1,1,m)j}^{(1)} H_{(g+1,1,m)j}^{(1)} \\ & + \sum_{s=2}^3 p_{(g+1,s)j}^{(1)} H_{(g+1,s)j}^{(1)} \\ & + p_{g+2,j}^{(1)} H_{g+2,j}^{(1)} + a(j) \\ & j = 1, \dots, h \end{aligned} \quad (5.132)$$

where:

$$\begin{aligned} a(j) = & a_j^{(0)} + a_j^{(1)} + \sum_{i=1}^{g+2} a_{ij}^{(1)} H_{ij}^{(1)} \\ & + \sum_{i=1}^g \sum_{s=1}^2 a_{(is)j}^{(1)} H_{(is)j}^{(1)} \\ & + \sum_{s=1}^3 a_{(g+1,s)j}^{(1)} H_{(g+1,s)j}^{(1)} \\ & + \sum_{m=1}^M a_{(g+1,1,m)j}^{(1)} H_{(g+1,1,m)j}^{(1)} \\ & j = 1, \dots, h \end{aligned} \quad (5.133)$$

The term $a(j)$ refers basically to the effects of technical change on the output price of sector j .

The second price relationship is:

$$\begin{aligned} \pi_j = & \sum_{i=1}^g \sum_{s=1}^2 p_{(is)j}^{(2)} H_{(is)j}^{(2)} + \sum_{i=1}^g a_{ij}^{(2)} H_{ij}^{(2)} \\ & + \sum_{i=1}^g \sum_{s=1}^2 a_{(is)j}^{(2)} H_{(is)j}^{(2)} + a_j^{(2)} \quad , \quad j = 1, \dots, h \end{aligned} \quad (5.134)$$

where π_j is the percentage change in the price of a unit of capital for industry j and the $H_{(is)j}^{(2)}$, $H_{ij}^{(2)}$ are cost shares.

Equation (5.134) is derived from:

$$\Pi_j Y_j = \sum_{i=1}^g \sum_{s=1}^2 P_{(is)j}^{(2)} X_{(is)j}^{(2)} \quad , \quad j = 1, \dots, h$$

in a similar way of the derivation of (5.132)-(5.133) from (5.128).

The third set of price equations is, in levels,

$$P_{(i2)}^{(0)} = P_{(i2)}^m \phi + G(i2,0) \quad , \quad i = 1, \dots, g \quad (5.135)$$

where $P_{(i2)}^{(0)}$ is the basic price of imported good i , $P_{(i2)}^m$ is the foreign currency c.i.f. price of imported units of good i , ϕ is the exchange rate, and $G(i2,0)$ is the tariff in cruzeiros per unit imported of good i .

To allow one to model tariffs in a variety of ways, the following equation is added:

$$\begin{aligned} G(i2,0) = & (\bar{G}(i2,0) \xi^{(3)}) h_1(i2,0) (T(i2,0) P_{(i2)}^m) h_2(i2,0) \\ & (V(i2,0)) h_3(i2,0) \\ & i = 1, \dots, g \end{aligned} \quad (5.136)$$

where the h 's and $\bar{G}(i2,0)$ are parameters, $\xi^{(3)}$ is the model consumer price index, and $T(i2,0)$ and $V(i2,0)$ are variables used to reflect ad valorem and specific rates of protection.

In percentage change form, (5.135) and (5.136) are:

$$p_{(i2)}^{(0)} = (p_{(i2)}^m + \phi) \zeta_1(i2,0) + g(i2,0) \zeta_2(i2,0)$$

$$i = 1, \dots, g$$

(5.137)

and:

$$g(i,2,0) = h_1(i2,0) \xi^{(3)} + h_2(i2,0) [t(i2,0) + p_{(i2)}^m + \phi]$$

$$+ h_3(i2,0) v(i2,0)$$

$$i = 1, \dots, g$$

(5.138)

where $\zeta_1(i2,0)$ and $\zeta_2(i2,0)$ are, respectively, the shares in the basic price of (i2) accounted for by the foreign currency price in cruzeiros, $P(i2)^{m\phi}$, and the tariff, $G(i2,0)$.

The set of equations that relates prices of domestic goods to f.o.b. export prices are given by:

$$P(i1)^{e\phi} = P(i1)^{(0)} + G(i1,4) + \sum_{r=1}^g A_{(r1)}^{(i1)} P_{(r1)}^{(0)} \quad (5.139)$$

$$i = 1, \dots, g$$

where $G(i1,4)$ is the export tax per unit export of (i1), and all the other variables are as defined before.

The LHS of (5.139) gives the price in cruzeiros of good (i1) paid by foreign at Brazilian ports (f.o.b. price); and the RHS is the basic price of good (i1) plus the cost of taxes and margins.

To allow flexibility in modelling export taxes, the following equation is introduced:

$$G(i1,4) = \text{sign}(i1,4) (\bar{G}(i1,4) \Xi^{(3)} h_1(i1,4) \\ (T(i1,4) P(i1)^{e\phi}) h_2(i1,4) (V(i1,4)) h_3(i1,4) \quad (5.140)$$

$$i = 1, \dots, g$$

where $\text{sign}(i1,4)$ is +1 in the case of a tax and -1 in the case of a subsidy, and all the other notation is similar to the one presented in (5.136).

In percentage change form, (5.139) and (5.140) become:

$$(p(i1)^e + \phi) = p(i1)^{(0)} \zeta_1(i1,4) + g(i1,4) \zeta_2(i1,4) \\ + (\sum_{r=1}^g M_{(r1)}^{(i1)} p_{(r1)}^{(0)}) \zeta_3(i1,4) \\ + (\sum_{r=1}^g M_{(r1)}^{(i1)} a_{(r1)}^{(i1)}) \zeta_3(i1,4)$$

$$i = 1, \dots, g$$

(5.141)

and:

$$\begin{aligned}
g(i1,4) &= h_1(i1,4) \xi^{(3)} + h_2(i1,4)[t(i1,4) + p(i1)^e + \phi] \\
&\quad + h_3(i1,4)v(i1,4) \\
i &= 1, \dots, g
\end{aligned} \tag{5.142}$$

where $\xi_1(i1,4)$, $\xi_2(i1,4)$ and $\xi_3(i1,4)$ are, respectively, the shares accounted for by the basic value, the export tax (negative for export subsidy), and the margins; and $M_{(r1)}^{(i1)4}$ is the share in the total cost of margin services involved in transferring good (i1) from domestic producers to the ports of exit represented by the use of good (r1).

In the case that the product is a nonexport one, the values of $h_1(i1,4)$ and $h_2(i1,4)$ are set at zero, $h_3(i1,4)$ at one, and $v(i1,4)$ and $g(i1,4)$ are allowed to be endogenous.

The fifth and final set of price equations, relating the various purchaser's prices paid by domestic users of good i from source s to its basic value is given by:

$$\begin{aligned}
P_{(is)j}^{(k)} &= P_{(is)}^{(0)} + G(is,jk) + \sum_{r=1}^g A_{(r1)}^{(is)jk} P_{(r1)}^{(0)} \\
i &= 1, \dots, g, \quad j = 1, \dots, h, \quad s, k = 1, 2
\end{aligned} \tag{5.143}$$

and:

$$\begin{aligned}
P_{(is)d}^{(3)} &= P_{(is)}^{(0)} + G(is,d3) + \sum_{r=1}^g A_{(r1)}^{(is)d3} P_{(r1)}^{(0)} \\
i &= 1, \dots, g, \quad s = 1, 2, \quad d = 1, \dots, D
\end{aligned} \tag{5.144}$$

where the G's are sale tax terms, and the other variables are as defined before.

Equations for purchaser's prices paid by the government and "other" demands could be included in the model, but they are not, as those demands are not modelled as being price sensitive.

In percentage change form (5.143) and (5.144) are:

$$\begin{aligned}
p(is)_j^{(k)} &= p(is)^{(0)} \zeta_1(is,jk) + g(is,jk) \zeta_2(is,jk) \\
&\quad + (\sum_{r=1}^g g_{M(r1)}(is)jk p_{(r1)}^{(0)}) \zeta_3(is,jk) \\
&\quad + (\sum_{r=1}^g g_{M(r1)}(is)jka_{(r1)}(is)jk) \zeta_3(is,jk) \\
i &= 1, \dots, g, \quad j = 1, \dots, h, \quad s, k = 1, 2
\end{aligned} \tag{5.145}$$

and:

$$\begin{aligned}
p(is)_d^{(3)} &= p(is)^{(0)} \zeta_1(is,d3) + g(is,d3) \zeta_2(is,d3) \\
&\quad + (\sum_{r=1}^g g_{M(r1)}(is)d3 p_{(r1)}^{(0)}) \zeta_3(is,d3) \\
&\quad + (\sum_{r=1}^g g_{M(r1)}(is)d3 a_{(r1)}(is)d3) \zeta_3(is,d3) \\
i &= 1, \dots, g, \quad d = 1, \dots, D, \quad s = 1, 2
\end{aligned} \tag{5.146}$$

where the definitions of the share coefficients (the ζ 's and M 's) follow the pattern established in (5.141).

Following the approach and notation of (5.142), one can add equations to allow flexible handling of the tax terms appearing in (5.145) and (5.146), i.e.,

$$\begin{aligned}
g(is,jk) &= h_1(is,jk) \xi^{(3)} + h_2(is,jk)[t(is,jk) + p(is)^{(0)}] \\
&\quad + h_3(is,jk)v(is,jk) \\
i &= 1, \dots, g, \quad j = 1, \dots, h, \quad s, k = 1, 2
\end{aligned} \tag{5.147}$$

and:

$$\begin{aligned}
g(is,d3) &= h_1(is,d3) \xi^{(3)} + h_2(is,d3)[t(is,d3) + p(is)^{(0)}] \\
&\quad + h_3(is,d3)v(is,d3) \\
i &= 1, \dots, g, \quad d = 1, \dots, D, \quad s = 1, 2
\end{aligned} \tag{5.148}$$

5.9. The Allocation of Investment Across Industries

This section addresses the question left open in section (5.3),

i.e., how many units of capital will be created for each industry.

The first step in doing so is to note that the current net rate of return on fixed capital in industry j is:

$$R_j(0) = (P_{(g+1,2)j}^{(1)} / \Pi_j) - d_j \quad (5.149)$$

where d_j is the rate of depreciation (assumed fixed) and $P_{(g+1,2)j}^{(1)}$ and Π_j are, as previously defined, the rental value and the cost of a unit of capital in industry j .

The second step is to assume that capital in industry j takes one period to install.

The third step is to assume that investors are cautious in assessing the effects of expanding the capital stock in industry j , i.e., they behave as if they expect that industry j 's rate of return schedule in one period's time will have the form:

$$R_j(1) = R_j(0) [K_j(1) / K_j(0)]^{-\beta_j} \quad (5.150)$$

where β_j is a positive parameter, $K_j(0)$ is the current level of capital stock in industry j and $K_j(1)$ is the level at the end of one period.

The fourth step is to assume that total endogenous investment expenditure, I , is allocated across industries so as to equate the expected rates of return. This means that there exists some rate of return Ω such that:

$$[K_j(1) / K_j(0)]^{-\beta_j} R_j(0) = \Omega, \quad j \in J \quad (5.151)$$

where J is the subset of $\{1, 2, \dots, h\}$ which contains the identifying numbers of those industries whose investment is treated as endogenous within the model.

The fifth step is to assume that:

$$K_j(1) = K_j(0)(1 - d_j) + Y_j, \quad j = 1, \dots, h \quad (5.152)$$

and

$$I = \sum_{j \in J} \Pi_j Y_j \quad (5.153)$$

Equation (5.152) implies that the capital stock at the end of one period is influenced only by the current capital stock and the current level of investment.

The sixth step is to explain the investment in the industries ($j \notin J$) for which the rate of return theory is considered inappropriate. For that, one uses the equations:

$$Y_j = (I_R)^{h_j^{(2)}} F_j^{(2)}, \quad j \notin J \quad (5.154)$$

where:

$$I_R = I / E^{(2)} \quad (5.155)$$

$E^{(2)}$ is the model capital-goods price index, I_R is the real level of private investment and $F_j^{(2)}$ (defined for $j \notin J$) is a shift variable.

Equations (5.149), (5.151)-(5.155) define the allocation of investment across industries, those equations in percentage change form may be expressed as follows:

$$r_j(0) = Q_j(p_{(g+1,2)}^{(1)} - \pi_j), \quad j = 1, \dots, h \quad (5.156)$$

$$-\beta_j[k_j(1) - k_j(0)] + r_j(0) = \omega, \quad j \in J \quad (5.157)$$

$$k_j(1) = k_j(0)(1 - G_j) + Y_j G_j, \quad j = 1, \dots, h \quad (5.158)$$

$$\sum_{j \in J} (\pi_j + Y_j) T_j = (\sum_{j \in J} T_j) i \quad (5.159)$$

$$Y_j = h_j^{(2)} i_r + f_j^{(2)}, \quad j \notin J \quad (5.160)$$

$$i_r = i - \xi^{(2)} \quad (5.161)$$

where $Q_j = [R_j(0) + d_j] / R_j(0)$, i.e., Q_j is the ratio of gross rate of return in industry j to the net rate of return. $G_j = Y_j / K_j(1)$,

i.e., G_j is the ratio of gross investment in industry j to its future capital stock, and T_j is the share of total fixed investment accounted for by industry j , i.e.:

$$T_j = (Y_j \Pi_j / \sum_{j=1}^h Y_j \Pi_j) \quad , \quad j = 1, \dots, h$$

5.10. The Market Clearing Equations

This section will present the equations which ensure that demand equals supply for domestically produced commodities and for primary factors of production: labor, capital and agricultural land.

The equations for domestic commodities are:

$$\begin{aligned} X_{(r1)}^{(0)} &= \sum_{j=1}^h X_{(r1)j}^{(1)} + \sum_{j=1}^h X_{(r1)j}^{(2)} + \sum_{d=1}^D X_{(r1)d}^{(3)} \\ &+ X_{(r1)}^{(4)} + X_{(r1)}^{(5)} + X_{(r1)}^{(6)} \\ &+ \sum_{i=1}^g \sum_{s=1}^2 \sum_{j=1}^h \sum_{k=1}^2 X_{(r1)}^{(is)jk} \\ &+ \sum_{i=1}^g \sum_{s=1}^2 \sum_{d=1}^D X_{(r1)}^{(is)d3} + \sum_{i=1}^g X_{(r1)}^{(i1)4} \\ &+ \sum_{i=1}^g \sum_{s=1}^2 \sum_{k=5}^6 X_{(r1)}^{(is)k} \\ r &= 1, \dots, g \end{aligned} \tag{5.162}$$

For primary factors, the equations are:

$$L_m = \sum_{j=1}^h X_{(g+1,1,m)j}^{(1)} \quad , \quad m = 1, \dots, M \tag{5.163}$$

$$K_j(0) = X_{(g+1,2)j}^{(1)} \quad , \quad j = 1, \dots, h \tag{5.164}$$

$$N_j = X_{(g+1,3)j}^{(1)} \quad , \quad j = 1, \dots, h \tag{5.165}$$

where L_m is the supply of labor of skill m , N_j is the supply of agricultural land in each industry, and all the other variables are as define before.

In percentage change form, equations (5.162)-(5.165) are:

$$\begin{aligned}
x_{(r1)}^{(0)} = & \sum_{j=1}^h x_{(r1)j}^{(1)} B_{(r1)j}^{(1)} + \sum_{j=1}^h x_{(r1)j}^{(2)} B_{(r1)j}^{(2)} \\
& + \sum_{d=1}^D x_{(r1)d}^{(3)} B_{(r1)d}^{(3)} \\
& + x_{(r1)}^{(4)} B_{(r1)}^{(4)} + x_{(r1)}^{(5)} B_{(r1)}^{(5)} \\
& + x_{(r1)}^{(6)} B_{(r1)}^{(6)} \\
& + \sum_{i=1}^g \sum_{s=1}^2 \sum_{j=1}^h \sum_{k=1}^2 x_{(r1)}^{(is)jk} B_{(r1)}^{(is)jk} \\
& + \sum_{i=1}^g \sum_{s=1}^2 \sum_{d=1}^D x_{(r1)}^{(is)d3} B_{(r1)}^{(is)d3} \\
& + \sum_{i=1}^g x_{(r1)}^{(i1)4} B_{(r1)}^{(i1)4} \\
& + \sum_{i=1}^g \sum_{s=1}^2 \sum_{k=5}^6 x_{(r1)}^{(is)k} B_{(r1)}^{(is)k} \\
& r = 1, \dots, g
\end{aligned} \tag{5.166}$$

$$l_m = \sum_{j=1}^h x_{(g+1,1,m)j}^{(1)} B_{(g+1,1,m)j}^{(1)} \quad , \quad m = 1, \dots, M \tag{5.167}$$

$$k_j(0) = x_{(g+1,2)j}^{(1)} \quad , \quad j = 1, \dots, h \tag{5.168}$$

$$n_j = x_{(g+1,3)j}^{(1)} \quad , \quad j = 1, \dots, h \tag{5.169}$$

where the B's in equation (5.166) are the shares of the sales of domestically produced goods which are absorbed by the various types of demands identified on the RHS of (5.162), and the B's in equation (5.167) are employment shares. All the other variables are as defined before.

5.11. Aggregate Imports, Exports and the Balance of Trade

Aggregate demand for imported good r , $r = 1, \dots, g$, is denoted by $X_{(r2)}^{(0)}$ and computed as:

$$\begin{aligned}
X_{(r2)}^{(0)} = & \sum_{k=1}^2 \sum_{j=1}^h x_{(r2)j}^{(k)} + \sum_{d=1}^D X_{(r2)d}^{(3)} \\
& + X_{(r2)}^{(5)} + X_{(r2)}^{(6)} \\
& r = 1, \dots, g
\end{aligned} \tag{5.170}$$

In percentage change form, this equation is:

$$\begin{aligned}
 x_{(r2)}^{(0)} = & \sum_{k=1}^2 \sum_{j=1}^h x_{(r2)j}^{(k)} B_{(r2)j}^{(k)} \\
 & + \sum_{d=1}^D x_{(r2)d}^{(3)} B_{(r2)d}^{(3)} \\
 & + x_{(r2)}^{(5)} B_{(r2)}^{(5)} + x_{(r2)}^{(6)} B_{(r2)}^{(6)} \\
 & r = 1, \dots, g
 \end{aligned} \tag{5.171}$$

where the B's are shares of total import flows.

In terms of foreign currency costs, the aggregate value of imports, M, is given by:

$$M = \sum_{r=1}^g P_{(r2)}^m x_{(r2)}^{(0)} \tag{5.172}$$

and in percentage change form, one has:

$$m = \sum_{r=1}^g (p_{(r2)}^m + x_{(r2)}^{(0)}) M_{(r2)} \tag{5.173}$$

where $M_{(r2)}$ is the share of the aggregate foreign currency cost of commodity imports which is accounted for by imports of good r.

The aggregate foreign currency receipts, E, from commodity exports is given by:

$$E = \sum_{r=1}^g P_{(r1)}^e x_{(r1)}^{(4)} \tag{5.174}$$

In percentage change form it is,

$$e = \sum_{r=1}^g (p_{(r1)}^e + x_{(r1)}^{(4)}) E_{(r1)} \tag{5.175}$$

where $E_{(r1)}$ is good r's share in aggregate export receipts.

The balance of trade on commodity account is expressed as:

$$B = E - M \tag{5.176}$$

That gives:

$$100\Delta B = Ee - Mm \tag{5.177}$$

where ΔB is the change (not percentage change) in B.

5.12. Macro Indices and Wage Indexation

The percentage change in the consumer price index is given by:

$$\xi(3) = \sum_{s=1}^2 \sum_{i=1}^g \sum_{d=1}^D w_{(is)d}^{(3)} p_{(is)d}^{(3)} \quad (5.178)$$

where $w_{(is)d}^{(3)}$ is the share of aggregate consumer spending devoted to good i from source s by income group d .

The percentage change in the capital goods price index is as follows:

$$\xi(2) = \sum_{j \in J} \tilde{T}_j \pi_j \quad (5.179)$$

where $\tilde{T}_j = T_j / \sum_{j \in J} T_j$, i.e., \tilde{T}_j is the share of total private investment expenditure accounted for by industry j .

The percentage change in aggregate employment is given by:

$$l = \sum_{m=1}^M l_m \psi_{lm} \quad (5.180)$$

where ψ_{lm} is the share of skill m in total employment.

The percentage change in the aggregate capital stock in base period value units is given by:

$$k(0) = \sum_{j=1}^h k_j(0) \psi_{2j} \quad (5.181)$$

where ψ_{2j} is the share of capital of type j (value at base period prices) in the total value of fixed capital for the economy.

The percentage change in the ratio of real private investment expenditure to real private consumption is given by:

$$f_r = i_r - c_r \quad (5.182)$$

Allowing wages and the prices of "other costs" to be indexed by the model consumer price index, one has that in percentage change form:

$$\begin{aligned}
 p_{(g+1,1,m)j}^{(1)} &= h_{(g+1,1,m)j}^{(1)} \xi^{(3)} + f_{(g+1,1)}^{(1)} + f_{(g+1,1)j}^{(1)} \\
 &\quad + f_{(g+1,1,m)}^{(1)} + f_{(g+1,1,m)j}^{(1)} \\
 m &= 1, \dots, M, \quad j = 1, \dots, h
 \end{aligned} \tag{5.183}$$

and:

$$p_{g+2,j}^{(1)} = h_{g+2,j}^{(1)} \xi^{(3)} + f_{g+2,j}^{(1)}, \quad j = 1, \dots, h \tag{5.184}$$

where the f's are variables and the h's are parameters.

5.13. The Complete Model

The model presented in this Chapter is constructed for 21 industries (sectors) and products (defined as h and g), 3 categories of labor (defined as M), and 3 income groups (defined as D). The names of the industries and products can be seen in Table A.7 in Appendix A. The 3 categories of labor and income groups are constructed in a way that the consumption basket of category of labor i is associated with the household consumption of income group i. Labor type 1 incorporates the workers in the income group 1 that receive between 0 and 5 minimum wages, type 2 incorporates the range from 5 to 20 minimum wages, and type 3 refers to range of more than 20 minimum wages.

The model assumes that each sector has some kind of margins associated with it (see section 5.7), but for the Brazilian case only sector 19 (Trade and Transport) has value of margins different from zero. The implication of this is a reduction in the number of equations, variables, parameters and coefficients in the model.

The subscripts referred as g+1 and g+2 in this chapter assume

the values of 22 and 23 respectively.

Before one goes on, it is necessary to make sure that the original source of ideas is acknowledged. Tables 5.1, and 5.2 are based respectively on Tables 23.1, 23.2 of DPSV (1982).

Tables 5.1 and 5.2 present, respectively, the model equations and the variables in the model with the ranges presented above. There are 8459 equations and 17695 variables, resulting in 9236 variables to be set exogenously. As many of the exogenous variables will have a value of zero, one does not need to be much worried about this number, however, the problem of closure in the model still remains and will be dealt with in Chapter 7.

Another problem that still remains is related to the large number of equations in the original system (presented in Table 5.1). To solve this, the original system will be transformed into a condensed system and the simulations will be done on the condensed system. The variables eliminated in the condensed system are marked with either a # or a & in Table 5.1, the condensed system is presented in Chapter 7.

Table 5.1

Model EquationsA Linear System in Percentage Changes

1. Demand for intermediate inputs, domestic and imported:

$$\begin{aligned} x_{(is)j}^{(1)} = & z_j - \sigma_{ij}^{(1)} (p_{(is)j}^{(1)} - \sum_s S_{(is)j}^{(1)} p_{(is)j}^{(1)}) \\ & + a_j^{(1)} + a_{ij}^{(1)} + a_{(is)j}^{(1)} \\ & - \sigma_{ij}^{(1)} (a_{(is)j}^{(1)} - \sum_s S_{(is)j}^{(1)} a_{(is)j}^{(1)}) \end{aligned} \quad (5.35)$$

$$i = 1, \dots, 21, \quad s = 1, 2, \quad j = 1, \dots, 21, \quad \text{N. of eqs.} = 882$$

2. Demand for labor by industry and skill group:

$$\begin{aligned} x_{(22,1,q)j}^{(1)} = & x_{(22,1)j}^{(1)} + a_{(22,1,q)j}^{(1)} \\ & - \sigma_{(22,1,q)j}^{(1)} (p_{(22,1,q)j}^{(1)} - \sum_q S_{(22,1,q)j}^{*(1)} p_{(22,1,q)j}^{(1)}) \\ & - \sigma_{(22,1,q)j}^{(1)} (a_{(22,1,q)j}^{(1)} - \sum_q S_{(22,1,q)j}^{*(1)} a_{(22,1,q)j}^{(1)}) \end{aligned}$$

$$q = 1, 2, 3, \quad j = 1, \dots, 21, \quad \text{N. of eqs.} = 63 \quad (5.66)$$

3. Industry demands for primary factors:

$$\begin{aligned} x_{(22,v)j}^{(1)} = & z_j + a_j^{(1)} + a_{22,j}^{(1)} + a_{(22,v)j}^{(1)} \\ & - \sigma_{(22,v)j}^{(1)} (p_{(22,v)j}^{(1)} - \sum_{v=1}^3 S_{(22,v)j}^{*(1)} p_{(22,v)j}^{(1)}) \\ & - \sigma_{(22,v)j}^{(1)} (a_{(22,v)j}^{(1)} - \sum_{v=1}^3 S_{(22,v)j}^{*(1)} a_{(22,v)j}^{(1)}) \end{aligned}$$

$$v = 1, 2, 3, \quad j = 1, \dots, 21, \quad \text{N. of eqs.} = 63 \quad (5.73)$$

4. Price to each industry of labor in general:

$$\begin{aligned} p_{(22,1)j}^{(1)} = & \sum_{q=1}^3 p_{(22,1,q)j}^{(1)} S_{(22,1,q)j}^{(1)} + \\ & \sum_{q=1}^3 a_{(22,1,q)j}^{(1)} S_{(22,1,q)j}^{(1)} \end{aligned} \quad (5.75)$$

$$j = 1, \dots, 21, \quad \text{N. of eqs.} = 21$$

Table 5.1 (continued)

5. Demand for "other costs":

$$x_{23,j}^{(1)} = z_j + a_j^{(1)} + a_{23,j}^{(1)} \quad (5.76)$$

$j = 1, \dots, 21 \quad , \quad \text{N. of eqs.} = 21$

6. Demand for inputs to capital creation:

$$x_{(is)j}^{(2)} = y_j + a_j^{(2)} + a_{ij}^{(2)} + a_{(is)j}^{(2)} \quad (5.80)$$

$$- \sigma_{ij}^{(2)} (p_{(is)j}^{(2)} - \sum_{s=1}^2 s_{(is)j}^{(2)} p_{(is)j}^{(2)})$$

$$- \sigma_{ij}^{(2)} (a_{(is)j}^{(2)} - \sum_{s=1}^2 s_{(is)j}^{(2)} a_{(is)j}^{(2)})$$

$i = 1, \dots, 21 \quad , \quad s = 1, 2 \quad , \quad j = 1, \dots, 21 \quad , \quad \text{N. of eqs.} = 882$

7. Household demands for commodities classified by source:

$$x_{(is)d}^{(3)} = x_{id}^{(3)} + a_{(is)d}^{(3)}$$

$$- \sigma_{id}^{(3)} (p_{(is)d}^{(3)} - \sum_{s=1}^2 s_{(is)d}^{(3)} p_{(is)d}^{(3)})$$

$$- \sigma_{id}^{(3)} (a_{(is)d}^{(3)} - \sum_{s=1}^2 s_{(is)d}^{(3)} a_{(is)d}^{(3)}) \quad (5.101)$$

$i = 1, \dots, 21 \quad , \quad s = 1, 2 \quad , \quad d = 1, 2, 3 \quad , \quad \text{N. of eqs.} = 126$

8. General price of each commodity to households in each income group:

$$p_{id}^{(3)} = \sum_{s=1}^2 s_{(is)d}^{(3)} p_{(is)d}^{(3)} \quad (5.103)$$

$i = 1, \dots, 21 \quad , \quad d = 1, 2, 3 \quad , \quad \text{N. of eqs.} = 63$

9. Household demands of income group d for commodities, undifferentiated by source:

$$x_{id}^{(3)} - q_d = \varepsilon_{id}(c_d - q_d) + \sum_{k=1}^{21} \eta_{ikq} p_{kd}^{(3)} + a_{id}^{(3)} \quad (5.104)$$

$$+ \sum_{k=1}^{21} \eta_{ikd} [a_{kd}^{(3)} + \sum_{s=1}^2 s_{(ks)d}^{(3)} a_{(ks)d}^{(3)}]$$

$i = 1, \dots, 21 \quad , \quad d = 1, 2, 3 \quad , \quad \text{N. of eqs.} = 63$

Table 5.1 (continued)

10. Aggregate consumer budget for each income group:

$$c_d = \sum_{(22,1,m)j \in J_d} (p_{(22,1,m)j}^{(1)} + x_{(22,1,m)j}^{(1)}) H_{(22,1,m)jd}^{(1)} - s_d H_d \quad (5.115)$$

$m = 1, 2, 3$, $j = 1, \dots, 21$, $d = 1, 2, 3$, N. of eqs = 3

11. Export demand functions:

$$p_{(i1)}^e = - \gamma_i x_{(i1)}^{(4)} + f_{(i1)}^e \quad (5.117)$$

$i = 1, \dots, 21$, N. of eqs. = 21

12. Government and "other" demands for commodities classified by source:

$$x_{(is)}^{(k)} = c_{rh(is)}^{(k)} + f_{(is)}^{(k)} \quad (5.118)$$

$i=1, \dots, 21$, $s=1, 2$, $k=5, 6$, N. of eqs. = 84

13. Real household expenditure:

$$c_r = \sum_{d=1}^3 c_d O_d - \xi^{(3)} \quad (5.119)$$

N. of eqs. = 1

14. Demands for margins to facilitate commodity flows to producers and to capital creators:

$$x_{(19,1)}^{(is)jk} = x_{(is)j}^{(k)} + a_{(19,1)}^{(is)jk} \quad (5.121)$$

$i = 1, \dots, 21$, $j = 1, \dots, 21$, $k, s = 1, 2$, N. of eqs. = 1764

15. Idem, to households:

$$x_{(19,1)}^{(is)d3} = x_{(is)d}^{(3)} + a_{(19,1)}^{(is)d3} \quad (5.125)$$

$i = 1, \dots, 21$, $d = 1, 2, 3$, $s = 1, 2$, N. of eqs. = 126

16. Idem, to ports prior to export:

$$x_{(19,1)}^{(i1)4} = x_{(i1)}^{(4)} + a_{(19,1)}^{(i1)4} \quad (5.126)$$

$i = 1, \dots, 21$, N. of eqs. = 21

Table 5.1 (continued)

17. Idem, to government and "other" demands:

$$x_{(19,1)}^{(is)k} = x_{(is)}^{(k)} + a_{(19,1)}^{(is)k} \quad (5.127)$$

$$i = 1, \dots, 21, \quad s = 1, 2, \quad k = 5, 6, \quad \text{N. of eqs.} = 84$$

18. Zero pure profits in production:

$$p_{(j1)}^{(0)} = \sum_{i=1}^{21} \sum_{s=1}^2 p_{(is)j}^{(1)} H_{(is)j}^{(1)} \quad (5.132)$$

$$+ \sum_{m=1}^3 p_{(22,1,m)j}^{(1)} H_{(22,1,m)j}^{(1)}$$

$$+ \sum_{s=2}^3 p_{(22,s)j}^{(1)} H_{(22,s)j}^{(1)}$$

$$+ p_{23,j}^{(1)} H_{23,j}^{(1)} + a_{(j)}$$

$$j = 1, \dots, 21, \quad \text{N. of eqs.} = 21$$

19. Weighted sums of the technical-change terms affecting the production functions of each industry:

$$a_{(j)} = a_{(j)}^{(0)} + a_{(j)}^{(1)} + \sum_{i=1}^{23} a_{ij}^{(1)} H_{ij}^{(1)} \quad (5.133)$$

$$+ \sum_{i=1}^{21} \sum_{s=1}^2 a_{(is)j}^{(1)} H_{(is)j}^{(1)}$$

$$+ \sum_{s=1}^3 a_{(22,s)j}^{(1)} H_{(22,s)j}^{(1)}$$

$$+ \sum_{m=1}^3 a_{(22,1,m)j}^{(1)} H_{(22,1,m)j}^{(1)}$$

$$j = 1, \dots, 21, \quad \text{N. of equations} = 21$$

20. Zero pure profits in capital creation:

$$\pi_j = \sum_{i=1}^{21} \sum_{s=1}^2 p_{(is)j}^{(2)} H_{(is)j}^{(2)} + \sum_{i=1}^{21} a_{ij}^{(2)} H_{ij}^{(2)} \quad (5.134)$$

$$+ \sum_{i=1}^{21} \sum_{s=1}^2 a_{(is)j}^{(2)} H_{(is)j}^{(2)} + a_{(j)}^{(2)}$$

$$j = 1, \dots, 21, \quad \text{N. of eqs.} = 21$$

21. Zero pure profits in importing:

$$p_{(i2)}^{(0)} = (p_{(i2)}^m + \phi) \zeta_1(i2,0) + g(i2,0) \zeta_2(i2,0)$$

$$i = 1, \dots, 21, \quad \text{N. of eqs.} = 21 \quad (5.137)$$

Table 5.1 (continued)

22. Flexible handling of tariff rates:

$$g(i2,0) = h_1(i2,0) \xi^{(3)} + h_2(i2,0)[t(i2,0) + p(i2)^m + \phi] \\ + h_3(i2,0)v(i2,0) \\ i = 1, \dots, 21, \quad N. \text{ of eqs} = 21 \quad (5.138)$$

23. Zero pure profits in exporting:

$$(p(i1)^e + \phi) = p(i1)^{(0)} \zeta_1(i1,4) + g(i1,4) \zeta_2(i1,4) \\ + (\sum_{r=1}^{21} M_{(r1)}^{(i1)4} p_{(r1)}^{(0)}) \zeta_3(i1,4) \\ + (\sum_{r=1}^{21} M_{(r1)}^{(i1)4} a_{(r1)}^{(i1)4}) \zeta_3(i1,4) \\ i = 1, \dots, 21, \quad N. \text{ of eqs} = 21 \quad (5.141)$$

24. Flexible handling of export taxes (subsidies):

$$g(i1,4) = h_1(i1,4) \xi^{(3)} + h_2(i1,4)[t(i1,4) + p(i1)^e + \phi] \\ + h_3(i1,4)v(i1,4) \\ i = 1, \dots, 21, \quad N. \text{ of eqs.} = 21 \quad (5.142)$$

25. Zero pure profits in the distribution of goods to producers and capital creators:

$$p(is)_j^{(k)} = p(is)^{(0)} \zeta_1(is,jk) + g(is,jk) \zeta_2(is,jk) \quad (5.145) \\ + (\sum_{r=1}^{21} M_{(r1)}^{(is)jk} p_{(r1)}^{(0)}) \zeta_3(is,jk) \\ + (\sum_{r=1}^{21} M_{(r1)}^{(is)jk} a_{(r1)}^{(is)jk}) \zeta_3(is,jk) \\ i = 1, \dots, 21, \quad j = 1, \dots, 21, \quad s, k = 1, 2, \quad N. \text{ of eqs.} = 1764$$

26. Zero pure profits in the distribution of goods to households:

$$p(is)_d^{(3)} = p(is)^{(0)} \zeta_1(is,d3) + g(is,d3) \zeta_2(is,d3) \quad (5.146) \\ + (\sum_{r=1}^{21} M_{(r1)}^{(is)d3} p_{(r1)}^{(0)}) \zeta_3(is,d3) \\ + (\sum_{r=1}^{21} M_{(r1)}^{(is)d3} a_{(r1)}^{(is)d3}) \zeta_3(is,d3) \\ i = 1, \dots, 21, \quad d = 1, 2, 3, \quad s = 1, 2, \quad N. \text{ of eqs.} = 126$$

Table 5.1 (continued)

27. Flexible handling of taxes (subsidies) to producers and capital creators:

$$g(is,jk) = h_1(is,jk) \xi^{(3)} + h_2(is,jk)[t(is,jk) + p(is)^{(0)}] + h_3(is,jk)v(is,jk) \quad (5.147)$$

$i = 1, \dots, 21$, $j = 1, \dots, 21$, $s, k = 1, 2$, N. of eqs. = 1764

28. Flexible handling of taxes (subsidies) to households:

$$g(is,d3) = h_1(is,d3) \xi^{(3)} + h_2(is,d3)[t(is,d3) + p(is)^{(0)}] + h_3(is,d3)v(is,d3) \quad (5.148)$$

$i = 1, \dots, 21$, $d = 1, 2, 3$, $s = 1, 2$, N. of eqs. = 126

29. Rates of return on capital in each industry:

$$r_j(0) = Q_j(p_{(22,2)}j^{(1)} - \pi_j) \quad (5.156)$$

$j = 1, \dots, 21$, N. of eqs. = 21

30. Equality of rates of return across industries:

$$-\beta_j[k_j(1) - k_j(0)] + r_j(0) = \omega \quad (5.157)$$

$j \in J$, N. of eqs. = 16

30. Capital accumulation:

$$k_j(1) = k_j(0)(1 - G_j) + y_j G_j \quad (5.158)$$

$j = 1, \dots, 21$, N. of eqs. = 21

32. Investment budget:

$$\sum_{j \in J} (\pi_j + y_j) T_j = (\sum_{j \in J} T_j) i \quad (5.159)$$

N. of eqs. = 1

Table 5.1 (continued)

33. Equations for handling exogenous investment:

$$y_j = h_j^{(2)} i_r + f_j^{(2)} \quad (5.160)$$

$j \notin J$, N. of eqs. = 5

34. Real private investment:

$$i_r = i - \xi^{(2)} \quad (5.161)$$

N. of eqs. = 1

35. Demand equals supply for domestically produced commodities:

$$\begin{aligned} x_{(r1)}^{(0)} = & \sum_{j=1}^{21} x_{(r1)j}^{(1)} B_{(r1)j}^{(1)} + \sum_{j=1}^{21} x_{(r1)j}^{(2)} B_{(r1)j}^{(2)} \\ & + \sum_{d=1}^3 x_{(r1)d}^{(3)} B_{(r1)d}^{(3)} \\ & + x_{(r1)}^{(4)} B_{(r1)}^{(4)} + x_{(r1)}^{(5)} B_{(r1)}^{(5)} \\ & + x_{(r1)}^{(6)} B_{(r1)}^{(6)} \\ & + \sum_{i=1}^{21} \sum_{s=1}^2 \sum_{j=1}^{21} \sum_{k=1}^2 x_{(r1)}^{(is)jk} B_{(r1)}^{(is)jk} \\ & + \sum_{i=1}^{21} \sum_{s=1}^2 \sum_{d=1}^3 x_{(r1)}^{(is)d3} B_{(r1)}^{(is)d3} \\ & + \sum_{i=1}^{21} x_{(r1)}^{(i1)4} B_{(r1)}^{(i1)4} \\ & + \sum_{i=1}^{21} \sum_{s=1}^2 \sum_{k=5}^6 x_{(r1)}^{(is)k} B_{(r1)}^{(is)k} \end{aligned}$$

$r = 1, \dots, 21$, N. of eqs. = 21 (5.166)

36. Demand equals supply for labor of each skill:

$$l_m = \sum_{j=1}^h x_{(22,1,m)j}^{(1)} B_{(22,1,m)j}^{(1)} \quad (5.167)$$

$m = 1, 2, 3$, N. of eqs. = 3

37. Demand equals supply for capital:

$$k_j^{(0)} = x_{(22,2)j}^{(1)} \quad (5.168)$$

$j = 1, \dots, 21$, N. of eqs. = 21

Table 5.1 (continued)

38. Demand equals supply for agricultural land:

$$n_j = x_{(22,3)} j^{(1)} \quad (5.169)$$

$$j = 1, \dots, 21, \quad \text{N. of eqs.} = 21$$

39. Import volumes:

$$x_{(r2)}^{(0)} = \sum_{k=1}^2 \sum_{j=1}^{21} x_{(r2)} j^{(k)} B_{(r2)} j^{(k)} \quad (5.171)$$

$$+ \sum_{d=1}^3 x_{(r2)} d^{(3)} B_{(r2)} d^{(3)}$$

$$+ x_{(r2)}^{(5)} B_{(r2)}^{(5)} + x_{(r2)}^{(6)} B_{(r2)}^{(6)}$$

$$r = 1, \dots, 21, \quad \text{N. of eqs.} = 21$$

40. Foreign currency value of imports:

$$m = \sum_{r=1}^{21} (p_{(r2)}^m + x_{(r2)}^{(0)}) M_{(r2)} \quad (5.173)$$

$$\text{N. of eqs.} = 1$$

41. Foreign currency value of exports:

$$e = \sum_{r=1}^{21} (p_{(r1)}^e + x_{(r1)}^{(4)}) E_{(r1)} \quad (5.175)$$

$$\text{N. of eqs.} = 1$$

42. The balance of trade:

$$100\Delta B = Ee - Mm \quad (5.177)$$

$$\text{N. of eqs.} = 1$$

43. Model consumer price index:

$$\xi^{(3)} = \sum_{s=1}^2 \sum_{i=1}^{21} \sum_{d=1}^3 w_{(is)} d^{(3)} p_{(is)} d^{(3)} \quad (5.178)$$

$$\text{N. of eqs.} = 1$$

44. Model capital-goods price index:

$$\xi^{(2)} = \sum_{j \in J} \tilde{T}_j \pi_j \quad (5.179)$$

$$\text{N. of eqs.} = 1$$

Table 5.1 (continued)

45. Aggregate employment:

$$l = \sum_{m=1}^3 l_m \Psi_{lm} \quad (5.180)$$

N. of eqs. = 1

46. Aggregate capital stock:

$$k(0) = \sum_{j=1}^{21} k_j(0) \Psi_{2j} \quad (5.181)$$

N. of eqs. = 1

47. Ratio of real investment to real consumption:

$$f_r = i_r - c_r \quad (5.182)$$

N. of eqs. = 1

48. Flexible handling of wages by occupation and industry:

$$p_{(22,1,m)j}^{(1)} = h_{(22,1,m)j}^{(1)} \xi^{(3)} + f_{(22,1)}^{(1)} + f_{(22,1)j}^{(1)} \\ + f_{(22,1,m)}^{(1)} + f_{(22,1,m)j}^{(1)}$$

$$m = 1, 2, 3 \quad , \quad j = 1, \dots, 21 \quad , \quad \text{N. of eqs.} = 63 \quad (5.183)$$

49. Indexing of the prices of "other costs":

$$p_{23,j}^{(1)} = h_{23,j}^{(1)} \xi^{(3)} + f_{23,j}^{(1)} \quad (5.184)$$

$$j = 1, \dots, 21 \quad , \quad \text{N. of eqs.} = 21$$

Table 5.2

Variables in the Model^a

Variable	Sub(super)script Range	Number	Description
$x_{(is)j}^{(k)}$	$i=1, \dots, 21$ $s=1, 2$ $j=1, \dots, 21$ $k=1, 2$	1764 [#]	Demands for inputs (domestic and imported) for current production and capital creation.
$x_{(22,1,q)j}^{(1)}$	$q=1, 2, 3$ $j=1, \dots, 21$	63 [#]	Demands for labor inputs by skill group and industry.
$x_{(22,v)j}^{(1)}$	$v=1, 2, 3$ $j=1, \dots, 21$	63 [#]	Industry demands for labor in general, capital and agricultural land.
$x_{23,j}^{(1)}$	$j=1, \dots, 21$	21 [#]	Demand for "other costs".
$x_{(is)d}^{(3)}$	$i=1, \dots, 21$ $s=1, 2$ $d=1, 2, 3$	126	Household demands for goods by type and source.
$x_{id}^{(3)}$	$i=1, \dots, 21$ $d=1, 2, 3$	63 [#]	Household demands for goods by type, undifferentiated by source.
$x_{(i1)}^{(4)}$	$i=1, \dots, 21$	21	Export volumes.
$x_{(is)}^{(k)}$	$i=1, \dots, 21$ $s=1, 2$ $k=5, 6$	84 [#]	Government and "other" demands for goods by type and source.
$x_{(19,1)}^{(is)jk}$	$i=1, \dots, 21$ $j=1, \dots, 21$ $s=1, 2$ $k=1, 2$	1764 [#]	Demand for margin services to facilitate commodity flows to production and capital creation.
$x_{(19,1)}^{(is)d3}$	$i=1, \dots, 21$ $s=1, 2$ $d=1, 2, 3$	126 [#]	Demand for margin services to facilitate the flow of goods to households.
$x_{(19,1)}^{(i1)4}$	$i=1, \dots, 21$	21 [#]	Idem, to ports for export.

Table 5.2 (continued)

Variable	Sub(super)script Range	Number	Description
$x_{(19,1)}^{(is)k}$	$i=1, \dots, 21$ $s=1, 2$ $k=5, 6$	84	Idem, to government and "other" users.
$x_{(r1)}^{(0)}$	$r=1, \dots, 21$	21	Total supply of domestic commodities.
$x_{(r2)}^{(0)}$	$r=1, \dots, 21$	21	Aggregate imports by commodity.
$p_{(is)j}^{(k)}$	$i=1, \dots, 21$ $s=1, 2$ $j=1, \dots, h$ $k=1, 2$	1764 [#]	Purchasers' prices for produced inputs for current production and capital creation.
$p_{(22,1,q)j}^{(1)}$	$q=1, 2, 3$ $j=1, \dots, 21$	63	Prices paid by industries for units of labor of different skill categories.
$p_{(22,v)j}^{(1)}$	$v=1, 2, 3$ $j=1, \dots, 21$	63 ^{&}	Prices paid by each industry for their labor in general, rental of capital and rental of agricultural land.
$p_{(is)d}^{(3)}$	$i=1, \dots, 21$ $s=1, 2$ $d=1, 2, 3$	126 [#]	Purchasers' prices paid for commodities by households.
$p_{id}^{(3)}$	$i=1, \dots, 21$ $d=1, 2, 3$	63 [#]	Purchasers' prices for consumer goods by type but not by source.
$p_{(i1)}^e$	$i=1, \dots, 21$	21	F.O.B. foreign currency export prices.
$p_{(is)}^{(0)}$	$i=1, \dots, 21$ $s=1, 2$	42	Basic prices of both domestic goods and imports.
$p_{23,j}^{(1)}$	$j=1, \dots, 21$	21 [#]	Price of "other costs" to each industry.
$p_{(i2)}^m$	$i=1, \dots, 21$	21	C.I.F. foreign currency import prices.

Table 5.2 (continued)

Variable	Sub(super)script Range	Number	Description
z_j	$j=1, \dots, 21$	21	Industry activity levels.
q_d	$d=1, 2, 3$	3	Number of households in each income group.
c_d	$d=1, 2, 3$	3	Aggregate household expenditure in each income group.
s_d	$d=1, 2, 3$	3	Aggregate residual value (income less consumption) in each income group.
c_r		1	Real aggregate household expenditure.
π_j	$j=1, \dots, 21$	21	Costs of units of capital.
ϕ		1	The exchange rate, Cr\$ per US\$, say.
$\xi(3)$		1	Model consumer price index.
$g(i2,0)$	$i=1, \dots, 21$	21 [#]	The g's are the tariffs per unit of imports. The t's and v's are variables allowing tariffs to be modelled as ad valorem or specific.
$t(i2,0)$		21	
$v(i2,0)$		21	
$g(i1,4)$	$i=1, \dots, 21$	21 [#]	Idem, for taxes, for exports.
$t(i1,4)$		21	
$v(i1,4)$		21	
$g(is,jk)$	$i=1, \dots, 21$	1764 [#]	Idem, for taxes, for sales of inputs to industries for current production and capital creation.
$t(is,jk)$	$j=1, \dots, 21$	1764	
$v(is,jk)$	$s=1, 2$	1764	
	$k=1, 2$		
$g(is,d3)$	$i=1, \dots, 21$	126 [#]	Idem, for taxes, for sales of commodities to households.
$t(is,d3)$	$s=1, 2$	126	
$v(is,d3)$	$d=1, 2, 3$	126	

Table 5.2 (continued)

Variable	Sub(super)script Range	Number	Description
$k_j^{(1)}$	$j=1, \dots, 21$	21 [#]	Future capital stocks.
$k_j^{(0)}$	$j=1, \dots, 21$	21	Current capital stocks.
$r_j^{(0)}$	$j=1, \dots, 21$	21	Current rates of return on fixed capital.
ω		1	Economy-wide expected rate of return on capital.
Y_j	$j=1, \dots, 21$	21	Capital creation by using industry.
i		1	Aggregate private investment expenditure.
i_r		1	Aggregate real private investment expenditure.
$f(i1)^e$	$i=1, \dots, 21$	21	Shifts in foreign export demands.
$f(is)^{(k)}$	$i=1, \dots, 21$ $s=1, 2$ $k=5, 6$	84	Shift terms for government and "other" demands.
$f_j^{(2)}$	$j \notin J$	5	Exogenous investment terms.
$\xi^{(2)}$		1	Model capital-goods price index.
l_m	$m=1, 2, 3$	3	Employment of labor by skill group.
n_j	$j=1, \dots, 21$	21	Use of agricultural land in each industry.
m		1	Foreign currency value of imports.
e		1	Foreign currency value of exports.
ΔB		1	The balance of trade.

Table 5.2 (continued)

Variable	Sub(super)script Range	Number	Description
l		1	Aggregate employment.
$k(0)$		1	Aggregate capital stock.
f_r		1	The ratio of real private investment expenditure to real household consumption expenditure.
$f_{(22,1)}^{(1)}$		1	General wage-shift variable.
$f_{(22,1)j}^{(1)}$	$j=1,\dots,21$	21	Variable used for simulating the effects of changes in the wages payable by particular industries relative to other industries.
$f_{(22,1,m)}^{(1)}$	$m=1,2,3$	3	Variable used in simulations involving changes in occupational wage relativities.
$f_{(22,1,m)j}^{(1)}$	$m=1,2,3$ $j=1,\dots,21$	63	Variable allowing changes in both occupational and industrial wage relativities.
$f_{23,j}^{(1)}$	$j=1,\dots,21$	21	Shift terms for allowing for changes in the real price of "other costs".
$a_j^{(0)}$	$j=1,\dots,21$	21	Neutral output-augmenting technical change.
$a_j^{(1)}$	$j=1,\dots,21$	21	Neutral input-augmenting technical change.
$a_j^{(2)}$	$j=1,\dots,21$	21	Idem, with respect to capital creation.
$a_{ij}^{(1)}$	$i=1,\dots,23$ $j=1,\dots,21$	483	Input-i-augmenting technical change.

Table 5.2 (continued)

Variable	Sub(super) script Range	Number	Description
$a_{ij}^{(2)}$	$i=1,\dots,21$ $j=1,\dots,21$	441	Idem, with respect to capital creation.
$a_{(is)j}^{(1)}$	$i=1,\dots,21$ $s=1,2$ $j=1,\dots,21$	882	Input-(is)-augmenting technical change.
$a_{(is)j}^{(2)}$	$i=1,\dots,21$ $s=1,2$ $j=1,\dots,21$	882	Idem, with respect to capital creation.
$a_{(22,1,q)j}^{(1)}$	$q=1,2,3$ $j=1,\dots,21$	63	Specific-skill-augmenting technical change.
$a_{(22,v)j}^{(1)}$	$v=1,2,3$ $j=1,\dots,21$	63	Labor-, capital- and agricultural-land-augmenting technical change.
$a_{id}^{(3)}$	$i=1,\dots,21$ $d=1,2,3$	63	Commodity-i-augmenting change in household preferences.
$a_{(is)d}^{(3)}$	$i=1,\dots,21$ $s=1,2$ $d=1,2,3$	126	Commodity-(is)-augmenting change in household preferences.
$a_{(19,1)}^{(is)jk}$	$i=1,\dots,21$ $s=1,2$ $j=1,\dots,21$ $k=1,2$	1764	Technical change associated with the use of services in facilitating input flows to industries for current production and capital creation.
$a_{(19,1)}^{(is)d3}$	$i=1,\dots,21$ $s=1,2$ $d=1,2,3$	126	Idem, for commodity flows to households.
$a_{(19,1)}^{(il)4}$	$i=1,\dots,21$	21	Idem, for the flow of exports from producers to the ports of exit.
$a_{(19,1)}^{(is)k}$	$i=1,\dots,21$ $s=1,2$ $k=5,6$	84	Idem, for commodity flows to government and "other" users.

Table 5.2 (continued)

Variable	Sub(super)script Range	Number	Description
a(j)	j=1,...,21	21 [#]	Weighted sums of the technical-change terms affecting the production functions for each industry

- Notes:

a. All variables are in percentage change with the exception of ΔB .

#. These variables are eliminated in the condensed system.

&. $p(22,v)j^{(1)}$ is eliminated for $v=1$ in the condensed system.

NOTES

1. In this model, $g = h$.
2. The first g factors refer to the g commodities, $g+1$ refer to primary factors, and $g+2$ refer to "other costs".
3. $s = 1$ refers to domestic commodity, while $s = 2$ refers to imported commodity.
4. The primary factor of type $s = 1$ refers to labor, $s = 2$ refers to fixed capital, and $s = 3$ refers to agricultural land.
5. For a discussion of CRESH functions see Hanoch (1971).
6. It will be assumed that none of the variables in the model will have a base period value of precisely zero. There is however an exception, the balance of trade, that will appear as change (ΔB) rather than a percentage change.
7. See Powell (1974), Klein and Rubin (1948-49), Stone (1954), and Geary (1950-51).
8. The model can be assumed to have the prices formed through a mark-up price theory, and variations in the mark-up rate can be studied through variations in the margins coefficients ($A_{(r1)}^{(is)jk}$, $A_{(r1)}^{(is)d3}$, $A_{(r1)}^{(i1)4}$, and $A_{(r1)}^{(is)5}$) introduced in section (5.7).
9. Differences across users in purchaser's prices are accounted for entirely by taxes and payments for margins.

CHAPTER 6

COEFFICIENTS AND PARAMETERS ESTIMATION

6.1. Introduction

This Chapter will present the data requirements as well as the way that the coefficients and parameters for the model, derived in Chapter 5, are estimated.

The coefficients and parameters in the model can come basically from 4 different sources: input-output data, econometric estimation, algebraic calculation, and user specification. The input-output data, the econometric estimation, and the algebraic calculation will be dealt with in this chapter.

Table 6.1 presents the coefficients and parameters in the model, as well as the method used to derive the coefficients and parameters estimated from the input-output data (the input-output requirements for the estimation are presented in section 6.2). Section 6.3 presents the econometric estimation and the algebraic calculation.

For the Brazilian case it is assumed that each industry produces only one type of commodity, in this way, the word sector throughout the text is used either to refer to industry or to product.

As for the original source of ideas, Table 6.1 is based on

Table 27.1 of DPSV (1982), and Figure 6.1 is based on Figure 25.1 of DPSV (1982). This chapter also draws from elements in Chapter 4 of DPSV (1982).

6.2. Input-Output Data

The input-output data used in the model are from the Brazilian input-output matrices for the year of 1975 (IBGE, 1984b). The data in its original form cannot be used to estimate the input-output derived coefficients and parameters in the model. To do so, some form of adjustments need to be made.

This section presents the data already adjusted. For the methodology used in adjusting the data see Appendix A.

The theoretical development is based on section 25 of DPSV (1982).

Figure 6.1 presents, in a schematic way, the input-output matrices needed to estimate the input-output derived coefficients and parameters.

Matrix A_1 shows the direct flows of domestic commodities into the production processes of the domestic industries. Matrix B_1 shows the direct flows of domestic commodities into capital formation. Matrix C_1 , and vectors D_1 , E_1 , and F_1 show, respectively, the flows to households, government, "other" demands, and exports.

The vector of "other" demands is used here as a way to eliminate the negative value of the gross operating surplus in sector 21 (Dummy). The only element different from zero in this

Figure 6.1
Input-Output Data Base for the Model

		Final Demands							
		Domestic industries (current production)	Domestic industries (capital formation)	Household consumption	Government	"Other" Demands	Exports		
Domestic Commodities		A ₁	B ₁	C ₁	D ₁	E ₁	F ₁	Row sums = total direct usage of domestic commodities	
Imports		A ₂	B ₂	C ₂	D ₂	0	0	Row sums = total imports (c.i.f.)	Duty Z ₂
Margin type 19 on dom. flows		G _{1,19}	H _{1,19}	I _{1,19}	J _{1,19}	0	L _{1,19}	Row sums = total of margin type 19 on sales of each domestic commodity	
Margin type 19 on imp. flows		G _{2,19}	H _{2,19}	I _{2,19}	J _{2,19}	0	0	Row sums = total of margin type 19 on sales of each imported commodity	
Tax on domestic flows		G _{1,22}	H _{1,22}	I _{1,22}	J _{1,22}	0	L _{1,22}	Row sums = total tax on sales of each domestic commodity	
Tax on imports flows		G _{2,22}	H _{2,22}	I _{2,22}	J _{2,22}	0	0	Row sums = total tax on sales of each imported commodity	
P I r n i p m . .	Labor	M	0	0	0	0	0		
	Capital	N							
	Land	P							
	Other costs	Q							
		Column sums = outputs of domestic industries at basic values	Column sums = investment expenditure by each industry	Column sums = household expenditure by each income group	Column sums = total of government expenditure	Column sums = total of "other" demands	Column sums = total of exports		

vector is in row 21. For more details about this vector see Appendix A.

Matrices A_1 , B_1 , C_1 , D_1 , E_1 , and F_1 contain only direct flows valued at basic prices. Flows of wholesale, transport and other margins services which are used to facilitate the transfer of commodities from producers to users are recorded elsewhere. In the notation of Chapter 5:

$$A_1 = [P(i1)^{(0)} X(i1)_j^{(1)}]_{21 \times 21}$$

$$B_1 = [P(i1)^{(0)} X(i1)_j^{(2)}]_{21 \times 21}$$

$$C_1 = [P(i1)^{(0)} X(i1)_d^{(3)}]_{21 \times 3}$$

$$D_1 = [P(i1)^{(0)} X(i1)^{(5)}]_{21 \times 1}$$

$$E_1 = [P(i1)^{(0)} X(i1)^{(6)}]_{21 \times 1}$$

$$F_1 = [P(i1)^{(0)} X(i1)^{(4)}]_{21 \times 1}$$

The first four nonzero matrices in the second row of Figure 6.1, A_2 , B_2 , C_2 , and D_2 , contain import flows valued at c.i.f. prices. The typical element of A_2 is the use of imported good i in production in sector j . B_2 shows the use of imports in capital creation, C_2 is the vector of imports for households consumption and D_2 is the vector of imports to government. In the notation of Chapter 5:

$$A_2 = [(P(i2)^{m\Phi}) X(i2)_j^{(1)}]_{21 \times 21}$$

$$B_2 = [(P(i2)^{m\Phi}) X(i2)_j^{(2)}]_{21 \times 21}$$

$$C_2 = [(P(i2)^{m\Phi}) X(i2)_d^{(3)}]_{21 \times 3}$$

$$D_2 = [(P(i2)^{m\Phi}) X(i2)^{(5)}]_{21 \times 1}$$

The final vector in the second row, Z_2 , shows the value of the import duty paid on commodities $1, 2, \dots, 21$. When one adds across the rows A_2 , B_2 , C_2 , D_2 , and Z_2 , one obtains the vector of commodity

imports valued at basic prices.

The third row of matrices, $G_{1,19}$, $H_{1,19}$, $I_{1,19}$, $J_{1,19}$, and $L_{1,19}$, are the flows of domestically produced commodity 19 (Trade and Transport) used as margins to facilitate each of the flows in matrices A_1 , B_1 , C_1 , D_1 , and F_1 . For example, the ij^{th} element of $G_{1,19}$ is the flow of good 19 used as a margin in the delivery of domestically produced intermediate input i to sector j . Hence,

$$G_{1,19} = [P_{(19,1)}^{(0)} X_{(19,1)}^{(i1)j1}]_{21 \times 21}$$

$$H_{1,19} = [P_{(19,1)}^{(0)} X_{(19,1)}^{(i1)j2}]_{21 \times 21}$$

$$I_{1,19} = [P_{(19,1)}^{(0)} X_{(19,1)}^{(i1)d3}]_{21 \times 3}$$

$$J_{1,19} = [P_{(19,1)}^{(0)} X_{(19,1)}^{(i1)5}]_{21 \times 1}$$

$$L_{1,19} = [P_{(19,1)}^{(0)} X_{(19,1)}^{(i1)4}]_{21 \times 1}$$

The nonzero matrices in the fourth row, $G_{2,19}$, $H_{2,19}$, $I_{2,19}$, and $J_{2,19}$, are the use of domestically produced good 19 as margins in the deliveries of imports from ports of entry to users. In notation of Chapter 5:

$$G_{2,19} = [P_{(19,1)}^{(0)} X_{(19,1)}^{(i2)j1}]_{21 \times 21}$$

$$H_{2,19} = [P_{(19,1)}^{(0)} X_{(19,1)}^{(i2)j2}]_{21 \times 21}$$

$$I_{2,19} = [P_{(19,1)}^{(0)} X_{(19,1)}^{(i2)d3}]_{21 \times 3}$$

$$J_{2,19} = [P_{(19,1)}^{(0)} X_{(19,1)}^{(i2)5}]_{21 \times 1}$$

The matrices in the fifth row, $G_{1,22}$, $H_{1,22}$, $I_{1,22}$, $J_{1,22}$, and $L_{1,22}$, are the tax (or subsidy if negative) associated with the delivery of domestically produced goods, respectively, to producers, capital creators, households, government, and exports, i.e.,

$$G_{1,22} = [G(i1,j1) X_{(i1)j}^{(1)}]_{21 \times 21}$$

$$H_{1,22} = [G(i1,j2) X_{(i1)j}^{(2)}]_{21 \times 21}$$

$$I_{1,22} = [G(i1,d3) X_{(i1)d}^{(3)}]_{21 \times 3}$$

$$J_{1,22} = [G(i1,5)X_{(i1)}^{(5)}]_{21 \times 1}$$

$$L_{1,22} = [G(i1,4)X_{(i1)}^{(4)}]_{21 \times 1}$$

The matrices in the following row, sixth, $G_{2,22}$, $H_{2,22}$, $I_{2,22}$, and $J_{2,22}$, are the tax (or subsidy if negative) associated with the delivery of imported goods, respectively, to producers, capital creators, households, and government. In Chapter 5 notation:

$$G_{2,22} = [G(i2,j1)X_{(i2)j}^{(1)}]_{21 \times 21}$$

$$H_{2,22} = [G(i2,j2)X_{(i2)j}^{(2)}]_{21 \times 21}$$

$$I_{2,22} = [G(i2,d3)X_{(i2)d}^{(3)}]_{21 \times 3}$$

$$J_{2,22} = [G(i2,5)X_{(i2)}^{(5)}]_{21 \times 1}$$

The next group of matrices, M , N , P , and Q , provide a breakdown of value added. The typical element of M is the purchase by industry j of labor of skill m , i.e.

$$M = [P_{(22,1,m)j}^{(1)}X_{(22,1,m)j}^{(1)}]_{3 \times 21}$$

The vector N contains the rental value of each sector's fixed capital, i.e.

$$N = [P_{(22,2)j}^{(1)}X_{(22,2)j}^{(1)}]_{1 \times 21}$$

Similarly, P shows the rental value of agricultural land used by each sector, i.e.

$$P = [P_{(22,3)j}^{(1)}X_{(22,3)j}^{(1)}]_{1 \times 21}$$

while Q is the "other costs" vector, i.e.

$$Q = [P_{23,j}^{(1)}X_{23,j}^{(1)}]_{1 \times 21}$$

The absence of labor, capital, land and "other costs" entries in the capital formation, household consumption, government, "other" final demands, and exports columns reflects the convention that primary factors are used only in current production.

Table A.7, in Appendix A, presents the values of the matrices

listed in Figure 6.1 and discussed above.

Table 6.1 lists the coefficients and parameters in the model, as well as the way that the coefficients and parameters derived from the input-output data are estimated (based on the matrices presented in Figure 6.1).

The values of the coefficients and parameters derived from the input-output data, and different from zero, are presented in Appendix C.

The next section will discuss the econometric estimation and the algebraic calculation.

6.3. Econometric Estimation and Algebraic Calculation

This section will present the methodology used in the construction of coefficients and parameters that require econometric estimation or algebraic calculation. This section will be divided in sub-sections, each sub-section will deal with the estimation of related coefficients and parameters.

6.3.1. Elasticity of Substitution Between Domestic and Foreign Sources of Supply

The elasticity of substitution between domestic and foreign sources of supply are assumed to be the same for all users, and to

Table 6.1Coefficients and Parameters in the Model1. $\sigma_{ij}^{(1)}$

- Elasticity of substitution between domestic and foreign sources of good i for use as a current input in the production of industry j .
- Econometric estimates.

2. $S_{(is)j}^{(1)}$

- Share of the purchasers-price value of good i from source s ($s=1$ for domestic, $s=2$ for imports) in industry j 's total purchases of good i for use as a current input to production.
- Input-output estimates. To form the $S_{(is)j}^{(1)}$, first sum the ij^{th} elements of matrices A_1 , A_2 , $G_{1,19}$, $G_{2,19}$, $G_{1,22}$, and $G_{2,22}$. This gives the total value, at purchaser's prices, of industry j 's current inputs of commodity i . Next sum the ij^{th} elements of A_1 , $G_{1,19}$, and $G_{1,22}$ to give the corresponding value of the domestically sourced input. $S_{(i1)j}^{(1)}$ is then the ratio of the second sum to the first. $S_{(i2)j}^{(1)}$ is computed as $1 - S_{(i1)j}^{(1)}$.

3. $\sigma_{(22,1,q)j}^{(1)}$

- CRESH parameter reflecting the degree of substitutability between labor of type q and labor of other types in the production process of industry j .
- Econometric estimates.

4. $S_{(22,1,q)j}^{(1)}$

- Share of labor of type q in the total cost of labor in industry j .
- Input-output estimates. To form $S_{(22,1,q)j}^{(1)}$ divide the qj^{th} element of matrix M by the sum of the j^{th} column of that matrix.

Table 6.1 (continued)5. $S_{(22,1,q)j}^{*(1)}$

- Modified share of labor of type q in the total cost of labor in industry j , defined according to (5.68) as a function of the unmodified shares ($S_{(22,1,q)j}^{(1)}$) and the CRESH substitution parameters ($\sigma_{(22,1,q)j}^{(1)}$).
- Estimated from previous econometric and input-output estimates.

6. $\sigma_{(22,v)j}^{(1)}$

- CRESH parameter reflecting the degree of substitutability between primary factor v and other primary factors in the production process of industry j .
- Econometric estimates.

7. $S_{(22,v)j}^{*(1)}$

- Modified share of primary factor v ($v=1$ for labor, $v=2$ for capital, $v=3$ for agricultural land) in the total cost of primary factors used in industry j , defined according to (5.71) as a function of the unmodified shares ($S_{(22,v)j}^{(1)}$) and the CRESH substitution parameters ($\sigma_{(22,v)j}^{(1)}$).
- Estimated from previous econometric estimates and the input-output data. To form $S_{(22,v)j}^{(1)}$ first sum down the j^{th} columns of matrices M , N , and P . Then $S_{(22,1)j}^{(1)}$ is the fraction of this sum represented by the sum of the j^{th} column of M , $S_{(g+1,2)j}^{(1)}$ is the fraction represented by the j^{th} component of N and $S_{(g+1,3)j}^{(1)}$ is the fraction represented by the j^{th} component of P .

8. $\sigma_{ij}^{(2)}$

- Elasticity of substitution between domestic and foreign sources of good i for use as an input to capital formation in industry j .
- Econometric estimates.

Table 6.1 (continued)9. $S_{(is)j}^{(2)}$

- Share of the purchasers value of good i from source s in industry j 's total purchases of i for input to capital creation.
- Input-output estimates. To form $S_{(i1)j}^{(2)}$, first sum the ij^{th} elements of B_1 , B_2 , $H_{1,19}$, $H_{2,19}$, $H_{1,22}$, and $H_{2,22}$. Next, sum the ij^{th} elements of B_1 , $H_{1,19}$, and $H_{1,22}$. $S_{(i1)j}^{(2)}$ is the ratio of the second to the first. $S_{(i2)j}^{(2)}$ is computed as $1 - S_{(i1)j}^{(2)}$.

10. $\sigma_{id}^{(3)}$

- Elasticity of substitution between domestic and foreign sources of good i for use by households in the income group d .
- Econometric estimates.

11. $S_{(is)d}^{(3)}$

- Share of the purchasers value of good i from source s in the total purchases of good i by households in the income group d .
- Input-output estimates. To form $S_{(i1)d}^{(3)}$ first sum the id^{th} elements of C_1 , C_2 , $I_{1,19}$, $I_{2,19}$, $I_{1,22}$, and $I_{2,22}$. Next sum the id^{th} elements of C_1 , $I_{1,19}$, and $I_{1,22}$. $S_{(i1)d}^{(3)}$ is the ratio of the second sum to the first. $S_{(i2)d}^{(3)}$ is computed as $1 - S_{(i1)d}^{(3)}$.

12. ϵ_{id}

- Household expenditure elasticity of demand for good i from either source.
- Econometric and input-output estimates. For details see section 6.3.5.

Table 6.1 (continued)13. η_{ikd}

- Household elasticities of demand for good i in general with respect to changes in the general household purchasers price for good k .
- Econometric and input-output estimates. For details see section 6.3.5.

14. $H_{(22,m)jd}^{(1)}$

- Share of individuals in the skill level m that work in industry j in the total aggregate income of income group d .
- Input-output estimates. For m different of d , it has a value equal to zero. For $m = d$, $H_{(22,m)jd}^{(1)}$ is the mj^{th} element of M expressed as a fraction of the total wage bill of labor in the skill level m , i.e., the sum over the m^{th} row of M .

15. H_d

- Share of the residual value (income less consumption) in the total aggregate income of income group d .
- Based on the national accounts and in the theory. The value of H_1 is zero, and the value of H_2 and H_3 is 0.3651 (see section A.11 in Appendix A).

16. O_d

- Share of income group d in the aggregate private consumption.
- Input-output estimates. First sum over the d^{th} columns of C_1 , C_2 , $I_{1,19}$, $I_{2,19}$, $I_{1,22}$, and $I_{2,22}$. O_d is the fraction of the sum of the d^{th} column in the sum over all components of these matrices.

17. γ_i

- Reciprocal of the foreign elasticity of demand for domestic good i .
- Econometric estimates. γ_i is stored as a positive number.

Table 6.1 (continued)18. $h_{(is)}^{(k)}$

- Indexing parameter. Fixes the relationship between movements in aggregate real private consumption and government ($k=5$) and "other" ($k=6$) demands for good i from source s .
- User specified.

19. $H_{(is)j}^{(1)}$

- Share of the purchasers value of inputs of good i from source s in the total costs of industry j .
- Input-output estimates. First compute the total cost in industry j by summing the j^{th} columns of matrices A_1 , A_2 , $G_{1,19}$, $G_{2,19}$, $G_{1,22}$, $G_{2,22}$, M , N , P , and Q . $H_{(i1)j}^{(1)}$ is the sum of the ij^{th} elements of A_1 , $G_{1,19}$, and $G_{1,22}$ expressed as a fraction of total costs. $H_{(i2)j}^{(1)}$ is the sum of the ij^{th} elements of A_2 , $G_{2,19}$, and $G_{2,22}$ expressed as a fraction of total costs.

20. $H_{(22,1,m)j}^{(1)}$

- Share of inputs of labor of type m in the total costs of industry j .
- Input-output estimates. $H_{(22,1,m)j}^{(1)}$ is the mj^{th} element of matrix M expressed as a fraction of the total costs in industry j .

21. $H_{(22,s)j}^{(1)}$

- Shares of inputs of capital ($s=2$) and land ($s=3$) in the total costs of industry j .
- Input-output estimates. $H_{(22,2)j}^{(1)}$ and $H_{(22,3)j}^{(1)}$ are computed by expressing the j^{th} elements of the vectors N and P as fractions of total costs in industry j .

22. $H_{23,j}^{(1)}$

- Share of "other costs" in the total costs of industry j .

Table 6.1 (continued)

- Input-output estimates. $H_{23,j}^{(1)}$ is the j^{th} element of the vector Q divided by the total costs in industry j .

23. $H_{ij}^{(1)}$

- For $i=1, \dots, 21$, $H_{ij}^{(1)}$ is the share of the purchasers value of intermediate inputs of good i in the total costs of industry j . For $i=22$, it is the share of primary factors (labor, capital and land) in total costs.
- Estimated from previous input-output estimates.

$$H_{ij}^{(1)} = \sum_{s=1}^2 H_{(is)j}^{(1)}, \quad i=1, \dots, 21, \text{ and}$$

$$H_{22,j}^{(1)} = \sum_{m=1}^3 H_{(22,1,m)j}^{(1)} + \sum_{s=2}^3 H_{(22,s)j}^{(1)}.$$

24. $H_{(is)j}^{(2)}$

- Share in the total costs of capital creation for industry j represented by the purchasers value of inputs of good i from source s .
- Input-output estimates. First compute the total costs of capital creation in industry j by summing the j^{th} columns of B_1 , B_2 , $H_{1,19}$, $H_{2,19}$, $H_{1,22}$, and $H_{2,22}$. $H_{(i1)j}^{(2)}$ is the sum of the ij^{th} elements of B_1 , $H_{1,19}$, and $H_{1,22}$ expressed as a fraction of the total costs of capital creation in industry j , and $H_{(i2)j}^{(2)}$ is the sum of the ij^{th} elements of B_2 , $H_{2,19}$, and $H_{2,22}$ expressed as a fraction of the total costs of j 's capital creation.

25. $H_{ij}^{(2)}$

- Share of the purchasers value of inputs of good i from both sources in the total costs of capital formation in industry j .
- Estimated from previous input-output estimates. $H_{ij}^{(2)}$ is computed as $\sum_{s=1}^2 H_{(is)j}^{(2)}$.

Table 6.1 (continued)26. $\zeta_1(i2,0)$

- Share of the landed, duty-free value in the basic value (i. e., the landed, duty-paid value) of imports of good i.
- Input-output estimates. First compute the landed, duty-free value of imports of good i by summing the i^{th} rows of matrices A_2 , B_2 , C_2 , and D_2 . The basic value is computed by adding the i^{th} element of vector Z_2 to this sum. $\zeta_1(i2,0)$ is then computed as the ratio of the duty-free value to the basic value.

27. $\zeta_2(i2,0)$

- Share of duty in the basic value of imports of good i.
- Estimated from previous input-output estimates. $\zeta_2(i2,0)$ is computed as $1 - \zeta_1(i2,0)$.

28. $h_1(i2,0)$

- Indexing parameter. Fixes the relationship between movements in the tariff per unit import of good i and the consumer price index.
- User specified.

29. $h_2(i2,0)$

- Parameter which allows the tariff per unit import of good i to be treated as ad valorem.
- User specified.

30. $h_3(i2,0)$

- Parameter which allows the tariff per unit import of good i to be treated as specific.
- User specified.

31. $\zeta_1(i1,4)$

- Share in the value of exports of good i at port of exit represented by the basic value of good i.

Table 6.1 (continued)

- Input-output estimates. First compute the at-port value of exports of good i by summing the i^{th} elements from the vectors F_1 , $L_{1,19}$, and $L_{1,22}$. $\zeta_1(i1,4)$ is the ratio of the i^{th} element of F_1 to this sum.
32. $\zeta_2(i1,4)$
- Share in the at-port value of exports of good i represented by export taxes or subsidies. $\zeta_2(i1,4)$ will be negative in the case of export subsidies.
 - Input-output estimates. $\zeta_2(i1,4)$ is the share of the i^{th} element of the vector $L_{1,22}$ in the at-port value of exports of good i .
33. $\zeta_3(i1,4)$
- Share of total margins (excluding export taxes) in the at-port export price of good i .
 - Input-output estimates. $\zeta_3(i1,4)$ is the share of the i^{th} element of the vector $L_{1,19}$ in the at-port value of exports of good i .
34. $M_{(r1)}(i1)4$
- Share of good r in the total cost of margins (excluding export taxes) required to transfer exports of good i from producers to the port of exit.
 - In the Brazilian case only sector 19 is used as a margin. So, for $r = 19$ and $(F_1)_i \neq 0$ one has that $M_{(r1)}(i1)4 = 1$, otherwise it has value zero.
35. $h_1(i1,4)$
- Indexing parameter. Fixes the relationship between movements in taxes (subsidies) per unit of export of good i and the consumer price index.
 - User specified.

Table 6.1 (continued)36. $h_2(i1,4)$

- Parameter which allows the export tax (subsidy) per unit of export of good i to be treated as ad valorem.
- User specified.

37. $h_3(i1,4)$

- Parameter which allows the export tax (subsidy) per unit of export of good i to be treated as specific.
- User specified.

38. $\zeta_1(is,jk)$

- Basic-value share in the purchasers value of good i from source s used as an input by industry j for purpose k ($k=1$ for current production, $k=2$ for capital formation).
- Input-output estimates. First compute the four purchasers values of good i flowing to industry j . The purchasers value of the domestic ($s=1$) flow for current purposes ($k=1$) is the sum $[(A_1)_{ij} + (G_{1,19})_{ij} + (G_{1,22})_{ij}]$. The purchasers value of domestic flow for capital purposes ($k=2$) is the sum $[(B_1)_{ij} + (H_{1,19})_{ij} + (H_{1,22})_{ij}]$. The two purchasers values of the imported flows are the sums $[(A_2)_{ij} + (G_{2,19})_{ij} + (G_{2,22})_{ij}]$ and $[(B_2)_{ij} + (H_{2,19})_{ij} + (H_{2,22})_{ij}]$. The basic value shares, $\zeta_1(is,jk)$, are shares of $(A_1)_{ij}$, $(B_1)_{ij}$, $(A_2)_{ij}$, and $(B_2)_{ij}$ in these four sums.

39. $\zeta_2(is,jk)$

- Share of commodity taxes in the purchasers value of inputs of good i from source s used by industry j for purpose k .
- Input-output estimates. The $\zeta_2(is,jk)$ are the shares of the ij^{th} elements of $G_{1,22}$ (for $s=1$, $k=1$), $H_{1,22}$ (for $s=1$, $k=2$), $G_{2,22}$ (for $s=2$, $k=1$), and $H_{2,22}$ (for $s=2$, $k=2$) in the four purchasers values of good i flowing to industry j .

Table 6.1 (continued)40. $\zeta_3(is, jk)$

- Share of total margins (excluding taxes) in the purchasers value of inputs of good i from source s used by industry j for purpose k .
- Input-output estimates. The $\zeta_3(is, jk)$ are the shares of the ij^{th} elements of $G_{1,19}$ (for $s=1, k=1$), $H_{1,19}$ (for $s=1, k=2$), $G_{2,19}$ (for $s=2, k=1$), and $H_{2,19}$ (for $s=2, k=2$) in the four purchasers value of good i flowing to industry j .

41. $M_{(r1)}(is)jk$

- Share of inputs of good r in the total cost of margins (excluding taxes) required to transfer flows of good i from source s from the producer (or port of entry) to user j for purpose k .
- In the Brazilian case only sector 19 is used as a margin. So, for $r = 19$ and for the cases where: $(A_1)_{ij} \neq 0$ (for $s=1, k=1$), $(B_2)_{ij} \neq 0$ (for $s=1, k=2$), $(A_2)_{ij} \neq 0$ (for $s=2, k=1$), and $(B_2)_{ij} \neq 0$ (for $s=2, k=2$), one has that $M_{(r1)}(is)jk = 1.0$, otherwise it has value zero.

42. $\zeta_1(is, d3)$

- Basic-value share in the purchasers value of good i from source s used by households in the income group d .
- Input-output estimates. The purchasers value of good i from domestic sources ($s=1$) flowing to households in the income group d is the sum $[(C_1)_{id} + (I_{1,19})_{id} + (I_{1,22})_{id}]$. The value for the corresponding import flow ($s=2$) is $[(C_2)_{id} + (I_{2,19})_{ij} + (I_{2,22})_{ij}]$. The basic value shares, $\zeta_1(is, d3)$, are the shares of $(C_1)_{id}$ and $(C_2)_{id}$ in these two purchasers values.

43. $\zeta_2(is, d3)$

- Share of commodity taxes in the purchasers value of good i from source s used by households in the income group d .

Table 6.1 (continued)

- Input-output estimates. The $\zeta_2(is, d3)$ are the shares of the id^{th} elements of $I_{1,22}$ (for $s=1$) and $I_{2,22}$ (for $s=2$) in the purchasers values for the two flows of good i to households in the income group d .

44. $\zeta_3(is, d3)$

- Share of total value of margins (excluding taxes) in the purchasers value of good i from source s used by households in the income group d .
- Input-output estimates. The $\zeta_3(is, d3)$ are the shares of the id^{th} elements of $I_{1,19}$ (for $s=1$) and $I_{2,19}$ (for $s=2$) in the purchasers values for the two flows of good i to households in the income group d .

45. $M_{(r1)}(is)d3$

- Share of inputs of good r in the total cost of margins (excluding taxes) required to transfer flows of good i from source s to households in the income group d .
- In the Brazilian case only sector 19 is used as a margin. So, for $r = 19$ and for the cases where: $(C_1)_{id} \neq 0$ (for $s=1$), $(C_2)_{id} \neq 0$ (for $s=2$), one has that $M_{(r1)}(is)d3 = 1.0$, otherwise it has value zero.

46. $h_1(is, jk)$

- Indexing parameter. Fixes the relationship between movements in the tax on the flow of good i from source s to industry j for purpose k and in the consumer price index.
- User specified.

47. $h_2(is, jk)$

- Parameter which allows taxes on intermediate and investment flows to be treated as ad valorem.
- User specified.

Table 6.1 (continued)48. $h_3(is, jk)$

- Parameter which allows taxes on intermediate and investment flows to be treated as specific.
- User specified.

49. $h_1(is, d3)$

- Indexing parameter. Fixes the relationship between movements in the tax on the flow of good i from source s to households in the income group d and in the consumer price index.
- User specified.

50. $h_2(is, d3)$

- Parameter which allows taxes on flows of good i to households to be treated as ad valorem.
- User specified.

51. $h_3(is, d3)$

- Parameter which allows taxes on flows of good i to households to be treated as specific.
- User specified.

57. Q_j

- Ratio of gross (before depreciation) to net (after depreciation) rate of return on capital in industry j with respect to increases in the planned stock of capital in industry j .
- See section 6.3.6 for method of estimation.

58. β_j

- Elasticity of the expected marginal rate of return on capital in industry j with respect to increases in the planned stock of capital in industry j .
- See section 6.3.6 for method of estimation.

59. G_j

- Ratio of industry j 's gross investment to its capital stock of the following year.
- See section 6.3.6 for method of estimation.

Table 6.1 (continued)60. T_j

- Share of total investment accounted for by industry j .
- Input-output estimates. First obtain the vector of industry investment expenditures by adding down the columns of B_1 , B_2 , $H_{1,19}$, $H_{2,19}$, $H_{1,22}$, and $H_{2,22}$. Then T_j is the share of the j^{th} entry of this vector in the sum of its elements.

61. J

- Set of integers identifying those industries for which the model is allowed to determine investment according to relative rates of return.
- User specified (see section 6.3.6).

62. $h_j(2)$

- Indexing parameter. Fixes the relationship between movements in aggregate real private investment and in investment in industry j where $j \notin J$.
- User specified.

63. $B_{(r1)j}^{(1)}$

- Share of total sales of domestic good r which is absorbed by industry j as a direct input into production.
- Input-output estimates. Computed as the rj^{th} element of A_1 divided by the total sales of domestic good r , i.e., the sum over the r^{th} rows of A_1 , B_1 , C_1 , D_1 , E_1 , and F_1 added to the sum of all entries in $G_{1,r}$, $G_{2,r}$, $H_{1,r}$, $H_{2,r}$, $I_{1,r}$, $I_{2,r}$, $J_{1,r}$, $J_{2,r}$, and $L_{1,r}$ (those matrices have values different from zero only for the case when $r = 19$).

64. $B_{(r1)j}^{(2)}$

- Share of total sales of domestic good r which is absorbed by industry j as a direct input to capital creation.

Table 6.1 (continued)

- Input-output estimates. Computed as the r_j^{th} element of B_1 divided by the total sales of domestic good r .

65. $B_{(r1)d}^{(3)}$

- Share of total sales of domestic good r which is absorbed by income group d as a direct input to household consumption.
- Input-output estimates. Computed as the r_d^{th} element of C_1 divided by the total sales of domestic good r .

66. $B_{(r1)}^{(4)}$

- Share of the total sales of domestic good r which is absorbed as a direct input to exports.
- Input-output estimates. Computed as the r^{th} element of F_1 divided by the total sales of domestic good r .

67. $B_{(r1)}^{(5)}$

- Share of the total sales of domestic good r which is absorbed as a direct input to government consumption.
- Input-output estimates. Computed as the r^{th} element of D_1 divided by the total sales of domestic good r .

68. $B_{(r1)}^{(6)}$

- Share of the total sales of domestic good r which is absorbed as a direct input to "other" demands.
- Input-output estimates. Computed as the r^{th} element of E_1 divided by the total sales of domestic good r .

69. $B_{(r1)}^{(is)jk}$

- Share of total sales of domestic good r which is absorbed as a margin on the sale of good i from source s to industry j for purpose k .

Table 6.1 (continued)

- Input-output estimates. For the Brazilian case, only for $r = 19$ $B_{(r1)}(is)jk$ has a value different from zero. $B_{(19,1)}(i1)j1$ is computed as the ij^{th} element of $G_{1,19}$ divided by the total sales of domestic good 19. $B_{(19,1)}(i2)j1$, $B_{(19,1)}(i1)j2$ and $B_{(19,1)}(i2)j2$ are respectively, the ij^{th} elements of $G_{2,19}$, $H_{1,19}$, and $H_{2,19}$ expressed as fractions of the total sales of domestic good 19.

70. $B_{(r1)}(is)d3$

- Share of total sales of domestic good r which is absorbed as a margin on the sale of good i from source s to households in the income group d .
- Input-output estimates. For the Brazilian case, only for $r = 19$ $B_{(r1)}(is)d3$ has a value different from zero. $B_{(19,1)}(i1)d3$ and $B_{(19,1)}(i2)d3$ are respectively, the id^{th} element of $I_{1,19}$ and $I_{2,19}$ expressed as fractions of the total sales of domestic good 19.

71. $B_{(r1)}(i1)4$

- Share of total sales of domestic good r which is absorbed as a margin on the transfer of exports of good i from producers to the ports of exit.
- Input-output estimates. For the Brazilian case, only for $r = 19$ $B_{(r1)}(i1)4$ has a value different from zero. $B_{(19,1)}(i1)4$ is computed as the ratio of the i^{th} element in $L_{1,19}$ to the total sales of domestic good 19.

72. $B_{(r1)}(is)k$

- Share of the total sales of domestic good r which is absorbed as a margin on the sale of good i from source s to government ($k=5$) and "other" demands ($k=6$).

Table 6.1 (continued)

- Input-output estimates. For the Brazilian case, only for $r = 19$ and $k = 5$ $B_{(r1)}^{(is)k}$ has a value different from zero. $B_{(19,1)}^{(i1)5}$ and $B_{(19,1)}^{(i2)5}$ are computed by expressing the i^{th} elements of $J_{1,19}$ and $J_{2,19}$ as fractions of the total sales of domestic good 19.

73. $B_{(22,1,m)j}^{(1)}$

- Share of the economy-wide employment in occupation m which is accounted for by industry j .
- Input-output estimates. First compute the economy-wide wage-bill for occupation m as the sum of the m^{th} row of matrix M . $B_{(22,1,m)j}^{(1)}$ is computed as the fraction of this row sum which is accounted for by the j^{th} element in the row.

74. $B_{(r2)j}^{(k)}$

- Share of total imports of good r which is absorbed by industry j for purpose k .
- Input-output estimates. First compute total imports of good r by summing across the r^{th} rows of A_2 , B_2 , C_2 and D_2 . $B_{(r2)j}^{(1)}$ and $B_{(r2)j}^{(2)}$ are respectively the shares of the rj^{th} element of A_2 and of B_2 in this sum.

75. $B_{(r2)d}^{(3)}$

- Share of total imports of good r which is absorbed by households in the income group d .
- Input-output estimates. $B_{(r2)d}^{(3)}$ is the share of the rd^{th} element of C_2 in total imports of good r .

76. $B_{(r2)}^{(5)}$

- Share of total imports of good r which is absorbed by government.

Table 6.1 (continued)

- Input-output estimates. $B_{(r2)}^{(5)}$ is the share of the r^{th} element of the vector D_2 in total imports of good r .

77. $B_{(r2)}^{(6)}$

- Share of the total imports of good r which is absorbed by "other" demands.
- For the Brazilian case, $B_{(r2)}^{(6)} = 0$ for every r .

78. $M_{(r2)}$

- Share in the foreign currency cost of total imports which is accounted for by imports of good r .
- Input-output estimates. The foreign currency value of total imports is computed as the sum of all elements of A_2 , B_2 , C_2 , and D_2 . $M_{(r2)}$ is the share in this total of the sum across the r^{th} rows of A_2 , B_2 , C_2 and D_2 .

79. $E_{(r1)}$

- Share of total export earnings which is accounted for by exports of good r .
- Input-output estimates. First form the vector $F_1 + L_{1,19} + L_{1,22}$. This show export earnings by commodity. $E_{(r1)}$ is the ratio of the r^{th} entry in this vector sum over all entries.

80. M

- Aggregate foreign currency value of imports.
- Input-output estimates. This is the sum of the elements in A_2 , B_2 , C_2 , and D_2 .

81. E

- Aggregate foreign currency value of exports.
- Input-output estimates. This is the sum of the elements in F_1 , $L_{1,19}$, and $L_{1,22}$.

Table 6.1 (continued)

82. $w_{(is)d}^{(3)}$

- Weight of good i from source s used in the income group d in the model consumer price index.
- Input-output estimates. First obtain the 42×3 matrix of households demands in purchasers prices by summing the matrices

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, \quad \begin{bmatrix} I_{1,19} \\ I_{2,19} \end{bmatrix}, \text{ and } \begin{bmatrix} I_{1,22} \\ I_{2,22} \end{bmatrix} \cdot w_{(il)d}^{(3)} \text{ is then the}$$

ratio of the id^{th} element of this matrix to the sum of all elements in the matrix. $w_{(i2)d}^{(3)}$ is the ratio of the $(21+i)^{\text{th}}$ element to the sum.

83. \tilde{T}_j

- Share of aggregate "private" investment represented by investment in industry j .
- Estimated from previous input-output estimates. The \tilde{T}_j are computed from the T_j (see discussion following eq. 5.179).

84. ψ_{1m}

- Share of occupation m in the economy-wide aggregate employment.
- Input-output estimates. ψ_{1m} is the share of the sum of the m^{th} row of M in the total sum of matrix M .

85. ψ_{2j}

- Share of capital employed in industry j in the economy's aggregate capital stock.
- Input-output estimates and data from the Brazilian industrial census. See section 6.3.7 for method of estimation.

86. $h_{(22,1,m)j}^{(1)}$

- Indexing parameter. Fixes the relationship between movements in the wage rate of occupation m and in the model consumer price index.
- User specified.

Table 6.1 (continued)87. $h_{23,j}^{(1)}$

- Indexing parameter. Fixes the relationship between movements in the price of "other costs" to industry j and in the model consumer price index.
- User specified.

be equal to 0.001 for all sectors, i.e.,

$$\sigma_{ij}^{(1)} = \sigma_{ij}^{(2)} = \sigma_{id}^{(3)} = \sigma_i = 0.001$$

$$i, j = 1, \dots, 21, \quad d=1, 2, 3$$

The value of 0.001 was chosen based on the following facts:

- 1) The rigidity in the composition of imports. As can be seen in Table 6.2 the share of 4 products/sectors (Fuels and Lubricants, Machinery and Equipments, Cereal, and Chemicals) in the import lists of products is approximately 65% in 1972 and near 84% in 1983;
- 2) The wide use of non-tariff restrictions, a fact always mentioned in studies about Brazilian external sector (see e.g., Tyler (1980, 1982, and 1983), Oliveira (1980), Dib (1981), Braga and Guimarães (1982), and Suzigan, et al. (1985)).

6.3.2. The Reciprocals of the Export Demand Elasticities

The reciprocals of the export demand elasticities, the γ_i 's, are assumed to have value of 0.05 for the sectors in which Brazil has a small share of the international trade, and the value of 0.16 for the sectors where Brazil has a share of 4% or more of the international trade. The value of 0.05 is borrowed from the ORANI model, as presented in DPSV (1982), while the value of 0.16 is derived from the work of Braga and Markwald (1983) that estimate the export demand elasticity for the exports of Brazilian manufactures.

Table 6.2

Share of Main Brazilian Imports (%)
1972-1983

Year	Fuels and Lubricants	Machinery and Equipments	Cereal	Chemicals	Sum of Cols. 1 to 4
1972	11.08	40.97	3.12	9.80	64.97
1973	12.42	34.59	5.65	9.33	61.99
1974	23.43	24.67	3.92	10.11	62.13
1975	25.39	32.22	3.05	8.89	69.55
1976	31.03	29.23	4.30	9.38	73.94
1977	33.94	25.79	2.32	9.93	71.98
1978	32.76	25.97	5.13	9.76	73.62
1979	37.45	20.87	5.44	10.00	73.76
1980	44.43	19.09	5.41	10.12	79.05
1981	51.34	18.21	4.87	6.67	81.09
1982	53.91	16.87	4.37	6.41	81.56
1983	55.79	16.24	5.87	6.24	84.14

Source: Conjuntura Econômica, Dec., 1984.

Table 6.3

Share of Main Brazilian Exports in the
International Trade (Market Economies) - 1973-82

Year	Agriculture		Mining	Metallurgy	Chemicals		
	072	222			081	423	512
1973	10.7	12.3	12.9	3.3	10.2	4.1	3.9
1974	14.3	11.5	15.5	3.3	7.6	1.3	3.4
1975	12.7	15.2	20.8	4.4	12.4	9.3	2.9
1976	12.4	15.0	19.9	5.7	15.2	13.5	1.6
1977	14.3	11.0	19.1	6.7	17.4	12.3	1.2
1978	15.6	2.4	21.8	8.0	15.4	11.5	1.6
1979	18.6	2.3	21.8	7.0	14.2	10.8	1.7
1980	15.0	4.7	24.0	7.2	15.4	13.5	4.2
1981	14.5	4.5	25.2	7.9	20.3	19.6	4.0
1982	12.0	1.5	27.2	8.1	17.4	13.8	4.1

Year	Text./Cloth./ Footwear	Food, Beverages and Tobacco Products				
	851	014	058	061	071	121
1973	3.4	6.3	4.6	12.5	29.2	3.7
1974	3.6	7.2	4.2	14.4	21.1	4.7
1975	4.2	6.0	5.5	9.7	21.3	6.1
1976	3.9	8.5	5.8	3.9	27.9	6.4
1977	3.3	8.4	8.0	5.9	19.6	6.9
1978	4.2	5.9	12.2	4.4	19.5	7.0
1979	4.1	6.3	9.3	4.3	18.1	8.5
1980	4.1	10.7	10.3	9.0	21.0	8.4
1981	6.1	13.1	17.6	8.3	19.0	8.9
1982	5.2	11.6	15.4	5.6	21.7	11.0

- Source: United Nations (1984)

- Description of the SITC codes:

014: Meat prepared, preserved, etc.

058: Fruit preserved, prepared

061: Sugar and honey

071: Coffee and substitutes

072: Cocoa

081: Feeding stuff for animal

121: Tobacco unmanufactured, refuse

222: Seeds for 'soft' fixed oil

281: Iron Ore and concentrates

423: Fixed vegetables oils, soft

512: Alcohols,
phenols, etc.

671: Pig iron, etc.

851: Footwear

Table 6.3 presents the shares of the products in which Brazil has some significance in international trade. This table also shows which sectors those products are part of. The values of the reciprocals of the export demand elasticities for the model sectors can be found in Table 6.4.

6.3.3. Elasticity of Substitution Between Primary Factors

The elasticity of substitution between primary factors ($\sigma_{(22,v)j}^{(1)}$) are assumed to be equal for $v=1,2,3$. This implies that the pairwise substitution elasticities between labor, land and capital are equal. In terms of Chapter 5, this is the same as assuming that:

$$h_{(22,v)j}^{(1)} = h_{22,j}^{(1)}$$

for all j , $v=1,2,3$

This implies that the CRESH function becomes a CES function, and that:

$$\sigma_{(22,1)j}^{(1)} = \sigma_{(22,2)j}^{(1)} = \sigma_{(22,3)j}^{(1)}$$

for all j

given that:

$$\sigma_{(22,v)j}^{(1)} = 1 / (1 - h_{(22,v)j}^{(1)})$$

$v = 1,2,3$

The values of the elasticity of substitution between primary factors for the Brazilian economy are based in Lysy and Taylor (1980) and are presented in Table 6.4. For a discussion of the way those elasticities are estimated see Macedo (1974, and 1975).

Table 6.4
Selected Coefficients and Parameters

	γ_1	$f(22, v_1)^{(1)}$	ϵ_{11}	ϵ_{12}	ϵ_{13}	β_1	β_2	β_3	ψ_{21}
1 Agriculture	.16	1.40	.8125	.6830	.4799	40.0000	.0964	1.2247	.0829
2 Mining	.16	.70	0.0000	0.0000	0.0000	-	-	-	.0112
3 Non-Metallic Min.	.05	.40	.6643	1.1379	1.6044	-	-	-	.0161
4 Metallurgy	.16	.60	1.1631	1.2514	.8514	-	-	-	.0360
5 Machinery	.05	.60	.7113	1.0858	1.6170	5.9623	.1714	1.5298	.0197
6 Electrical Eq.	.05	.50	1.2254	.9057	.8598	8.1679	.1714	1.4188	.0066
7 Transport Eq.	.05	.80	2.3558	1.5970	1.6683	5.9623	.1714	1.5298	.0111
8 Wood Products	.05	.70	.6980	1.0994	1.6078	6.3319	.1638	1.5369	.0094
9 Paper Products	.05	.60	.7849	.7943	.8102	2.0000	.1771	2.2308	.0094
10 Rub./Leat./Plas.	.05	.60	.8575	1.1525	1.6364	4.3033	.1771	1.6154	.0080
11 Chemicals	.16	.80	1.4687	1.1639	.8615	6.7847	.1771	1.4651	.0361
12 Cosm./Pharm.	.05	.80	1.0363	1.1639	.8438	9.4174	.1501	1.4651	.0030
13 Text./Cloth./Foot.	.16	.60	1.2121	1.0671	.8539	2.0000	.1501	3.0513	.0154
14 Food/Bev./Tobacco	.16	.60	.8309	.7027	.4833	7.4851	.1501	1.5442	.0310
15 Printing	.05	.40	.4865	.9026	1.5742	8.3291	.1638	1.4469	.0042
16 Other Ind. Prod.	.05	.60	.4865	.9026	1.5742	3.8427	.1638	1.6897	.0021
17 Public Utilities	.05	.80	.7635	.6850	.4833	-	-	-	.1269
18 Civil Construction	.05	1.20	0.0000	0.0000	0.0000	7.1686	.1638	1.5000	.0635
19 Trade and Transp.	.05	1.40	1.1518	1.1587	.8497	2.0000	.1638	2.1594	.3404
20 Services	.05	1.30	1.7283	1.0890	.8732	3.8427	.1638	1.6897	.1670
21 Dummy	.05	1.20	0.0000	0.0000	0.0000	-	-	-	.0000

Source: See Chapter 6

6.3.4. Elasticity of Substitution Between Labor Occupations

Following the ORANI model (see DPSV, 1982, pp. 190-191) the elasticity of substitution between labor occupations ($\sigma_{(22,1,q)j}^{(1)}$) are assumed to be equal to 0.9 for $q=1,2,3$ and $j=1,\dots,21$.

6.3.5. Household Expenditure and Price Elasticities of Demand

The household expenditures elasticities (ϵ_{id}) for the Brazilian economy are presented in Table 6.4. They were estimated from the unweighted Engel's elasticities presented in Bonelli and da Cunha (1981) (those elasticities are presented in Table B.1 in Appendix B).

Using the shares of the consumer budget from the different classes (calculated from matrices C_1 , C_2 , $I_{1,19}$, $I_{2,29}$, $I_{1,22}$, and $I_{2,22}$), and the unweighted elasticities, one can obtain the weighted Engel's elasticities (household expenditure elasticities). Those elasticities will have the property that:

$$\sum_{i=1}^{21} S_{id}^{(3)} \epsilon_{id} = 1, \quad d=1,2,3$$

Where:

$S_{id}^{(3)}$ = share of product i in the consumption basket of income group d

ϵ_{id} = Engel's elasticity of product i in income group d

The price elasticities of demand (η_{ikd}) are estimated by using the Frisch formula (see Frisch, 1959) and the household expenditure elasticities, i. e.,

$$\eta_{ikd} = - \varepsilon_{id} S_{kd}^{(3)} (1 + (\varepsilon_{kd} / \check{w}_d)) + \delta_{ik} (\varepsilon_{id} / \check{w}_d)$$

Where:

$\delta_{ik} = 1$ for $i=k$ and 0 otherwise

\check{w}_d is the Frisch parameter for income group d

$i, k = 1, \dots, 21$, $d = 1, 2, 3$

All the other variables are as defined before.

The Frisch parameter (see Frisch, 1959) has the value of -6.6 for $d=1$, -4.0 for $d=2$, and -1.7 for $d=3$. Those values are based on calculations made by Lluch, Powell, and Williams (1977, pp.74-81) for different countries, with different levels of per-capita income.

The estimated values of the price elasticities of demand are presented in Appendix C.

6.3.6. The Investment Parameters and Coefficients

The investment parameters and coefficients are: β_j (elasticity of the expected rate of return schedules), Q_j (ratio of the gross to the net rates of return of fixed investment), and G_j (ratio of annual gross investment to future capital stocks). In addition, one has that with the exceptions of sectors 2 (Mining), 3 (Non-Metallic Minerals), 4 (Metallurgy), 17 (Public Utilities), and 21 (Dummy) that are under strong influence of the government policies (see Trebat, 1980) or/and the rate-of-return theory of investment does not apply, the other sectors in the model belong to the set J of industries to which the rate-of-return theory of investment is to apply.

β_j , Q_j , and G_j are estimated as follows (see DPSV, 1982, p. 197):

$$\beta_j = (\ln(\text{Av}(R_j(0))) - \ln \Omega) / (\ln(\text{Av}(K_j(1)/K_j(0))))$$

$$Q_j = ((\text{Av}(R_j(0))) + d_j) / \text{Av}(R_j(0))$$

$$G_j = 1 - (\text{Av}(K_j(0)/K_j(1))) (1 - d_j)$$

$$j \in J$$

Where:

$\text{Av}(R_j(0))$ is the average of the rate of return over time in sector j

$\text{Av}(K_j(1)/K_j(0))$ is the average of growth factors in sector j

Ω is an economy-wide real rate of interest

d_j is the rate of depreciation on fixed capital in sector j

$K_j(0)$ is the current level of capital stock in sector j

$K_j(1)$ is the level of capital stock at the end of one period, in sector j

Table 6.4 presents the values of β_j , Q_j , and G_j . The average of the rates of return used in the calculation of those values are from Langoni (1974) and are presented in Table B.2 in Appendix B; the data for the stock of capital in the agricultural sector are from Langoni (1974) and are presented in Table B.3 in Appendix B; the data for the stock of capital in the other sectors are from Neves (1978) and are presented in Table B.4 in Appendix B. It was not possible to find data on stock of capital for each sector separately, so, the data presented in Table B.4 refer to proxies that are applied to groups of sectors; the economy-wide real rate of interest is assumed to be 8%; and the rate of depreciation on fixed capital in each sector is assumed to be 8% (see Langoni, 1974). The

period to which the capital stock refers is assumed to be one year.

For some sectors, the value of β_j were either too high or negative. When this happens, the following procedure was carried out:

- a) For too high values, a value of 40 was assigned to β_j (sector 1);
- b) For negative values, a value of 2 was assigned to β_j (sectors 9, 13 and 19).

6.3.7. The Sectors Shares in the Aggregate Capital Stock

As there are no data for the stock of capital in the Brazilian economy, the sectors shares in the aggregate capital stock (Ψ_{2j}) are estimated by making use of the input-output data and the 1980 Industrial Census (see IBGE, 1984a).

The procedure to construct Ψ_{2j} works as follows:

- a) Using matrices B_1 , B_2 , $H_{1,19}$, $H_{2,19}$, $H_{1,22}$, and $H_{2,22}$ sum down the columns of these matrices to construct a vector with the total investment in each sector (proxy for the stock of capital), then calculate the share of each sector (the j^{th} element in the sum vector) in total investment (sum over all the elements of these matrices);
- b) Add the values of the shares for the industrial sectors (sectors 2 through 16). Then, create new shares for the industrial sectors by distributing the total share of the industrial sector by distributing the total share of the industrial sector to each industrial sector according to

the share of each industrial sector in the industry stock of capital as presented in the 1980 Industrial Census.

The values of the sectors shares in the aggregate capital stock are presented in Table 6.4.

CHAPTER 7

MODEL ESTIMATION

7.1. Introduction

Chapter 5 presented the multisectoral model for the Brazilian economy, while Chapter 6 presented the estimation of the data base and the coefficients and parameters needed for the estimation of the Brazilian model. This chapter deals with issues related with the estimation of the model, which are: a) the method used in the model solution; b) how the model can be closed; and c) how the model can be reduced to a workable size.

The above issues are discussed in separated sections of this chapter: Section 7.2 will present a discussion of the methods that can be used in solving the model; the problem of closure is discussed in Section 7.3; while in Section 7.4 the reduction of the original model to a workable size is presented.

7.2. Solution Methods

Following Pearson and Rimmer (1983, pp. 1-2), the equations of a Computable General Equilibrium (CGE) model can be written as

$$F(Z) = 0 \quad (7.1)$$

where F is, in general, a nonlinear function. Johansen-type models are solved by first linearizing F , near a known solution of (7.1), in terms of percentage changes in the variables Z . (7.1) is then replaced by the matrix equation

$$Dz = 0 \quad (7.2)$$

where z is a vector of percentage change and D is an $m \times n$ matrix. In general

$$n > m$$

so that values for $(n - m)$ components of z must be set exogenously, and (7.2) may be solved to obtain values for the remaining m quantities. z can be partitioned into a vector z_2 of exogenous variables, and a vector z_1 of endogenous variables, then (7.2) can be written as

$$Az_1 = -Bz_2 \quad (7.3)$$

where A is an $m \times m$ matrix, z_1 is $m \times 1$, B is $m \times p$ ($p = n - m$) and z_2 is $p \times 1$. The solution for the model is then giving by

$$z_1 = -A^{-1}Bz_2 \quad (7.4)$$

So, the model constructed here for the Brazilian economy can be solved only with matrix algebra. However, in the process of linearization, of the initial system, errors occurred, and the result given in (7.4) is only an approximation of the true result (see Johansen, 1974; DPSV, 1982; and Rimmer, 1981).

Other methods for the model solution, which reduce the linearization errors are available, as the Euler's method (see DPSV, 1982). However, the costs implied in the solution of those methods do not always compensate for the precision of its results (see Section 47 in DPSV, 1982). The papers by Rimmer (1981), and Pearson

and Rimmer (1983) discuss ways of improving the estimates of Johansen's method, in precision of the results and in computer time.

7.3. Closing the Model

Giving the way that the model was constructed, it allows for flexibility in choosing between endogenous and exogenous variables, i. e., flexibility in choosing the way that the model is going to be closed.

The model can be closed such that results can be given for the short- or for the long-run period. In the short-run closure, the vector of capital stocks is exogenous to the system. In the long-run closure it is endogenous to the system, and either the vector of the rates of return or the vector of rentals on capital becomes exogenous to the system.

The short-run period is approximately 2 years for the ORANI model; tests need to be made with the Brazilian model to determine the time interval of the model solutions.

Section 23 in DPSV (1982) presents a discussion of a typical list of variables that are set exogenously to the system, this discussion can be applied to the Brazilian model and the reader is referred to that Section. A guideline, adapted from DPSV (1982), of the rules that needed to be followed when closing the models is:

"It is not true that the model can be closed by the exogenous setting of any p variables. For example, at least one monetary variable should be included in the

exogenous list... Similarly, some care is necessary to avoid inconsistencies. For example, if an attempt were made to set all three variables, f_r , i_r and c_r exogenously, then (5.182) would be violated. Although we can offer no formal theory to guide the model users in their choice of exogenous variables, as a working rule, if a price appears on the exogenous list, then a corresponding quantity should be on the endogenous list and vice versa. If wages are exogenous, then employment will be endogenous; if export taxes are endogenous, then export volumes will be exogenous; if tariffs are exogenous, then imports will be endogenous; and if sales taxes are endogenous, then consumption will be exogenous" (DPSV, 1982, p. 148).

Further discussion about closure in the ORANI model, and that can be applied to the Brazilian model, can be found, e. g., in Cooper and McLaren (1980 and 1981), Powell, Cooper, and McLaren (1983), Cronin (1985), and Powell (1985).

7.4. Reducing the Model to a Workable Size

The reduction of the original model (8459 equations and 17695 variables, as can be seen in Chapter 5) is made by choosing variables that will always be endogenous to the system and then eliminating these variables by substituting them in the other equations of the system. This does not pose a problem in getting

solutions for the eliminated variables when a simulation of the model is made, the solutions for these variables are getting by first solving for the reduced model and then solving for the eliminated endogenous variables.

Following the ORANI model (see section 32 in DPSV, 1982) the endogenous variables eliminated from the model are the intermediate-input and capital-input flows, commodity flows to households, demands for margin services and purchaser's prices. After the elimination of these variables one is left with a system of 437 equations and 1091 variables (790 original variables and 301 composite variables,¹ the b's presented in Table 7.1).

The way to get to the equations of the reduced system and the equations of the reduced system are presented below. No explicit derivation of the equations is made here. The reader is referred to section 32.1 in DPSV (1982) for an example of the derivation of one equation, the first of the reduced system.

7.4.1. Derivation of the Reduced System

The following derivation of the reduced system is based on Table 32.1 in DPSV (1982).

The coefficients, parameters and variables of the reduced system are presented in a way different from the one of the original system. They are presented in matrix form. A description of the vector variables presented in the reduced system is showed in Table 7.1 (based on Table 32.2 in DPSV, 1982).

The equations of the reduced system are as follows:

1. Costs of Units of Capital:

Substitute (5.145) and (5.147) into (5.134) to get:

$$\pi = A_1 p_1 + A_2 p_2 + A_3 \xi^{(3)} + b_1 \quad (7.5)$$

N. of eqs. = 21

2. Zero Pure Profits in Production:

Substitute (5.145), (5.184), (5.133), and (5.147) into (5.132) to get:

$$\begin{aligned} B_1 p_1 = & B_2 p_2 + B_3 p_{22,1} + B_4 p_{22,2} + B_5 p_{22,3} \\ & + B_6 f_{23} + B_7 \xi^{(3)} + b_2 \end{aligned} \quad (7.6)$$

N. of eqs. = 21

3. Investment by Investing Industry:

From (5.157), (5.158), and (5.160) one gets:

$$y = C_1 \kappa(0) + C_2 r(0) + C_3 \omega + C_4 i_r + C_5 f^{(2)} \quad (7.7)$$

Where:

$$C_1 = C_2 = C_3 = 0 \quad \text{when } j \notin J$$

$$C_4 = C_5 = 0 \quad \text{when } j \in J$$

N. of eqs. = 21

4. Household Demands for Domestically Produced Commodities:

Using (5.146), (5.148), (5.103), and (5.104) into (5.101) gives:

$$x_{1d}^{(3)} = D_{1d} p_1 + D_{2d} p_2 + D_{3d} c_d + D_{4d} q_d + D_{5d} \xi^{(3)} + b_{3d} \quad (7.8)$$

$d = 1, 2, 3$

N. of eqs. = 63

5. Households Demands for Imported Commodities:

Using (5.146), (5.148), (5.103), and (5.104) into (5.101) gives:

$$x_{2d}^{(3)} = E_{1d}p_1 + E_{2d}p_2 + E_{3d}c_d + E_{4d}q_d + E_{5d}\xi^{(3)} + b_{4d} \quad (7.9)$$

$$d = 1, 2, 3$$

$$N. \text{ of eqs. } = 63$$

6. Supply Equals Demand for Labor by Skill:

Derived by using (5.66), (5.73), and (5.75) into (5.167):

$$\lambda = F_1z + F_2p_{22,1} + F_3p_{22,2} + F_4p_{22,3} + b_5 \quad (7.10)$$

$$N. \text{ of eqs. } = 3$$

7. Supply Equals Demand for Capital by Industry:

Substituting (5.73), and (5.75) into (5.168) gives:

$$\kappa(0) = z + G_1p_{22,1} + G_2p_{22,2} + G_3p_{22,3} + b_6 \quad (7.11)$$

$$N. \text{ of eqs. } = 21$$

8. Supply Equals Demand for Agricultural Land by Industry:

Substituting (5.73), and (5.75) into (5.169) gives:

$$n = z + H_1p_{22,1} + H_2p_{22,2} + H_3p_{22,3} + b_7 \quad (7.12)$$

$$N. \text{ of eqs. } = 21$$

9. Supply Equals Demand for Domestically Produced Commodities:

Making use of (5.35), (5.80), (5.145), (5.147), (5.118), (5.121), (5.125), (5.126) and (5.127) into (5.166) one gets:

$$\begin{aligned} x(1) = & J_1z + J_2y + J_{3,1}x_{1,1}^{(3)} + J_{3,2}x_{1,2}^{(3)} + J_{3,3}x_{1,3}^{(3)} \\ & + J_{4,1}x_{2,1}^{(3)} + J_{4,2}x_{2,2}^{(3)} + J_{4,3}x_{2,3}^{(3)} + J_5x^{(4)} \\ & + J_6f_1^{(5)} + J_7f_1^{(6)} + J_8f_2^{(5)} + J_9f_2^{(6)} \\ & + J_{10}c_r + J_{11}p_1 + J_{12}p_2 + J_{13}\xi^{(3)} + b_8 \end{aligned} \quad (7.13)$$

$$N. \text{ of eqs. } = 21$$

10. Import Volumes:

Using (5.35), (5.80), (5.145), (5.147), and (5.118) into (5.171) gives:

$$\begin{aligned}
x(2) = & K_1 z + K_2 y + K_{3,1} x_{2,1}^{(3)} + K_{3,2} x_{2,2}^{(3)} \\
& + K_{3,3} x_{2,3}^{(3)} + K_4 f_2^{(5)} + K_5 f_2^{(6)} \\
& + K_6 c_r + K_7 p_1 + K_8 p_2 + K_9 \xi^{(3)} + b_9
\end{aligned} \tag{7.14}$$

N. of eqs. = 21

11. Foreign Currency Value of Imports:

This is a rewrite of (5.173):

$$m = L_1 x(2) + L_1 p^m \tag{7.15}$$

N. of eqs. = 1

12. Foreign Currency Value of Exports:

This is a rewrite of (5.175):

$$e = M_1 x^{(4)} + M_1 p^e \tag{7.16}$$

N. of eqs. = 1

13. Change in the Balance of Trade:

This is a rewrite of (5.177):

$$\Delta B = N_1 e - N_2 m \tag{7.17}$$

N. of eqs. = 1

14. Basic Prices of Imports:

Substituting (5.138) into (5.137) gives:

$$p_2 = P_1 p^m + P_2 \phi + P_3 t(2) + P_4 \xi^{(3)} + b_{10} \tag{7.18}$$

N. of eqs. = 21

15. Export Prices Related to Basic Prices:

Substituting (5.142) into (5.141) yields:

$$p^e = -\frac{1}{\pi} \phi + Q_1 p_1 + Q_2 v(4) + Q_3 \xi^{(3)} + b_{11} \tag{7.19}$$

N. of eqs. = 21

16. Model Consumer Price Index:

Using (5.146), and (5.148) into (5.178) gives:

$$\begin{aligned}\xi^{(3)} = & R_{1,1}p_1 + R_{1,2}p_1 + R_{1,3}p_1 + R_{2,1}p_2 \\ & + R_{2,2}p_2 + R_{2,3}p_2 + b_{12}\end{aligned}\quad (7.20)$$

N. of eqs. = 1

17. Model Capital Goods Price Index:

This is a rewrite of (5.179):

$$\xi^{(2)} = S_1\pi \quad (7.21)$$

N. of eqs. = 1

18. Real Household Expenditure:

This is a rewrite of (5.119)

$$c_r = O_1c - \xi^{(3)} \quad (7.22)$$

N. of eqs. = 1

19. Real Private Investment Expenditure:

This is a rewrite of (5.161):

$$i_r = i - \xi^{(2)} \quad (7.23)$$

N. of eqs. = 1

20. Money Wages:

This is a rewrite of (5.183):

$$\begin{aligned}p_{22,1} = & T_1 \xi^{(3)} + \underline{1}f_{(22,1)}^{(1)} + T_2f_{(22,1)}^{(ind)} \\ & + T_3f_{(22,1)}^{(occ)} + f_{(22,1)}^{(o/i)}\end{aligned}\quad (7.24)$$

N. of eqs. = 63

21. Ratio of Real Private Investment to Real Consumption Expenditure:

This is a rewrite of (5.182):

$$f_r = i_r - c_r \quad (7.25)$$

N. of eqs. = 1

22. Aggregate Employment:

This is a rewrite of (5.180):

$$1 = U_1 \lambda \quad (7.26)$$

N. of eqs. = 1

23. Aggregate Capital Stock:

This is a rewrite of (5.181):

$$k(0) = V_1 \kappa(0) \quad (7.27)$$

N. of eqs. = 1

24. Foreign Currency Export Prices:

This is a rewrite of (5.117):

$$p^e = w_1 x^{(4)} + f^e \quad (7.28)$$

N. of eqs. = 21

25. Investment Budget:

This is a rewrite of (5.159):

$$X_1 i = X_2 \pi + X_2 Y \quad (7.29)$$

N. of eqs. = 1

26. Rates of Return on Capital:

This is a rewrite of (5.156):

$$r(0) = Y_1 p_{22,2} - Y_1 \pi \quad (7.30)$$

N. of eqs. = 21

27. Aggregate Consumer Budget for Each Income Group:

Using (5.66), (5.73), and (5.75) into (5.115) gives:

$$\begin{aligned} c_d = & Z_{1d} z + Z_{2d} p_{22,1} + Z_{3d} p_{22,2} + Z_{4d} p_{22,3} \\ & - Z_{5d} s_d + b_{13d} \end{aligned} \quad (7.31)$$

$d = 1, 2, 3$, N. of eqs. = 3

TABLE 7.1

Vector Variables in the Reduced System

Vector Variables	Dimension	Typical Element	Definition
π	21	π_j	Prices of capital units
p_1	21	$p(i1)^{(0)}$	Basic prices of domestic commodities
p_2	21	$p(i2)^{(0)}$	Basic prices of imported commodities
$\xi^{(3)}$	1	-	Model index of consumer prices
$p_{22,1}$	63	$p(22,1,m)_j^{(1)}$	Occupation-industry wage rates
$p_{22,2}$	21	$p(22,2)_j^{(1)}$	Rentals on capital
$p_{22,3}$	21	$p(22,3)_j^{(1)}$	Rentals on agricultural land
f_{23}	21	$f_{23,j}^{(1)}$	Shift terms associated with the price of other costs
y	21	y_j	Capital creation by using industry
$\kappa^{(0)}$	21	$k_j^{(0)}$	Current capital stocks
$r^{(0)}$	21	$r_j^{(0)}$	Current rates of return on fixed capital
ω	1	-	Economy-wide expected rate of return on capital
i_r	1	-	Aggregate real private investment
$f^{(2)}$	5	$f_j^{(2)}, j \neq J$	Exogenous investment terms

Table 7.1 (continued)

Vector Variables	Dimension	Typical Element	Definition
$x_{1d}^{(3)}$	21	$x_{(i1)d}^{(3)}$	Household demands for domestic commodities in each income group, $d=1,2,3$
c	3	c_d	Aggregate household expenditure
q_d	1	-	Number of households in each income group, $d=1,2,3$
$x_{2d}^{(3)}$	21	$x_{(i2)d}^{(3)}$	Household demands for imported commodities in each income group, $d=1,2,3$
λ	3	l_m	Employment of labor by occupational group
z	21	z_j	Industry activity level
n	21	n_j	Use of agricultural land in each industry
$x^{(1)}$	21	$x_{(r1)}^{(0)}$	Total supplies of domestic commodities
$x^{(4)}$	21	$x_{(i1)}^{(4)}$	Export demands
$f_1^{(5)}$	21	$f_{(i1)}^{(5)}$	Shift term associated with government demands for domestic commodities
$f_1^{(6)}$	21	$f_{(i1)}^{(6)}$	Shift term associated with other demands for domestic commodities
$f_2^{(5)}$	21	$f_{(i2)}^{(5)}$	Shift term associated with government demands for imported commodities
$f_2^{(6)}$	21	$f_{(i2)}^{(6)}$	Shift term associated with other demands for imported commodities

Table 7.1 (continued)

Vector Variables	Dimension	Typical Element	Definition
c_r	1	-	Real aggregate household expenditure
$x(2)$	21	$x_{(r2)}^{(0)}$	Aggregate imports by commodity
m	1	-	Foreign currency value of imports
p^m	21	$p_{(i2)}^m$	C.I.F. foreign currency import prices
e	1	-	Foreign currency value of exports
p^e	21	$p_{(i1)}^e$	F.O.B. foreign currency export prices
ΔB	1	-	Change in the balance of trade
ϕ	1	-	The exchange rate, Cr\$ per US\$
$t(2)$	21	$t_{(i2,0)}$	Ad valorem tariffs
$v(4)$	21	$v_{(i1,4)}$	Specific export tax or subsidy terms
i	1	-	Aggregate private investment expenditure
$\xi(2)$	1	-	Model capital-goods price index
$f_{(22,1)}^{(1)}$	1	-	General wage shift variable
$f_{(22,1)}^{(ind)}$	21	$f_{(22,1)j}^{(1)}$	Variable used for simulating the effects of changes in the wages payable by particular industries relative to other industries

Table 7.1 (continued)

Vector Variables	Dimension	Typical Element	Definition
$f_{(22,1)}^{(occ)}$	3	$f_{(22,1,m)}^{(1)}$	Variable used in simulations involving changes in occupational wage relativities
$f_{(22,1)}^{(o/i)}$	63	$f_{(22,1,m)j}^{(1)}$	Variable allowing changes in both occupational and industrial wage relativities
f_r	1	-	Ratio of real private investment expenditure to real household consumption expenditure
l	1	-	Aggregate employment
$k(0)$	1	-	Aggregate capital stock
f^e	21	$f_{(i1)}^e$	Shifts in foreign export demands
s_d	1	-	Aggregate residual value (income less consumption) in each income group, $d=1,2,3$
b_1	21	$(b_1)_j$	$(b_1)_j$ is a function of $a_j^{(2)}$, $a_{ij}^{(2)}$, $a_{(is)j}^{(2)}$, $a_{(r1)(is)j2}$, $t(is,j2)$ and $v(is,j2)$ for $i,r=1,\dots,21$ and $s=1,2$

Table 7.1 (continued)

Vector Variables	Dimension	Typical Element	Definition
b_2	21	$(b_2)_j$	$(b_2)_j$ is a function of $a_{(r1)}(is)j1$, $t(is,j1)$ and $v(is,j1)$ for $i,r=1,\dots,21$ and $s=1,2$, and of all the technical change variables affecting industry j 's production function, i.e., the variables on the RHS of (5.133)
b_{3d}	21	$(b_{3d})_t$	$(b_{3d})_t$ is a function of $a(is)d^{(3)}$, $a_{id}^{(3)}$, $a_{(r1)}(is)d3$, $t(is,d3)$, and $v(is,d3)$ for $i,r=1,\dots,21$ and $s=1,2$; $d=1,2,3$
b_{4d}	21	$(b_{4d})_t$	$(b_{4d})_t$ is a function of $a(is)d^{(3)}$, $a_{id}^{(3)}$, $a_{(r1)}(is)d3$, $t(is,d3)$, and $v(is,d3)$ for $i,r=1,\dots,21$ and $s=1,2$; $d=1,2,3$
b_5	3	$(b_5)_m$	$(b_5)_m$ is a function of $a_{(22,1,q)}j^{(1)}$, $a_j^{(1)}$, $a_{22,j}^{(1)}$, $a_{(22,v)}j^{(1)}$ for $q=1,2,3$, $j=1,\dots,21$ and $v=1,2,3$

Table 7.1 (continued)

Vector Variables	Dimension	Typical Element	Definition
b_6	21	$(b_6)_j$	$(b_6)_j$ is a function of $a_j^{(1)}$, $a_{22,j}^{(1)}$, $a_{(22,v)j}^{(1)}$ and $a_{(22,1,q)j}^{(1)}$ for $v, q=1, 2, 3$
b_7	21	$(b_7)_j$	$(b_7)_j$ is a function of $a_j^{(1)}$, $a_{22,j}^{(1)}$, $a_{(22,v)j}^{(1)}$ and $a_{(22,1,q)j}^{(1)}$ for $v, q=1, 2, 3$
b_8	21	$(b_8)_r$	$(b_8)_r$ is a function of $a_j^{(k)}$, $a_{ij}^{(k)}$, $a_{(is)j}^{(k)}$, $t_{(is,jk)}$, $v_{(is,jk)}$ and $a_{(t1)}^{(is)jk}$ for $j=1, \dots, 21$; $k, s=1, 2$ and $i, t=1, \dots, 21$. It is also a function of $a_{(r1)}^{(i1)4}$, $a_{(r1)}^{(is)d3}$, and $a_{(r1)}^{(is)k}$ for $i=1, \dots, 21$; $s=1, 2$ and $k=5, 6$

Table 7.1 (continued)

Vector Variables	Dimension	Typical Element	Definition
b_9	21	$(b_9)_r$	$(b_9)_r$ is a function of $a_j^{(k)}$, $a_{rj}^{(k)}$, $a_{(rs)j}^{(k)}$, $t(rs,jk)$, $v(rs,jk)$ and $a_{(t1)}^{(rs)jk}$ for $j, t=1, \dots, 21$, and $k, s=1, 2$
b_{10}	21	$(b_{10})_i$	$(b_{10})_i$ is a function of $v(i2,0)$
b_{11}	21	$(b_{11})_i$	$(b_{11})_i$ is a function of $a_{(r1)}^{(i1)4}$ and $t(i1,4)$ for $r=1, \dots, 21$
b_{12}	1	-	b_{12} is a function of $a_{(r1)}^{(is)d3}$, $t(is,d3)$, $v(is,d3)$ for $i, r=1, \dots, 21$, $s=1, 2$, and $d=1, 2, 3$
b_{13d}	1	-	b_{13d} is a function of $a_j^{(1)}$, $a_{22,j}^{(1)}$, $a_{(22,v)j}^{(1)}$, and $a_{(22,1,q)j}^{(1)}$ for $q, v=1, 2, 3$, and $j=1, \dots, 21$; $d=1, 2, 3$

NOTES

1. 8967 original variables are used in the construction of the 301 composite variables.

CHAPTER 8

CONCLUSION

The way in which we suppose the increase in aggregate demand to be distributed between different commodities may considerably influence the volume of employment.

John Maynard Keynes, The General Theory of Employment, Interest, and Money.

In constructing a model for the Brazilian economy, among the various types of model, a multisectoral model was chosen. And as so, the input-output framework was considered as the best kind of approach to be used in the construction of this kind of model.

The next decision to be faced was what kind of multisectoral model should be built. As was seen in Chapter 2, using input-output analysis, one can construct input-output models, dynamic input-output models, static-linear programming models, dynamic optimizing models, and economy-wide models. The choice was to construct a general purpose economy-wide model.

Still, another decision arises: should the economy-wide model

be constructed such that its solutions are to be given in levels or in growth rates (Johansen-type model). The decision was in favor of a model solved for growth rates.

An advantage of a model solved for growth rates over a model solved for levels is that the former can be solved only with matrix algebra, while the latter is usually a non-linear system and each model requires its own solution algorithm. In this way, it is easier to shift between exogenous and endogenous variables in the model solved for growth rates.

In constructing a general purpose economy-wide multisectoral model for the Brazilian economy, the ORANI model for the Australian economy (see DPSV, 1982) was chosen as the starting point and was modified in a way that it can reflect and can be used to study the Brazilian reality. The main differences between both models are that in the Brazilian model:

- a) A special treatment is giving for the government sector;
- b) The demand for household consumption is broken down by different income groups, and an equation linking the workers income with their expenditure is introduced; allowing in this way for the study of income distribution problems;
- c) An industry by industry framework is used, opposing to an industry by commodity framework used in the ORANI model;
- d) Prices are assumed to be formed through a mark-up price theory, while the ORANI model assumes that prices are formed by maximizing profits.

Given the above choices about specifications for the Brazilian

model, it was constructed for: a) 21 industries; b) 3 types of primary factors: 3 categories of labor, fixed capital (building, plant, and machinery), and agricultural land; c) one type of other costs (production taxes, costs of holding liquidities, costs of holding inventories, and other miscellaneous production costs); d) 2 sources of products (domestic and imported); e) 6 types of product use (inputs to current production, inputs to capital formation, commodity flows to household consumption, exports, government demands, and other demands); and f) 3 income groups. The model also presents a detailed specification for trade margins and taxes.

Concerning the data base needed for the estimation of the coefficients and parameters, there have been a great improvement in the collection and publication of data, allowing in this way for the construction of a model complex as the one presented in this work, however, much more need to be done.

Looking at the basic input-output data used in the estimation of parameters and coefficients in the model one can see that they refer to the 1975 input-output data for the Brazilian economy (see IBGE, 1984b). This shows that improvements need to be done in the updating system of the input-output data, as this kind of data are the main core of the model being constructed here.

Improvements also need to be made in the estimation of the coefficients and parameters in the model, as some of the coefficients and parameters are best guesses of the author (based in the characteristics of the Brazilian economy and of the model constructed here), and some were estimated based in secondary sources of information.

In conclusion, what is needed is an improvement in the quality and the quantity of data used in the model. It is expected that this kind of improvement will occur as the need for this kind of data arise, and as the Brazilian institutions responsible for data collection improve their process of collection and presentation of data.

A discussion of problems related to some of the specifications used in the ORANI model, and that also apply to the Brazilian model, is presented in Chapter 8 of DPSV (1982), and the reader is referred to this work for further details.

The model constructed here can be expanded and improved in a series of way, such as:

- a) It can be expanded to give results at the regional level (see DPSV, 1982, Dixon and Parmenter, 1980, Liew, 1982, and Parmenter, Pearson, and Jagielski, 1985), a highly desirable quality due to the great disparities among regions in Brazil;
- b) It can be integrated with a macroeconometric model, in such a way that the influence of monetary over real variables can be studied (see Cooper and McLaren, 1980 and 1981, and Powell, 1981);
- c) And it can also be combined with a demographic model, such that variations in the labor supply, and their impact over other economic variables can be studied (see DPSV, 1982).

The model presented in this work represents the first effort of constructing a general purpose economy-wide multisectoral model for the Brazilian economy that can be used in the planning process. The

problems and critics of this model are yet to come, and they will probably appear only when the first results for this model will come out of a computer. This is expected and accepted as a way to improve a job just started in this dissertation.

It is the hope of the author that the model constructed here can be used to better understand the Brazilian reality and problems. And as so, it can be used in the process of solving the problems and disparities faced by the Brazilian people.

APPENDIX A

METHODOLOGY USED IN THE CONSTRUCTION OF THE
INPUT-OUTPUT DATA USED IN THE MODELA.1. Introduction

The objective of this Appendix is to present the methodology used in the construction of the input-output data used in the model.

It will start with a discussion of the source of data and an overview of how the data were adjusted to meet the needs of this work. After which, the process used in the construction of each matrix presented in Figure 6.1 will be discussed (the value of these matrices are presented in Table A.7).

The input-output data used in the model are for the year 1975, the last year for which this kind of data were available. It was calculated by Fundação Instituto Brasileiro de Geografia e Estatística (IBGE, 1984b) and it is not yet published.

The IBGE matrices present the input-output relations in a commodity by industry framework. They present 261 commodities and 123 real and 4 dummy industries.

The input-output matrices used here, from IBGE (1984b), are:

- a) Inputs to production and flows of goods to final demand matrix (Matrix 2) (285x149);
- b) Imported inputs to production and imported goods to final

demand matrix (Matrix 3) (262x148);¹

c) Net indirect taxes matrix (Matrix 4) (263x15);

d) Trade margins matrix (Matrix 5) (263x15);

These 4 matrices were the starting point to construct the matrices presented in Figure 6.1.

The row and column labels of IBGE's Matrices 2, 3, 4, and 5, are presented respectively in Tables A.1A and A.1B, A.2A and A.2B, A.3A and A.3B, and A.4A and A.4B.

As can be seen in those tables, and as mentioned above, the original IBGE matrices have 261 products, and 123 real and 4 dummy sectors (columns 125 to 128 in Tables A.1B and A.2B). Those were aggregated in such a way as to get 20 real products and sectors and 1 dummy product and sector.

The way the sectors were aggregated are presented in Table A.5. The products aggregation are presented in Table A.6.

More adjustments were made to construct the matrices presented in Figure 6.1, but these will be discussed in the next sections that will present the way that each matrix was constructed.

A.2. Adjustments on Matrix 2

The adjustments made on Matrix 2 were as follows:

- a) The sector Supply of Scrap Iron and Recyclable residues (col. 128) has negative values for its inputs. To solve this problem, these values were distributed to the real sectors (col. 1 to 123) according to the proportion of these inputs

Table A.1ARow Labels of Matrix 2

Row Number	IBGE Codes	Description
1	0101001	
.	.	
.	.	261 Products
.	.	
261	6001001	
262	6009020	Imported products
263	7020001	IPI (tax over industrialized products)
264	7030001	ICM (tax over commodities)
265	7040001	Subsidies to the product
266	7050001	Duty on imported products
267	7060001	ISS (tax over services)
268	7070001	Other indirect taxes
269	7900000	Intermediate usage (total down rows 1 to 268)
270	8001020	Wages of presidents, directors and managers
271	8001030	Wages of administrative workers
272	8001040	Wages of workers occupied in production
273	8001050	Gratification paid to employees
274	8001060	Share in profits
275	8001000	Total of wages (total down rows 270 to 274)
276	8002010	INPS (social security)
277	8002020	FGTS (job-tenure guarantee fund)
278	8002030	Other expenditures with employees
279	8002000	Total down rows 276 to 278
280	8003000	Self-employed
281	8004001	Capitalist surplus
282	8004002	Familiar surplus
283	8005000	Subsidies to the activity
284	8000000	Value added (sum of rows 275, and 279 to 283)
285	9000000	Total production (sum of rows 269 and 284)

Source: IBGE (1984b)

Table A.1B

Column Labels of Matrix 2

Column Number	IBGE Code	Description
1	01010	
.	.	
.	.	123 Sectors
.	.	
123	56010	
124	59000	Intermediate usage (total across cols 1 to 123)
125	60011	Financial (dummy)
126	60012	Machinery location (dummy)
127	60020	Enterprises
128	60030	Supply of scrap iron and recyclable residues
129	71010	Household consumption in the income group between 0 and 2 minimum wages
130	71020	Household consumption in the income group between 2 and 5 minimum wages
131	71030	Household consumption in the income group between 5 and 10 minimum wages
132	71040	Household consumption in the income group between 10 and 20 minimum wages
133	71050	Household consumption in the income group with more than 20 minimum wages
134	71000	Total of household consumption (total across cols. 129 to 133)
135	72010	Non-monetary consumption (dummy)
136	72020	Personal consumption (dummy)
137	73010	Government - General Administration
138	73020	Government - Health and Social Security
139	73030	Government - Education
140	73040	Government - Defense
141	73000	Government - Total (total across cols. 137 to 140)
142	74011	Gross Fixed Capital Expenditure (GFCE) in machinery and equipments
143	74012	GFCE in other items
144	74010	Total GFCE (total across cols. 142 and 143)
145	74020	Variation in inventories
146	75010	Exports of goods and services
147	79000	Final demand (sum of cols. 134, 135, 136, 141, 144, 145, and 146)
148	80000	Total production (sum of cols. 124, 125, 126, 127, 128, 147, and 149), rows 1 to 261.
149	90000	Errors and omissions

Source: IBGE (1984b)

Table A.2ARow Labels of Matrix 3

Row Number	IBGE Code	Description
1	0101001	
.	.	
.	.	261 Products
.	.	
261	6001001	
262	6009020	Imported products (total down the 261 rows)*

* The values of this row are the same as the values presented in row 262 of Matrix 2.

Source: IBGE (1984b)

Table A.2BColumn Labels of Matrix 3

Column Number	IBGE Code	Description
1	01010	
.	.	
.	.	Same labels as columns 1 to 145 in Matrix 2
.	.	
145	74020	
146	79000	Final demand of imports (sum of cols. 134, 135, 136, 141, 144, and 145)
147	80000	Total of imports (sum of cols. 124, 125, 126, 127, 128, 146, and 148)
148	90000	Errors and omissions

Source: IBGE (1984b)

Table A.3ARow Labels of Matrix 4

Row Number	IBGE Code	Description
1	0101001	
.	.	
.	.	261 products
.	.	
261	6001001	
262	6009020	Imported products
263	7000000	Total of net indirect taxes (total down the 262 rows)*

* This total refers to the sum of rows 263 to 268 of Matrix 2.

Source: IBGE (1984b)

Table A.3BColumn Labels of Matrix 4

Column Number	IBGE Code	Description*
1	04000	Agriculture (cols. 1 to 10)
2	05000	Mining (cols. 11 to 14)
3	10000	Transformation industry (cols. 15 to 105 and 123)
4	40000	Public utilities services (cols. 106 and 107)
5	42010	Civil construction (col. 108)
6	51000	Trade and transport (cols. 109 to 114)
7	53010	Communications (col. 115)
8	54000	Finance (cols. 116 and 117)
9	55000	Services (cols. 118 to 122)
10	60000	Dummies (cols. 125 to 128)
11	71000	Household consumption (col. 134)
12	73000	Government (col. 141)
13	74010	GFCE (col. 144)
14	75010	Exports (col. 146)
15	81000	Total (sum across cols. 1 to 14 plus cols. 136 and 145 in Matrix 2)

* The reference of columns between parenthesis refers to the correspondence of the sectors presented here and the ones shown in Matrix 2.

Source: IBGE (1984b)

Table A.4ARow Labels of Matrix 5

Row Number	IBGE Code	Description
1	0101001	
.	.	
.	.	261 products
.	.	
261	6001001	
262	6009020	Imported products
263	5102001	Total of trade margins (total down the 262 rows)*

* The value of this row correspond to the values presented in row 244 of Matrix 2.

Source: IBGE (1984b)

Table A.4BColumn Labels of Matrix 5

Column Number	IBGE Code	Description
1	04000	
.	.	
.	.	Same as columns 1 to 15 in Matrix 4
.	.	
15	81000	

Source: IBGE (1984b)

Table A.5

Sector Aggregation

Sector Number	Sector Name	Range of Columns	IBGE Codes
1	Agriculture	1- 10	01010, 01020, 02020, 02030, 02040, 02050, 02910, 03010, 03020, 04990
2	Mining	11- 14	05010, 05020, 05030, 05040
3	Non-Metallic Minerals	15- 20	10010, 10020, 10030, 10040, 10050, 10910
4	Metallurgy	21- 31	11011, 11012, 11020, 11031, 11032, 11040, 11050, 11060, 11070, 11080, 11910
5	Machinery	32- 39	12010, 12020, 12030, 12040, 12050, 12060, 12070, 12080
6	Electrical Equipment	40- 47	13010, 13020, 13030, 13040, 13050, 13060, 13070, 13080
7	Transport Equipment	48- 53	14010, 14020, 14030, 14040, 14050, 14910
8	Wood Prod.	54- 57	15010, 15020, 16010, 16020
9	Paper Prod.	58- 60	17010, 17020, 17030
10	Rubber, Leather and Plastics	61- 63 and 76-77	18010, 18020, 19990, 23010, 23020
11	Chemicals	64- 73	20010, 20020, 20031, 20032, 20040, 20050, 20060, 20070, 20080, 20910
12	Cosmetics / Pharmaceut.	74- 75	21990, 22990
13	Textiles, Clothing and Footwear	78- 84	24010, 24020, 24030, 24040, 24910, 25010, 25020
14	Food, Beverages, and Tobacco	85-102	26010, 26020, 26030, 26040, 26050, 26076, 26080, 26090, 26100, 26110, 26120, 26130, 26140, 26150, 26910, 27010, 27020, 28990
15	Printing	103-104	29010, 29020
16	Other Ind. Products	105 and 123	30990, 56010
17	Public Utilities	106-107 and 115	40010, 41010, 53010
18	Civil Const.	108	42010
19	Trade and Transport Services	109-114	51010, 51020, 52010, 52020, 52030, 52040
20		116-122	54010, 54020, 55010, 55021, 55022, 55030, 55040
21	Dummy	125-127	60011, 60012, 60020

Source: IBGE (1984b)

Table A.6

Product Aggregation

Product Number	Product Name	Range of Columns	IBGE Codes			
1	Agriculture	1- 28	0101001, 0101002, 0101003, 0101091, 0102001, 0201001, 0202001, 0203001, 0204001, 0205001, 0205002, 0206001, 0291001, 0291002, 0291003, 0291004, 0291005, 0291006, 0291007, 0291008, 0291091, 0301001, 0301002, 0302001, 0302002, 0303001, 0303091, 0410001			
2	Mining	29- 33	0501001, 0501091, 0502092, 0503001, 0504001			
3	Non-Metallic Minerals	34- 42	1001001, 1002001, 1002002, 1002091, 1003001, 1004001, 1005091, 1005102, 1091091			
4	Metallurgy	43- 72	1101191, 1101206, 1101207, 1102020, 1102021, 1102022, 1102023, 1102024, 1102025, 1102026, 1102027, 1102028, 1102029, 1103101, 1103201, 1104001, 1104002, 1104091, 1105002, 1105003, 1105091, 1106001, 1106091, 1107001, 1108001, 1191001, 1191002, 1191003, 1191007, 1191091			
5	Machinery	73- 86	1201001, 1202001, 1202002, 1202091, 1203001, 1204001, 1204002, 1204091, 1205001, 1206001, 1206002, 1207001, 1207002, 1208001			
6	Electrical Equipment	87-101	1301001, 1301002, 1301091, 1302001, 1303001, 1304001, 1305001, 1305002, 1305003, 1306001, 1306002, 1307001, 1307002, 1308001, 1308002			
7	Transport Equipment	102-113	1401001, 1402001, 1403001, 1403002, 1404001, 1404002, 1404003, 1405001, 1405002, 1405003, 1491001, 1491002			
8	Wood Products	114-120	1501101, 1501201, 1502001, 1502002, 1502003, 1601001, 1602001			
9	Paper Products	121-126	1701001, 1702001, 1702002, 1702091, 1703001, 1703091			
10	Rubber, Leather and Plastics	127-130 and 168-172	1801001, 1802001, 1802002, 1999001, 2301001, 2302001, 2302002, 2302003, 2302091			
11	Chemicals	131-163	2001001, 2001002, 2001003, 2001092, 2002001, 2003101, 2003102, 2003103, 2003104, 2003105, 2003106, 2003107, 2003193, 2003203, 2003204, 2003209, 2004092, 2005001, 2005002, 2005003, 2005004, 2005005, 2006001, 2006008			

Table A.6 (continued)

Product Number	Product Name	Range of Columns	IBGE Codes
11	Chemicals		2006010, 2006011, 2007001, 2007002, 2008001, 2008002, 2091001, 2091002, 2091092
12	Cosmetics / Pharmaceut.	164-167	2199001, 2199002, 2299001, 2299002
13	Textiles, Clothing and Footwear	173-191	2401101, 2401102, 2401201, 2402001, 2402002, 2402003, 2402004, 2403001, 2403002, 2404001, 2404002, 2491001, 2491002, 2491003, 2491006, 2491009, 2501001, 2501003, 2502091
14	Food, Beverages, and Tobacco	192-227	2601001, 2602001, 2603001, 2604001, 2605001, 2605002, 2606001, 2607101, 2607104, 2607191, 2608001, 2608002, 2608003, 2608004, 2609001, 2610001, 2610002, 2610091, 2611001, 2611002, 2611093, 2612001, 2613001, 2613002, 2614001, 2614002, 2614003, 2615001, 2691001, 2691091, 2701101, 2701201, 2701301, 2702001, 2801001, 2802001
15	Printing	228-232	2901001, 2901002, 2901004, 2902001, 2902005
16	Other Ind. Products	233-239 and 261	3099001, 3099091, 3100011, 3100012, 3100013, 3100014, 3100015, 6001001
17	Public Utilities	240-241 and 248	4001001, 4101001, 5301001
18	Civil Const.	242	4201001
19	Trade and Transport	243 and 245-247	5101001, 5201001, 5202001, 5204002
20	Services	249-260	5401001, 5402001, 5501001, 5502001, 5502002, 5503001, 5503002, 5503003, 5503004, 5503005, 5503006, 5504001
21	Dummy		Row of zeros as inputs to other products. Only element = 0 refers to the demand of this product by "other" demands.

Source: IBGE (1984b)

in the real sector. The value of self-employed was distributed according to the distribution of the inputs;

- b) A dummy sector including columns 125 (Financial, dummy), 126 (Machinery Location, dummy), and 127 (Enterprises) was created. The new sector has a negative value for the capitalist surplus. This value was then allocated to the vector of "other demands" (to be created), and those sectors start to have positive production value, instead of a zero value;
- c) The final demand Personal Consumption (dummy), col. 136, was distributed to the final demand of Household Consumption (cols. 129 to 134) according to the proportion of these goods in the consumption of the different household classes of income;
- d) The following columns of Matrix 2 were not taken in consideration when constructing the matrices presented in Figure 6.1: 135 (Non-Monetary Consumption), 145 (Variation in Inventories), and 149 (Errors and Omissions).

A.3. Adjustments on Matrix 3

The adjustments made on Matrix 3 were as follows:

- a) A dummy sector including columns 125, 126 and 127 was created;
- b) Idem to item (c) in section A.2;
- c) The following columns of Matrix 3 were not taken into

consideration when constructing the matrices presented in Figure 6.1: 135 (Non-Monetary Consumption), 145 (Variation in Inventories), and 148 (Errors and Omissions).

A.4. Derivation of Matrices A_1 , A_2 , C_1 , C_2 , D_1 , D_2 , E_1 , and F_1

Matrix A_1 (A_2) is derived by aggregating the sectors and products of the adjusted Matrix 2 (3) according to, respectively, the aggregations presented in Tables A.5 and A.6. Row 21, Dummy product, is filled with zeros.

Matrix C_1 (C_2) is derived from rows 1 to 261 of Matrix 2 (3) that are aggregated according to Table A.6, and from columns 129 to 133 in this matrix. Those columns are aggregated in three columns as follows:

- a) Column 1, household consumption in the income group between 0 and 5 minimum wages, is the result of the aggregation of columns 129 and 130;
- b) Column 2, household consumption in the income group between 5 and 20 minimum wages, is the result of the aggregation of columns 131 and 132;
- c) Column 3, household consumption in the income group with more than 20 minimum wages, corresponds to column 133.

Vector D_1 (D_2) is derived from rows 1 to 261 of Matrix 2 (3) that are aggregated according to Table A.6, and from column 141 in this matrix.

Vector E_1 has rows 1 to 20 filled to zero, row 21 has the value

discussed in section A.2, item (b).

Vector F_1 is derived from rows 1 to 261 of Matrix 2 that are aggregated according to Table A.6, and from column 146 in this matrix.

A.5. Derivation of Matrices $G_{1,19}$, $G_{2,19}$, $G_{1,22}$, and $G_{2,22}$

Matrices $G_{1,19}$, and $G_{2,19}$ are derived from IBGE's Matrix 5, while matrices $G_{1,22}$ and $G_{2,22}$ are derived from Matrix 4.

To derive matrix $G_{1,19}$ ($G_{1,22}$), the rows of Matrix 5 (4) are aggregated according to Table A.6, the columns of those matrices are transformed into new columns, as follows:

- a) Column 1: becomes column 1;
- b) Column 2: becomes column 2;
- c) Column 3: is to be disaggregated into columns 3 to 16 according to the proportions derived from columns 3 to 16 in matrix A_1 , i.e., each element in a row of columns 3 to 16 in matrix A_1 is divided by the row sum of those columns.
- d) Column 4: becomes column 17;
- e) Column 5: becomes column 18;
- f) Column 6: becomes column 19;
- g) Column 7: is added to column 17;
- h) Column 8: becomes column 20;
- i) Column 9: is added to column 20;
- j) Column 10: becomes column 21.

To derive matrix $G_{2,19}$ ($G_{2,22}$) one first constructs a "distribution" matrix from matrix A_2 as follows:

- a) For columns 1, 2, and 17 to 21: divide the ij^{th} element of matrix A_2 by the sum of j^{th} column;
- b) For columns 3 to 16: divide the ij^{th} element of matrix A_2 by the sum all over those columns.

Then, one uses row 262 (imported products) of Matrix 5 (4) and the values of the "distribution" matrix to construct matrix $G_{2,19}$ ($G_{2,22}$). The process works in the following way:

- a) Column 1 of row 262 is disaggregated according to the values of column 1 in the "distribution" matrix;
- b) Column 2 is disaggregated according to the values of column 2;
- c) Column 3 is disaggregated according to the values of columns 3 to 16;
- d) Columns 4 and 7 are added together and the resulting value is disaggregated according to the values of column 17;
- e) Column 5 is disaggregated according to the values of column 18;
- f) Column 6 is disaggregated according to the values of column 19;
- g) Columns 8 and 9 are added together and the resulting value is disaggregated according to the values of column 20;
- h) Column 10 is disaggregated according to the values of column 21.

A.6. Derivation of Matrices B_1 , B_2 , $H_{1,19}$, $H_{2,19}$, $H_{1,22}$, and $H_{2,22}$

The matrices referring to capital formation are constructed by using the investment matrix (20x20) constructed for the year of 1970 by Bonelli and da Cunha (1981), and IBGE's Matrices 2, 3, 4, and 5.

Before one can use the investment matrix, one needs to adjust this matrix to characteristics of the model being developed here.

The adjustments are as follows:

a) For the rows one has:

Bonelli and da Cunha (1981)	Brazilian Model
Row	Becomes Row
1	1
4	4
5	5
6	6
7	7
8	8
15	16
17	18
18	20
20	19

All the other rows in the Brazilian model are filled with zeros.

b) After the rows were adjusted, the columns of this adjusted matrix were adjusted in the following way:

1) Columns 1 to 14: no changes;

- 2) Column 15: 64.71% of the value of each element in this column goes to a new column 15, 35.29% goes to column 16;
- 3) Column 16: becomes column 17;
- 4) Column 17: becomes column 18;
- 5) Column 18: 1.77% of the value of each element in this column is added to column 17, 98.23% goes to column 20;
- 6) Column 19: 83.63% of the value of each element in this column goes to column 19, 16.37% is added to column 17;
- 7) Column 20: it is added to column 19;
- 8) A new column 21, filled with zeros, is created.

The new adjusted investment matrix (21x21), created according to the procedure above, is used to distribute the values of GFCE that are presented in Matrices 2, 3, 4, and 5.

To do so, the adjusted investment matrix is used to construct a "distribution" matrix by dividing each element of the i^{th} row by the sum of the i^{th} row.

Matrix B_1 (B_2) is constructed by first aggregating the rows of column 144 in Matrix 2 (3) according to Table A.6 and then distributing the values of each element of the aggregated column 144 according to the "distribution" matrix.

Matrix $H_{1,19}$ ($H_{1,22}$) is constructed by first aggregating the rows of column 13 in Matrix 5 (4) according to Table A.6 and then distributing the values of each element of the aggregated column 13 according to the "distribution" matrix.

To construct $H_{2,19}$ ($H_{2,22}$) one first distribute the values of imports presented in row 262, column 13, of Matrix 5 (4) according

to the proportions derived from the aggregated column of GFCE in Matrix 3, i. e., the share of each element in the aggregated column of GFCE in the column total. Then, the resulting vector has the values of each of its rows distributed according to the "distribution" matrix.

A.7. Derivation of Matrices $I_{1,19}$, $I_{2,19}$, $I_{1,22}$, and $I_{2,22}$

The derivation of matrices $I_{1,19}$, $I_{2,19}$, $I_{1,22}$, and $I_{2,22}$ starts by distributing, respectively, the value of row 244 (trade margins) and the sum of rows 263-268 (indirect taxes) of column 135 (personal consumption, dummy) in Matrix 2 to column 11 (household consumption) in Matrices 5 and 4. Those values are distributed according to the participation of the i^{th} element of column 11 in the sum of this column (rows 1 to 262).

Matrix $I_{1,19}$ ($I_{1,22}$) is constructed by aggregating the adjusted column 11 of Matrix 5 (4) according to Table A.6 and then, distributing the resulting column to the proportions derived from matrix C_1 , i. e., the ij^{th} element of Matrix C_1 is divided by the sum of the i^{th} row.

Matrix $I_{2,19}$ ($I_{2,22}$) is constructed by distributing the value of imports (row 262) in column 11 of Matrix 5 (4) according to the proportions derived from matrix C_2 , i. e., the ij^{th} element of matrix C_2 is divided by the sum all over this matrix.

A.8. Derivation of Vectors $J_{1,19}$, $J_{2,19}$, $J_{1,22}$, and $J_{2,22}$

Vector $J_{1,19}$ ($J_{1,22}$) is constructed by aggregating column 12 of Matrix 5 (4) according to the procedure presented in Table A.6.

Vector $J_{2,19}$ ($J_{2,22}$) is constructed by distributing the value of imports (row 262) in column 12 of Matrix 5 (4) according to the proportions derived from vector D_2 , i. e., the i^{th} element of vector D_2 is divided by the vector sum.

A.9. Derivation of Vectors $L_{1,19}$ and $L_{1,22}$

Vector $L_{1,19}$ ($L_{2,19}$) is derived by aggregating column 14 of Matrix 5 (4) according to the procedure presented in Table A.6.

A.10. Derivation of Vector Z_2

To construct vector Z_2 one makes use of the adjusted IBGE's Matrices 2 and 3. From Matrix 2 one uses row 266 (duty on imported products), and from Matrix 3 one uses rows 1 to 261. The following columns are used from both matrices: columns 1 to 123, 125 to 127, 129 to 133, 141, and 144.

From Matrix 3, a "distribution" matrix is constructed by dividing the ij^{th} element of the selected rows and columns by the sum of the j^{th} column. The "distribution" matrix is then used to construct a "duty on imported products matrix" by distributing the

j^{th} element, of the selected columns, of row 266 of Matrix 2, according to the "distribution matrix".

Vector Z_2 is constructed by adding the columns of the "duty on imported products matrix" and aggregating the rows of the resulting sum vector according to Table A.6.

A.11. Derivation of Matrices M, N, P, and Q

Matrix M is derived by making use of the input-output data (IBGE's Matrix 2), the 1980 Industrial (see IBGE, 1984a) and Agricultural (see IBGE, 1983) Brazilian Census, and the 1975 National Accounts (see Conjuntura Econômica, June, 1984).²

The steps used in the construction of matrix M are as follows:

- 1) The value of gratification paid to employees (row 273) and share in profits (row 274) are distributed to rows 270 to 272 (wages) according to the participation of ij^{th} element of these rows in the sum of the j^{th} column (rows 270 to 272);
- 2) For the non-agricultural sectors, the value of self-employed (row 280) is distributed to rows 271 and 272 (wages) according to the participation of the ij^{th} element of these rows in the sum of the j^{th} column (rows 271 and 272);
- 3) The columns in rows 270 to 272, 280, 281, and 282 are aggregated according to Table A.5;
- 4) Using data from the 1980 Agricultural Census, one arrives to the following procedures for the agricultural sector (sector

- 1):
 - a) The sum of the incomes presented in rows 270 to 272 in the agricultural sector are assigned to the workers in income group 1 (row 1 of matrix M);
 - b) Row 280 (self-employed) is assigned to the workers in income group 1;
 - c) 77.80% of the familiar surplus (row 282) is assigned to the works in income group 1;
 - d) 22.20% of row 282 is assigned to the works in income group 2 (row 2 of matrix M);
- 5) Using data from the 1980 Industrial Census, for the industrial sectors (sectors 2 through 16), the adjusted values in row 270 are assigned to the workers in income groups 1, 2, and 3 (the same is true for rows 271 and 272);
- 6) For the non-agricultural and non-industrial sectors (sectors 17 through 21), the weighted average of the distribution of the incomes in the industrial sectors is used as a proxy to distribute the values in rows 270 through 272 to the workers in income groups 1, 2, and 3.
- 7) The value of the total income (the sum of wages, salaries, transfer payments, profits, etc.) should now be adjusted to be equal or greater than the total value of consumption in the different income groups (sum down the columns of matrices C_1 , C_2 , $I_{1,19}$, $I_{2,19}$, $I_{1,22}$, and $I_{2,22}$). The total income received by income group 1 should be equal to the consumption made in this income group, as it is assumed that the workers in income group 1 spend all of their earnings in

consumption. The total of income received by income groups 2 and 3 should be 1.575 greater than total consumption made in these income groups, the value of 1.575 is derived from the 1975 national accounts. This adjustment is made by allocating part of the capitalist surplus (row 281) to the incomes of the different income groups.

After matrix M was constructed, the remaining value of the capitalist surplus is distributed to vectors N, P, and Q according to the shares, respectively, of fixed capital, agricultural land and working capital in the total capital stock of each sector. The data for the agricultural sector (sector 1) comes from the 1980 Agricultural Census, the data for the industrial sectors (sector 2 through 16) comes from the 1980 Industrial Census, and the data for the non-agricultural and non-industrial sectors (sector 17 through 21) is derived by using the values already allocated to the industrial sectors.

The values of rows 279 (total down rows 276 to 278) and 283 (subsidies to the activity) are aggregated according to Table A.5 and are added to vector Q.

Table A.7 (continued)

Sector	Vector										
	D ₁	D ₂	J _{1,19}	J _{2,19}	J _{1,22}	J _{2,22}	E ₁	F ₁	L _{1,19}	L _{1,22}	Z ₂
1 Agriculture	245	2	46	46	3	1	0	7417	1501	0	358
2 Mining	0	0	0	0	0	0	0	4079	2890	231	514
3 Non-Metallic Min.	104	4	13	13	30	2	0	248	23	0	66
4 Metallurgy	234	2	25	25	55	1	0	1738	228	0	994
5 Machinery	141	40	5	5	22	21	0	1928	84	0	1868
6 Electrical Eq.	354	65	52	52	76	28	0	1791	89	0	1041
7 Transport Eq.	474	15	90	90	77	6	0	3362	0	0	497
8 Wood Products	38	0	0	0	8	0	0	756	238	0	7
9 Paper Products	323	6	51	51	90	3	0	429	22	0	88
10 Rub./Leat./Plas.	117	10	39	39	36	4	0	632	32	0	66
11 Chemicals	1598	24	344	344	622	10	0	8273	417	-341	551
12 Cosm./Pharm.	1824	11	16	16	326	5	0	117	18	0	194
13 Text./Cloth./Foot.	247	8	1	1	49	3	0	5238	102	0	86
14 Food/Bev./Tobacco	535	0	106	106	79	0	0	14309	1586	-692	316
15 Printing	2269	38	82	82	291	16	0	126	5	0	60
16 Other Ind. Prod.	732	103	94	94	171	44	0	458	12	0	302
17 Public Utilities	2106	0	0	0	74	0	0	0	0	0	0
18 Civil Construction	0	0	0	0	0	0	0	0	0	0	0
19 Trade and Transp.	1647	19	0	0	6	8	0	12762	0	0	365
20 Services	22104	0	0	0	220	0	0	0	0	0	35
21 Dummy	0	0	0	0	0	0	199840	0	0	0	0

Table A.7 (continued)

Matrices	Sector										
	1	2	3	4	5	6	7	8	9	10	11
M: Income Group 1	41614	773	2915	7256	6294	2177	4043	3095	1377	1772	1708
M: Income Group 2	33700	2826	6860	14691	11636	7289	6987	5108	2863	5401	15777
M: Income Group 3	16514	1600	3728	7053	5138	3554	3448	2835	1379	2781	8696
N	2294	1331	3093	5669	4330	2961	2808	2273	1226	2369	7254
P	10355	0	0	0	0	0	0	0	0	0	0
Q	6923	568	1761	4145	3883	1780	2454	1669	634	1215	2409

Matrices	Sector									
	12	13	14	15	16	17	18	19	20	21
M: Income Group 1	671	5821	5015	1942	759	6360	21146	35420	35749	7185
M: Income Group 2	5077	10669	17103	4387	2348	10779	15169	76069	79142	6822
M: Income Group 3	2719	5591	9632	2184	1166	5098	5395	37243	39615	313
N	2212	4968	8105	1873	996	4286	4494	30934	33089	0
P	0	0	0	0	0	0	0	0	0	0
Q	865	2888	3803	1086	517	3043	5107	13958	15704	4965

Source: See Appendix A

NOTES

1. In the construction of Matrix 3 it was assumed that there is no direct flows of imports to exports, i. e., imports need to be domestically processed before they can be exported.
2. The process used in the creation of Matrix M is the result of a work conducted with Manuel A. R. da Fonseca (see Fonseca and Guilhoto, 1986).

APPENDIX B

DATA USED IN THE ECONOMETRIC ESTIMATION
AND THE ALGEBRAIC CALCULATION

Table B.1

Engel Elasticities (Unweighted)

Sector	ϵ_{i1}^a	ϵ_{i2}^b	ϵ_{i3}^c
1. Agriculture	0.795	0.656	0.571
2. Mining	0.000 ^d	0.000 ^d	0.000 ^d
3. Non-Metallic Min.	0.650	1.093	1.909
4. Metallurgy	1.138	1.202	1.013
5. Machinery	0.696	1.043	1.924
6. Electrical Eq.	1.199	0.870	1.023
7. Transport Eq.	2.305	1.534	1.985
8. Wood Products	0.683	1.056	1.913
9. Paper Products	0.768	0.763	0.964
10. Rub./Leat./Plas.	0.839	1.107	1.947
11. Chemicals	1.437	1.118	1.025
12. Cosm./Pharm.	1.014	1.118	1.004
13. Text./Cloth./Foot.	1.186	1.025	1.016
14. Food/Bev./Tobacco	0.813	0.675	0.575
15. Printing	0.476	0.867	1.873
16. Other Ind. Prod.	0.476	0.867	1.873
17. Public Utilities	0.747 ^e	0.658 ^e	0.575 ^e
18. Civil Construction	0.000 ^d	0.000 ^d	0.000 ^d
19. Trade and Transp.	1.127 ^f	1.113 ^f	1.011 ^f
20. Services	1.691 ^g	1.046 ^g	1.039 ^g
21. Dummy	0.000 ^d	0.000 ^d	0.000 ^d

- Source: Bonelli and da Cunha (1981) (BC)

- Notes:

- a. Based on the income group from 2 to 5 minimum wages in BC;
- b. Based on the income group from 5 to 10 minimum wages in BC;
- c. Based on the income group with + 10 minimum wages in BC;
- d. The share of this item in the consumption of a given income group is zero or near zero;
- e. Elasticity based on the Electrical Energy sector in BC;
- f. Elasticity based on the Trade sector in BC;
- g. Elasticity based on the Service sector in BC.

Table B.2Average Rates of Return (1954-67)

Sector	Average Rates
1. Agriculture	0.356 ^a
2. Mining	-
3. Non-Metallic Min.	-
4. Metallurgy	-
5. Machinery	0.151 ^b
6. Electrical Eq.	0.191
7. Transport Eq.	0.151 ^b
8. Wood Products	0.149 ^c
9. Paper Products	0.065
10. Rub./Leat./Plas.	0.130 ^d
11. Chemicals	0.172
12. Cosm./Pharm.	0.172 ^e
13. Text./Cloth./Foot.	0.039 ^f
14. Food/Bev./Tobacco	0.147 ^g
15. Printing	0.179
16. Other Ind. Prod.	0.116 ^h
17. Public Utilities	-
18. Civil Construction	0.160
19. Trade and Transp.	0.069 ⁱ
20. Services	0.116 ^h
21. Dummy	-

- Source: Langoni (1974) (L);

- Notes:

- a. Period 1948-69;
- b. From the Equipments and Instruments sector in L;
- c. Average of Wood and Wood Products sectors in L;
- d. From the Rubber sector in L;
- e. From the Chemicals sector in L;
- f. From the Textile sector in L;
- g. From the Food sector in L;
- h. Weighted average of the industrial sectors in L;
- i. From the Transport Sector in L.

Table B.3Stock of Capital in the Agricultural Sector
(Cr\$ 1000 of 1953)

Year	Stock of Capital
1948	86149
1949	87767
1950	89401
1951	91052
1952	92721
1953	94408
1954	96113
1955	97838
1956	99583
1957	101344
1958	103135
1959	104943
1960	106774
1961	108627
1962	110504
1963	112405
1964	114331
1965	116282
1966	118259
1967	120264
1968	122295
1969	125663

Source: Langoni (1974)

Table B.4

Stock of Capital in the Non-Agricultural Sectors
(Cr\$ Million of 1970)

Year	Durable Consumer and Capital Goods ^a	Intermediary Goods ^b	Non-Durable Consumer Goods ^c	Total of the Industrial Sectors ^d
1955	3796	5924	7901	22.5
1956	3877	6405	8357	23.8
1957	4250	6885	8879	25.3
1958	4755	7620	8377	27.3
1959	5742	8436	10007	30.0
1960	6521	10007	11204	34.6
1961	7400	11797	12532	39.5
1962	8138	13309	13639	43.5
1963	8671	14761	14486	47.0
1964	9159	16043	15363	50.1
1965	9637	17553	16251	53.3
1966	10346	19347	17111	57.1
1967	11185	20760	18091	60.9
1968	12154	22385	19238	65.4
1969	13246	24204	20694	70.6
1970	14519	26354	22404	76.7
1971	16337	29497	24611	84.9
1972	18974	35021	27748	98.2
1973	22522	40666	31270	113.8
1974	26445	47845	35254	132.8
1975	31348	55858	39350	153.8

- Source: Neves (1978)

- Notes:

- a. Used as a proxy to sectors 5, 6, and 7;
- b. Used as a proxy to sectors 9, 10, and 11;
- c. Used as a proxy to sectors 12, 13, and 14;
- d. Cr\$ billion of 1970, used as a proxy to sectors 8, 15, 16, 18, 19, and 20.

APPENDIX C

VALUES OF THE COEFFICIENTS AND PARAMETERS IN THE MODEL

This appendix presents the values of the coefficients and parameters in the model that are different from zero and are not presented in Chapter 6. For a description of the coefficients and parameters see Table 6.1.

Table C.1 (continued)

$S_{(22,1,q)j}^{*(1)}$			$S_{(22,v)j}^{*(1)}$			$S_{(22,1,q)j}^{(1)}$			
1	2	3	1	2	3	1	2	3	
1	.453171	.366990	.179839	.878933	.021957	.099111	.453171	.366990	.179839
2	.148738	.543543	.307720	.796130	.203870	0.000000	.148738	.543543	.307720
3	.215854	.508025	.276122	.813628	.186372	0.000000	.215854	.508025	.276122
4	.250215	.506572	.243213	.836481	.163519	0.000000	.250215	.506572	.243213
5	.272835	.504425	.222739	.841951	.158049	0.000000	.272835	.504425	.222739
6	.167201	.559854	.272945	.814701	.185299	0.000000	.167201	.559854	.272945
7	.279259	.482587	.238154	.837569	.162431	0.000000	.279259	.482587	.238154
8	.280384	.462774	.256842	.829233	.170767	0.000000	.280384	.462774	.256842
9	.244989	.509556	.245455	.820853	.179147	0.000000	.244989	.509556	.245455
10	.178014	.542623	.279364	.807715	.192285	0.000000	.178014	.542623	.279364
11	.065221	.602633	.332146	.783050	.216950	0.000000	.065221	.602633	.332146
12	.079306	.599606	.321088	.792868	.207132	0.000000	.079306	.599606	.321088
13	.263638	.483158	.253203	.816345	.183655	0.000000	.263638	.483158	.253203
14	.157956	.538677	.303367	.796641	.203359	0.000000	.157956	.538677	.303367
15	.228114	.515349	.256536	.819639	.180361	0.000000	.228114	.515349	.256536
16	.177679	.549447	.272874	.810919	.189081	0.000000	.177679	.549447	.272874
17	.286010	.484742	.229249	.838422	.161578	0.000000	.286010	.484742	.229249
18	.506982	.363669	.129349	.902729	.097271	0.000000	.506982	.363669	.129349
19	.238148	.511449	.250403	.827826	.172174	0.000000	.238148	.511449	.250403
20	.231374	.512230	.256396	.823613	.176387	0.000000	.231374	.512230	.256396
21	.501747	.476385	.021868	1.000000	0.000000	0.000000	.501747	.476385	.021868

$S_{(11)d}^{(3)}$			$S_{(12)d}^{(3)}$			$H_{(22,1,1)j1}^{(1)}$	$H_{(22,1,2)j2}^{(1)}$	$H_{(22,1,3)j3}^{(1)}$	
1	2	3	1	2	3				
1	.987155	.922729	.898666	.012845	.077271	.101334	.215513	.098913	.099675
2	1.000000	1.000000	0.000000	0.000000	0.000000	0.000000	.004005	.008294	.009656
3	1.000000	1.000000	.933763	0.000000	0.000000	.066237	.015094	.020134	.022503
4	.994770	.978937	.994366	.005230	.021063	.005634	.037579	.043118	.042571
5	1.000000	.850133	.823150	0.000000	.149867	.176850	.032594	.034153	.031012
6	.978464	.955334	.935812	.021536	.044666	.064188	.011274	.021395	.021449
7	.930194	.997780	.984394	.069806	.002220	.015606	.020938	.020507	.020810
8	1.000000	1.000000	1.000000	0.000000	0.000000	0.000000	.016029	.014994	.017113
9	1.000000	1.000000	1.000000	0.000000	0.000000	0.000000	.007129	.008403	.008324
10	1.000000	1.000000	1.000000	0.000000	0.000000	0.000000	.009176	.015852	.016783
11	.997403	.998916	.999634	.002597	.001084	.000366	.008843	.046309	.052485
12	.997431	.997756	.997382	.002569	.002244	.002618	.003477	.014901	.016408
13	1.000000	.996044	.992270	0.000000	.003956	.007730	.030149	.031314	.033746
14	.989705	.986720	.980504	.010295	.013280	.019496	.025973	.050199	.058135
15	.992470	.963031	.941072	.007530	.036969	.058928	.010056	.012875	.013180
16	.957959	.915566	.873929	.042041	.084434	.126071	.003932	.006891	.007037
17	1.000000	1.000000	1.000000	0.000000	0.000000	0.000000	.032939	.031639	.030770
18	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	.109513	.044522	.032563
19	.997148	.994138	.989456	.002852	.005862	.010544	.183438	.223272	.224788
20	1.000000	1.000000	1.000000	0.000000	0.000000	0.000000	.185138	.232293	.239102
21	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	.037210	.020023	.001890

Table C.1 (continued)

	Ψ_{1a}	H_d	O_d	$H_{(22,1,v)_J}^{(1)}$			$H_{(22,v)_J}^{(1)}$		$H_{23,J}^{(1)}$	$H_{22,J}^{(1)}$
				1	2	3	2	3		
1	.276052	0.000000	.375223	.269056	.217888	.106774	.014832	.066949	.044763	.675498
2	.487082	.365100	.420359	.074697	.272970	.154538	.128603	0.000000	.054913	.630808
3	.236865	.365100	.204418	.093222	.219403	.119250	.098926	0.000000	.056324	.530801
4	-	-	-	.069997	.141713	.068039	.054687	0.000000	.039986	.334436
5	-	-	-	.104419	.193053	.085247	.071843	0.000000	.064419	.454562
6	-	-	-	.056245	.188332	.091817	.076511	0.000000	.045997	.412905
7	-	-	-	.059239	.102371	.050519	.041138	0.000000	.035955	.253267
8	-	-	-	.106393	.175601	.097460	.078142	0.000000	.057366	.457595
9	-	-	-	.075257	.156528	.075400	.067042	0.000000	.034667	.374226
10	-	-	-	.057213	.174396	.089786	.076511	0.000000	.039245	.397906
11	-	-	-	.013999	.129352	.071293	.059469	0.000000	.019751	.274114
12	-	-	-	.034306	.259378	.138897	.113009	0.000000	.044215	.545590
13	-	-	-	.073395	.134507	.070490	.062631	0.000000	.036407	.341022
14	-	-	-	.033117	.112938	.063603	.053520	0.000000	.025114	.263178
15	-	-	-	.110328	.249250	.124074	.106427	0.000000	.061712	.590080
16	-	-	-	.047703	.147513	.073260	.062600	0.000000	.032509	.331076
17	-	-	-	.168249	.285156	.134859	.113368	0.000000	.080496	.701632
18	-	-	-	.127138	.091199	.032437	.027022	0.000000	.030704	.277796
19	-	-	-	.144238	.309766	.151660	.125968	0.000000	.056839	.731632
20	-	-	-	.150045	.332179	.166272	.138884	0.000000	.065913	.787381
21	-	-	-	.035954	.034137	.001567	0.000000	0.000000	.024845	.071657

 $\zeta_1(i2,0) \quad \zeta_2(i2,0) \quad \zeta_1(i1,4) \quad \zeta_2(i1,4) \quad \zeta_3(i1,4) \quad H_{(19,1)}^{(1)4}$

1	.919454	.080546	.831689	0.000000	.168311	1.000000
2	.979576	.020424	.566528	.032083	.401389	1.000000
3	.878926	.121074	.915129	0.000000	.084871	1.000000
4	.917380	.082620	.884028	0.000000	.115972	1.000000
5	.903223	.096777	.958250	0.000000	.041750	1.000000
6	.878919	.121081	.952660	0.000000	.047340	1.000000
7	.919846	.080154	1.000000	0.000000	0.000000	1.000000
8	.890050	.109950	.760563	0.000000	.239437	1.000000
9	.922571	.077429	.951220	0.000000	.048780	1.000000
10	.865995	.134005	.951807	0.000000	.048193	1.000000
11	.949652	.050348	.990897	-.040843	.049946	1.000000
12	.843801	.156199	.866667	0.000000	.133333	1.000000
13	.827358	.172642	.980899	0.000000	.019101	1.000000
14	.871447	.128553	.941196	-.045517	.104322	1.000000
15	.805077	.194923	.961832	0.000000	.038168	1.000000
16	.859904	.140096	.974468	0.000000	.025532	1.000000
17	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
18	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
19	.953640	.046360	1.000000	0.000000	0.000000	1.000000
20	.896082	.103918	0.000000	0.000000	0.000000	0.000000
21	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table C.1 (continued)

$B_{(r_1)d}^{(3)}$			$B_{(r_1)}^{(4)}$		$B_{(r_1)}^{(5)}$	$B_{(r_1)}^{(6)}$	$B_{(19,1)}^{(11)d3}$		
1	2	3					1	2	3
1	.067855	.049244	.013326	.064349	.002126	0.000000	.016738	.012147	.003287
2	.003071	.000424	0.000000	.316816	0.000000	0.000000	.000035	.000005	0.000000
3	.018265	.017510	.009798	.008844	.003709	0.000000	.000182	.000175	.000098
4	.014479	.010815	.003070	.017708	.002384	0.000000	.000399	.000298	.000085
5	.019154	.023539	.009036	.028560	.002089	0.000000	.000997	.001225	.000470
6	.068450	.078389	.049070	.051523	.010184	0.000000	.003011	.003449	.002159
7	.004588	.116029	.138632	.050480	.007117	0.000000	.000458	.011572	.013827
8	.089997	.122879	.072291	.027551	.001385	0.000000	.003424	.004675	.002750
9	.010610	.024151	.011034	.026949	.020290	0.000000	.000418	.000951	.000435
10	.024358	.042292	.029698	.024553	.004545	0.000000	.000142	.000247	.000173
11	.026006	.069328	.041110	.071888	.013886	0.000000	.004884	.013021	.007721
12	.351973	.287847	.082226	.006183	.096386	0.000000	.012769	.010442	.002983
13	.144670	.173357	.090967	.077567	.003658	0.000000	.014904	.017859	.009371
14	.334727	.240759	.065239	.096773	.003618	0.000000	.034472	.024794	.006719
15	.056129	.164387	.098230	.008305	.149552	0.000000	.001009	.002956	.001766
16	.078141	.111558	.069996	.023712	.037898	0.000000	.002380	.003398	.002132
17	.115805	.198793	.097336	0.000000	.062060	0.000000	0.000000	0.000000	0.000000
18	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
19	.113252	.110184	.040737	.041960	.005415	0.000000	0.000000	0.000000	0.000000
20	.037291	.087330	.059593	0.000000	.090442	0.000000	0.000000	0.000000	0.000000
21	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.000000	0.000000	0.000000

$B_{(19,1)}^{(12)d3}$			$B_{(19,1)}^{(11)d4}$		$B_{(19,1)}^{(11)d5}$	
1	2	3			1	2
1	.000109	.000509	.000185	.004935	.000151	.000000
2	0.000000	0.000000	0.000000	.009502	0.000000	0.000000
3	0.000000	0.000000	.000014	.000076	.000043	.000001
4	.000006	.000018	.000001	.000750	.000082	.000000
5	0.000000	.000275	.000129	.000276	.000016	.000012
6	.000055	.000135	.000124	.000293	.000171	.000016
7	.000027	.000020	.000170	0.000000	.000296	.000004
8	0.000000	0.000000	0.000000	.000783	0.000000	0.000000
9	0.000000	0.000000	0.000000	.000072	.000168	.000001
10	0.000000	0.000000	0.000000	.000105	.000128	.000002
11	.000009	.000010	.000002	.001371	.001131	.000006
12	.000021	.000015	.000005	.000059	.000053	.000003
13	0.000000	.000048	.000049	.000335	.000003	.000002
14	.000460	.000428	.000171	.005215	.000349	0.000000
15	.000006	.000084	.000082	.000016	.000270	.000009
16	.000072	.000216	.000212	.000039	.000309	.000025
17	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
18	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
19	.000059	.000117	.000078	0.000000	0.000000	.000005
20	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
21	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

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- a) Baer, W., J. J. M. Guilhoto, and M. A. R. da Fonseca, "Structural Changes in Brazil's Industrial Economy: 1960-1980";
- b) Fonseca, M. A. R. da, and J. J. M. Guilhoto, "Simulations of Government Policies in the Brazilian Economy: Intersectoral Flows and Income Distribution".