Aggregate demand for narrow and broad money: a study for the brazilian economy (1970-1983)

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AGGREGATE DEMAND FOR NARROW AND BROAD MONEY: A STUDY FOR THE BRAZILIAN ECONOMY (1970-1983)*

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ABSTRACT

To study the aggregate demand for narrow and broad money for the Brazilian economy in its most recent period, 1970 to 1983, a basic model was developed.

From this model, which is a restricted one, an unrestricted model was derived. Using information from both models, the unrestricted model was used to derive a common factor model as well as a first differences model.

The best results are attained with the common factor model.

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RESUMO

Para estudar a demanda agregada por moeda na sua forma mais restrita e mais abrangente, para a economia brasileira no período de 1970 a 1983, um modelo básico foi desenvolvido.

A partir deste modelo básico, que é um modelo restrito, um modelo não restrito foi desenvolvido. Usando informações dos dois modelos, o modelo não restrito foi usado para derivar um modelo com um fator comum, assim como um modelo em primeiras diferenças.

Os melhores resultados são obtidos com o modelo do fator comum.

1. INTRODUCTION

Previous work on the demand for money in Brazil has concentrated on the demand for narrow money (M1), and is limited to results for the period before 1979[1], when (it has been suggested) there may have occurred a structural change. This paper provides results for both narrow (M1) and broad (M3) money, and tests whether the post 1978 period is different.

A discussion of the theory of demand for money can be found in Goldfeld (1973 and 1976), and Laider (1977). Feige and Pearce (1977) presents a survey of empirical studies for the U.S.A.; a survey for European countries, Australia and Japan is presented in Fase and Kuné (1975); and Barbosa (1978) presents a survey of studies made for Brazil.

The methodology presented by Blommestein and Palm (1982) was chosen as the basis for studying the aggregate demand for money for the Brazilian economy. The choice of this work was made because it allows the derivation of different models from one initial model. These derivations are made using information from the theory as well as from the data set.

The methodology consists basically of the construction of a restricted model, from which an unrestricted model is derived. Using information from both models, the unrestricted model is used

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1 See Barbosa (1978), and more specifically Pastore (1973), da Silva (1973), Silveira (1973), Contador (1974), and Cardoso (1981).

to derive a common factor and a first differences model. An ARIMA model is also constructed for comparison with the models presented above.

The period to be studied is from 1970, IV to 1983, IV and it is broken down into two subperiods: 1970, IV to 1978, IV, and 1979, I to 1983, IV. A Chow test is conducted to test for the hypothesis of structural change between the two subperiods.

A test to verify the existence of monetary illusion in the aggregate demand for narrow and broad money is also conducted.

The work is organized as follows: in the next section the restricted and the unrestricted models are presented; in section 3, the empirical analysis of the models is made, and the common factor model and the model in first differences are also derived; conclusions are made in section 4.

2. THE MODEL

The aggregate demand for money is defined in the following way (all variables are natural logarithms and the time _t_ is given in quarters):

$$M_t^* = \alpha_0 + \alpha_1 Y_t^* + \alpha_2 R_t + \alpha_3 P_t^* \tag{2.1}$$

Where:

M_t^* = desired amount of liquidities (nominal) at the end of period _t_

Y_t^* = expected income (real) for period _t_

R_t = a representative interest rate (nominal) at the end of period _t_

P_t^* = the expected price level at the end of period $\underline{t}$

The unobserved variables M_t^*, Y_t^*, and P_t^* are defined as follows: M_t^* uses a partial adjustment process of the form:

$$M_t - M_{t-1} = \theta\ (M_t^* - M_{t-1})\ ,\quad 0 < \theta \leq 1 \tag{2.2}$$

Y_t^* and P_t^* use an adaptive expectations mechanism as:

$$Y_t^* - Y_{t-1}^* = \lambda\ (Y_{t-1} - Y_{t-1}^*)\ ,\ 0 < \lambda \leq 1 \tag{2.3}$$

and

$$P_t^* - P_{t-1}^* = \kappa\ (P_{t-1} - P_{t-1}^*)\ ,\ 0 < \kappa \leq 1 \tag{2.4}$$

Using the lag operator L, defined as $L\ x_t = x_{t-1}$, one can write equations (2.2), (2.3) and (2.4) as:

$$M_t^* = \frac{1}{\theta}\ M_t - \frac{(1 - \theta)}{\theta}\ L\ M_t \tag{2.5}$$

$$Y_t^* = \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i\ L^i\ Y_{t-1} = \frac{\lambda\ Y_{t-1}}{1-(1-\lambda)L} \tag{2.6}$$

$$P_t^* = \kappa \sum_{i=0}^{\infty} (1 - \kappa)^i\ L^i\ P_{t-1} = \frac{\kappa\ P_{t-1}}{1-(1-\kappa)L} \tag{2.7}$$

Substituting equations (2.5), (2.6), and (2.7) into equation (2.1) gives:

$$M_t = \alpha_0\,\theta + \frac{\alpha_1\theta\lambda\, Y_{t-1}}{} + \alpha_2\theta\, R_t + \frac{\alpha_3\theta\kappa\, P_{t-1}}{1-(1-\kappa)L} \quad (2.8)$$

Premultiplying equation (2.8) by the polynomials in L in the denominator gives the following equation (the restricted model):

$$\tilde{M}_t = \gamma_0 + \gamma_1\,\tilde{M}_{t-1} + \gamma_2\,Y'_{t-1} + \gamma_3\,\tilde{R}_t + \gamma_4\,P'_{t-1} \quad (2.9)$$

Where:

$$\gamma = \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \end{bmatrix} = \begin{bmatrix} \alpha_0\ \theta\ \lambda\ \kappa \\ (1-\theta) \\ \alpha_1\ \theta\ \lambda \\ \alpha_2\ \theta \\ \alpha_3\ \theta\ \kappa \end{bmatrix} \quad (2.10)$$

A variable (e.g. X_t) with a (~) is defined as:

$$\tilde{X}_t = [1-(1-\lambda)L][1-(1-\kappa)L]\ X_t \quad (2.11)$$

and

$$Y'_{t-1} = [1-(1-\kappa)L]\ Y_{t-1} \quad (2.12)$$

and

$$P'_{t-1} = [1-(1-\lambda)L]\ P_{t-1} \quad (2.13)$$

The α's (excluding α_0) in equation (2.1) are long term elasticities, given that in the long run $M_t^* = M_t$, $Y_t^* = Y_t$, and $P_t^* = P_t$. The γ's (excluding γ_o) in equation (2.9) are short term elasticities.

In order to estimate equation (2.9), one needs to assign previous values to κ and λ; however, this equation can be modified in such a way that one does not need to worry about these previous values. The resulting equation (the unrestricted model) is:

$$M_t = \beta_0 + \beta_1 M_{t-1} + \beta_2 M_{t-2} + \beta_3 M_{t-3} + \beta_4 Y_{t-1} + \beta_5 Y_{t-2} + \\ + \beta_6 R_t + \beta_7 R_{t-1} + \beta_8 R_{t-2} + \beta_9 P_{t-1} + \beta_{10} P_{t-2} \qquad (2.14)$$

Where:

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \\ \beta_{10} \end{bmatrix} = \begin{bmatrix} \alpha_0 \ \theta \ \lambda \ \kappa \\ (3 - \lambda - \kappa - \theta) \\ -(1-\kappa)(1-\lambda)-(2-\kappa-\lambda)(1-\theta) \\ (1-\lambda)(1-\kappa)(1-\theta) \\ \alpha_1 \ \theta \ \lambda \\ -\alpha_1 \ \theta \ \lambda \ (1 - \kappa) \\ \alpha_2 \ \theta \\ -\alpha_2 \ \theta \ (2 - \lambda - \kappa) \\ \alpha_2 \ \theta(1 - \kappa)(1 - \lambda) \\ \alpha_3 \ \theta \ \kappa \\ -\alpha_3 \ \theta \ \kappa \ (1 - \lambda) \end{bmatrix} \qquad (2.15)$$

To estimate the γ's of equation (2.9) and the β's of equation (2.14), one adds a disturbance term, u_t . u_t is assumed to have expectation zero, constant variance, zero serial correlation, and independence of the explanatory variables.

In addition, to estimate equation (2.9), onde needs to assign previous values to κ and λ.

3. EMPIRICAL RESULTS

3.1 Data Description

The data used in the analysis are quarterly figures adjusted for seasonality.

Estimated quarterly figures of GNP, measured in 1977 prices, were used for income (Y)[2]. These figures were derived from yearly ones through the method presented in Harberger (1963), in which the resulting estimates are by definition free of seasonal fluctuations (as remarked by Driehuis, 1972).

All other variables used in the analysis were seasonally adjusted using the X-11 method. A multiplicative adjustment process was assumed.

For money, two concepts were used: narrow (M1) and broad (M3)[3]. M1 is defined as currency held by the public plus demand deposits in the "Banco do Brasil" and commercial banks. M3 is defined as the sum of M1, demand deposits in savings banks, fixed time deposits, and savings deposits. All figures refer to the end of the quarter.

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[2] The source for GNP is Fundação Getúlio Vargas.

[3] The source for

The interest rate (% year) paid on three-month Treasury Bills was used for the interest rate (R)[4]. For the price level (P)[5], the general price index (internal disposability) with the basis equal to 100 in 1977 was used. The values of both variables were measured at the end of the quarter.

3.2 Results for the Models

In this section, the results for the aggregate demand for narrow (M1) and broad money (M3) are presented for both the restricted and unrestricted models for the periods: a) 1970,IV to 1983,IV (whole period); b) 1970,IV to 1978,IV (first subperiod); c) 1979, I to 1983,IV (second subperiod). These results will provide the information to construct a third model, a common factor model.

The division of the original period of analysis into two subperiods allows for the comparison of the aggregate demand for money in distinct periods of the Brazilian economy.

The first subperiod, 1970,IV to 1978,IV, was a relatively stable one. The GNP grew during all those years; and inflation was maintained most of the time between the 20% and the 40% levels. In the second subperiod, 1979,I to 1983,IV, due to a crisis in the external sector, Brazil started a phase of deaccelerated growth, negative growth rates of GNP occurred in some years, and the inflation level skyrocketed to the 200% level.

In order to estimate the aggregate demand for money for the restricted model, using ordinary least squares (OLS), it is necessary to choose values for λ and κ. The criterion adopted here assumes values of (0.1, 0.2, ..., 1.0) for λ and κ, then selects those values that maximize the value of the likelihood function and generate a θ in the interval $0 < \theta \leq 1$.

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[4] The source for R is Banco Central do Brasil.

[5] The source for P is Fundação Getúlio Vargas.

The values of λ and κ that satisfy these conditions are:

For M1:

a) $\lambda = \kappa = 1.0$ for the whole period and the first subperiod;
b) $\lambda = 1.0$ and $\kappa = 0.8$ for the second subperiod.

For M3:

a) $\lambda = \kappa = 1.0$ for the whole period, the first and the second subperiods[6].

Table 1 shows the results of the regression for the restricted model[7]. It also presents the results for $\lambda = \kappa = 1.0$, for M1, for subperiod 2; these results will be used to calculate a Chow test between subperiods 1 and 2; however, it should be noted that the value of θ falls outside the interval pre-defined. Table 2 presents the regression results for the unrestricted model.

Before discussing the elasticities resulting from the above models, it is necessary to analyze the various statistics of the models, to compare the different models and then to choose the one that best fits the data and presents the least number of statistical problems. In the chosen model, the elasticities will be discussed and then compared with previous estimates for the Brazilian economy.

The Durbin-Watson (DW) statistics for regressions number 1 to 13, presented in Tables 1 and 2, are inconclusive in relation to first order serial correlation; but since the models have lagged endogenous variables in the regression, the DW statistic is biased towards the non-detection of serial correlation.

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[6] The values of the likelihood functions and of the θ s for the different values of λ and κ are not presented here, but they are available upon request to the author.

[7] One should note than when $\lambda = 1$, $P'_{t-1} = P_{t-1}$ (see equation 2.13), when $\kappa = 1$, $Y'_{t-1} = Y_{t-1}$ (see equation 2.12), and when $\lambda = \kappa = 1$, $\tilde{X}_t = X_t$ (see equation 2.11).

TABLE 1

Ordinary Least Squares Applied to the Restricted Model
Results for M1 and M3

Money	M1	M1	M1	M1	M3	M3	M3
Period	1970,IV 1983,IV	1970,IV 1978,IV	1979,I 1983,IV	1979,I 1983,IV	1970,IV 1983,IV	1970,IV 1978,IV	1979,I 1983,IV
λ	1.0	1.0	1.0	1.0	1.0	1.0	1.0
κ	1.0	1.0	1.0	0.8	1.0	1.0	1.0
$\hat{\theta}$	0.103	0.251	-0.079	0.019	0.23	0.160	0.236
Const.	-1.224	-4.161	7.999	7.118	0.891	-2.317	21.619
	(-0.958)	(-1.333)	(0.871)	(0.721)	(0.947)	(-0.786)	(3.279)
$\tilde{M}_{t-1}$	0.897	0.749	1.079	0.981	0.977	0.839	0.764
	(9.190)	(4.649)	(3.853)	(3.016)	(17.700)	(6.051)	(5.489)
Y'_{t-1}	0.173	0.505	-0.640	-0.637	-0.053	0.290	-1.449
	(1.067)	(1.444)	(-0.835)	(-0.639)	(-0.501)	(0.915)	(-2.764)
$\bar{R}_t$	-0.059	-0.049	0.002	0.005	-0.028	-0.036	-0.009
	(-2.698)	(-1.217)	(0.021)	(0.056)	(-1.326)	(-0.875)	(-0.132)
P'_{t-1}	0.110	0.178	-0.053	0.012	0.069	0.175	0.263
	(1.522)	(1.656)	(-0.232)	(0.056)	(1.354)	(1.361)	(1.666)
R2	0.99	0.99	0.99	0.99	0.99	0.99	0.99
DW	2.424	2.291	2.126	2.148	2.262	2.453	3.105
F-val.	30811.81	5208.03	1585.83	914.06	46806.73	7849.14	5309.69
DF	45	25	12	12	45	25	12
ln(L)	107.45	68.51	32.76	31.89	107.86	68.65	37.91
$\hat{a}_1$	1.669	2.011	8.060	-32.793	-2.34	1.801	-6.150
$\hat{a}_2$	-0.570	-0.196	-0.020	0.253	-1.226	-0.227	-0.038
$\hat{a}_3$	1.066	0.709	0.676	0.771	3.025	1.090	1.118
nr.	1	2	3	4	5	6	7

Notes: t-values between parentheses;
DW is the Durbin-Watson statistic;
DF is the number of degrees of freedom in the regression;
ln(L) is the value of the log-likelihood;
nr. is the regression number.

TABLE 2

Ordinary Least Squares Applied to the Unrestricted Model
Results for M1 and M3

Money	M1	M1	M1	M3	M3	M3
Period	1970,IV 1983,IV	1970,IV 1978,IV	1979,I 1983,IV	1970,IV 1983,IV	1970,IV 1978,IV	1979,I 1983,IV
Const.	-0.160	-0.686	34.754	1.098	-3.257	42.169
	(-0.113)	(-0.133)	(2.316)	(1.028)	(-0.597)	(5.29)
M_{t-1}	0.583	0.366	0.071	0.828	0.428	-0.403
	(2.947)	(1.443)	(0.110)	(5.126)	(1.703)	(-1.239)
M_{t-2}	0.463	0.500	0.949	0.469	0.454	1.1018
	(2.218)	(1.645)	(1.776)	(2.166)	(1.639)	(4.316)
M_{t-3}	-0.071	0.125	0.066	-0.301	-0.038	-0.3654
	(-0.325)	(0.440)	(0.151)	(-1.576)	(-0.137)	(-1.359)
Y_{t-1}	0.493	2.552	-3.444	-0.780	1.490	-1.057
	(0.627)	(2.202)	(-2.552)	(-1.033)	(1.111)	(-0.928)
Y_{t-2}	-0.458	-2.489	0.833	0.699	-1.134	-1.635
	(-0.578)	(-1.986)	(0.595)	(0.947)	(-1.089)	(-1.133)
R_t	-0.042	-0.033	0.093	-0.067	0.024	-0.015
	(-0.966)	(-0.346)	(0.918)	(-1.523)	(0.258)	(-0.266)
R_{t-1}	-0.080	-0.096	-0.156	0.002	-0.044	-0.081
	(-1.192)	(-0.98[illegible])	(-1.403)	(0.034)	(-0.428)	(-1.649)
R_{t-2}	0.065	0.083	-0.018	0.060	0.022	-0.004
	(1.475)	(0.854)	(-0.140)	(1.335)	(0.228)	(-0.071)
P_{t-1}	-0.062	-0.207	-0.561	0.274	-0.487	0.407
	(-0.403)	(-0.466)	(-2.060)	(1.833)	(-1.004)	(3.248)
P_{t-2}	0.132	0.263	0.557	-0.245	0.647	0.349
	(0.835)	(0.648)	(1.409)	(-1.499)	(1.396)	(1.213)
R2	0.99	0.99	0.99	0.99	0.99	0.99
DW	2.044	2.098	2.628	2.059	2.163	2.728
F-val.	13659.57	2197.10	1118.64	19976.00	3116.25	9219.69
DF	39	19	6	39	19	6
nr.	8	9	10	11	12	13

Notes: t-values between parentheses;
DW is the Durbin-Watson statistic;
DF is the number of degrees of freedom in the regression;
nr. is the regression number.

TABLE 3a

Durbin h Test for the Restricted and Unrestricted Models
Results for M1 and M3

Regression Number	Value of the h test	Nr. of Obs.	Serial Correlation
1	-2.7642	50	Yes
2	-1.7924	30	No
5	-1.0215	50	No
6	-1.9031	30	No
7	-3.3286	17	Yes

Note: The test for serial correlation was conducted at the 5% level (two-tailed test).

TABLE 3b

Alternative of the Durbin h Test for the Restricted and Unrestricted Models - Results for M1 and M3

Regression Number	t-value	DF	Serial Correlation
1	-2.595	43	Yes
2	-0.947	23	No
3	-2.111	10	No
4	-2.443	10	Yes
5	-0.942	43	No
6	-1.586	23	No
7	-5.036	10	Yes
8	-0.672	37	No
9	0.199	17	No
10	0.379	4	No
11	-0.563	37	No
12	-0.898	17	No
13	-0.528	4	No

Note: The test for serial correlation was conducted at the 5% level (two-tailed test).

To solve the above problem, the Durbin h test and an alternative test for when this can not be calculated were made. The results are presented in Tables 3a and 3b. The values of these tests suggest the existence of a first order serial correlation in regressions number 1, 4 and 7[8], which can be an indicator of problems with the restricted model when applied to M1 and M3. There seems to be no problem of first order serial correlation in the unrestricted model.

Table 4 presents the results of a Chow test applied to both the restricted and the unrestricted models, and for M1 and M3. The objective of the test is to verify the hypothesis of no structural change between subperiods 1 and 2. The results show that, at the 5% level test, the null hypothesis (no structural change) can not be rejected for M1 (restricted and unrestricted models), but can be rejected in relation to M3 (restricted and unrestricted models).

Table 4

CHOW TEST FOR THE RESTRICTED AND UNRESTRICTED MODELS
RESULTS FOR M1 and M3

Regressions Numbers	F-value	DF
2 and 3	1.603	5,40
6 and 7	4.126	5,40
9 and 10	2.035	11,28
12 and 13	2.993	11,28

A test to verify the inexistence of monetary illusion in equation (2.1), test for $\alpha_3 = 1$, was conducted for regressions number 1 to 7 (restricted model). This test was constructed using a first

8 It should be noted that the power of this test is not good below 30 observations.

order Taylor series approximation. While this is not a very powerful test, due to its asymptotic properties, it seems to be the best one available. The results for this test, as presented in Table 5, show the inexistence of monetary illusion in the aggregate demand for narrow and broad money, for the different periods studied here.

Table 5

TEST FOR THE INEXISTENCE OF MONETARY ILLUSION
(α_3 = 1 IN THE RESTRICTED MODEL)

Regression Number	t-value	DF	Monetary Illusion
1	0.2046	45	No
2	-1.5482	25	No
3	-0.5158	12	No
4	-0.1066	12	No
5	0.3911	45	No
6	0.2708	25	No
7	1.2609	12	No

Note: The test was conducted at the 5% level (two-tailed test).

To compare the restricted model with the unrestricted model, a F-test was conducted assuming that the restricted model is the correct specification. To construct this test one should note that: a) for the case when $\lambda = \kappa = 1.0$, to test for the hypothesis that the restricted model is the right one is the same as to test for the hypothesis that $\beta_2 = \beta_3 = \beta_5 = \beta_7 = \beta_8 = \beta_{10} = 0$ in the unrestricted model; b) for the case when $\lambda = 1.0$ and $\kappa = 0.8$, it is the same as to test for the hypothesis that $\beta_3 = \beta_8 = \beta_{10} = 0$ in the unrestricted model.

The results of the test, see Table 6, show that the hypothesis that the restricted model is the correct specification, when compared to the unrestricted model, can not be rejected except for 2 cases (M1 and M3 for the second subperiod).

Table 6

TEST TO COMPARE THE RESTRICTED AND THE UNRESTRICTED MODELS

H_0 : THE RESTRICTED MODEL IS THE CORRECT SPECIFICATION

Regressions Numbers	F-value	DF	Can Reject H_0
1 and 8	1.81	6,39	No
2 and 9	1.23	6,19	No
3 and 10	2.52	6, 6	No
4 and 10	5.86	3, 6	Yes
5 and 11	1.50	6,39	No
6 and 12	0.97	6,19	No
7 and 13	7.68	6, 6	Yes

Note: The test was conducted at the 5% level.

From the above, one can see that there are problems with the unrestricted and the restricted models. The unrestricted model shows too many exogenous variables; the test that compares this model with the restricted model shows that some of those variables can be eliminated from the model. On the other hand, the restricted model presents problems of first order serial correlation.

But one can use this information to derive a new model. If one looks at the restricted model, one can see that $\lambda = \kappa = 1.0$ for regressions number 1 and 5 (whole period for M1 and M3, respectively); which, besides showing a high response of adjustment for income and prices, suggests the existence of a common factor in the unrestricted model.

By imposing the restriction $\lambda = \kappa = 1.0$ in the unrestricted model, the parameter space can be reduced (see Hendry and Mizon, 1978), and led to a specification like (common factor model):

$$M_t = \delta_0+\delta_1 M_{t-1}+\delta_2 M_{t-2}+\delta_3 Y_{t-1}+\delta_4 R_t+\delta_5 R_{t-1}+\delta_6 P_{t-1}+\frac{u_t}{1-\rho L} \qquad (3.1)$$

Since one has lagged endogenous variables in model (3.1), which clearly presents first order serial correlation, the parameters were estimated by a method presented in Johnston (1972). The method uses instrumental variables to replace M_{t-1} and M_{t-2} and then applies the two-stages full transform method, as installed in the package SAS (1982), which estimates the parameter values of the regression.

In order to construct the instrumental variables for M_{t-1} and M_{t-2}, M_t was regressed (using OLS) against Y_{t-1}, Y_{t-2}, R_t, R_{t-1}, R_{t-2}, P_{t-1}, and P_{t-2}. The estimated value of M_t was then calculated and it was used to compute the instruments for M_{t-1} and M_{t-2}.

The results are as follows (t-values between parentheses):

For M1, period of 1970,IV-1983,IV:

$$
\begin{array}{llllll}
M_t = & -\ 5.462 & -\ 0.031\ M_{t-1} & + & & Y_{t-1} \\
 & (-1.823) & (-0.093) & (2.268) & (2.035) & \\
 & -\ 0.030\ R_t & -\ 0.108\ R_{t-1} & + & & \dfrac{\hat{u}_t}{1 + 0.639\ L} \\
 & (-0.596) & (-1.982) & (2.574) & &
\end{array}
\qquad (3.2)
$$

$$
\begin{array}{l}
R2 = 0.99\ ,\ \rho = -0.639\ ,\ DF = 42\ ,\ nr. = 14 \\
\qquad\qquad\qquad (-5.813)
\end{array}
$$

For M3, period of 1970,IV-1983,IV:

$$
\begin{array}{llllll}
M_t = & -\ 5.964 & + & + & + & \\
 & (-1.674) & (0.631) & (1.287) & (2.121) & \\
 & -\ 0.073\ R_t & -\ 0.027\ R_{t-1} & + & & \dfrac{\hat{u}_t}{1 + 0.729\ L} \\
 & (-1.197) & (-0.427) & (3.330) & &
\end{array}
\qquad (3.3)
$$

$$
\begin{array}{l}
R2 = 0.99\ ,\ \rho = -0.729\ ,\ DF = 42\ ,\ nr. = 15 \\
\qquad\qquad\qquad (-7.458)
\end{array}
$$

A test statistic comparing the common factor model (when applied to M1 and M3 for the whole period) with the unrestricted model, becomes harder since the method used to calculate the parameters of the common factor model produces estimates that "are usually similar to ordinary least-squares estimates, but the standard errors may be very different, affecting significance tests" (SAS, 1982, p. 187). One is therefore going to assume, for the moment, that this model is the best one. A later comparison with the other models will prove or disprove that.

As an alternative to the models with the common factor restriction, equation (2.14) can be rewritten as a first differences model:

$$\Delta M_t = \mu_0 + \mu_1 \Delta M_{t-1} + \mu_2 \Delta M_{t-2} + \mu_3 (M_{t-1} - Y_{t-1} - P_{t-1}) + \mu_4 \Delta Y_{t-1} + \mu_5 \Delta R_t + \mu_6 R_{t-2} + \mu_7 \Delta P_{t-1} + \varepsilon_t \tag{3.4}$$

The explanation of equation (3.4) is given by Blommestein and Palm (1982) as:

> "Equation [(3.4)] explains the growth rate of nominal money balances as a function of the lagged money growth rates, an error learning mechanism $(M_{t-1}-Y_{t-1}-P_{t-1})$ being equal to the logarithm of the velocity of money in t-1 with respect to the transactions, the [lagged] growth rate of expenditures in constant prices, the change in the interest rate, the lagged interest rate and the [lagged] rate of change in prices". (pp. 373-374).

For M1, the results are as follows for the period 1970,IV--1983,IV:

$$\begin{aligned}\Delta M_t = &- \underset{(-1.216)}{0.281} - \underset{(-0.295)}{0.050}\, \Delta M_{t-1} + \underset{(2.124)}{0.418}\, \Delta M_{t-2} - \underset{(-1.266)}{0.064}\, (M_{t-1}-Y_{t-1}-P_{t-1}) \\ &- \underset{(-0.147)}{0.112}\, \Delta Y_{t-1} - \underset{(-0.942)}{0.041}\, \Delta R_t + \underset{(0.107)}{0.002}\, R_{t-2} - \underset{(-0.652)}{0.103}\, \Delta P_{t-1} + \hat{\varepsilon}\end{aligned} \tag{3.5}$$

DW = 1.989 , R2 = 0.49 , F = 5.841 , DF = 42 , nr. = 16

Equation (3.5) has a steady state solution that is given by:

$$M = A + Y + P + 0.031\ R \tag{3.6}$$

Where:

$$A = 15.63\ (-0.281 - 0.632\ \Delta M - 0.112\ \Delta Y - 0.041\ \Delta R - 0.103 \Delta P)$$

One can get the inverse of the steady state transaction velocity of money as a function of the interest rate (r) by taking the antilogs of equation (3.6):

$$\frac{m}{p\ y} = \alpha\ r^{0.031} \tag{3.7}$$

Where $\ln \alpha = A$ and m, r, p and y are antilogs of M, R, P and Y, respectively.

In this case, one can see that the steady state velocity varies inversely with the interest rate, "a finding that one would not expect from theoretical consideration" (Blommestein and Palm, 1982, p. 376). This might be an indication that there are problems with the model in first differences when applied to M1.

The likelihood ratio test for comparing regression nr. 16 (first differences model for M1, whole period) with regression nr. 8 (unrestricted model for M1, whole period) shows a value of 13.59 and has an asymptotic χ^2 - distribution with 3 degrees of freedom, which indicates that at the 5% level test the hypothesis that the first differences model is the correct specification is rejected. Once more, this shows problems
differences when applied to M1, whole period.

Only for illustrative purposes, a slightly different version of equation (3.5) for M1, period 1970,IV-1983,IV, is presented here (note that ΔR_{t-1} is used instead of R_{t-2}).

$$\Delta M_t = \underset{(-1.147)}{-0.235} - \underset{(-0.708)}{0.113}\ \Delta M_{t-1} + \underset{(1.806)}{0.338}\ \Delta M_{t-2} - \underset{(-1.496)}{0.058}\ (M_{t-1}-Y_{t-1}-P_{t-1})$$

$$- \underset{(-0.464)}{0.321}\ \Delta Y_{t-1} - \underset{(-0.092)}{0.004}\ \Delta R_t - \underset{(-2.434)}{0.102}\ \Delta R_{t-1} - \underset{(-0.019)}{0.003}\ \Delta P_{t-1} + \hat{\varepsilon}_t \qquad (3.8)$$

DW = 2.082 , R2 = 0.56 , F = 7.507 , DF = 42 , nr. = 17

For M3, the results for model (3.4), for the period 1970,IV--1983,IV, are:

$$\Delta M_t = \underset{(-1.203)}{-0.242} + \underset{(0.055)}{0.007}\ \Delta M_{t-1} + \underset{(3.032)}{0.461}\ \Delta M_{t-2} - \underset{(-1.513)}{0.056}\ (M_{t-1}-Y_{t-1}-P_{t-1})$$

$$- \underset{(-1.224)}{0.778}\ \Delta Y_{t-1} - \underset{(-2.064)}{0.086}\ \Delta R_t + \underset{(0.518)}{0.010}\ R_{t-2} + \underset{(1.930)}{0.231}\ \Delta P_{t-1} + \hat{\varepsilon}_t \qquad (3.9)$$

DW = 2.174 , R2 = 0.75 , F = 18.309 , DF = 42 , nr. = 18

The steady state solution for equation (3.9), M3, is given by:

$$M = A + Y + P + 0.179\ R \qquad (3.10)$$

Where:

$$A = 17.86\ (- 0.242 - 0.532\ \Delta M - 0.778\ \Delta Y - 0.086\ \Delta R + 0.231\ \Delta P)$$

For M3, the inverse of the steady state transaction velocity of money as a function of the interest rate can be calculated by taking antilogs from equation (3.10):

$$\frac{m}{p\ y} = \alpha\ r^{0.179} \qquad (3.11)$$

All variables are defined as before.

Once more, contrary to the theory, the steady state velocity varies inversely with the interest rate. This might be an indication that there are problems with the model in first difference when applied to M3.

The value of the likelihood ratio test for comparing regression nr. 18 (first differences model for M3, whole period) with regression nr. 11 (unrestricted model for M3, whole period) is 4.49 and has an asymptotic χ^2 -distribution with 3 degrees of freedom. This indicates that at the 5% level test the hypothesis that the first differences model is the correct specification can not be rejected.

Repeating the same procedure used for M1, a slightly different variation of equation (3.9) is presented here for M3, period 1970, IV-1983,IV:

$$\begin{aligned}\Delta M_t = &- \underset{(-1.203)}{0.202} - \underset{(-0.175)}{0.023}\,\Delta M_{t-1} + \underset{(3.311)}{0.453}\,\Delta M_{t-2} - \underset{(-1.594)}{0.055}\,(M_{t-1}-Y_{t-1}-P_{t-1}) \\ &- \underset{(-2.431)}{1.122}\,\Delta Y_{t-1} - \underset{(-1.865)}{0.073}\,\Delta R_t - \underset{(-1.528)}{0.063}\,\Delta R_{t-1} + \underset{(2.424)}{0.289}\,\Delta P_{t-1} + \hat{\varepsilon}_t\end{aligned} \tag{3.12}$$

DW = 2.186 , R2 = 0.76 , F = 19.498 , DF = 42 , nr. = 19

To complete the analysis, an ARIMA model was fitted for M1 and M3.

Of the several ARIMA models estimated for M1, the one chosen was:

$$(1 + \underset{(-2.63)}{0.558}L - \underset{(8.02)}{0.895}L^2 - \underset{(4.47)}{0.659}L^3)(1 - L)M_t = (1 + \underset{(-3.83)}{0.853}L)\,\varepsilon_t \tag{3.13}$$

Q(6)	= 4.71	DF	= 2
Q(12)	= 10.88	DF	= 8
Q(18)	= 14.49	DF	= 14
Q(24)	= 25.43	DF	= 20

Number of Observations = 53
t-values between parentheses

For M3, of the several ARIMA models estimated, the one chosen was:

$$\underset{(-5.18)}{(1 + 0.616L)}(1 - L)^2 M_t = \varepsilon_t \tag{3.14}$$

Q(6) =	5.20	DF =	5
Q(12) =	20.37	DF =	11
Q(18) =	25.16	DF =	17
Q(24) =	32.16	DF =	23

Number of Observations = 53

t-value between parentheses

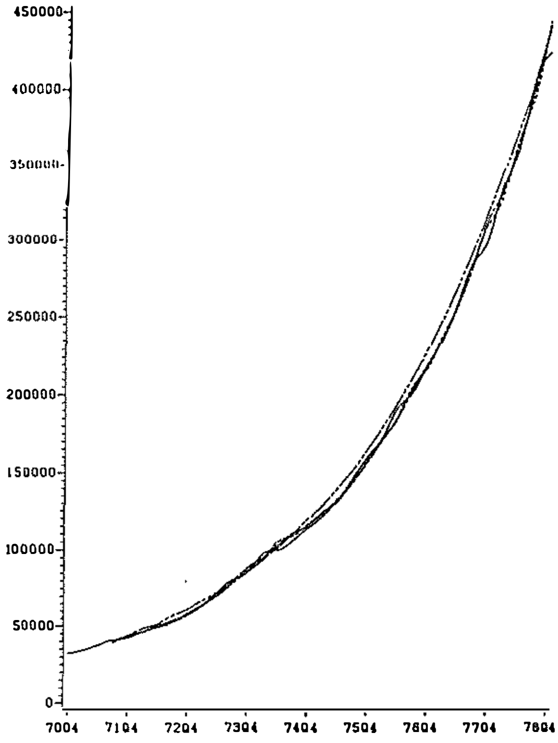

FIGURE 1A: NOMINAL VALUES OF M1 IN MILLIONS OF CRUZEIROS: 1970–IV TO 1978–IV
SEASONALLY ADJUSTED
REGRESSION NR. 1
REGRESSION NR. 8
ARIMA(3,1,1)

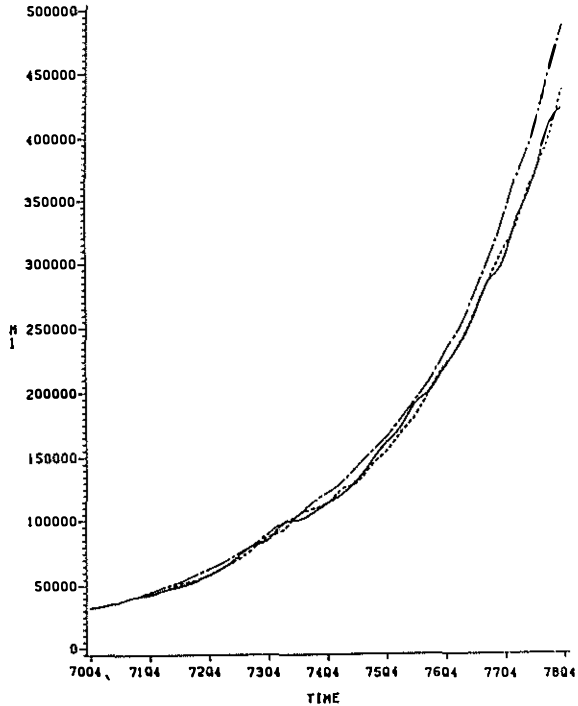

FIGURE 1B: NOMINAL VALUES OF M1 IN MILLIONS OF CRUZEIROS: 1970–IV TO 1978–IV
SEASONALLY ADJUSTED
REGRESSION NR. 14
REGRESSION NR. 18

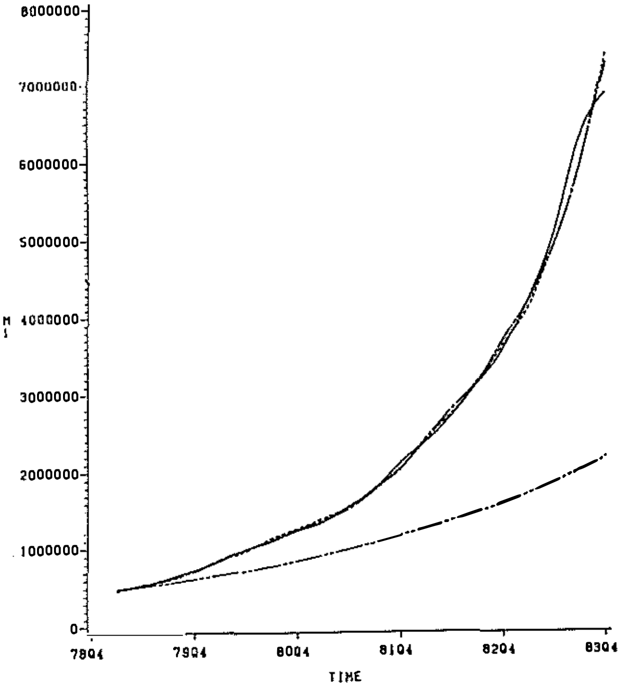

FIGURE 1C: NOMINAL VALUES OF M1 IN MILLIONS OF CRUZEIROS: 1979-I TO 1983-IV

SEASONALLY ADJUSTED
REGRESSION NR. 1
REGRESSION NR. 8
ARIMA(3,1,1)

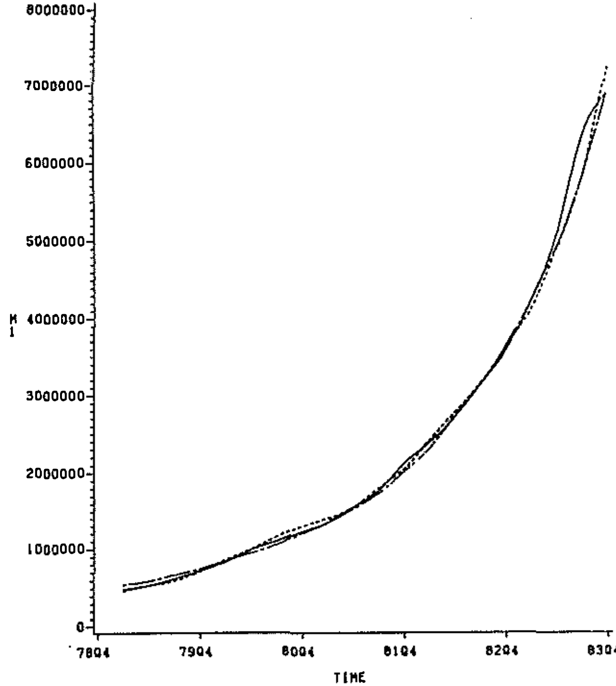

FIGURE 1D: NOMINAL VALUES OF M1 IN MILLIONS OF CRUZEIROS: 1979-I TO 1983-IV

SEASONALLY ADJUSTED
REGRESSION NR. 14
REGRESSION NR. 18

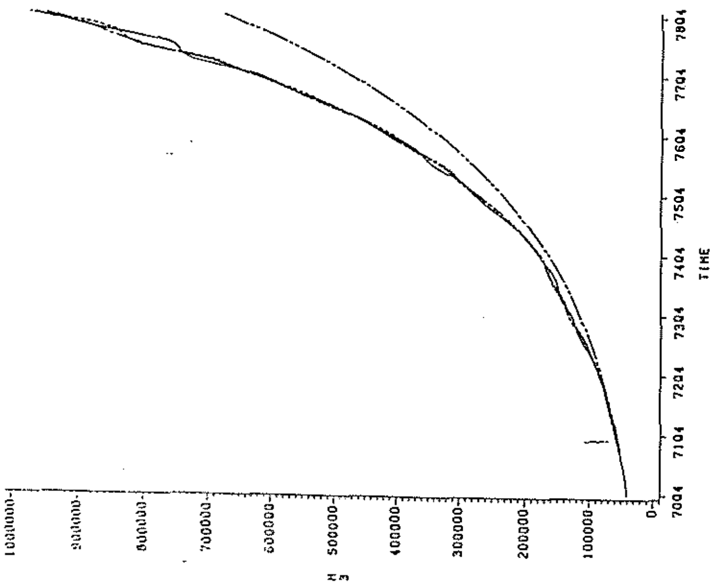

FIGURE 2A: NOMINAL VALUES OF M3 IN MILLIONS OF CRUZEIROS: 1970–IV TO 1978–IV
——— SEASONALLY ADJUSTED
– – – – REGRESSION NR. 6
– – – – REGRESSION NR. 11
– – – —— ARIMA(1,2,0)

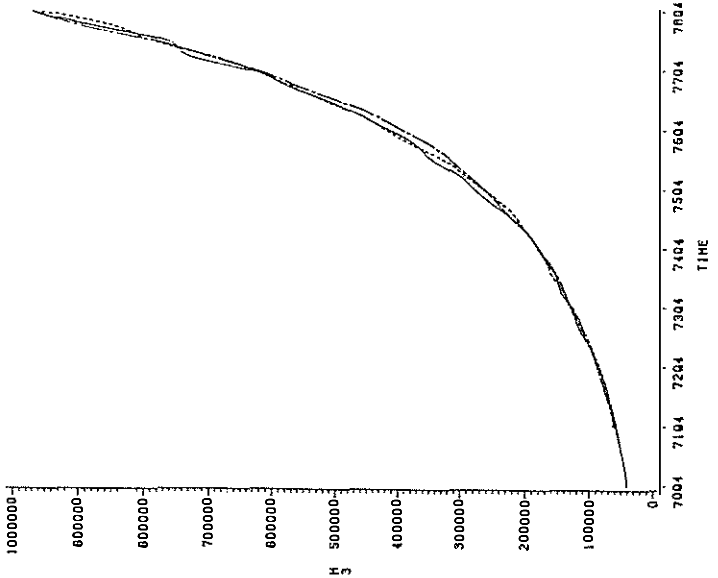

FIGURE 2B: NOMINAL VALUES OF M3 IN MILLIONS OF CRUZEIROS: 1970–IV TO 1978–IV
——— SEASONALLY ADJUSTED
– – – – REGRESSION NR. 15
– – – – REGRESSION NR. 18

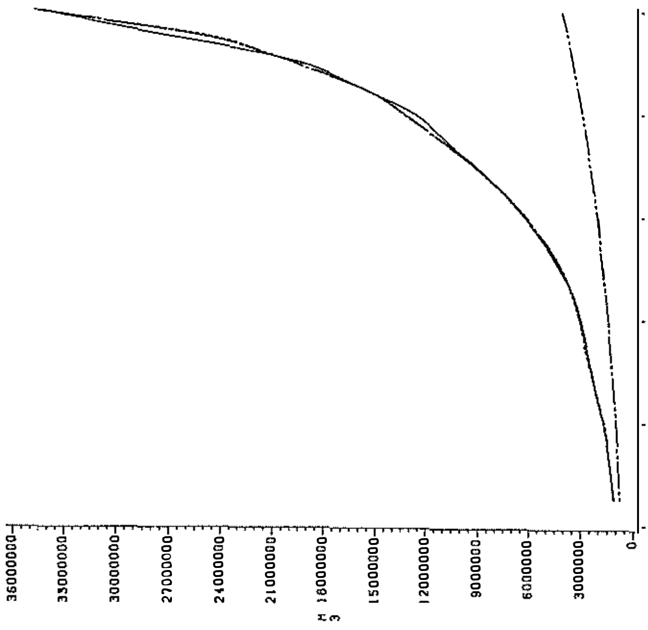

FIGURE 2C: NOMINAL VALUES OF M3 IN MILLIONS OF CRUZEIROS: 1979-I TO 1983-IV
SEASONALLY ADJUSTED
REGRESSION NR. 6
REGRESSION NR. 11
ARIMA(1,2,0)

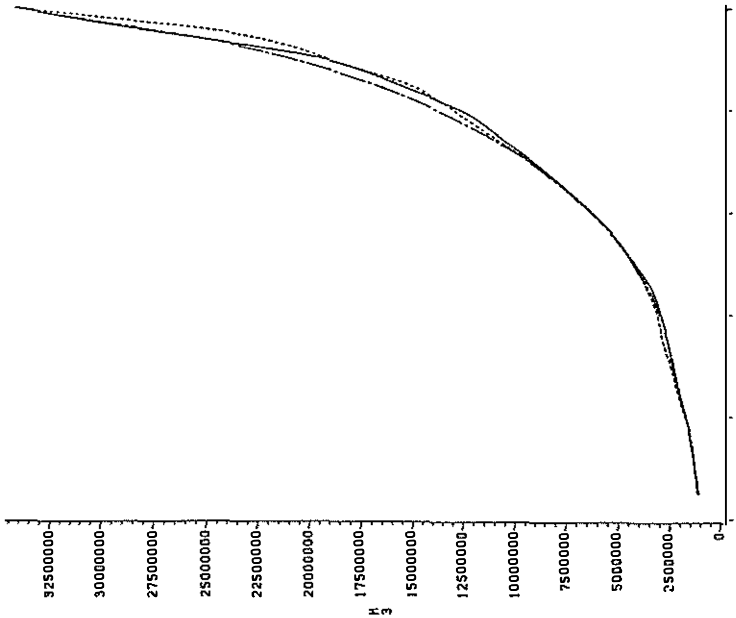

FIGURE 2D: NOMINAL VALUES OF M3 IN MILLIONS OF CRUZEIROS: 1979-I TO 1983-IV
SEASONALLY ADJUSTED
REGRESSION NR. 15
REGRESSION NR. 18

In order to compare the prediction power of the different models calculated for the demand function for narrow and broad money, the models (restricted, unrestricted, common factor, first differences, and ARIMA) were plotted with the seasonally adjusted values for M1 and M3.

Figures 1A through 1D show the plotted values for M1. From the analysis of these figures, one can see that the ARIMA model is reasonable for the first subperiod (1970,IV-1978,IV), but misleading for the second one (1979,I-1983,IV). The restricted model (regression nr. 1) and the unrestricted model (regression nr. 8) are very close in their prediction values. Between the common factor model (regression nr. 14) and the first differences model (regression nr. 16), one has to choose the common factor model. This also seems to be the best model, in terms of prediction values, of the several models presented in Figures 1A through 1D.

In Figures 2A through 2D, the values for M3 were plotted. The ARIMA model is clearly the worst in terms of predicting the values for M3. The restricted model (regression nr. 5) and the unrestricted model (regression nr. 11) have prediction values very close to one another. A comparison between the common factor model (regression nr. 15) and the first differences model (regression nr. 18) now becomes harder than the comparison for M1, but one is inclined to select the common factor model. Indeed, this model seems to perform better than the other models, for M3.

From the visual analysis made above, one can automatically discard the ARIMA models as they clearly are the ones that present the worst prediction power of the models plotted. From the statistical point of view, problems were seen in the unrestricted model (too many variables), in the restricted model (serial correlation), and in the first differences model (the steady state velocity varies inversely with the interest rate). The statistical tests comparing the different models also indicate that a reduction in the parameters space of the unrestricted model is the right thing to do (see analysis of Table 6), and that the model in first differences can be rejected as being the right specification. All of this information, plus the fact that the common factor model uses information from the restricted and the unrestricted models, and that its estimates of the elasticities present the right signs, leads one to believe that the common factor model is the one that best explains the aggregate demand for narrow and broad money for the Brazilian economy.

The next paragraphs in this section make an analysis of the results attained in the common factor model, for M1 and M3.

From regression nr. 14, for M1, one has the right sign for the income elasticity of money, as well as for the interest and price elasticities. If one compares these elasticies with previous studies made for the Brazilian economy (see a survey in Barbosa, 1978), usually using M1 as the definition for money, one can see that the value of 0.741 for the income elasticity conforms with those studies which, in general, present values between 0.7 and 1.0. For the interest elasticity, the value of -0.03 for time t and the value of -0.108 for time t-1 also agree with previous studies. In relation to the price elasticity, it presents a value of 0.394. This regression also shows a negative relation (-0.031) with the quantity of money in time t-1 and a positive relation (0.58) to its quantity in time t-2. It should be noted that the coefficients of M_{t-1} and R_t are not significantly different from zero.

From regression nr. 15, for M3, it can be seen that all the elasticities present the right signs. As one rarely sees a study of aggregate demand for money for the Brazilian economy using M3as the definition for money, it is not possible to compare the results for M3 with previous studies. The results attained for this regression are: an income elasticity of 0.825; an interest elasticity of -0.073 in relation to time t and of -0.027 in relation to time t-1; a price elasticity of 0.591; and a positive relation with the quantity of money in time t-1 (0.174) and in time t-2 (0.272). Looking at the regression results, one sees low t-values for the coefficients of M_{t-1} and R_{t-1}.

A comparison between the regression results obtained for M1 and M3 leads one to observe that there is a closeness between the elasticity coefficients of both regressions. One can then conclude that there is not much difference in measuring the aggregate demand in terms of narrow or broad money.

4. CONCLUSION

In this work, results were obtained for different specifications of the aggregate demand for narrow and broad money for the Brazilian economy for the period 1970,IV - 1983,IV, and subperiods 1970,IV - 1978,IV and 1979,I - 1983,IV.

All the specifications were basically derived from an initial model, which is a restricted one. From this model an unrestricted model was derived. Using information from these models, a third one, a common factor model, was derived. This last model was also expressed in the form of first differences. Also, an ARIMA model was calculated for comparison with the above models.

From statistical as well as graphical analysis of the different models, the common factor model appeared to be best in explaining the aggregate demand for both narrow and broad money.

The test for structural change between the two subperiods, using the restricted and the unrestricted models, showed that the hypothesis of no structural change can not be rejected for M1, but can be rejected for M3.

Tests also showed that the hyphothesis of inexistence of monetary illusion in the aggregate demand for M1 and M3 can not be rejected, implying that the demand for money for the Brazilian case is for real balances.

It was also seen that the elasticities resulting from the chosen model (common factor) for M1, do not disagree with those obtained in previous studies made for the Brazilian economy.

A comparison between the elasticities in the common factor model, using M1 or M3 as the definition for money, shows that there is not much difference in the results attained using either definition. But, given that the hypothesis of no structural change was rejected for M3, the results for M3 may be better.

REFERENCES

Banco Central do Brasil, *Boletim do Banco Central do Brasil*, various issues.

BARBOSA, F.H. "A Demanda de Moeda no Brasil": Uma Resenha da Evidência Empírica", in *Pesquisa e Planejamento Econômico*, Vol. 8, nº 1, pp.33-82, April, 1978.

BLOMMESTEIN, H.J. and PALM, F.C. "Econometric Specification Analysis - An Application to the Aggregate Demand for Money in the Netherlands", in Deistler, M., E. Furst, and G. Schwödiauer (eds.), *Games, Economics Dynamics, and Time Series Analysis*, Wien; Wurzburg: Physica-Verlag, 1982.

CARDOSO, E.A. "Uma Equação para a Demanda de Moeda no Brasil", in *Pesquisa e Planejamento Econômico, 11*(3), pp. 617-636, December, 1981.

CONTADOR, C.R. "Desenvolvimento Financeiro, Liquidez e Substituição entre Ativos no Brasil; A Experiência Recente", in *Pesquisa e Planejamento Econômico, 4*(2), pp. 245-284, June, 1974.

DRIEHUIS, W. *Fluctuations and Growth in a Near Full Employment Economy*, Rotterdam, Rotterdam University Press, 1972.

FASE, M.M.G. and KUNÉ, J.B. "The Demand for Money in Thirteen European and Non-European Countries: A Tabular Survey", in *Kredit und Kapital*, 3, pp. 410--419, 1975.

FEIGE, E.L. and PEARCE, D.K. "The Substitutability of Money and Near-Monies: A Survey of the Time-Series Evidence", in *The Journal of Economic Literature, XV*(2), pp. 439-469, June, 1977.

Fundação Getúlio Vargas, *Conjuntura Econômica*, various issues.

GOLDFELD, S.M. "The Demand for Money Revisited", in *Brookings Papers on Economic Activity*, 3, pp. 577-638, 1973.

GOLDFELD, S.M. "The Case of the Missing Money", in *Brookings Papers on Economic Activity*, 3, pp. 683-739, 1976.

GRANGER, C.W.J. and NEWBOLD, P. *Forecasting Economic Time Series*. New York, Academic Press, 1977.

HARBERGER, A.C. "The Dynamics of Inflation in Chile", in Christ, C. (ed.), *Measurement in Economics*, Stanford, Stanford University Press, 1963.

HENDRY, D.F. and MIZON, G.E. "Serial Correlation as a Convenient Simplification, Not a Nuisance: A Comment on a Study of the Demand for Money by the Bank of England", in *The Economic Journal*, Vol. 88, pp. 549-563, September, 1978.

JOHNSTON, J. *Econometric Methods*, 2nd, ed., New York, McGraw-Hill, 1972.

LAIDER, D. *The Demand for Money*, New York, Harper & Row, 1977.

PASTORE, A.C. "Aspectos da Política Monetária Recente no Brasil", in *Estudos Econômicos*, 3(3), pp. 7-58, Sep.-Dec., 1973.

PINDYCK, R.S. and RUBINFELD, D.L. *Econometric Models and Economic Forecasts*, 2nd ed., New York, McGraw-Hill, 1981.

SAS Institute Inc. *SAS/ETS User's Guide, 1982 Edition*, Cary, NC: SAS Institute Inc., 1982.

SILVA, A.M. da "Demanda de Moeda e Taxa Esperada de Inflação: Um Estudo Empírico de Argentina, Brasil, Chile e E.U.A.", in *Estudos Econômicos*, 3(3), pp. 59-101, Sep.-Dec., 1973.

SILVEIRA, A.M. "The Demand for Money: The Evidence from the Brazilian Economy", in *Journal of Money, Credit and Banking*, 5(1), pp. 113-140, 1973.