Generalized Impulse Response Analysis: General or Extreme?

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Abstract

This note discusses a pitfall of using the generalized impulse response function (GIRF) in vector autoregressive (VAR) models (Pesaran and Shin, 1998). The GIRF is *general* because it is invariant to the ordering of the variables in the VAR. The GIRF, in fact, is *extreme* because it yields a set of response functions that are based on extreme identifying assumptions that contradict each other, unless the covariance matrix is diagonal. With a help of empirical examples, the present note demonstrates that the GIRF may yield quite misleading economic inferences.

Keywords: Generalized Impulse Response Function; Orthogonalized Impulse Response Function; Vector Autoregressive Models

JEL Classification: C13; C32; C51

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1 Introduction

Notwithstanding its popularity, the orthogonalized impulse response function (OIRF; Sims, 1980) analysis of structural vector autoregressive (VAR) models is subject to the so-called Wold-ordering problem.[1] That is, when one changes the order of the VAR with an alternative identifying assumption, she may obtain dramatically different response functions (Lütkepohl, 1991).

Pesaran and Shin (1998) propose the generalized impulse response function (GIRF), an ordering-invariant approach, based on the work of Koop *et al.* (1996). The GIRF has been employed by many researchers: Boyd *et al.* (2001), Cheung *et al.* (2004), and Huang *et al.* (2008), to name a few. This note shows, however, that the GIRF may results in misleading inferences caused by its extreme identification schemes.

The remainder of this note is organized as follows. Section 2 analytically demonstrates why the GIRF actually can be considered extreme in perspective of its identification method. In Section 3, I provide two empirical examples that highlights substantial differences between the GIRF and the OIRF. Section 4 concludes.

2 A Pitfall of the GIRF

This section provides a simple analytical explanation on a pitfall in using the GIRF, which demonstrates potentially serious problems of using the GIRF.

Let $\psi^g_{y_j}(n)$ and $\psi^o_{y_j}(n)$ denote the GIRF and the OIRF at time $t+n$, respectively, when there is one standard error shock at time t to the j^{th} variable in an m-variate VAR with $\mathbf{y}_t = [y_{1,t}\ y_{2,t}\ \cdots\ y_{m,t}]'$. Pesaran and Shin's (1998) Proposition 3.1 implies $\psi^g_{y_1}(n) = \psi^o_{y_1}(n)$.[2] Define $\tilde{\psi}^o_{y_j}(n)$ as the OIRF when $y_{j,t}$ is ordered first in $\mathbf{y}_t$. By construction, $\psi^g_{y_1}(n) = \tilde{\psi}^o_{y_1}(n)$. Now re-order the vector so that $\mathbf{y}_t = [y_{2,t}\ y_{1,t},\ y_{3,t}\ \cdots\ y_{m,t}]'$, which yields $\psi^g_{y_2}(n) = \tilde{\psi}^o_{y_2}(n)$ by the proposition

[1] The OIRF recursively identifies the structural shocks by using the Choleski decomposition factor of the covariance matrix, which yields a unique lower triangular matrix. This scheme, therefore, assumes that the variable ordered first in the VAR is contemporaneously unaffected by all other variables.

[2] That is, the GIRF and the OIRF coincide for the shock to the first variable in $\mathbf{y}_t$.

and because the GIRF is invariant to the ordering of the variables in $\mathbf{y}_t$. Repeat this procedure until we get $\psi^g_{y_m}(n) = \tilde{\psi}^o_{y_m}(n)$. Collecting these response functions, the GIRF for the entire system is,

$$\psi^g(n) = \left\{ \tilde{\psi}^o_{y_1}(n)\ \tilde{\psi}^o_{y_2}(n)\ \cdots\ \tilde{\psi}^o_{y_m}(n) \right\}$$

The GIRF, therefore, is *not general* in effect because it employs extreme identifying assumptions that each variable is ordered first. More seriously, $\tilde{\psi}^o_{y_i}(n)$ and $\tilde{\psi}^o_{y_j}(n)$ are not consistent with each other when $i \neq j$ unless the covariance matrix is diagonal. For instance, $\tilde{\psi}^o_{y_i}(n)$ assumes that $y_{i,t}$ is not contemporaneously affected by all other variables including $y_{j,t}$, while $\tilde{\psi}^o_{y_j}(n)$ needs an assumption that $y_{j,t}$ is not contemporaneously affected by all other variables including $y_{i,t}$.[3] Hence, the GIRFs conflict each other. This result also trivially applies to vector error correction models. In the next section, I show that such inconsistency may lead to misleading economic inferences.

3 Empirical Examples

This section provides empirical illustration to compare the implications of the GIRF with those of the OIRF.

I first use a quadvariate VAR model of the US per capita investment (i), consumption (c), real GDP (y), and the government expenditure share (g) relative to the real GDP, measured in logarithms. The data frequency is quarterly and the observations span from 1948Q1 to 2008Q4, obtained from the Federal Reserve Bank of St. Louis FRED data base.

I report the ordering-free GIRF and the OIRF with an ordering $[i\ c\ g\ y]$ to a government expenditure shock in Figure 1.[4] Both response functions display a decrease in i, which may be consistent with the crowding-out effect. However, responses of c and y exhibit noticeably different dynamic adjustments. For example, the GIRF implies a significant *decrease* in y at the 5% level over a year, while the OIRF implies significantly *positive* responses of y for about a year. Since y

[3] If it is diagonal, there is no gain of using a structural VAR model, because it coincides with a reduced-form VAR, in other words, equation-by-equation least squares estimations.

[4] 95% confidence bands are obtained from 5,000 nonparametric bootstrap simulations.

includes not only private but also public sector outputs, it is surprising to see a substantial decreases in y in the short-run as we seen from the GIRF.[5] Similarly, the GIRF implies a significant decrease in c for about a year, while the OIRF displays slow positive adjustments of c which are insignificant.

Figure 1 about here

I implement another example to better understand why there may be such differences between the GIRF and the OIRF. For this, I use a trivariate VAR model of i, c, and y from the previous exercise, which was also employed by Pesaran and Shin (1998).

Note that the GIRFs to (one standard error) investment shock (Panel 2-a in Figure 2) coincide with the OIRFs to an i-shock when i is assumed to be contemporaneously unaffected by other two variables, c and y (Panel 2-b). Note also that under this assumption, the OIRFs to a y-shock are very different from the corresponding GIRFs. However, the GIRFs to a y-shock are identical to the OIRFs when y is ordered first in the VAR (Panel 2-d) by construction. Again, the other OIRFs under that assumption are quite different from the corresponding GIRFs. Likewise, the GIRFs to a c-shock are identical only to the OIRFs to a c-shock when c is ordered first (Panel 2-c).

Based on these findings, it seems important to estimate and report response functions based on the underlying economic model. For example, if one interprets y-shocks as an output (supply) shock, while i-shocks and c-shocks are treated as expenditure (demand) shocks, she may employ an ordering $[y\ i\ c]$ assuming that y does not contemporaneously respond to demand shocks. Then, she will report the response functions to an i-shock, for instance, that are very different from the GIRFs both quantitatively and qualitatively. If one believes that i is primarily driven by *animal spirit*, she may employ $[i\ y\ c]$ instead and report quite smaller responses of i to a y-shock than the corresponding GIRF.

Figure 2 about here

[5] For this, private sector activities should decrease by more than an increase in the increases in the public sector.

4 Conclusion

This note points out that there is a pitfall in using the GIRF. Economic inferences based on the GIRF can be misleading because the GIRF employs a set of extreme identifying assumptions that contradict each other unless the covariance matrix is diagonal. Our empirical example demonstrates that this is by no means a negligible matter. In such cases, it would be more reasonable to use identifying assumptions that consistently describes the underlying economic model.

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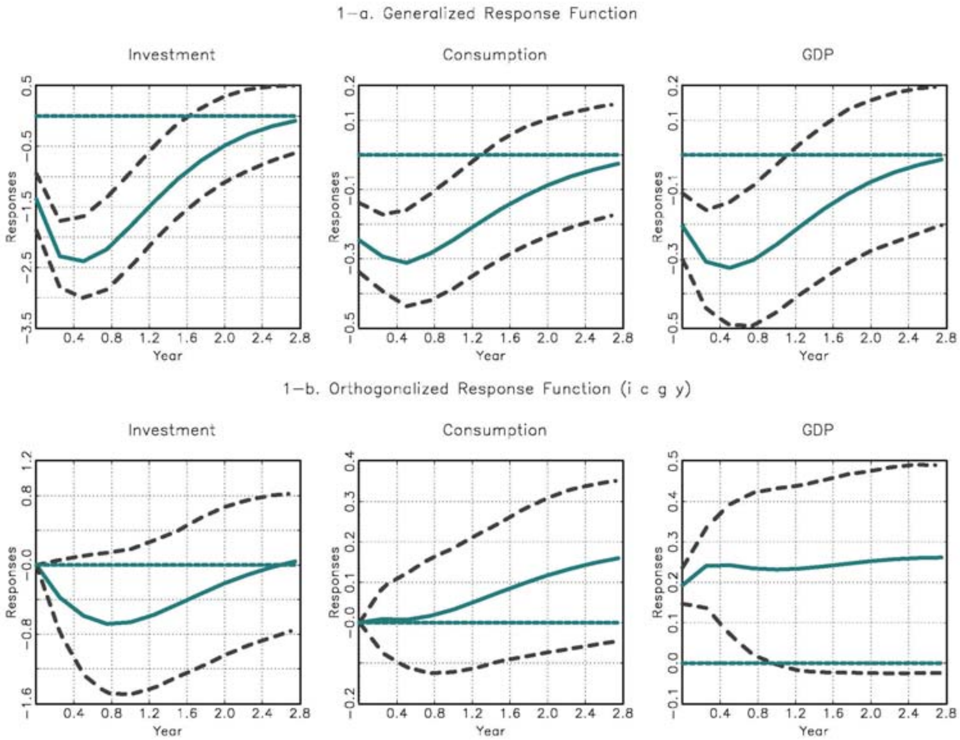
Firgue 1. Response Functions to a Government Share Shock



Note: Solid lines are point estimates. Dashed lines are 95% nonparametric confidence bands from 5,000 bootstrap simulations.

Firgue 2. Further Comparisons of Impulse Response Functions