Reliability of the Translog Cost Function: Some Theory & an Application to the Demand of Energy in French Manufacturing

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December 1995

Online at http://mpra.ub.uni-muenchen.de/53987/
MPRA Paper No. 53987, posted 8. March 2014 14:30 UTC
n° 9547

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Abstract

We investigate the behaviour of three versions of the translog cost function: the standard *Log-quadratic* version and two *nonlinear* versions with *exponential* and *linear* technical progress biases, respectively. Their respective performances are assessed according to three criteria: (1) on theoretical ground, (2) on their ability to provide plausible estimates of economic and technological characteristics, and finally, (3) on their ability to fit input shares, input-output ratios and unit cost. The most important result is that the standard form exhibits some weakness in fitting. We show via a series of experiments that those imprecisions are due to a lack of flexibility of the *Log-quadratic* form. Therefore, our results suggest that caveats should be attached to several previous studies.

**Keywords**: translog cost function, reliability of fit, flexibility.

Résumé

Nous étudions le comportement de trois versions de la fonction de coût translog: la version *log-quadratique* usuelle et deux autres versions *non linéaires* avec progrès technique *exponentiel* et *linéaire*, respectivement. Leurs performances sont appréciées selon trois critères: (1) leurs capacités à satisfaire les propriétés théoriques, (2) leurs capacités à fournir des estimations vraisemblables des caractéristiques techniques et économiques, et finalement, (3) la précision obtenue dans l’ajustement des parts, des ratios input-output, et du coût unitaire. Le résultat le plus frappant est que la forme usuelle révèle certaines faiblesses dans l’ajustement. Nous montrons, par une série d’expériences, que ces imprécisions sont dues à un manque de flexibilité de la forme *log-quadratique*. Nos résultats mettent aussi en question la fiabilité de nombreuses études antérieures.

**Mots-clés**: fonction de coût translog, précision de l’ajustement, flexibilité.
1 Introduction

The analysis of industrial factor demands, particularly energy, has been a popular subject to applied economists. The duality between production and cost functions provides a convenient framework to model the industrial production process and to investigate the substitution possibilities among factors for more than two-factor technologies. Following the dual approach, the early literature often adopts flexible functional forms to represent the underlying minimum-cost function. These functional forms are capable of providing local second-order approximations of any arbitrary cost function. They, present the advantage to not impose too many a priori restrictions on the various technological characteristics, e.g., the elasticity of substitution must not be equal to one (as in Cobb-Douglas) or constant (as in CES).\(^1\) On the other hand there remains the question of the extent to which the estimated flexible forms will reflect the cost minimizing behaviour over the range of observations. Still, the functional form that has gained the widest acceptance by far in empirical investigation is the translog, due to Christensen, Jorgenson and Lau (1973).\(^2\)

The choice of the translog form by analysts is justified by two principal reasons: (a) its logarithmic form "facilitates" the derivation of economic indicators (elasticities, returns to scale, productivity); (b) this specification is used as a benchmark since it is the most widely used in empirical investigations: this phenomenon is self-reinforcing since the researcher is strongly induced to compare directly its estimations to some reference group, based on the nature of the data, the approach (primal or dual) pursued, modelling characteristics and hypothesis concerning the adjustment of inputs.

Whereas, our comparative study of the sensitivity of the estimations to the functional form used casts some doubt on the reliability of the translog model. The precision of fit of translog (henceforth TL) may seems satisfactory if one limits

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\(^1\) See, for example, Diewert (1971a), Lau (1974), and Diewert and Wales (1987), for formal definitions about flexibility.

\(^2\) Berndt (1991) quote, as an example, from J. Johnston (1984): "A currently favored specification is the transcendental logarithmic (or translog) function. This is a very flexible form, capable of approximating a wide variety of functional forms."
oneself to scrutinize the relative shares, that is the dependent variables of the direct regression system, but it is no longer so if the unit cost or the input-output ratios are also scrutinized. Meanwhile, the results stemming from two other commonly used flexible forms (Generalized Leontief (GL) and symmetric Generalized McFadden (MF)) are satisfactory in terms of the goodness of fit, when the three preceding criteria are gathered.³

The shortcomings of TL in fitting factor's demand and unit cost, in comparison to GL and MF, seem independent to the assumptions about adjustments of inputs and to the type of the data. It reveals some weakness of the TL specification itself.⁴

Surprisingly, this problem has never been treated, not even signalled, to our knowledge, in the vast empirical literature based on the TL cost function. The contribution of our paper is twin: to reveal the weakness of this specification and, even more important, to locate the origin of these shortcomings, following two lines of research.

The first line of research consists in exploring some aspects of the modelling essentially linked to the logarithmich specification, which associates a multiplicative stochastic error to the estimated cost and imposes an exponential structure to the technical progress. We investigate two modelling characteristics: first, alternative stochastic specifications (additive or muliplicative); second, alternative specifications of the technical progress (exponential or linear). On this purpose, we develop two non-linear versions of the translog cost function with an exponential or additive technical progress biases.

The second line of research comes back to the parameterization used for the

³In my thesis, based on both partial and static equilibrium models, along with the translog, two commonly used flexible forms of the cost function have been experimented for examining the substitution possibilities between capital, labor and energy for the three industrial branches of French manufacturing: the Generalized Leontief form and the Symmetric Generalized McFadden.

⁴That is, the imprecisions of the translog model in the fitting has been confirmed for the three sets of data, first, following a static equilibrium approach. second, assuming a temporary equilibrium (when capital is assumed to be fixed in the short run). The three models have been all estimated by imposing linear homogeneity and symmetry of the Hessian matrix on the cost function. The hypothesis of constant returns to scale and a nonneutral technical change are also maintained for the estimation.
estimated model. It consists in exploring different degrees of flexibility of the standard log-quadratic version, by varying the constraints imposed on returns to scale and technical progress parameters.

Our econometric models have been fitted to annual data for the three industrial branches of French manufacturing: intermediate goods, equipement goods and consumer goods. The data contains informations on output level, on prices and quantities for three inputs: capital \((K)\), labor \((L)\) and energy \((E)\). The period of study runs from 1970 to 1989, so that the most important fluctuations of energy prices are included. This framework is based on a static equilibrium approach. Our choice is justified by two principal reasons: (1) it is the simplest and the more often used; (2) our comparative study of the impact of the specification to the quality of fit has reached the same conclusion for the two approaches (static or temporary equilibrium).

2 Variants of the translog Specification

In this set-up, we estimate the cost-minimizing demand of inputs in three versions: in terms of shares derived from the logarithm of the cost for \(TL\), that is the standard log-quadratic version (with a multiplicative error term and exponential technical progress), and in terms of input-output ratios derived from the cost, for the two non-linear versions (with an additive error term and exponential or linear technical progress). Constant returns to scale in addition to homotheticity is imposed on the three estimated versions \((TL, TLE, TLA)\).

The first non-linear cost function, denoted by \('TLE'\), has an additive error term and an exponential technical progress. It is from an analytical point of view strictly equivalent to the usual log-quadratic function, the difference laying in the stochastic specification. As in the \(TL\) form, the technical change parameters with respect to input prices sum to zero.\(^5\) This version has, consequently, the same degree of

\(^5\)Imposing this kind of restrictions on the parameters of the input share equations, including those for the technical progress, is unavoidable for assuring linear homogeneity of the \(TL\) cost function.
flexibility as the standard version $T_L$.

The second non-linear cost function, denoted by $T_{LA}$, has an additive error term and a linear technical progress affecting independently each input. The interest of this modified translog is that the restrictions assuring homogeneity do not affect the technical change parameters. This specification is indeed more flexible than $T_L$ and $T_{LE}$ (both are with exponential technical progress). We indicate in table 1 the number of estimated parameters for each specification. This allows us to compare their degrees of flexibility.\(^6\)

The first interest of these two non-linear versions of translog is that the derived input demand are expressed in terms of input-output ratios. The second interest comes from the fact that they make us able to dissociate the effect of stochastic specification (comparison between $T_L$ and $T_{LE}$) from the pure effects of technical change modelling (comparison between $T_{LE}$ and $T_{LA}$) on the reliability of estimates.

The performances of our three models, in approximating the true technology, are evaluated with three principal criteria: (1) from a theoretical point of view, by testing if the regularity conditions which validate the interpretation of the technology by means of a cost function; then, (2) from an economic point of view, by examining values, derivatives and temporal evolutions of price elasticities and Allen substitution elasticities, and finally (3) from a statistical point of view, by comparing the quality of fitting input shares, input-output ratios and unit cost.

### 2.1 Regularity Conditions

It is necessary that any demand system able to serve as a simulation tool satisfies the constraints of theory. Linear homogeneity implies that demand is determined by relative prices and not nominal prices. The symmetry of the parameters guarantees the symmetry of the Hessian matrix. Monotonicity and concavity in the prices derive from the assumption that costs are minimized.

\(^6\)Flexibility here is meant in the sense of Diewert. It is measured in terms of the number of free parameters in the model. See Diewert and Wales (1987).
**Homogeneity and Symmetry**  Homogeneity and symmetry constraints are imposed for the estimation procedure. As these two conditions, in particular homogeneity, are seldom checked in empirical studies, we test the symmetry of the parameters for a clarification. The likelihood ratio statistics for testing the symmetry restrictions for the three versions of translog are given in table 2. Assuming homogeneity, the symmetry restrictions are acceptable in $TL$: with a 5% threshold for intermediate goods and consumer goods and only with a 1% threshold for equipment goods. In $TLE$, this hypothesis is acceptable with a 5% threshold for the three branches of French manufacturing. In contrast, this property is more fragile with $TLA$: acceptable for a 1% threshold only for consumer goods, it is rejected under the usual criteria for intermediate goods and equipment goods. We note that even if the symmetry conditions are not satisfied, this has little consequence for the remainder of the paper since we impose this condition *a priori* in all the estimations (T. J. Wales 1977).

**Monotonicity and Concavity**  The imposition of monotonicity and concavity constraints is often very difficult to implement and limits the flexibility of the translog cost function. We limit here ourselves to the verification of these properties on each point of the sample.

Under the homogeneity and symmetry constraints, monotonicity is well verified for the three versions of translog: the relative shares of the cost for $TL$ and the input-output ratios for the non-linear versions ($TLE$ and $TLA$) are positive over all the range of data.

Table 3, which counts the number of points fulfilling the concavity condition, suggests that this property is almost not verified on our data for the three versions of translog. Nevertheless, we note in all cases an improvement when the two non-linear versions are used.

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7The imposition of homogeneity and symmetry conditions is very easy in practice.
8We have tested linear homogeneity on our data with the Cobb-Douglas cost function. The results show that this hypothesis is clearly rejected.
9In our experience with $GL$ and $MF$, concavity is verified throughout the historic period for equipment and consumer goods, but is rejected all the same for intermediate goods.
2.2 Economic Performances

The point that deserves examination, for a comparative analysis of the economic performances of the three versions of translog, is the coherence between the three types of estimations in terms of economic implications. This comparison is greatly facilitated by the evaluation of the elasticities of demand and of substitution derived from each of the three models. The analysis is done in three steps. We examine first their levels at the sample mean point, then their derivatives, and finally, for a deeper understanding, we focus our attention on their evolutions over the historic period.

Elasticities at the sample mean point  Tables 4, 5 and 6 report the price elasticities of demand and partial elasticities of substitution derived from the three models ($TL$, $TLE$, $TLA$) and estimated for the three branches of French manufacturing (intermediate goods, equipment goods and consumer goods).

At the sample mean point, the own-price elasticities of demand are all negative: concavity is well respected for the three models in this point (which is not observed), though it is only partially respected over the sample itself (see table 3).\textsuperscript{10}

For equipment goods (table 5) and consumer goods (table 6), the elasticities calculated from the estimated parameters of the two non-linear versions ($TLE$ and $TLA$) keep the same sign as those evaluated with the standard version ($TL$). These elasticities are all, with the exception of the capital-energy elasticity, statistically significant. We systematically observe substitutability between all the inputs for equipment goods, while complementary between capital and energy emerges for consumer goods. Still, the elasticity of substitution between these two inputs ($\sigma_{KE}$) is little informative: significant with $TLA$ only for the equipment goods (table 5) and with $TL$ and $TLE$ only for consumer goods (table 6).

\textsuperscript{10}From an economic viewpoint the own price elasticity of demand is of vital importance. It gives a measure of input's conservation proportionate to an increase in that input's price, when output remains constant. The greater the substitutability of other factors for the input in question, the greater the opportunity for resource conservation ($\kappa_i = -\sum_{j \neq i} \kappa_{ij}$).
As for intermediate goods (table 4), a branch which is a great consumer of energy, the elasticities obtained by means of the three types of estimations (TL, TLE, TLA) reveal more clearly the sensitivity of the estimates to the alternative modelling aspects. Though the price-elasticities estimated using TL are almost all statistically significant, the elasticities related to energy (ke, lE and eE) are, in general, not significant for the non-linear versions TLE and TLA. The relation between energy and the other factors of production seems to be ambiguous in this branch. We conclude to a weak separability between energy and the two other factors: capital and labor. This result does not seem to be offensive if one takes into account the nature of the allocation of resources which is at the origin of the "autonomy" of energy in this branch. The relative cost share of energy in the intermediate goods is indeed neatly above its share in the other branches.

Though the elasticities from TLE and TL have, on the three data sets, matching signs (they predict the same evolution of the productive combination), the values are greater for TLE than for TL and TLA. Thus the structure of the estimated technology appears slightly more flexible when one employs the non-linear version with an exponential technical progress (TLE). This phenomenon is particularly striking for the consumer goods. Moreover, capital is more affected than the other inputs.

On the other hand, the results derived from TLA contradicts, in certain cases, the results derived from TL concerning the signs of the elasticities. More precisely, it concerns the elasticities of substitution capital-energy and labor-energy evaluated for intermediate goods. In the meanwhile, these results obtained with TLA are in accordance with our previous results obtained with MF.

To summarize, the stochastic specification of the model (multiplicative or additive error term associated to the cost) has limited consequences on the sign of the elasticities. But, the results show sensitivity to the modelling of the technical progress (comparison between TLE and TLA). For these reasons, combining the two effects (stochastic specification effect and technical progress effect: comparison between TL and TLA), leads to contradictions as far as technical relations between capital and the other inputs for intermediate goods are concerned (signs of
the elasticities of substitution capital-energy and labor-energy).

Variations of the Elasticities When one limits one's scope to the levels of the elasticities evaluated at the approximation point, the results of the three types of estimations seems agreeing, at least for equipment goods and consumer goods. But the results reveals an inconsistency if not a contradiction between the estimates for the intermediate goods. In particular, only the elasticities derived from the non-linear versions show some sort of disconnection between energy and the other inputs and point out the autonomy of this input.

The elasticities are calculated with the first and second derivatives of the cost. Thus they are unable to inform us on the evolution of the technological relations between factors in face of exogenous shocks on the prices.\(^{11}\) This analysis is rendered possible by the evaluation of the derivatives of the elasticities relatively to the input prices. We make use of the third derivatives of the cost function.

We put in table 7 the levels and variations of the price-elasticities relative to energy (capital-energy, labor-energy and energy-energy) at the 1980 point, for the consumer goods, basing the calculations on the first, second and third order derivatives of the cost function. The sense of variation is thus determined.\(^{12}\)

The values and the derivatives of the elasticities are, for the three types of estimations, all statistically significant at a 5% confidence level. In addition they have the same signs and, except for two, the same magnitude.

The own price elasticities of energy for the three forms are very sensitive to the price variation of this input. This elasticity appears particularly sensitive to the price of labor with TLA. The variations of the capital-energy elasticity suggest that the complementarity between capital and energy becomes larger after an increase

\(^{11}\) In terms of substitutability and complementarity.

\(^{12}\) At the approximation point, the elasticities are determined as functions of the exogenous variables which correspond to the average logarithm of the prices in TL. Thus the analysis of their variations relatively to the prices of the inputs at this point does not allow a correct interpretation of the results. We have arbitrarily chosen the 1980 point which corresponds to the reference year of the sample : the input prices are all unitary.
in the price of labor. Conversely, these two inputs tend to be more substitutable when their prices are increased. Though the relation of substitution between labor and energy seems the less sensitive to exogenous shocks on the prices of inputs, it is magnified after the increase of the energy price. The curve of $\epsilon_{LE}$ jumps in 1973–74 and 1979–80 as a consequence of the two Oil Shocks.

To sum up, the derivatives of the elasticities estimated from the three versions of translog suggest that the variations of the elasticities (given by the sign of the derivatives) is unsensitive to the structure of the technical progress and/or to the nature of the error term associated with the cost. On the other hand, the levels of the derivatives of the elasticities are larger when one makes use of the version with linear technical progress ($TLA$). The demands of these inputs appear more variable relatively to the prices when variants of the stochastic specification of the model and the structure of technical progress are combined.

**Temporal Evolutions of the Elasticities** For representing the sensitiveness of the estimates to the stochastic specification of the model and to the structure of technical progress, we have drawn on the same graphics the three curves corresponding to the three versions of translog for the same elasticity. We limit ourselves to the analysis of elasticities involved in the evolution of the productive combination as a response to the variation of the price of energy.

Figure 1, 2 and 3 show the evolutions of, respectively, the capital-energy, labor-energy and energy-energy price-elasticities, as derived from $TL, TLE$ and $TLA$, for the consumer goods branch.

At the first sight, one can observe that the curves associated to the two non-linear versions $TLE$ and $TLA$ keep the same shape as those estimated from the usual log-quadratic. In terms of level, the spread between them decreases from 1980 onwards. In particular, the labor-energy elasticity (figure 2) and energy-energy elasticity (figure 3) estimated with $TL$ and $TLA$ cross at this point.

Anyway, these elasticities reveal some temporal variability concerning the extent to which inputs are substitutable or complementary and a neat variability for the
own-price elasticity of energy. However, they keep the same sign throughout the period under study, at least for consumer goods detailed here.\textsuperscript{13}

The variations of the elasticity levels are more important for TLA (non-linear with linear technical progress). It appears, again, that the values taken by the elasticities are more sensitive to the specification of the technical progress than to the stochastic specification of the model.

In addition, for the three elasticities considered here, which measure the variations of the levels of the demand of each input as a function of the price of energy, we note patent reactions to the Oil Shocks. The shocks on the energy price will therefore amplify the reactions of the productive combination without modifying the structure of the technological relations between factors (inversion of the sign), at least for the consumer goods branch.

Quality of Fit After this detailed study of the sensitivity of the estimates to the version of translog, first, in terms of mathematical performances (by testing the basic theoretical properties), then in terms of economic performances (thorough a minutely analysis of price and Allen substitution elasticities), we address now the core of the problem: is the quality of fit improved when one uses one or the other non-linear form?

In the absence of an appropriate statistical test, the impact on the quality of fit of the model implemented is assessed by the comparison between the observed values and the fitted values of shares, input-output ratios and unit cost of the three specifications. On this purpose, we draw them on the same graphics. Figures 4, 5 and 6 illustrates the apparent influence of the stochastic specification of the model, as well as the modelling of the technical progress.

Observation shows that the fit of the cost share of the energy input in the branch of consumer goods (figure 4) seems of good quality in the TL case, as well as in the TLA case. In the meanwhile, the results are less satisfactory for TLE: the

\textsuperscript{13}On the other hand, for the intermediate goods branch, variations in magnitudes are accompanied with variations in signs.
share of energy is overestimated before 1979, while it is underestimated afterwards. This result is surprising since the quality of the fit poses no particular problem for the cost share, even when the estimated demand system was expressed in terms of input-output ratios (cases TLA, GL and MF). For this reason, specifying an exponential technical progress and an additive error term associated to the cost, is not, in our opinion, satisfactory.

Figure 5 shows the evolution of the energy-output ratio for the same industry and reveals the bad performance of TL. The improvement of the results observed for TLE seems insignificant, while the fit is obviously better for TLA.

The same conclusion comes out of figure 6. Once again, the quality of fit seems unsensitive to the sole stochastic specification: it is disappointing either for TL and TLE. In contrast, the use of TLA greatly improves the results.

We conclude that the little gain obtained by TLE in the fitting of input-output ratios is counterbalanced by a comparable loss in the fitting of shares. In the three graphics, the curve obtained by TLA reproduces more closely the observed values. This results indicates that the exponential structure of the technical progress is at the origin of the shortcomings. Our analysis supports the idea that the estimation of the demand system in terms of input-output ratios associated with an exponential specification for the technical progress does not resolve the problem.

3 Extensions on the Flexibility Regarding the Technical Progress and Concluding Remarks

We complete now our search of the origin of the shortcomings of translog by reconsidering the parameterization used for the estimation of the model. We explore in this section different degrees of flexibility of the usual log-quadratic form, by varying the restrictions imposed to the parameter of the technical progress. We study in particular the gain in precision obtained when the degree of flexibility relatively to the temporal trend is increased.
Remember that the cost function we have estimated till now is "flexible" up to the second order only for the input prices, while it is "semi-flexible" for the temporal trend and "non-flexible" for the level of the output. The semi-flexibility relatively to the temporal trend comes from the non-existence of first order and second order autonomous terms for the technical progress.\textsuperscript{14} The non-flexibility relatively to the level of the output comes the constant returns to scale. They imply homogeneity of degree zero, for the level of output, of the unit cost and of the derived demand function.

Under these two constraints, it is possible, in the translog case, to identify all the parameters of the cost function from the input demand system. It is to be stressed that in the usual version of the translog, the only identifiable parameters from the derived demand system are those in interaction with the logarithm of the prices. In addition, for estimating or testing restrictions on the returns to scale and on the autonomous technical progress, the cost equation must be incorporated to the regression system. Unfortunately, most empirical studies are based on the estimation of the system of shares.

It happens that the usual translog specification generates a system of demand in terms of the relative shares of the cost. The property of additivity of the shares reflects the homogeneity in the price of the cost function and implies supplementary restrictions on the parameters associated to the exogenous variables interacting with the prices of the inputs, the parameters of the technical progress included. Thus, the translog model is, by construction, less flexible (with less parameters) than the non-logarithmic model GL and MF.

This is apparent in the lack of precision in the fitting of the input-output ratios and of the unit cost.\textsuperscript{15}

\textsuperscript{14}See Diewert and Wales (1987 and 1992) for more details about the notions of flexibility and semi-flexibility.

\textsuperscript{15}The unit cost function is a linear combination of the endogenous variables of the systems of demand GL and MF expressed in terms of input-output ratios. In contrast, it is impossible to recalculate directly the unit cost from the system of demand of TL, expressed in terms of relative shares of the cost.
temporal trend appearing in each equation of demand sum to zero. For this reason, it is impossible to estimate a technical progress that affects independently each input in the translog case. This lack of flexibility apparently affects the quality of the estimates. As all the trend parameters appear in the cost function, the effect of the temporal trend will not be fully taken into account, a fact that can explain the bad fits of the cost and, consequently, of the input-output ratios: passing from the relative share to an input-output ratio (TL) or the converse (GL, MF), necessitates the use of the cost recalculated from the estimated parameters. The bad fit of the observed input-output ratio observed for translog can came from a bad fit of the cost.

The question at hand is the following: what is the impact of the addition to the cost function of autonomous parameters for the technical progress on the precision of the usual translog in the fitting of input-output ratios and the unit cost?

We estimate different models by imposing or relaxing restrictions to the parameters associated to the temporal variable. The three estimated models are: (1) a model with constant returns to scale and without technical progress (TL0), (2) a model with constant returns to scale with a technical progress interacting with prices only (the reference model TL), (3) model (2) with relaxing the restriction imposed to the technical progress autonomous parameter of order 1 (TL1).

We limit ourselves to the examination of the evolutions on the sampling period of input-output ratios and of the unit cost derived from the three preceding models. The corresponding curves are given in figure 7, 8 and 9, respectively.

A look suffices to be convinced that the quality of fit of the unit cost and of the input-output ratio of energy is neatly improved when the first order autonomous term is incorporated. In constrast, the fitting is of poor quality in the other two cases.\textsuperscript{16}

The major purpose of this study is to determine the sensivity of empirical results

\textsuperscript{16}Though the parameter estimated for the autonomous technical progress of order 2 is always statistically significant, it has no effect on the precision of translog when the first order autonomous parameter is estimated. A second experience relaxing the restriction on the returns to scale has been tempted. The gain in precision is modest.
and the reliability of fit to the alternative modelling aspects when estimations are based on a translog cost function: (1) alternative stochastic and technical change specification by means of two non-linear versions developed here; (2) the problem of properly specifying the parameter structure in estimating procedure by means of the standard version.

Our results show that the stochastic specification of the model and the restrictions on the returns to scale play indeed a minor role in the quality of fit. Contrarily, the degree of flexibility of the estimated cost function, relatively to the temporal variable (technical change), plays a significant role in the precision of the model, in particular through the autonomous parameters. This is confirmed by the results found when one uses two non-linear versions.
Appendix 1: Theory

1 The true technology

Let us consider a firm (or a sector) producing a single output \( y \geq 0 \) from \( n \) dimensional vector of variable inputs \( x = (x_1, ..., x_n)^T \geq 0 \), according to a technology represented by a well-behaved production function \( y = f(x, t) \), where we introduce a time trend \( t \) as a proxy to technological change.\(^{17}\) Denote by \( p = (p_1, ..., p_n) \) the vector of input prices. Given that firm (or sector) is price-taker, Shephard duality theorem states that technology may be equivalently represented by a dual cost function \( C(p, y, t) \) relevant to cost minimizing behaviour in competitive input markets

\[
C(p, y, t) = \min_{x \geq 0} \left\{ \sum_{i=1}^{n} p_i x_i \mid f(x, t) \geq y \right\}
\]

(1)

Standard duality properties establish theoretical regularity conditions on the resulting minimum cost function \( C(.) \) inherited from the well-behaved production function \( f(.) \). For our purpose, the most important conditions are: \( C \) must be positive real-valued and monotonous non-decreasing in \( p \) and \( y \); linear homogeneous, symmetric with respect to second partial derivatives, and concave in \( p \).\(^{18}\) Let \( C(.) \) be twice continuously differentiable with respect to its arguments. These regularity conditions may be conveniently expressed in terms of the cost function derivatives given in table a.

We also use a result due to Hotelling (1932), Hicks (1946), Samuelson (1947), and Shephard (1953): the cost minimizing input demand for the \( i \)th factor can be obtained simply by partially differentiating the cost function with respect to the \( i \)th factor price, provided that the cost function satisfies regularity conditions of the Shephard duality theorem

\[
x_i(p, y, t) = \frac{\partial C}{\partial p_i}, \quad \forall i \quad \text{(Shephard lemma)}
\]

(2)

\(^{17}\)The production function \( f(x, t) \) is said well-behaved if it exhibits some desired theoretical regularity conditions. The most important are: for all \( x > 0 \), \( f(x, t) \) is positive, twice continuously differentiable, monotonic increasing, and concave. Under those conditions, the maximal output which the firm (or sector) can produce, at one period of time, from the input bundle \( x \) is assumed to be given by \( y = f(x, t) \), where \( f \) summarizes the technological possibilities open to the firm.

\(^{18}\)See Shephard (1953) or Diewert (1971) for the duality properties between production and cost functions.
Table a: Theoretical consistency properties of the cost function

<table>
<thead>
<tr>
<th>1. Domain.</th>
<th>$C(p, y, t)$ is a positive real-valued function.</th>
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<tbody>
<tr>
<td></td>
<td>$C(p, y, t) \geq 0, \forall \ p, y &gt; 0$ and $C(p, 0, t) = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1. Monotonicity.</th>
<th>$C(p, y, t)$ is non-decreasing in $p$ and $y$.</th>
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<tbody>
<tr>
<td></td>
<td>$\frac{\partial C}{\partial p_i} \geq 0$ or $x_i \geq 0, \forall \ i$ and $\frac{\partial C}{\partial y} \geq 0$</td>
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</table>

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<tr>
<th>2. Homogeneity.</th>
<th>$C(p, y, t)$ is linear homogeneous in $p$. Hence Euler Theorem yields</th>
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<tr>
<td></td>
<td>$\sum_{i=1}^{n} p_i \frac{\partial C}{\partial p_i} = C$ (2.a), $\sum_{i=1}^{n} p_i \frac{\partial^2 C}{\partial p_i \partial p_j} = 0, \forall \ j$ (2.b)</td>
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<tr>
<td></td>
<td>$\sum_{i=1}^{n} p_i \frac{\partial^2 C}{\partial p_i \partial y} = \frac{\partial C}{\partial y}$ (2.c), $\sum_{i=1}^{n} p_i \frac{\partial^2 C}{\partial p_i \partial t} = \frac{\partial C}{\partial t}$ (2.d)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. Symmetry.</th>
<th>$C(p, y, t)$ is twice continuous differentiable, Young theorem implies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{\partial^2 C}{\partial p_i \partial p_j} = \frac{\partial^2 C}{\partial p_j \partial p_i}$ or $\frac{\partial x_i}{\partial p_i} = \frac{\partial x_j}{\partial p_j}, \forall \ i, j$</td>
</tr>
<tr>
<td></td>
<td>i.e. that the Hessian matrix must be symmetric.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. Concavity.</th>
<th>$C(p, y, t)$ is concave in $p$ if</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\left[\frac{\partial^2 C}{\partial p_i \partial p_j} \right]_{i,j}$ is a negative semidefinite matrix,</td>
</tr>
<tr>
<td></td>
<td>i.e. that the Hessian matrix must be negative semidefinite.</td>
</tr>
</tbody>
</table>
Usually, at a point of cost minimization, the direct structure of cost-minimizing input demands can be expressed in terms of input-output ratios (denoted by \(a_i\)) derived from the cost or in terms of cost shares (denoted by \(S_i\)) derived from the logarithm of cost.

1. **Simple case**: \(C(p_1, p_2, \cdots, p_n, y, t)\)
   \[
a_i = \frac{1}{y} \frac{\partial C}{\partial p_i} = \frac{z_i}{y}
   \quad (3)
   \]

2. **Logarithmic case**: \(\log C(\log p_1, \log p_2, \cdots, \log p_n, \log y, t)\)
   \[
   S_i = \frac{\partial \log C}{\partial \log p_i} = \frac{p_i z_i}{C}
   \quad (4)
   \]

For an empirical investigation, a structural model is needed to specify the dual cost function.

## 2 The functional forms

In this paper we make an experiment with three versions of the translog cost function. The **standard log-quadratic version** (TL), due to Christensen, Jorgensen and Lau (1973), and two **nonlinear** versions (TLE and TLA) developed here. Along with the translog, two other specifications are invoked: the generalized Leontief (GL) due to Diewert (1971) and the symmetric generalized McFadden (MF) due to McFadden (1978), and Diewert and Wales (1987). The three standard structural models considered (TL, GL, MF) belong to the generalized quadratic family of flexible functional forms.\(^\text{19}\)

### 2.1 The translog cost function

**Standard version (TL)** The analytic expression of the standard version of the translog form is *Log-quadratic*. It associates however a *multiplicative* stochastic error with the estimated cost function and imposes an *exponential* structure on the technical progress.

\(^{19}\)A functional form is flexible if it provides a second order differential approximation to an arbitrary twice continuously differentiable function at any point in an admissible domain. For variant definitions of the flexibility property, see Barnett (1983), Lau (1986), and Diewert and Wales (1987). Diewert and Wales (1987), measure of flexibility is given in terms of the number of free parameters present in the structural model.
Its general formula allowing for non-neutral technical change and non-homotheticity is given by

\[
\log C(p, y, t)_{TL} = a_0 + \sum_{i=1}^{n} a_i \log p_i + a_y \log y + a_t t + 0.5 \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \log p_i \log p_j \\
+ \sum_{i=1}^{n} a_{iy} \log y \log p_i + \sum_{i=1}^{n} a_{it} \log p_i + 0.5a_{yy} \log y^2 \\
+ a_{yt} \log y + 0.5a_{tt} t^2, \quad a_{ij} = a_{ji}, \quad \forall \ i, j
\]  

(5)

where \(a_0, a_i, a_y, a_t, a_{ij}, a_{iy}, a_{it}, a_{yy}, a_{yt}, \) and \(a_{tt}\) are parameters to be estimated, while the symmetry of the Hessian matrix require the symmetry of the second order parameters. Note that the translog cost function defined by (5) has \(n(n + 1)/2 + 3n + 6\) parameters.

Optimal demands for inputs are obtained in terms of cost shares by logarithmic differentiation of the cost function

\[
S_i = a_i + \sum_{j=1}^{n} a_{ij} \log p_j + a_{iy} y + a_{it} t
\]  

(6)

Since, the cost function must be linear homogeneous in input prices, the cost shares must sum to unity

\[
\sum_{i=1}^{n} S_i = 1
\]

(7)

Hence, linear homogeneity in input prices yields \((n + 3)\) adding-up restrictions relating the parameters of cost share equations

\[
\sum_{i=1}^{n} a_i = 1, \quad \sum_{i=1}^{n} a_{ij} = 0, \quad \forall \ j \\
\sum_{i=1}^{n} a_{iy} = 0, \quad \sum_{i=1}^{n} a_{it} = 0
\]

(8)

However the homogeneity properties reduce the number of free parameters to \(n(n + 1)/2 + 2n + 3\).
The degree of returns to scale is measured by the inverse of the cost flexibility, defined as the elasticity of cost with respect to output

\[ r = \frac{1}{\epsilon_C y} \]

where

\[ \epsilon_C y = \alpha_y + \sum_{i=1}^{n} \alpha_{iy} \log p_i + \alpha_{yy} \log y + \alpha_{yt} t \]  \hfill (9)

The cost function would be homothetic if it could be written as a separable function of output and factor prices. Thus the following \((n - 1)\) additional parameter restrictions \(\alpha_{iy} = 0\) (\(\forall i\)) must be added to impose homotheticity on the translog cost function.

Constant returns to scale occurs, so the dual cost function is linear homogeneous in output \(y\), when the following \((n + 2)\) additional linear restrictions are satisfied by the parameters

\[ 1 - \alpha_y = \alpha_{yt} = \alpha_{yy} = \alpha_{iy} = 0, \forall i \]  \hfill (10)

Hence, by imposing the above restrictions on the parameters, the translog unit cost function takes the simpler form

\[ \log C(p, t)_{TL} = \alpha_0 + \sum_{i=1}^{n} \alpha_i \log p_i + \alpha_{it} t + 0.5 \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} \log p_i \log p_j \]
\[ + \sum_{i=1}^{n} \alpha_{it} t \log p_i + 0.5 \alpha_{tt} t^2 \]  \hfill (11)

However the constant returns to scale property reduce the number of free parameters to \((n + 2)(n + 1)/2\).

The rate of technical change is conceptually the rate of reduction in the unit cost when factor prices are constant. For the translog unit cost function, the negative of the rate of technical change takes the following form

\[ \frac{\partial \log C(p, t)}{\partial t} = \alpha_t + \sum_{i=1}^{n} \alpha_{it} \log p_i + 0.5 \alpha_{tt} t^2 \]  \hfill (12)
The technical change is said to be unbiased (or share neutral) if it leaves relative cost shares undistributed. So under constant returns to scale, we need only to impose \((n - 1)\) additional restrictions

\[
\alpha_{it} = 0, \quad \forall \ i
\]  

(13)

It occurs with a constant rate if \(\alpha_{it} = 0\), where the first order neutral component \(\alpha_t\) shows the autonomous reduction in unit cost in response to improved technology.

The central purpose of estimating the parameters of a flexible cost function is to estimate the technical input substitutability. The most widely used measure of input substitutability is the partial Allen-Uzawa elasticities of substitution, which can be obtained from the first and second order derivatives of the cost function (Uzawa 1962)

\[
\sigma_{ij} = \frac{CC_{ij}}{C_i^2}, \quad \text{where} \quad \begin{cases} 
C_i = \frac{\partial C}{\partial p_i} \\
C_{ij} = \frac{\partial^2 C}{\partial p_i \partial p_j}
\end{cases}
\]  

(14)

For the translog cost function these elasticities turn out to be equal to

\[
\sigma_{ij} = \frac{\alpha_{ij} + S_i S_j}{S_i S_j}, \quad \forall \ i \neq j
\]  

(15)

Since the price elasticities of demand are employed to evaluate the direct price effects on the change in factors use

\[
\epsilon_{ij} = \frac{\partial \log x_i}{\partial \log p_j}, \quad \forall \ i, j
\]  

(16)

It follows that for the translog cost function, these elasticities can be expressed as

\[
\epsilon_{ij} = \frac{\alpha_{ij} - \delta_{ij} S_i + S_i S_j}{S_i}, \quad \text{where} \quad \delta_{ij} = \begin{cases} 
1 & \text{for } i = j \\
0 & \text{for } i \neq j
\end{cases}
\]  

(17)

From an economic point of view, the price elasticities are of vital importance. The cross elasticities indicate which inputs are substitutes or complements of each other.
A positive (negative) value of \( \epsilon_{ij} \) indicate that inputs \( i \) and \( j \) are substitute (complement). Furthermore, if \( \epsilon_{ij} = 0 \), inputs \( i \) and \( j \) are independent. The own elasticities of demand measure the change in factor use in response to its own price, when output and the other factor prices remains constant. Finally, stable cost minimizing input demands require that each \( \epsilon_{ii} \) be nonpositive.

Before presenting the two other versions of the translog model, we give some remarks about the analytical structure of the standard version \( TL \)

1. It is not possible to identify all the characteristics of the underlying technology from the system of cost share equations: the parameter \( \alpha_0 \), and furthermore the first and the second order autonomous parameters for returns to scale \( (\alpha_y, \alpha_{yy}) \) and those for technical change \( (\alpha_t, \alpha_{tt}) \) don't appear in the share equations.

2. The parameters \( \alpha_{ii} \) can be interpreted as constant share biases of technical change. For the translog cost function, these parameters must sum to zero \( (\sum_{i=1}^{n} \alpha_{ii} = 0) \). This means that the bias of technical change with respect to the price of one input are restricted to be the negative of the sum of biases technical change with respect to the prices of the other inputs \( (\alpha_{ii} = -\sum_{j \neq i} \alpha_{ij}) \). So that, we can rule out the possibility of either positivity or negativity for all the \( \alpha_{ii} \). This structure casts some doubt in the reliability of those parameters to obtain a good interpretation, and this difficulty increases with the number of inputs considered.

3. The estimated share elasticities describes the implications of patterns of substitutions among inputs for relative resources allocation. The first order parameters \( \alpha_i \) can be interpreted as average value shares of each input. The second order parameters \( \alpha_{ij} \) can be interpreted as constant share elasticities with respect to input prices. This property is in fact specific to the translog model. Therefore, positive share elasticities imply that the value shares increase in input prices. However, the share of each input should be nonincreasing in the price of the input itself. Then, the parameters \( \alpha_{ii} \) should be nonpositive.

The first "non-linear" version (TLE) The first nonlinear version of the translog model is from an analytical point of view strictly equivalent to the standard Log-quadratic version, the difference laying in the stochastic specification. Its general formula allowing for non-neutral technical change and non-homotheticity is given by

\[
C(p, y, t)_{TLE} = \alpha_0 \cdot \exp \{ \alpha_t t + \alpha_{tt} t^2 \} \cdot y \left( \alpha_y + 0.5 \alpha_{yy} \log y + \alpha_{yt} t \right) \\
\times \prod_{i=1}^{n} p_i \left( \alpha_t + 0.5 \sum_{j=1}^{n} \alpha_{ij} \log p_j + \alpha_{iy} \log y + \alpha_{it} t \right)
\]

where \( \alpha_{ij} = \alpha_{ji}, \ \forall \ i, \ j. \)
All the restrictions assuring linear homogeneity in input prices for the standard version of the translog cost function still available for this version. So, the two versions with exponential technical progress (TL and TLE) have the same degree of flexibility.

Assuming constant returns to scale, unit cost is given by

\[ C(p,t)_\text{TLE} = \alpha_0 \cdot \exp\{\alpha_t t + \alpha_{tt} t^2\} \cdot \prod_{i=1}^{n} p_i \left(\alpha_i + 0.5 \sum_{j=1}^{n} \alpha_{ij} \log p_j + \alpha_{it} t\right) \]  \hspace{1cm} (19)

Optimal demands for inputs are derived in terms of input-output ratios.

\[ a_i(p,t)_\text{TLE} = \left(\frac{\alpha_0}{p_i}\right) \cdot \exp\{\alpha_t t + \alpha_{tt} t^2\} \cdot \left(\alpha_i + \sum_{j=1}^{n} \alpha_{ij} \log p_j + \alpha_{it} t\right) \prod_{i=1}^{n} p_i \left(\alpha_i + 0.5 \sum_{j=1}^{n} \alpha_{ij} \log p_j + \alpha_{it} t\right) \]  \hspace{1cm} (20)

The specification of the cost-minimizing demand in terms of input-output ratios allows however to determine all the parameters present in the cost function from the derived factor demands.

The second "non-linear" version (TLA) With this second nonlinear version of the translog cost function (TLA), stochastic specification and technical progress modelling are simultaneously altered, in comparison with the standard version TL.

Its general formula allowing for non-neutral technical change and non-homotheticity is given by

\[ C(p,y,t)_\text{TLA} = \alpha_0 \cdot \exp\{\alpha_t t + \alpha_{tt} t^2\} \cdot y(\alpha_y + 0.5\alpha_{yy} \log y + \alpha_{yt} t) \prod_{i=1}^{n} p_i \left(\alpha_i + 0.5 \sum_{j=1}^{n} \alpha_{ij} \log p_j \right) + \sum_{i=1}^{n} \lambda_{iy} p_i \log y + \sum_{i=1}^{n} \lambda_{it} p_i t \]  \hspace{1cm} (21)

where \( \alpha_{ij} = \alpha_{ji}, \forall i,j. \)
Homogeneity in input prices yields the following \((n + 1)\) adding-up restrictions on the parameters

\[
\sum_{i=1}^{n} \alpha_i = 1, \quad \sum_{j=1}^{n} \alpha_{ij} = 0, \quad \forall \ i
\]

(22)

However, with this version of translog, the linear homogeneity conditions do not impose additive restrictions on the parameters allowing for non-homotheticity \((\lambda_{iy})\) and on technical progress biases parameters \((\lambda_{it})\). So, the cost function defined by (21) and (22) has \(n(n + 1)/2 + 2n + 5\) free parameters. Then, the specification with linear technical progress \((TLA)\) has more parameters than the two other specifications with exponential technical progress \((TL\) and \(TLE)\). Hence, \((TLA)\) is more flexible than \(TL\) and \(TLE\).

Assuming constant returns to scale, the unit cost formula is equivalent to

\[
C(p,t)_{TLA} = a_0 \cdot \exp \{\alpha_t t + \alpha_{it} t^2\} \cdot \prod_{i=1}^{n} p_i \cdot (\alpha_i + 0.5 \sum_{j=1}^{n} \alpha_{ij} \log p_j)
+ \sum_{i=1}^{n} \lambda_{it} p_i t
\]

(23)

However, the constant returns to scale property reduce the number of free parameters to \((n + 2)(n + 1)/2 + 1\).

Demands for inputs are derived in terms of input-output ratios

\[
a_i = \frac{a_0}{p_i} \cdot \exp \{\alpha_t t + \alpha_{it} t^2\} \cdot (\alpha_i + \sum_{j=1}^{n} \alpha_{ij} \log p_j)
\prod_{i=1}^{n} p_i \cdot (\alpha_i + 0.5 \sum_{j=1}^{n} \alpha_{ij} \log p_j)
+ \lambda_{it} t
\]

(24)

We turn now to surround from a purely analytic point of view some differences between the two nonlinear version tied to the technical progress modelling aspects (exponential or linear).
If we denote by:

\[ C(p) = \alpha_0 \cdot \prod_{i=1}^{n} p_i^{(\alpha_{ij} + 0.5 \sum_{j=1}^{n} \alpha_{ij} \log p_j)} \]  
(25)

\[ \Phi(t) = \exp\{\alpha_i t + \alpha_{ii} t^2\} \]  
(26)

\[ S_i(p) = \alpha_i + \sum_{j=1}^{n} \alpha_{ij} \log p_j \]  
(27)

Assuming constant returns to scale, the two nonlinear versions of the translog unit cost function can be expressed as

\[ C(p, t)_{TLE} = \Phi(t) \cdot C(p) \cdot \prod_{i=1}^{n} p_i^{(\alpha_{ii} t)} \]  
(28)

\[ a_i(p, t)_{TLE} = \Phi(t) \cdot (S_i(p) + \alpha_{ii} t) \cdot \frac{C(p)}{p_i} \cdot \prod_{i=1}^{n} p_i^{(\alpha_{ii} t)} \]  
(29)

\[ S_i(p, t)_{TLE} = S_i(p) + \alpha_{ii} t \]  
(30)

\[ C(p, t)_{TLA} = \Phi(t) \cdot C(p) + \sum_{i=1}^{n} \lambda_{ii} p_i t \]  
(31)

\[ a_i(p, t)_{TLA} = \Phi(t) \cdot S_i(p) \cdot \frac{C(p)}{p_i} + \lambda_{ii} t \]  
(32)

\[ S_i(p, t)_{TLA} = (\Phi(t) \cdot S_i(p) \cdot C(p) + \lambda_{ii} p_i t) / C(p, t)_{TLA} \]  
(33)

The price elasticities derived from the two versions can be written as

\[ \tau_{ij}^{TLE} = \frac{\alpha_{ij} - \delta_{ij} S_i(p, t)_{TLE} + S_i(p, t)_{TLE} S_j(p)_{TLE}}{S_i(p, t)_{TLE}} \]  
(34)

\[ \tau_{ij}^{TLA} = \frac{\varphi(t) \cdot C(p) \cdot \alpha_{ij} - \delta_{ij} S_i(p) + S_i(p) S_j(p)}{C(p, t)_{TLA}} \]  
(35)
3 The generalized Leontief cost function (GL)

For simplifying the exposition, we limit ourself to give the constant returns to scale unit cost formula for the GL model

\[
C(p,t)_{GL} = \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \sqrt{p_i p_j} + \sum_{i=1}^{n} \beta_{it} p_i t
\]  

(36)

which is linearly homogeneous in input prices by construction, while the symmetry of the Hessian matrix requires the symmetry of the second order parameters: \( \beta_{ij} = \beta_{ji}, \quad \forall \ i, \ j \).

Cost-minimizing input demands are derived in terms of input-output ratios, and are given by

\[
a_i = \frac{x_i}{y} = \sum_{j=1}^{n} \beta_{ij} \sqrt{p_j / p_i} + \beta_{it} t
\]  

(37)

3.1 The symmetric generalized McFadden cost function (MF)

Assuming constant returns to scale, we limit our self to give the unit cost formula for the Symmetric Generalized McFadden model

\[
\frac{C}{y} = g(p) + \sum_{i=1}^{n} \gamma_i p_i + \sum_{i=1}^{n} \gamma_{it} p_i t
\]  

(38)

where

\[
g(p) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \delta_{ij} p_i p_j}{\tilde{p}}, \quad \text{with} \quad \tilde{p} = \sum_{i=1}^{n} p_i \cdot S_i^0
\]  

(39)

The auxiliary exogenous variable \( \tilde{p} \) is introduced here to impose analytically the linear homogeneity on the \( g(\cdot) \) in a symmetric way. The MF cost function is however homogenous in input prices by construction, while symmetry implies that \( \delta_{ij} = \delta_{ji}, \quad \forall \ i \neq j \).
Thus in order to identify all the parameters, some additional restrictions are needed

\[
\sum_{i=1}^{n} \delta_{ij} = 0
\]  (40)

We are able now to derive the cost-minimizing input demands by means of Shephard lemma, which are given here in terms of input-output ratios

\[
a_i = \gamma_i + \frac{\sum_{j=1}^{n} \delta_{kj} P_j}{\bar{p}} - 0.5S_{i}^{0} \cdot \frac{\sum_{k=1}^{n} \sum_{j=1}^{n} \delta_{kj} P_k P_j}{\bar{p}} + \gamma_{it} t
\]  (41)

The most important property of the McFadden cost function is that its Hessian matrix allows globally imposing curvature conditions without loss of flexibility.
Bibliographie


Appendix 2: Empirical illustration

Table 1: Degrees of flexibility for $TL$, $TLE$, $TLA$

<table>
<thead>
<tr>
<th></th>
<th>$TL$</th>
<th>$TLE$</th>
<th>$TLA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of parameters</td>
<td>9</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Symmetric restrictions</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\times$</td>
<td>8</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 2: Likelihoods ratio test for the symmetry of the parameters, given homogeneity (probability of rejection in parenthesis)

<table>
<thead>
<tr>
<th></th>
<th>$TL$</th>
<th>$TLE$</th>
<th>$TLA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intermediate goods</td>
<td>0.38</td>
<td>2.21</td>
<td>11.94</td>
</tr>
<tr>
<td></td>
<td>(0.46)</td>
<td>(0.86)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>Equipment goods</td>
<td>5.59</td>
<td>0.47</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>(0.98)</td>
<td>(0.51)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>Consumer goods</td>
<td>0.81</td>
<td>0.45</td>
<td>4.75</td>
</tr>
<tr>
<td></td>
<td>(0.63)</td>
<td>(0.50)</td>
<td>(0.97)</td>
</tr>
<tr>
<td>$k$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: Concavity

<table>
<thead>
<tr>
<th></th>
<th>$TL$</th>
<th>$TLE$</th>
<th>$TLA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intermediate goods</td>
<td>0</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>Equipment goods</td>
<td>11</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>Consumer goods</td>
<td>12</td>
<td>19</td>
<td>14</td>
</tr>
</tbody>
</table>

Number of observations where concavity is respected (out of 20).
Table 4: Price and Allen elasticities computed at sample mean point for intermediate goods

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>TL</th>
<th>TLE</th>
<th>TLA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{KK}$</td>
<td>$-0.068^*$</td>
<td>$-0.194^*$</td>
<td>$-0.105^*$</td>
</tr>
<tr>
<td>(0.010)</td>
<td>(0.021)</td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{KL}$</td>
<td>$0.110^*$</td>
<td>$0.214^*$</td>
<td>$0.103^*$</td>
</tr>
<tr>
<td>(0.012)</td>
<td>(0.027)</td>
<td>(0.018)</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{KE}$</td>
<td>$-0.042^*$</td>
<td>$-0.020$</td>
<td>$0.002$</td>
</tr>
<tr>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.013)</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{LK}$</td>
<td>$0.035^*$</td>
<td>$0.066^*$</td>
<td>$0.032^*$</td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.008)</td>
<td>(0.005)</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{LL}$</td>
<td>$-0.054^*$</td>
<td>$-0.083^*$</td>
<td>$-0.030^*$</td>
</tr>
<tr>
<td>(0.008)</td>
<td>(0.012)</td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{LE}$</td>
<td>$0.019^*$</td>
<td>$0.017$</td>
<td>$-0.002$</td>
</tr>
<tr>
<td>(0.007)</td>
<td>(0.009)</td>
<td>(0.006)</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{EK}$</td>
<td>$-0.041^*$</td>
<td>$-0.019$</td>
<td>$0.001$</td>
</tr>
<tr>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.013)</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{EL}$</td>
<td>$0.057^*$</td>
<td>$0.053$</td>
<td>$-0.006$</td>
</tr>
<tr>
<td>(0.020)</td>
<td>(0.028)</td>
<td>(0.023)</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{EE}$</td>
<td>$-0.016$</td>
<td>$-0.034$</td>
<td>$-0.005$</td>
</tr>
<tr>
<td>(0.029)</td>
<td>(0.027)</td>
<td>(0.021)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{KL}$</td>
<td>$0.182^*$</td>
<td>$0.350^*$</td>
<td>$0.224^*$</td>
</tr>
<tr>
<td>(0.020)</td>
<td>(0.045)</td>
<td>(0.036)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{KE}$</td>
<td>$-0.211^*$</td>
<td>$-0.103$</td>
<td>$0.009$</td>
</tr>
<tr>
<td>(0.074)</td>
<td>(0.073)</td>
<td>(0.085)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{LE}$</td>
<td>$0.094^*$</td>
<td>$0.087$</td>
<td>$-0.013$</td>
</tr>
<tr>
<td>(0.034)</td>
<td>(0.046)</td>
<td>(0.043)</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parenthesis

* : significant values at the level 5%
Table 5: Price and Allen elasticities computed at sample mean point for equipment goods

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>$TL$</th>
<th>$TLE$</th>
<th>$TLA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{KK}$</td>
<td>$-0.077^*$</td>
<td>$-0.145^*$</td>
<td>$-0.069^*$</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.031)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>$\epsilon_{KL}$</td>
<td>$0.076^*$</td>
<td>$0.130^*$</td>
<td>$0.050^*$</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.032)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>$\epsilon_{KE}$</td>
<td>$0.001$</td>
<td>$0.015$</td>
<td>$0.019^*$</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>$\epsilon_{LK}$</td>
<td>$0.011^*$</td>
<td>$0.018^*$</td>
<td>$0.007^*$</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>$\epsilon_{LL}$</td>
<td>$-0.029^*$</td>
<td>$-0.033^*$</td>
<td>$-0.015^*$</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$\epsilon_{LE}$</td>
<td>$0.018^*$</td>
<td>$0.015^*$</td>
<td>$0.008^*$</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>$\epsilon_{EK}$</td>
<td>$0.004$</td>
<td>$0.044$</td>
<td>$0.058^*$</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.023)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>$\epsilon_{EL}$</td>
<td>$0.368^*$</td>
<td>$0.325^*$</td>
<td>$0.187^*$</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.053)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>$\epsilon_{EE}$</td>
<td>$-0.372^*$</td>
<td>$-0.369^*$</td>
<td>$-0.245^*$</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.046)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>$\sigma_{KL}$</td>
<td>$0.091^*$</td>
<td>$0.154^*$</td>
<td>$0.097^*$</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.038)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>$\sigma_{KE}$</td>
<td>$0.030$</td>
<td>$0.376$</td>
<td>$0.757^*$</td>
</tr>
<tr>
<td></td>
<td>(0.180)</td>
<td>(0.189)</td>
<td>(0.339)</td>
</tr>
<tr>
<td>$\sigma_{LE}$</td>
<td>$0.440^*$</td>
<td>$0.386^*$</td>
<td>$0.337^*$</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.063)</td>
<td>(0.070)</td>
</tr>
</tbody>
</table>

Standard errors in parenthesis

*: significant values at the level the level 5%
Table 6: Price and Allen elasticities computed at sample mean point for consumer goods

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>$TL$</th>
<th>$TLE$</th>
<th>$TLA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_{kk}$</td>
<td>$-0.091^*$</td>
<td>$-0.155^*$</td>
<td>$-0.067^*$</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.024)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>$\xi_{kl}$</td>
<td>$0.126^*$</td>
<td>$0.197^*$</td>
<td>$0.094^*$</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.027)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>$\xi_{ke}$</td>
<td>$-0.035^*$</td>
<td>$-0.042^*$</td>
<td>$-0.027$</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>$\xi_{lk}$</td>
<td>$0.021^*$</td>
<td>$0.032^*$</td>
<td>$0.015^*$</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$\xi_{ll}$</td>
<td>$-0.052^*$</td>
<td>$-0.068^*$</td>
<td>$-0.041^*$</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>$\xi_{le}$</td>
<td>$0.031^*$</td>
<td>$0.036^*$</td>
<td>$0.026^*$</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$\xi_{ek}$</td>
<td>$-0.087^*$</td>
<td>$-0.100^*$</td>
<td>$-0.065$</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.036)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>$\xi_{el}$</td>
<td>$0.452^*$</td>
<td>$0.538^*$</td>
<td>$0.398^*$</td>
</tr>
<tr>
<td></td>
<td>(0.076)</td>
<td>(0.079)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>$\sigma_{kl}$</td>
<td>$0.156^*$</td>
<td>$0.243^*$</td>
<td>$0.168^*$</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.033)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>$\sigma_{ke}$</td>
<td>$-0.634^*$</td>
<td>$-0.756^*$</td>
<td>$-0.764$</td>
</tr>
<tr>
<td></td>
<td>(0.257)</td>
<td>(0.274)</td>
<td>(0.439)</td>
</tr>
<tr>
<td>$\sigma_{le}$</td>
<td>$0.560^*$</td>
<td>$0.661^*$</td>
<td>$0.744^*$</td>
</tr>
<tr>
<td></td>
<td>(0.094)</td>
<td>(0.097)</td>
<td>(0.104)</td>
</tr>
</tbody>
</table>

Standard errors in parenthesis
* : significant values at the level 5%

33
Table 7: Derivatives of own and cross elasticities of demand with respect to the price of energy evaluated at 1980 for consumer goods

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>( TL )</th>
<th>( TLE )</th>
<th>( TLA )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_{KE} )</td>
<td>(-0.042^*)</td>
<td>(-0.050^*)</td>
<td>(-0.033)</td>
</tr>
<tr>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.017)</td>
<td></td>
</tr>
<tr>
<td>( \partial_{KE} )</td>
<td>(0.078^*)</td>
<td>(0.077^*)</td>
<td>(0.082^*)</td>
</tr>
<tr>
<td>(0.014)</td>
<td>(0.012)</td>
<td>(0.019)</td>
<td></td>
</tr>
<tr>
<td>( \partial_{LE} )</td>
<td>(-0.100^*)</td>
<td>(-0.093^*)</td>
<td>(-0.078^*)</td>
</tr>
<tr>
<td>(0.011)</td>
<td>(0.009)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>( \partial_{KE} )</td>
<td>(0.021^*)</td>
<td>(0.016^*)</td>
<td>(0.029^*)</td>
</tr>
<tr>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>( \epsilon_{LE} )</td>
<td>(0.037^*)</td>
<td>(0.042^*)</td>
<td>(0.036^*)</td>
</tr>
<tr>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>( \partial_{KE} )</td>
<td>(-0.015^*)</td>
<td>(-0.015^*)</td>
<td>(-0.015^*)</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>( \partial_{LE} )</td>
<td>(-0.016^*)</td>
<td>(-0.013^*)</td>
<td>(-0.067^*)</td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>( \partial_{LE} )</td>
<td>(0.031^*)</td>
<td>(0.028^*)</td>
<td>(0.048^*)</td>
</tr>
<tr>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>( \epsilon_{EE} )</td>
<td>(-0.415^*)</td>
<td>(-0.476^*)</td>
<td>(-0.434^*)</td>
</tr>
<tr>
<td>(0.055)</td>
<td>(0.057)</td>
<td>(0.047)</td>
<td></td>
</tr>
<tr>
<td>( \partial_{KE} )</td>
<td>(0.094^*)</td>
<td>(0.086^*)</td>
<td>(0.125^*)</td>
</tr>
<tr>
<td>(0.017)</td>
<td>(0.015)</td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>( \partial_{LE} )</td>
<td>(0.149^*)</td>
<td>(0.102^*)</td>
<td>(0.802^*)</td>
</tr>
<tr>
<td>(0.048)</td>
<td>(0.042)</td>
<td>(0.022)</td>
<td></td>
</tr>
<tr>
<td>( \partial_{EE} )</td>
<td>(-0.243^*)</td>
<td>(-0.188^*)</td>
<td>(-0.493^*)</td>
</tr>
<tr>
<td>(0.054)</td>
<td>(0.049)</td>
<td>(0.040)</td>
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</tr>
</tbody>
</table>

Standard errors in parenthesis

* : significant values at the level 5%
Table 8: Degrees of flexibility for $TL_0$, $TL$, $TL_1$

<table>
<thead>
<tr>
<th></th>
<th>$TL_0$</th>
<th>$TL$</th>
<th>$TL_1$</th>
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<tbody>
<tr>
<td>Number of parameters</td>
<td>7</td>
<td>9</td>
<td>10</td>
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<tr>
<td>Symmetric restrictions</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 9: Degrees of flexibility for $TL$, $GT$, $MF$ (technical progress crossed with prices)

<table>
<thead>
<tr>
<th></th>
<th>$TL$</th>
<th>$GL$</th>
<th>$MF$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of parameters</td>
<td>9</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Symmetric restrictions</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 10: Degrees of flexibility for $TL_0$, $GL_0$, $MF_0$ (without technical progress)

<table>
<thead>
<tr>
<th></th>
<th>$TL_0$</th>
<th>$GL_0$</th>
<th>$MF_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of parameters</td>
<td>7</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Symmetric restrictions</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>
FIGURE 3
EVOLUTION OF THE OWN PRICE ELASTICITY OF DEMAND FOR ENERGY
ESTIMATED WITH THE THREE VERSIONS OF TRANSLOG
—CONSUMER GOODS—

FIGURE 4
ADJUSTMENT OF THE COST SHARE OF ENERGY
FOR THE THREE VERSIONS OF TRANSLOG
—CONSUMER GOODS—
FIGURE 1
ADJUSTMENT OF THE ENERG-OUTPUT
FOR THE THREE VERSIONS OF TRANSLOG
— CONSUMER GOODS —

FIGURE 6
ADJUSTMENT OF THE UNIT COST
FOR THE THREE VERSIONS OF TRANSLOG
— CONSUMER GOODS —
GRAPHIQUE

AJUSTEMENT DU COUT UNITAIRE
QUAND ON releve LES RESTRICTIONS IMPOSEES AUX PARAMETRES
DE RENDEMENTS D'ECHELLE ET DE PROGRES TECHNIQUE POUR TRANSLUC

LEGENDRE :

- Rns : PAS DE RESTRICTIONS SUR LES RENDEMENTS D'ECHELLE
- Tns : PAS DE RESTRICTIONS SUR LE PROGRES TECHNIQUE

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| ORS | R≤< | R=1 | T=> |

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