Is Economics Entering its Post-Witchcraft Era?

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Abstract

Recently, an awareness has been emerging in economics of the fact that important problems are not solvable algorithmically, that is, by any finite number of steps. This statement can be made mathematically exact and this paper reviews the contributions that have been made in this regard, all related to standard topics in economics.

1 Introduction: on the Medieval Horror of Malleus Maleficarum

Quite a long time ago, when one of the authors was in school, he was asked to read the infamous late 15th century inquisitorial book *Malleus Maleficarum* and, beyond the endless gory details of tortures to be applied to witches, to comment on what he found as by far the most important feature of that text. As it happened, what he found was the—most surprising—perfect identity in their respective views, which both the inquisitors and the witches manifestly shared with one another, of the way the world functions. *Shared* indeed, and did so of course, without in the very least being aware of that monumentally important fact.

What they shared was nothing else but their fundamental—and never ever questioned— belief that for absolutely every problem in life there exists a clear cut and unquestionable solution, and on top of it, that solution is given by a finite number of equally clear and well defined steps which are to be implemented in one and only one well specified order. Needless to say, the inquisitors who wrote that book used various finite sets of strictly theological and legal arguments, plus—of course, and so fashionable at the time—thoroughly described tortures, each consisting of a finite number of clearly described but horrible procedures. In their turn, the witches, in a perfectly similar manner, were using a large, and often hilarious—if not in fact, ridiculous—variety of procedures, each of them consisting of a well defined finite number of steps, to be executed in one and only one strictly specified order. Witches and inquisitors were, therefore, united in their respective belief that a formulaic solution to their respective problem *exists*. This reminds one of the authors of what he has always held to be the somewhat questionable guiding philosophy behind extramarital affairs[1]: if one intimate relationship does not make one happy, then two will perhaps ensure felicity. So much for an algorithmic view of the ways the world works.

It is worth noting that the algorithmic approach has a long, universal and venerated presence in human civilization, as manifested in the surprisingly rich variety of rather dramatic actions of both heroes and villains which populate folk tales—including those for children. The difference between the *Malleus* and

[1] As opposed to pure adultery.

such actions or algorithms, however, is that the authors of all those tales, no matter how ancient, clearly manifested a meta-algorithmic or trans-algorithmic wisdom. Indeed, in none of those tales is it sufficient either for the hero or the villain to perform one single, no matter how extraordinary algorithm. On the contrary, the ever fascinating suspense—with the breath-taking dynamics of its many sudden and unexpected turns in those tales—comes precisely from the fact that, for a long long time, and just until the happy end, none of the algorithms performed by the hero seems to suffice for his or her ultimate success. And certainly, the algorithms of the villains will eventually fail, no matter how diabolical they may be. It is, therefore, most clearly in that way that the authors of all such tales, including those for children, teach us a wisdom which completely failed the inquisitors and witches of older times, or for that matter, so many of the economists or politicians of our days... It is thus remarkable that even children's stories turn out to know much better than such algorithm-bound personages as the inquisitors and witches of *Malleus Maleficarum*. But how about the practitioners of much of present day economics, or for that matter, politics? Do we indeed witness, as we have done throughout long ages, an ongoing massive regression of any number of adults to infancy ? The 8 year old son of one of the authors often comes up with jokes he himself invents. A recent one is the following :

> How do extra-terrestrials call our human children ? Answer : juvenile humanoids. And how do extra-terrestrials call our adult humans ? Answer : extreme juvenile humanoids ...

2 The Dismal Applied Science

Mathematical economics is undeniably a beautiful theory with considerable epistemic value. Recall, however, that it was also a most practical thing when American soldiers were told to deliver food and other goods to the island of Vanuatu during World War II.However, is it a good thing that islanders today worship a deity called *John Frum* ("John from Illinios/Kansas/Texas") who is supposed to return one day with salvation and... more cargo? Economics is not the only science blighted by insistent application of simplistic, if not in fact, atavistic medieval views. Statistics, among other sciences, is often equally damned. The government of South Africa takes comfort, for example, in the finding of a recent report [14], regarding possible perverse incentives in the Child Support Grant, or CSG, according to which

> the quantitative analysis suggests that the take-up rate of the CSG by teenage mothers remains low. Teenagers, that is, younger than 20 years, represent 5% of all CSG recipients registered at October 2005.

The report concludes, apparently largely on the basis of this single statistic[2], that there is no perverse incentive in the grant. The consideration that teenage mothers are sooner or later mothers of 30 with a 14 year old child, still receiving the grant, is absent. Buried in the same report, one will read that

> the proportion of mothers who were teenagers at the birth—not necessary at the application for CSG—of their eldest child to be registered on SOCPEN from 2000–2004, increased from 23% in 2000 to 32% in 2004.

This telling figure appears not to have influenced the conclusions of the report. And unless the minister for Social Development has read it very carefully, he will now sleep soundly at night. The activists who are proposing extending the payment of the grant until age 18—thereby creating an enabling, if not in fact, encouraging environment for two successive generations of one family to simultaneously receive it—will also be encouraged by the Department of Social Development's conclusion rather than by the facts. Will our descendants look upon a social policy that actively promoted and rewarded a quarter of a million teenage pregnancies per annum in a medium-sized country with the same horror with which we regard the *Malleus*?

Mathematical economics still contributes much to the way that we see the world. Take the fashionable business of carbon emissions trading as an example. Carbon emission caps, and the tradability of the gap between an enterprise's actual emissions and its cap, have created a new asset class, called carbon accounting units, the market in which might be expected to approach a Walrasian equilibrium. By neoclassical economics we know that a equilibrium price exists, but we do not necessarily know whether there is a single equilibrium or many equilibria—nor whether there are any stable equilibria. Furthermore,

[2] This is what the great physicist Richard Feynman has called [15] *cargo cult science*.

there is no reason to believe that we can be more sure whether carbon trading units will be cheap or expensive next year, than we can be sure about the price of pork bellies in Chicago. It is rather amusing that individuals in favour of carbon caps (and trading in these new commodities) are sometimes stridently against so-called speculators and other investors. For some people carbon credits are clearly already very affordable, allowing Mr Albert Gore—for example—to effortlessly be a green prophet with a heated swimming pool.

Carbon trading can be better understood through a quotidian analogy: consider the possibility of trading highway speeding. A person could earn speed credits by driving, say, at 90km/h from Pretoria to Durban, and then sell these on the market—for example, to two people who want to each drive at an average of 135km/h from Durban to Pretoria. Monitoring will be necessary, but there will be a good incentive to drive slower than the speed limit of 120km/h. And there will be no need for speeding tickets, although, obviously, strict but non-violent monitoring of the road network will be required. The market will simply establish a price for speeding that matches supply and demand. This should at least be an improvement over the incorrect pricing observed so spectacularly in a study [13] of fines at Israeli day-care centers.

The existence proofs of mathematical economics have imbued theoretical economics with a semblence of the supposed rigid veracity of physics. The belief, erroneous in the minds of the authors, that the world in which we live is identical with an appropriate (nay, existing) model in physics has—in some places—also spread to economics. Without impugning the achievements of mathematical economics, this papers attemps to highlight some recent results—inspired by similar questions asked in mathematics—that reflect on the limits of the practical applications of the theory.

The founders of modern economics were well aware of problems related to the tractability of the mathematical models that they developed. Vilfredo Pareto has pointed out in his 1927 *Manuel d'économie politique* that the equations for a real economy would be so many as to be impossibly complex to solve. Of course he did not have computers in mind but it turns out that his intuition about this absolutely correct, as we shall explain later. In 1945 Friedrich Hayek remarked:

> The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess.

Hayek's comments can be interpreted to refer to availability of data only if one thinks of the "dispersed bits" that he refers to as in geographic dispersion. However, if one takes it to say simply that the necessary information or results cannot be succinctly described then his remark echoes the non-computability results of Richter and Wong which we briefly review below.

3 Enter Modern Economics

The need for a formal definition of *algorithm* or *computation* became clear to the scientific community of the early twentieth century due mainly to two problems posed by David Hilbert:

- the *Entscheidungsproblem*—can an algorithm be found that, given a statement in first-order logic (for example Peano arithmetic), determines whether it is true in all models of a theory; and
- Hilbert's tenth problem—does there exist an algorithm which, given a Diophantine equation, determines whether is has any integer solutions?

The *Entscheidungsproblem*, or *decision problem*, can be traced back in some form at least to Leibniz and was successfully and independently resolved in the negative during the mid-1930s by Alonzo Church and Alan Turing. Unless it is indicated or directly implied otherwise, "computation' and "compute" in the paper will refer to the action of and functions computed by an appropriate Turing machine. The notion of formal computability enters economics in two very obvious ways.

- The computability of model outcomes: any practical applications would clearly require that the result be computable, i.e. that it could—even just in principle, because computers improve over time—be produced by a computer.

- As a limit on the actions of the agents: instead of allowing arbitrary preferences or utility functions, one could—reasonably—want to restrict the theory to computable objects, for example computable preferences on computable bundles of goods. A computable preference is simply one which can be calculated in a finite amount of time using a machine with a finite amount of resources and a computable bundle would be any bundle that can be coherently represented using only a finite number of symbols.

If it is assumed that the economic agents are anything like real human beings then it makes perfect sense for all the objects in the theory to be taken to be computable. For example, computable prices are the only prices that one can conceivable use in an MS-Excel[3] or Gnumeric[4] spreadsheet.

3.1 Computability in equilibrium theory

The general equilibrium theorem of Kenneth Arrow and Gérard Debreu, establishing the existence of a competitive equilibrium in an economy with a finite number of consumers and a finite number of goods, is proved using the Brouwer fixed-point theorem (BFPT). BFPT is of course also used in game theory and other areas of economics. In this section I denotes the standard unit interval $[0, 1]$ of real numbers.

Theorem 1 (Brouwer) *Any continuous function $f : I^2 \to I^2$ has a fixed point, i.e. there exists an $x \in I^2$ such that $f(x) = x$.*

The fact is that the BFPT is very non-constructive. In fact, there are functions f for which the fixed points themselves are *all* non-constructive when viewed as solitary objects. Given a function f, we do might not only not know how to find the fixed point x such that

$$f(x) = x$$

but it can also be a point which cannot be defined in an *effective* way at all. By this we mean that there exist functions f for which there exists no algorithm (using finite resources, e.g. a Turing machine) that can approximate any fixed point of f, given a margin of error ε, up to the given margin of error.

The exact definitions of computable real numbers and of computable functions on the natural numbers appear in Alan Turing's pioneering work of the 1930s. A *computable function* from the natural numbers ($\mathbb{N}_0$) to the natural numbers is a function that can be computed in principle by a Turing machine (which, for the purposes of this paper, the reader can assume to be a desktop computer with infinite memory). A computable real number is any number which can in principle be approximated by a Turing machine.

Definition 1 *A real number x is called* computable *if there exist computable functions $\phi_1^x, \phi_2^x, \phi_3^x : \mathbb{N}_0 \to \mathbb{N}_0$ such that for all $k \in \mathbb{N}_0$*

$$\left| x - (-1)^{\phi_1^x(k)} \frac{\phi_2^x(k)}{\phi_3^x(k)} \right| \quad \leq \quad 2^{-k}.$$

The approriate definition of computability for a real-valued function of real variables is beyond the scope of this paper and the reader is referred to the paper of Brattka [8], for example. In this paper, a computable real function f will be represented as a Turing-computable function that maps descriptions of computable reals x to descriptions of the computable reals $f(x)$. That is, f is called computable if there exists a computer program that transforms an approximation procedure for x to an approximation procedure for $f(x)$. This is a rather natural notation but the observant reader will remark that—as there are only countably many computable reals—such an f need not necessarily be defined outside a set of measure zero. Nevertheless, all of the continuous functions usually used in economics courses are computable in this sense. These functions are often called *computably coded*.

The computable counterexample to the Brouwer theorem is due to Orevkov [23] (cited i.a. in [17]) who worked in the Russian school of constructivism. Baigger [1] rewrote the Orevkov construction to define a computable function on I^2 which has no computable fixed point. A similar construction was described by Richter and Wong [30] in their paper that established the following

Theorem 2 (Richter and Wong [30, 29]) *For any $\ell \geq 3$ there exists a computable economy E with a finite number of agents and with ℓ goods for which no computable equilibrium exists.*

[3]The authors are not affiliated with Microsoft Corporation but would not reject donations or grants from Microsoft.
[4]Even Linux is limited by the theory of formal computability.

This theorem is a direct application of the Orevkov counter example and therefore says not only that the equilibrium cannot be found but also that the equilibrium price vector is an uncomputable one. The non-computability of the price vector implies not only that it cannot be found, but that it has no finite description in any formal computing scheme.

One can consider the situation where some additional information about the equilibria of an exchange economy are known, for example If f is a computable coded function with computable modulus of uniform continuity mapping computable points in I^2 to computable points in I^2 then f can be extended to a total function $f^*(x) : I^2 \to I^2$ and the following version of the BFPT can be obtained.

Theorem 3 (Hirst [17]) *If f^* has finitely many fixed points, then f has a computable fixed point.*

This result, together with Debreu's proof [11] that *regular* economies have finitely equilibria, appears to allow one to salvage a computable general equilibrium theorem. Certainly, if f^* has one fixed point then there appears to be a procedure for finding it. However, if f^* has more than one, but still finitely many fixed points, then the proof only infers the existence of a fixed point which is a computable real number, but in a non-constructive way. That is, it proves the *existence* of a computable x such that

$$f^*(x) = x$$

but we still have no indication of how to find it.

3.2 Computability in individual choice

In a recent interview[5], 2002 Nobel laureate Vernon L Smith—asked which economists had influenced him most—replied:

> At Harvard, Wassily Leontief, who had a certain amount of skepticism about economics and a great sense of humor. After two weeks of studying utility theory, student raised his hand and asked, "Professor, what is utility good for?" Leontief responded "It's good for teaching."

It would nevertheless obviously be nice of computable (in some sense) preferences should give rise to computable utility functions. The following theorems appeared to establish that this is the case.

Theorem 4 (Richter and Wong [31, 29]) *If $\succ \subseteq \mathbb{R}^\ell_+ \times \mathbb{R}^\ell_+$ is a computable preference relation, then there exists a computable function $u : \mathbb{R}^\ell_+ \to \mathbb{R}^\ell_+$ that rationalizes $\succ$.*

However, Richard and Wong seem to have defined a computable preference relation through the characteristic function. This is in fact a bad definition since all computable functions are continuous and hence sets with computable characteristic functions are both open and closed (clopen). In Euclidean space however the only clopen sets are the empty set and the entire space. The preceding result (although correctly derived) appears to contradict an earlier result of Bridges and Richman [9] but only because of this specific use of the definition of a computable preference relation. The result of Bridges and Richman did in fact establish that a computable preference relation can give rise to an non-computable utility function.

3.3 Computability in social choice

The final remark concern Fishburn's treatment of the Arrow impossibility theorem.

Theorem 5 (Arrow) *If a decision-making body has at least two (but finitely many) members and at least three (but finitely many) options to decide among, then it is impossible to design a universal (it should create a deterministic, complete societal preference order from every set of individual preferences), non-imposing (every societal preference should be achievable) social choice function that satisfies all of the following three conditions.*

- *non-dictatorship;*
- *unanimity (if all individuals prefer x to y then so does the society); and*

[5]With Russ Roberts, `http://www.econtalk.org/archives/2007/05/vernon_smith_on.html#highlights` (accessed 2007-05-23).

- *independence of irrelevant alternatives.*

Peter Fishburn, recipient of the 1996 INFORMS von Neumann Theory Prize, showed in 1970 [12] that the three conditions in Arrow's Impossibility Theorem can in fact be satisfied by a social choice function if there are *infinitely* many decision makers. H. Reiju Mihara has however proved that Arrow's Theorem does in fact still hold for the case of a countably) infinite society if all coalitions are required to be computable and we require a pairwise-computable social welfare function [19]. In this case computability seems a highly reasonable requirement as it simply means that, for example, one can in practice identify the coalitions and that any two different options can be ranked in practice.

4 The historical background in relation to Game Theory

Let us review more significant moments in the vagaries of the reflex type and never questioned total belief in the algorithmic approach to a variety of human problems, as they manifested themselves in more recent times. Algorithms are of course supposed to solve appropriate classes of problems. Many of such problems, not seldom the more critical ones, may involve optimisation. And when the interests of several parties are at play, as happens in human society, they can typically lead to the need for conflict resolution. No wonder, therefore, that in Economics, as well as in a variety of other fields of applicative importance there has been a significant interest in the development of whole ranges of rigorous methods, that is algorithms, for dealing with such situations. Several such modern fields, as well as their interconnections are reviewed in [25]. A significant and major confluence and interaction between Economics and Game Theory happened to come about during WW II, with the publication by John von Neumann and Oskar Morgenstern of their massive and influential book Theory of Games and Economic Behaviour, Princeton University Press, 1944.

Now, after more than six decades, it may be worth recalling a few features of the respective relevant background. The first major result in Game Theory was obtained by von Neumann at the age of 25, in 1928, when he solved the two person zero sum game, which of course is the simplest possible nontrivial game. As it happened, Borel, a French mathematician of high standing at the time, studied games not long earlier. However, he did not manage to obtain that fundamental result, and in fact, proposed a related conjecture which later proved to be wrong. The remarkable fact about von Neumann being involved at the time in Game Theory is that during the same period he contributed fundamental results to the axiomatisation of Set Theory and also set up the first rigorous mathematical treatment based on Hilbert space operators of the just emerging modern Quantum Mechanics. In this regard, one may wonder whether his dealing with games at the time was no more than a mere active form of relaxation, given what would appear to be the incomparably higher importance of the other two subjects he was involved in.

A proper answer to that question requires a better look at von Neumann's biography. Born in 1903 into a wealthy and privileged family in Budapest, he grew up, until 1914 when WW I erupted, in a glorious period not only of his own country, the Austro-Hungarian empire, but of much of the rest of Europe as well. Indeed, after WW I, Europe would not reach a comparable state of prosperity, freedom or security for another seven or eight decades, and then do so mainly due to the European Union. Soon after WW I erupted, life changed dramatically for the worse all over Europe, and did so no matter how privileged one had earlier been. Added to that came the ongoing misery, not only economic and social, but also political, which for many years would engulf inter-war Europe. All that dramatic, sudden and unexpected change became the life experience of von Neumann—starting from his early teenage years. A similar experience was to be the fate of the well known philosopher Karl Popper, born in 1902 in Vienna, who came from a comparably privileged family. Consequently, von Neumann and Popper became two of the most active and representative members of their European generation who wanted not only to understand how such a catastrophe like WW I could happen in civilized Europe, but also how possibly to prevent it from ever happening again.

Popper's answer led him to his development of the concept of Open Society, while von Neumann turned, among other things, to Game Theory. In von Neumann's view Game Theory would for the first time in human affairs offer a systematic scientific approach to conflict management between rational and civilised parties. And indeed, the ultimate horror of WW I appeared to von Neumann, Popper and other thinkers of their generation as coming precisely from the fact that the European political class which had been responsible for it had been constituted from people coming from some of the most privileged old families, people whom quite likely more than any other ones, one could assume to be civilised and rational. It is not, therefore, so surprising that in 1928, while fully involved in foundational issues of Set Theory and Quantum Mechanics, von Neumann consecrated time and effort to the solution of the two person zero sum game. And during the next decade and half, until the publication of that first ever and massive treaties on The Theory of Games and Economic Behaviour, von Neumann considered it so important to further pursue significant research in Game Theory. And he did so while at the same time he happened to be involved in a considerable number of other crucially important projects, such as the theoretical development and effective building of the first electronic computer, the Los Alamos project related to the atom bomb, a variety of urgently needed applications of mathematics required by the American military during WW II, and endless advisory work on highest level government committees.

In this way, the most special importance and priority von Neumann attributed to Game Theory during those decades becomes particularly clear. With the end of WW II, however, von Neumann's view of Game Theory did not change. Indeed, barely one year later, starting with 1946, the world had to face what would come to be known as the Cold War. And in that newly emerged, and what at the time appeared most likely a long lasting and dangerous conflict, many influential people besides von Neumann would see a crucial role played by Game Theory in both tactical and strategic thinking. The leaders of the Soviet Union, the Cold War enemy of the West, were of course much unlike the old European political class which had led to WW I. However, one could still rely on their rationality in view of the following two facts. Their by far highest priority was to secure the outcome of their Bolshevik Revolution of 1917. At the same time, they were fully aware of the highly fragile, thus significantly insecure state of their empire. Consequently, they were tremendously risk averse, and thus not at all prone to adventurism. That was the period when the famous think tank at Rand Corporation would invest considerable effort in a large range of applications of Game Theory, while at places like Princeton the work on furthering and deepening the theoretical basis of Game Theory appeared as one of the most crucial contributions to the salvation of human civilization.

It was in this atmosphere, so compellingly described by Sylvia Nasar in her book *A Beautiful Mind* [21], that young stars like John Nash emerged with major new contributions to Game Theory. And that theory appeared at the time to go on and on, from strength to strength... As it happened, and as viewed by von Neumann and the others involved in it, the strength, and thus value of Game Theory was precisely in the fact that it could deliver rigorous algorithmic ways for solving large varieties of conflicts between rational agents. Strangely enough, and quite likely out of wishful thinking, no one seemed to be concerned about the possible limitations of such an algorithmic approach. On the contrary, the perception kept persisting that those games which were still without solutions would sooner or later find one, given enough time for the development of Game Theory. Such a view was indeed strange, and should have been found to be like that ever since Turing's Halting Problem of the late 1930s showed so clearly the limits of the algorithmic approach. Needless to say, the earlier, 1931 Incompleteness results of Gödel should have already rang the warning bells in this regard. Also, there were results in Pure Mathematics itself, such as the impossibility of algorithmic solution of the Word Problem in semigroups, or later in groups, which were pointing in the same direction. Yet none of these seemed to register at the time with the Game Theory community.

A major turning point, nearer to the concerns of Game Theory, yet again far from being recognized as such at the time, came in 1951, with Arrow's Impossibility Theorem. Kenneth J Arrow was interested in Social Choice Theory and came across the fact that, under rather general, natural and minimal conditions, individual choices cannot be aggregated into a social choice, unless such an aggregation is done by a dictator. The importance of that most shocking negative result was immediately recognized and Arrow would be given the Nobel Prize in 1972. However, its deeper and wider meanings were not. And indeed, how could anyone in a genuinely democratic society like that of America venture unbiased and freely into the exploration of the possible deeper consequences of that impossibility involving, as it did, nothing less than a dictator?

And yet, one obvious implication of Arrow's result, one that would by no means constitute a challenge to democracy, was that it again pointed so sharply and clearly, and in a context of significant social importance, to the inevitable limitations inherent in any algorithmic approach. Certainly, the aggregation, which Arrow proved not to exist, constituted nothing but yet another algorithm. And no matter how reasonable and mild the respective requirements, such an algorithm would simply not exist. As it happened at the time, the message of Arrow's result remained confined to Social Choice Theory. Consequently, it would not reach the Game Theory community which for a while longer would still be fascinated by the algorithmic approach.

A further indication of the long ongoing fascination Game Theory has held, a fascination which could, and to a certain extent, would cloud its more proper perception and assessment, is the following fact related to the contributions Nash made to it in the late 1940s and early 1950s. As widely known among specialists, and also stated more recently in [20], the systematic distinction between cooperative and non-cooperative games was introduced by Nash. Furthermore, what is considered to be his major contribution to Game Theory, and earned him the 1994 Nobel Prize in Economics, is the so called Nash Equilibrium Theorem in non-cooperative games. However, as a rather simple analysis shows [26], the very concept of equilibrium which Nash uses in order to prove its existence by his celebrated theorem does in fact rely on a particularly strong cooperative assumption, even if that cooperation is by an overall tacit agreement of the players not to rock the boat too much, rather than by doing certain specific things.

Strangely enough, no one seems to have noted that fact so far. Instead, the Nash Equilibrium Theorem is considered as the fundamental result in Non-cooperative Game Theory, and as such, the true generalization to $n > 2$ players of the 1928 solution of von Neumann to the two person zero sum game. Needless to say, economists have ever since its formulation nearly six decades ago embraced that Equilibrium Theorem as a fundamental and normative paradigm in non-cooperation, as illustrated by the Nobel prize Nash was given for it. Such a saga should, however, not surprise anybody unduly. After all, it is but one instance among not a few others in which. . . science is not done scientifically. In case one may be interested in other more recent and by no means less important such instance, it suffices to read the recent book by the physicist Lee Smolin, entitled *The Trouble with Physics* [28].

In 1968 William F Lucas described a game with 10 players which had no solution [18]. That result was certainly quite shocking for anybody strongly committed to the algorithmic view of the world. And yet, it would take about three more decades until Binmore, [3, 4, 5, 6, 7], would prove the fact that, rather typically, games do not have solutions ... Following that negative result by Lucas, even if rather tacitly, the views on Game Theory changed significantly. There would hardly at all be further major conceptual or theoretical developments, except perhaps for the introduction of the so called oceanic games, in which there are infinitely many, and in fact, a continuum of players.

By the 1980s, a certain revival of interest in Game Theory would occur. However, it would no longer be expected to save civilization, and instead, it would merely be one of the many tools used in Economics, [24]. A somewhat unexpected turn away from the earlier highly regarded Game Theory towards distinctly practical concerns happened gradually in the 1960s and 1970s, when Multiple Criteria Decision Making, or in short MCDM, came to the fore. That clearly corresponded to the pressing day to day needs of a large variety executives in management who were supposed simultaneously to optimize several rather conflicting criteria or objectives. There had obviously been much earlier many such situations, but they had been dealt with on a case to case basis, with the specifics of the decision situation and the idiosyncrasies of the decision makers having mostly determined the outcomes. However, a clear enough awareness about the existence of a more general pattern, and hopefully, principles and solution methods as well, had been lacking. Typically, in the presence of multiple criteria one would use a priority based method, or one would simply attach weights to the criteria, and thus reduce their multiplicity to a single one.

One of the cases which helped turn the attention to the usefulness of a more systematic and general approach was the designing in the late 1950s of the Phantom fighter-bomber American jet. In that instance, those involved in the design soon understood that the issue was not so much about technology or cost, but how to reconcile the following critically important no less than five parameters of the plane : minimum weight and size in order to make it highly maneuverable, maximum engine power, fuel tanks and payload carrying weapons and ammunition. What was noted clearly, was that MCDM problems were not quite problems of Game Theory, although they involved rather tough conflicts. Indeed, the executives which faced MCDM problems, either as individuals, or as members of a small group, were not playing one against the other. That fact, which was even more clear in the case of a single executive having to deal with an MCDM problem did in fact bring with it a significant advantage, compared with games, when it comes to finding solutions.

5 Conclusion

The intention of the preceding remarks is not to deny that, for example, markets function in practice. Indeed. prominent skeptics of equilibrium theory like experimental economist Vernon Smith [27] have shown how, in many instances, a kind of stability does emerge in simulated transactional environments. Words in Hayek's famous 1945 paper [16], already referred to above, remain appropriate.

> If we can agree that the economic problem of society is mainly one of rapid adaptation to changes in the particular circumstances of time and place, it would seem to follow that the ultimate decisions must be left to the people who are familiar with these circumstances, who know directly of the relevant changes and of the resources immediately available to meet them.

The question arises why an awareness of the futility of always and exclusively relying on algorithmic solutions is still so much missing in management, not to mention the executive and legislative branches of government, and in general, in politics as such.

References

[1] Baigger, Günter. 1985. Die Nichtkonstruktivität des Brouwerschen Fixpunktsatzes. *Arch. Math. Logik Grundlag.*, 25(3-4):183–188.

[2] BBC NEWS (Asia-Pacific): Vanuatu cargo cult marks 50 years. http://news.bbc.co.uk/2/hi/asia-pacific/6363843.stm (accessed March 31, 2007).

[3] Binmore, K. 1987. Modelling rational players. Part I. *Economics and Philosophy*, 3:179-214.

[4] Binmore, K. 1988. Modelling rational players. Part II. *Economics and Philosophy*, 4:9-55.

[5] Binmore, Ken. 1998. Game Theory and the Social Contract, Vol. 2: Just Playing. The MIT Press.

[6] Binmore, Ken. 1990. Essays on the foundations of game theory. Oxford: Basil Blackwell.

[7] Binmore, Ken. 1991. Fun and Games: A Text on Game Theory. D.C. Heath.

[8] Brattka, Vasco. 2001. Recursion and Computability over Topological Structures. *Electronic Notes in Theoretical Computer Science* 40:5.

[9] Bridges, Douglas S., and Fred Richman. 1991. A recursive counterexample to Debreu's theorem on the existence of a utility function. *Mathematical Social Sciences* 21, no. 2 (April): 179-182.

[10] Canning, David. 1992. Rationality, Computability, and Nash Equilibrium. *Econometrica* 60, no. 4 (July): 877-888.

[11] Debreu, Gerard. 1970. Economies with a Finite Set of Equilibria. *Econometrica* 38, no. 3 (May): 387-392.

[12] Fishburn, Peter C. 1970. Arrow's impossibility theorem: Concise proof and infinite voters. *Journal of Economic Theory* 2, no. 1 (March): 103-106.

[13] Gneezy, U., and A. Rustichini. 2000. A Fine is a Price. *The Journal of Legal Studies* 29 (1): 1-18. http://citeseer.ist.psu.edu/gneezy98fine.html (accessed March 29, 2007)

[14] Government of South Africa. 2006. Report on Incentive Structures of Social Assistance Grants in South Africa. Pretoria: Department of Social Development. http://www.welfare.gov.za/documents/2006/gps.pdf (accessed March 31, 2007).

[15] Feynman, Richard. 1974. Cargo Cult Science. http://calteches.library.caltech.edu/51/02/CargoCult.htm (accessed March 31, 2007).

[16] Hayek, F. A. 1945. The Use of Knowledge in Society.
The American Economic Review 35, no. 4 (September): 519-530.

[17] Jeffry L. Hirst. Notes on Reverse Mathematics and Brouwer's Fixed Point Theorem.
available online:- `http://www.mathsci.appstate.edu/~jlh/snp/pdfslides/bfp.pdf`.

[18] Lucas, W. F. 1968. A game with no solution.
Bull. Amer. Math. Soc. 74:237-239.

[19] Mihara, H. Reiju. 1999. Arrow's theorem, countably many agents, and more visible invisible dictators.
Journal of Mathematical Economics 32, no. 3 (November): 267-287.

[20] Milnor, John. 1998. John Nash, and *A beautiful mind* [Simon Schuster, New York, 1998] by S. Nasar.
Notices of the American Mathematical Society 45, no. 10: 1329-1332.

[21] Nasar, S. (1998). *A Beautiful Mind : A Biography of John Forbes Nash, Jr.* Simon & Schuster.

[22] Neumann, John Von, and Oskar Morgenstern. 1944.
Theory of Games and Economic Behavior. Princeton University Press.

[23] Orevkov, V.P. A constructive map of the square into itself, which moves every constructive point.
Dokl. Akad. Nauk SSSR, 152:55–58, 1963.

[24] Rasmusen, Eric. 2006. Games and Information: An Introduction to Game Theory. Blackwell Publishing Limited.

[25] Rosinger, Elemér E. 2004. Reconsidering Conflict and Cooperation.
math/0405065. http://arxiv.org/abs/math/0405065 (accessed July 1, 2007).

[26] Rosinger, Elemer E. 2005. The Nash-Equilibrium Requires Strong Cooperation.
math/0507013. http://arxiv.org/abs/math/0507013 (accessed July 1, 2007).

[27] Rassenti, Stephen, Stanley S. Reynolds, Vernon L. Smith, and Ferenc Szidarovszky. 2000. Adaptation and convergence of behavior in repeated experimental Cournot games.
Journal of Economic Behavior & Organization 41, no. 2 (February): 117-146.

[28] Smolin, L. (2007). *The Trouble With Physics: The Rise of String Theory, The Fall of a Science, and What Comes Next.* Mariner Books.

[29] Wong, Kam-Chau and Richter, Marcel K. 1996. Bounded rationalities and computable economies. Discussion paper 297, Minnesota - Center for Economic Research, December 1996. Available online:- `http://ideas.repec.org/p/fth/minner/297.html` [accessed 2004-12-30].

[30] Wong, Kam-Chau and Richter, Marcel K. 1999. Non-computability of competitive equilibrium.
Economic Theory, 14(1):1–27.

[31] Wong, Kam-Chau and Richter, Marcel K. 1999. Computable preference and utility.
Journal of Mathematical Economics, 32(3):339–354.