Resource Depletion, Growth, Collapse, and the Measurement of Capital

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28 January 2014

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February 28, 2014

Abstract

Growth is often treated as something like a general property of any well-managed economic system, but the sustainability of this has been called into question since the 1970s. The current paper argues that the main problem with growth statistics - measured in income or capital - lies in the way the measures are constructed. Any measure of the total value of capital relies on a common denominator of that value, a numeraire, the choice of which also determines the dynamic development of the value statistics. In some cases the resulting patterns may differ sharply. One such case is the depletion of natural resources. The current paper develops a simple model of a 4-good economy (two resources, two final products) with the slow (exogenous) depletion of resources. It is shown that the choice of the numeraire determines the form of the capital statistics. This result is confirmed for both Walrasian, heuristic, and local pricing models in a computer simulation.

Keywords: capital, economic collapse, resource depletion, price level, limits to growth

1 Introduction

It has been forty years since Meadows et al. (2004) first presented their report "The Limits to Growth" to the Club of Rome. Although the central

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message (the collapse of the economic system is to be expected in the first half of the 21st century) was received with mixed feelings, especially among economists, the question drew much attention, has spurrned several follow-up studies, and continues to be the topic of numerous scientific conferences.

Growth is often treated as something like a general property of any well-managed economic system and the essential purpose of economic policy. Though counterweighted by considerable inflation, growth is measurable. Chained price GDP per capita data for developed and developing countries indicate positive though fluctuating growth rates, hence exponential growth. The pattern is interrupted by occasional crises from which the exponential growth, however, quickly recovers. This implicit assumption of never-ending growth has been criticized by both Marxian economists and ecological economists. Both approaches employ a similar critique: the finiteness of labor (or 'variable capital') and energy or natural resources respectively which are not reflected in both the expectations for further growth and the volume of the nominal capital stock (and in the most recent decades also and especially that of the financial derivatives).

The core of the problem, however, may lie in the very foundation of how growth (and capital and wealth) is measured. In order to define a general measure of the total value of capital (or of consumption, or of production), a common denominator of that value has first to be chosen, a numeraire. In static systems, e.g. for the computation of Walrasian equilibria, this is perfectly appropriate. If the dynamic perspective is considered, however, the outcome differs greatly depending on which numeraire is chosen. In the real world this is generally accomplished with complex combinations of actual goods and bonds, i.e. in official currencies, the dynamic development of the value of which is watched over and steered by official bodies. The role of currencies in modern economies is certainly much more complex, but one aspect of it remains to provide a measure of value that is backed by, ultimately, natural ressources or some form of bonds and securities that are nevertheless not independent of them.

However, this is not the whole problem: As the development differs with the choice of different numeraires there may be systematically wrong assessments depending on what numeraire has been chosen. Trying to measure capital in US Dollar or some other currency simply sets this currency (in turn depending on some stock market index) as the numeraire. The core of the problem, however, lies not in slight shifts in values of stocks in the past but in (the incalculability of) future costs. This is especially striking - as will be shown in the current paper - with the depletion of resources necessary for the production of other goods. It may result in erratic movements of the capital statistics.

Separately, both issues discussed in this paper have already been raised in the literature: As detailed in the literature section below, many scholars have criticised the measurement of capital and many others contended that an economic collapse induced by overuse and eventual depletion of resources may follow in a not too far future. What is puzzling about this is the sharp contrast to the prodigious rates of economic growth much of the developed world has been witnessing for many decades with merely temporary interruptions. The current paper brings the two points - unreliable value statistics and price and capital statistics changes triggered by depletion resources - together. It may be speculated that some (though certainly not all[1]) of the tremendous growth of the past reflects not so much a real increase in wealth or capital but rather the way in which it was measured.

The current paper develops a simple model of an exchange economy that roughly follows the ecological economists' idea of limited resources: Agents acquire resources with the quantity of extracted resources gradually falling in time. Those resources are then transformed into final goods which provide 'utility' to the agents and are durable but have a stochastic lifetime, i.e. a small probability of deterioration for any time step. The final good is both a consumption and an investment good; the efficiency of final good production increases slowly but steadily over time. Further, in every time step agents have the opportunity to trade resources against final goods or vice versa; for this exchange procedure, both a neoclassical setting with a central Walrasian competitive equilibrium and a heterodox setting with agent-based heuristic decision making were studied. For both settings the established relative market prices (of resource and final good) may be used to study the development of total wealth (or total capital) in the model economy. With a resource as the numeraire, total capital follows the scarcity of that resource. The time series of total wealth with the final good as numeraire, however, exhibits continuous exponential growth - interrupted only by short periods of decline whenever the resource becomes a little more scarce - which goes on until the natural resource is exhausted at which point the economy - naturally

[1]There has been real growth in the economic capacity of the world (in some countries more than in others) and the technological progress since the rise of modern information technology was arguably unprecedented.

- collapses. The model was studied using a Python computer simulation.

Section 2 will review the relevant literature on both the critique of capital measures, the discussion of resource depletion, and economic growth. A theoretical model will be developed in section 3 which will form the basis for the simulation study in section 4 followed by a few concluding remarks in section 5.

2 Literature Review

The issue of resource depletion and the possible collapse of the economic system as a consequence of the depletion of resources is unsurprisingly a contentious one. While ecological economists like Georgescu-Roegen (1975b,a) and Daly (1968, 1993) (and many others) argue in favor of a planned reduction of economic resource exploitation (and therefore production), the economic mainstream (neoclassical, new-classical, and new-Keynesian approaches) held that either significant resource depletion was not imminent or that the market's price mechanism would guarantee an optimal resource exploitation path. Stiglitz sarcastically stated that there are "oscillations in the general views on the prospects for the future, alternating between the despair of imminent and inevitable doom and the euphoria of an impending new millenium, [which] have a remarkable regularity." (Stiglitz, 1980)

The two "views" mentioned by Stiglitz represent two different, even conflicting, approaches to the outlook on the future economic development. However, while a number of issues are strongly contested between these approaches, the one thing that has very rarely been called into question recently is the basis of measurement of what the entire discussion revolves around, of capital, of income, of economic output, and of prices in general. This is what the first part of this literature review will focus on. It will be followed by two more parts discussing the literature on limited or unlimited economic growth and on long-term forecasts of resource depletion and economic collapse.

2.1 Measuring Value: GDP, Capital, and Wealth

Virtually every modern model of economic growth is constructed to include total capital (in the economy) as a measure of productive capacity, of high-tech productive capacity, or at least - in the Marxian case - of exploitation. This abundance of models employing capital terms is contrasted by an unfortunate lack of data on total capital. While some progress has been made in recent decades and there are some datasets (e.g. the EU KLEMS database, see O'Mahony and Timmer (2009)), they are still only isolated records, regionally and temporally not necessarily consistent among each other. As Robinson put it, for statistics on capital (and profits) "it is necessary to define the value of the stock of capital, and that no one seemed able to do." (Robinson, 1978, p. xvi)

As capital statistics are hence often not available, the closest alternative which may be used to substitute them is income, i.e. GDP or GDP per capita, the flow variable corresponding to the stock variable capital. GDP is indeed widely used to determine the wealth and welfare of regions and countries, hence it is, in effect, taken as a proxy for capital. The question that arises from this would be: Why is the stock value so difficult to come by if we have the flow data series? Is there something wrong with it? As the following sections of the paper show, there is.

The basic problem of capital measurement is that neither is it homogenous nor is its composition - a problem famously discussed by Sraffa (1960). There are many different kinds of capital and the patterns and quantities in which they are arranged and employed change over time. In order to obtain a consistent data series on the development of total capital, one capital good (real or combined) has to be defined as the common unit of measurement, the numeraire. This choice also determines the dynamics which may deviate considerably from alternative numeraires (see the following sections). Pasinetti (1993) discussed this problem as the "two degrees of freedom" of prices: The prices as such offer one degree of freedom since they are only determined in relative terms but not in absolute values, i.e. the value measure or price of exactly one good may be arbitrarily chosen. In economic models, usually the price of the numeraire is set to 1. Another degree of freedom results from the dynamic price change, since for the next period, the prices are again only determined in relative terms, i.e. for the next period exactly one price may again be arbitrarily chosen. In economic models, the price of the numeraire is usually again set to 1, in turn defining the value of the numeraire to be constant.

Since capital is not used in empirical studies to that extent anyway, so far, the problem may be a purely theoretical one.[2] Unfortunately, the same goes

[2]One might stress that stock values are, however, some form of - widely available and non-ambiguous - capital statistics.

for the GDP (and the GDP per capita) which is indeed widely used not only as a measure for the "wealth of nations" but also also as the essential key statistic for assessing the past and future economic development of countries and regions.

Interestingly, Kuznets (1934), in the very report to the US senate, in which he proposed how to measure "national income" which would later form the basis of the today well-established GDP computation (Vanoli, 2008), stated: "(...) the definiteness of the result suggests, often misleadingly, a precision and simplicity in the outlines of the object measured. Measurements of national income are subject to this type of illusion and resulting abuse, especially since they deal with matters that are the center of conflict of opposing social groups where the effectiveness of an argument is often contingent upon oversimplification." (Kuznets, 1934, p. 5-6) He specifies a number of limitations including that not all economic activity can be captured in this measure (housework), that part of the measurement does not constitute true economic activity (rents), and most importantly price changes. The latter qualification is equivalent to the limitation discussed for capital above and in more detail in the following sections. Kuznets goes on to note that "Notions of productivity or welfare as understood by the user of the estimates are often read by him into the income measurement, regardless of the assumptions made by the income estimator in arriving at the figures." (Kuznets, 1934, p. 7) And: "All that the national income estimator can say is that this or the other part of the national total has increased or declined more than the others." Kuznets (1934, p. 7) However, today, in complete defiance of the warning by its inventor, the GDP is the single most widely used measure or basis of computation for both welfare, productivity, wealth, and capital. It still suffers from all the pitfalls stated by Kuznets but is, for the lack of a better alternative, used nevertheless.

The problem is widely acknowledged among neo-Ricardians and Sraffians (compare Mongiovi (2003)) who defy the mainstream economic theory of value and prices anyway. (Sraffa, 1926, 1960) The same is true for Pasinetti (1993) and other post-Keynesians and some other heterodox economic traditions (compare Fine (2003)). But it has yet to make its way into the mainstream of the profession, while much of growth theory and ecological economics (that will be discussed in the remainder of this section) also remain ignorant of it.

2.2 Economic Growth and the Finiteness of Resources

There are a number of approaches to economic growth which with respect to their predictions regarding the long-term development of the system fall roughly into three categories: the Solow-Swan (Solow, 1956, 1957, Swan, 2002 [1956]) theory and similar approaches see growth as a development towards a unique and stable steady-state equilibrium which is continuously shifted (elevated) by a steady technological progress that is exogenous to the theory. The AK-theory, also by Solow (1956) but rooted in earlier theories by Harrod (1939), Domar (1946), and others, and later refined into the theories of endogeous growth (Uzawa, 1965, Cass, 1965, Lucas, 1988) on the other hand leads to a never-ending exponential growth. A third group sees the observed growth as partly or entirely illusive on the grounds that production is directly linked to certain factor inputs (labor or natural resources including energy) which in turn are finite. This category noteably comprises ecological economic (Georgescu-Roegen, 1975b,a, Daly, 1968, 1993) theories but also Marxian (Marx, 1963 [1885]) economics. Specifically ecological economics concludes that the economic system does (or will in the near future) exceed the capacity of its environment and will consequently collapse as soon as the available resources are exhausted. Others[3] (e.g. Minsky (1980), Keen (1995), Michl and Foley (2007)) have proposed more unconventional models with much more complex dynamics which may - contrary to less complex approaches - have the power to conveniently explain economic crises, cycles, and other irregular patterns that are observed empirically.

What can be observed from real-world growth data (such as the GDP per capita growth of the US over the last decades) is a general overlinear growth pattern interrupted by occasional crises (the most evident being 1929 and 2008, and possibly some smaller events in for instance 1975, 1987, 2001) and overlayed by a pattern of growth waves or business cycles.[4] It is obvious that particularly AK-type theories may be able to fit this pattern relatively

[3] Still other approaches such as recent econometric advances investigating the growth effect of knowledge, spillovers, and many other aspects (see for instance Frenken et al. (2007) or, for a more recent overview, McCann and Ortega-Argilés (2013)), or Schumpeterian evolutionary economics Nelson and Winter (1974, 1982), Silverberg and Lehnert (1993) focus on other aspects like microfoundations, the role of technology, geography, etc. and do not assume a homogeneous view on the long term development.

[4] On the debate on the existence, form and causes of business cycles, see Samuelson (1939), Goodwin (1967), Lucas (1981), Chen (2002), Freeman and Perez (1988), de Groot and Franses (2008, 2009).

well. Other theories sometimes accomplish this by attributing large parts of the observed growth to a growing exponential (technology) parameter - in effect, including an AK-type feature into the model. What remains largely unexplained is the occurence of crises and how unlimited exponential growth could possibly be achieved in the real world.[5]

Ecological economist's concerns were more practical: Both Georgescu-Roegen (1975b,a) and Daly (1968, 1993) pointed out that the current mode of economic production relies heavily on non-renewable (or slowly-renewable) resources and will prove impossible to be sustained as the resources start to run out. More generally, economics is not independent from physics; hence the second law of thermodynamics applies, every activity creates entropy and consumes energy - a process that is limited to the amount of energy available (Georgescu-Roegen, 1975b,a). As entropy increases, it will be impossible to sustain economic activity or any life at all. While Georgescu-Roegen did not claim that this event was in any way imminent, the depletion of some of the most important resources for the contemporary economy is.

Taking note of these debates and the ones taking place at the same time in ecology (see below), most economists - while admitting the theoretical possibility that resources run out eventually and that this causes severe problems - held that this was likely not imminent. As it was put in 1980 by Stiglitz (1980), "although there are undoubtedly market failures leading to inefficiencies in resource allocation, there is no reason to believe they are worse here than elsewhere." Some theoretical models of resource depletion tended to ignore the concerns harboured by ecologists and ecological economists and followed one of several possible equilibrium approaches, the most important one being the model by Hotelling (1931). This model assumes a present value function P_0 for the profits obtained from an exhaustable resource at time t (PV_t) of $PV_0 = PV_t e^{-\gamma t}$ where e^{-t} is a regular exponential time preference function calibrated with parameter γ to include both preferences and the interest rate. As the fair price of the resource is equivalent to the present value, the price follows a growing exponential function $p_t = p_0 e^{\gamma t}$ which grows with factor $\frac{\partial p_t / \partial t}{p_t} = \gamma$. This prediction is derived from assumptions regarding rational individuals, their time preferences and the interest rate. The increase of the price with increasing scarcity is probably realistic but the at-

[5]Solow famously slammed endogenous growth theory models as "very unrobust" since they "cannot survive without exactly constant returns to capital" and would otherwise converge to a steady state (like the Solow-Swan model) or "exceed any stated bound before Christmas" (Solow, 1994).

tempted quantification of this increase may argueably be doomed to fail (as also recognized by more recent theorists (Krautkraemer, 1998, Krautkraemer and Toman, 2003)). Models of this type (with some modifications) continue to be used (Krautkraemer, 1998, Krautkraemer and Toman, 2003), the concept is known as the Hotelling rule. The fact that the proponents of this approach are unable to see the predicted price dynamics leads them to conclude: "These other factors, particularly the discovery of new deposits and technological progress that lowers the cost of extracting and processing nonrenewable resources, appear to play a relatively greater role than finite availability in determining observed empirical outcomes." (Krautkraemer and Toman, 2003) It is, however, generally not considered that uncertainty, interaction dynamics, "animal spirits", and the very assumption that resource depletion is not imminent may alter the price dynamics significantly.

On the other hand, there are models of the transition from the current system based on energy from non-renewable sources to a future more sustainable economic system. One example that is also particularly close to the analysis in the current paper is that by Michl and Foley (2007). They develop a complete macro-economic model and investigate both the development and the distribution of income and capital. Modelling the price development of oil with a Hotelling rule similar to the one discussed above they find - among other conclusions - that the depletion of resources leads to a peak and subsequent slight fall of the wealth statistic which entails portfolio effects, a change in the structure of capital, and shifts in the distribution of income which is the main focus of the article. Some of those dynamics, namely the development of the wealth statistic, are reproduced with a different mechanism in the model presented in the following section. However, the focus of the model in the current paper is different.

2.3 Ecological Economics and Ecology

As mentioned, many ecological economists, first and foremostly Georgescu-Roegen (1975b,a) and Daly (1968, 1993), have argued on the grounds of limited natural resources and energy as well as the thermodynamic entropy laws that growth is also limited. They went on to note that any ecological carrying capacity of the planet would also be limited to a certain (possibly large) timespan, contrary to what had been considered in the study Meadows et al. (2004) had conducted for the Club of Rome (Georgescu-Roegen, 1975b). As any carrying capacity would be exceeded more rapidly by an

exponentially growing economy, the obvious suggestion is the transition to a steady state economy through sparing use of depletable resources (Daly, 1993).[6] Georgescu-Roegen (1975b) concedes that the mainstream models-generate a price mechanism that allows to temporarily ease resource scarcity and allocate resources efficiently within the precepts of a growing economy but suggests that this does not affect the main point of ecological economics.
In ecology on the other hand there is a long tradition that has discussed the implications of the finiteness of resources for a long time. Bardi (2011a) provides an extensive overview starting with Hubbert's model more than ten years before Meadows et al. published their report, covering Meadows et al.'s study from 1973 as well as the ensuing debate and more recent findings. Meadows at al. (2004) found the observed resource consumption of the global economy to be beyond earths long term carrying capacity. Though this has been disputed more recent studies (Turner, 2008, Bardi, 2011a) hold that the predicted dynamics may be roughly accurate. The predicted collapse includes a significant raduction in available resources in the near future, a decrease in industrial output and food production after about 2025 and as a direct consequence a decline of the world's population.
As pointed out by Bardi (2011b), some models predict the development of resource extraction, as well as production and several other variables as following regular, often symmetric bell shapes. This, however, is not likely, as several real world examples show and as Meadows et al.'s (2004) report correctly anticipates. Bardi observes that this is due to delayed consequences (in Bardi's argument the consequence of pollution resulting from contemporary economic production); he terms this the "Seneca effect".
Bardi (2011a,b) (following earlier studies including that by Meadows et al.) shows that for the production and depletion of resources, the development is indeed bell shaped as evidenced from several empirical examples including whales, oil in the US, gold and ore in South Africa.
One effect, however, has largely been neglected: the economic system is not a lose collection of unrelated production processes, but in fact highly integrated. If, for instance, resources run out unanticipatedly this will cause a massive disruption of the entire system. Early anticipation may help to substitute the resource in question by others or to concentrate economic effort on developing other parts of the system and making it independent from the resource that will run out. If there is, however, not sufficient time or willing-

[6] I.e., the time this carrying capacity lasts should be extended as far as possible.

ness to accomplish this, a massive downturn will ensue anyway as soon as this becomes obvious creating an effect similar to the "Seneca effect" mentioned above but unrelated to pollution (perhaps preceeding or aggravating the pollution effect). In the following sections, a simple model with an example of such an effect manifesting itself in the financial system will be presented.
Finally, it should be mentioned that - different from what the simple benchmark case in the following section might suggest - resource depletion does not have to be a continuous process in time. In fact, it almost never is. Investors and decision makers in firms and in regulatory bodies are aware of the possibility of the final depletion of a resource, they will try to substitute the resource for others and to replace the technologies dependent on the depleting resource by others. At the same time, extraction of what is left of the resource tends to become more costly, the price of the resource tends to increase as well as the price of the final goods from it - but so will the value of the capital stock invested in such goods. This may create a deceptive illusion of an increase in total capital. The dynamics induced by this are not immediately obvious; they will be studied in more detail in the following section.[7]

3 A Simple Model of Resource Depletion, Prices, and the Capital Stock

In order to investigate the effects of ressource depletion on growth and specifically measurement of capital and output this section will outline a simple Walrasian competitive exchange model in an economy with four goods. Four regimes will be considered covering a spectrum from fairly neoclassical settings to non-neoclassical agent-based models (the latter ones discussed in detail in section 4, the simulation section): (1) centralized Walrasian competitive exchange with a neoclassical auctioneer and accurate expectations, (2) centralized Walrasian competitive exchange with a neoclassical auctioneer and overstated expectations, (3) decentralized Walrasian competitive exchange on a ring topology, and (4) exchange with heuristic price setting on a

[7]Further, exploitation of resources is often (if not always) subject to heavy economies of scale making decisions of large players - specifically governments of souverain states - much more important in this system. As a discussant at the 2012 EAEPE conference, where an earlier version of the current work was presented, put it, the availability of depletable resources is essentially a function of policy.

ring topology. The fourth regime is completely independent from the following Walrasian exchange model and serves the purpose of showing that the results are generalizable and specifically that they are preserved under much less restrictive assumptions.
Consider the following basic setting:
There are four goods, two exhaustable resources R_1 and R_2, two final goods X_1 (made of resource R_1) and X_2 (made of resource R_2). Resources are extracted from the environment and immediately (from one period t to the next) used up for production of the respective final good. The number of final goods produced per resource is determined by an economy-wide and slowly increasing[8] technology level τ such that for any agent $X_{1,t+1} = \tau R_{1,t}$ and $X_{2,t+1} = \tau R_{2,t}$ (the values are rounded so that agents do only produce whole numbers of goods). Final goods may exist for several periods, but for any instance of a final good there is a probability of 10% per time period that it degrades (and vanishes). The number of resources extracted by an agent is subject to small stochastic fluctuations (so that trading makes sense). The resource is slowly depleted, i.e. the expected value of extracted resources per period decreases, specifically, R_2 depletes faster than R_1.
The following models use neoclassical standard utility functions of current consumption opportunities (the quantities of final goods) and future production of final goods (the quantities of resources weighted by the technology level τ). This is a simple form of intertemporal optimization without, however, using an infnite horizon optimization.[9] They do, however, not include perfect information, that is, the agents are only informed about the quantities of goods that are currently available in the economy; they are not aware of the eventual depletion of the resources before it occurs.

3.1 Centralized Walrasian Competitive Exchange

The agents' utility function is assumed to be logarithmic. As τR_i goods X_i can be produced from R_i units of resources, in the setting with accurate

[8]For the below simulations, an exponential function with constant growth rate $\tau = 1.001^t$ is used.

[9]There are plenty of reasons to avoid infinite dimensional optimization models, the most important ones being that people do not have the capacity to solve such problems as they go and that studies have observed usage of heuristics (including systematic errors) instead of exact calulations.Tversky and Kahneman (1972), Kahneman and Tversky (1973)

expectations, agents follow the utility function[10]

$$U(R_1, R_2, X_1, X_2) = log(X_1) + log(X_2) + log(\tau R_1) + log(\tau R_2) \tag{1}$$

With endowments W_{Ri} and W_{Xi} (with $\sum_i p_{Ri}W_{Ri} + \sum_i p_{Xi}W_{Xi} = \overline{W}$), and prices p_{R1}, p_{R2}, p_{X1}, and p_{X2}, this yields a Lagrangian

$$\mathcal{L}(X_i, R_i, W_{Ri}, W_{Xi}, \lambda) = \sum_i (log(X_i) + \sum_i log(\tau R_i)) - \lambda(\sum_i (p_{Ri}R_i + p_{Xi}X_i) - \overline{W}) \tag{2}$$

yielding as first order conditions

$$\begin{array}{ll} \frac{\partial \mathcal{L}}{\partial R_i} = 0 & = \frac{1}{R_i} - \lambda p_{Ri} \\ \frac{\partial \mathcal{L}}{\partial X_i} = 0 & = \frac{1}{X_i} - \lambda p_{Xi} \end{array} \tag{3}$$

as well as the above budget constraint. Resolving one first order consdition (for instance that for R_1) for λ and substituting into the others yields

$$p_{R1}R_1 = p_{R2}R_2 = p_{X1}X_1 = p_{X2}X_2 \tag{4}$$

which, substituted into the budget constraint gives the offer curves

$$\begin{array}{ll} R_1 = & \frac{\overline{W}}{4p_{R1}} \\ R_2 = & \frac{\overline{W}}{4p_{R2}} \\ X_1 = & \frac{\overline{W}}{4p_{X1}} \\ X_2 = & \frac{\overline{W}}{4p_{X2}} \end{array} \tag{5}$$

and as the prices are the same for everyone the aggregate demand for R_1 (for R_2, X_1, and X_2 analogous)

$$R_1 = \frac{\sum \overline{W}}{4p_{R1}} = \frac{1}{4}\left(\frac{p_{R1}}{p_{R1}}\sum W_{R1} + \frac{p_{R2}}{p_{R1}}\sum W_{R2} + \frac{p_{X1}}{p_{R1}}\sum W_{X1} + \frac{p_{X2}}{p_{R1}}\sum W_{X2}\right) \tag{6}$$

[10]The idea being that current consumption possibilities X_i and future consumption possibilities τR_i are multiplied. Utility function (1) is equivalent to $U(R_1, R_2, X_1, X_2) = log(X_1X_2\tau R_1 \tau R_2)$. Note that the quantities of the resource in this utility function represent a preference for the capacity of future production of final goods, not for the posession or consumption of the resource as such.

As the market is required to be in equilibrium, this (demand) has to equal supply, $\sum W_{R1}$, yielding

$$p_{R2}\sum W_{R2} + p_{X1}\sum W_{X1} + p_{X2}\sum W_{X2} = 3p_{R1}\sum W_{R1} \tag{7}$$

and analogous for R_2, X_1, and X_2. Thus, the equilibrium prices are indirectly proportional to the sum of endowments, i.e. after choosing one of the goods, say R_1 as numeraire (setting $p_{R1} = 1.0$) the other prices are obtained as

$$\begin{array}{rl} p_{R2} = & \frac{\sum W_{R1}}{\sum W_{R2}} \\ p_{X1} = & \frac{\sum W_{R1}}{\sum W_{X1}} \\ p_{X2} = & \frac{\sum W_{R1}}{\sum W_{X2}} \end{array} . \tag{8}$$

As demand equals supply ($R_i = \sum W_{Ri}$, $X_i = \sum W_{Xi}$), this can also be written as

$$\begin{array}{rl} p_{R2} = & \frac{R_1}{R_2} \\ p_{X1} = & \frac{R_1}{X_1} \\ p_{X2} = & \frac{R_1}{X_2} \end{array} . \tag{9}$$

3.2 Centralized Walrasian Competitive Exchange - Modified Utility Function That Allows Zero Quantities

It should be noted, however, that the above utility function (1) is not defined for quantities of 0 of any of the four goods.[11] It can thus strictly speaking not be used to model resource depletion. Nevertheless, a slight transformation which preserves all important properties of the model leads to a function that behaves smoothly for good's quantities becoming 0 with a utility function

$$U(R_1, R_2, X_1, X_2) = log(X_1+1)+log(X_2+1)+log(\tau(R_1+1))+log(\tau(R_2+1)) \tag{10}$$

The budget constraint remains unchanged ($\sum_i p_{Ri}W_{Ri} + \sum_i p_{Xi}W_{Xi} = \overline{W}$); the Lagrangian becomes

$$\begin{array}{rl} \mathcal{L}(X_i, R_i, W_{Ri}, W_{Xi}, \lambda) = \sum_i (log(X_i+1) & +\sum_i log(\tau R_i + 1)) \\ & -\lambda(\sum_i (p_{Ri}R_i + p_{Xi}X_i) - \overline{W}) \end{array} \tag{11}$$

[11] Since $log(0) \rightarrow -\infty$.

yielding as first order conditions

$$\begin{array}{rl} \frac{\partial \mathcal{L}}{\partial R_i} = 0 & = \frac{1}{R_i+1} - \lambda p_{Ri} \\ \frac{\partial \mathcal{L}}{\partial X_i} = 0 & = \frac{1}{X_i+1} - \lambda p_{Xi}. \end{array} \tag{12}$$

This results in the price-quantity relations (4) becoming

$$p_{R1}(R_1+1) = p_{R2}(R_2+1) = p_{X1}(X_1+1) = p_{X2}(X_2+1) \tag{13}$$

and the offer curves

$$\begin{array}{rl} R_1 = & \frac{\overline{W}}{4p_{R1}} - 3p_{R1} + p_{R2} + p_{X1} + p_{X2} \\ R_2 = & \frac{\overline{W}}{4p_{R2}} - 3p_{R2} + p_{R1} + p_{X1} + p_{X2} \\ X_1 = & \frac{\overline{W}}{4p_{X1}} - 3p_{X1} + p_{R1} + p_{R2} + p_{X2} \\ X_2 = & \frac{\overline{W}}{4p_{X2}} - 3p_{X2} + p_{R1} + p_{R2} + p_{X1} \end{array} \tag{14}$$

Substituting the budget constraint for $\overline{W}$ (as in the model above), applying the equilibrium condition (demand must equal supply, $R_1 = W_{R1}$ and equivalently for R_2, X_1, and X_2), and resolving the system for the prices (setting R_1 as numeraire) yields

$$\begin{array}{rlll} p_{R1} = & 1 & & \\ p_{R2} = & \frac{\sum W_{R1}+1}{\sum W_{R2}+1} & = & \frac{R_1+1}{R_2+1} \\ p_{X1} = & \frac{\sum W_{R1}+1}{\sum W_{X1}+1} & = & \frac{R_1+1}{X_1+1} \\ p_{X2} = & \frac{\sum W_{R1}+1}{\sum W_{X2}+1} & = & \frac{R_1+1}{X_2+1} \end{array}. \tag{15}$$

3.3 The Capital Stock with Prices Obtained through Centralized Walrasian Competitive Exchange with Accurate Expectations

For the first and more simple model it follows immediately (from equations (4) and (9)) that the overall capital stock, measured in monetary units with R_1 as numeraire is

$$C_{R1} = R_1 + \frac{R_1}{R_2}R_2 + \frac{R_1}{X_1}X_1 + \frac{R_1}{X_2}X_2 = 4R_1. \tag{16}$$

The system is symmetric, thus, if the capital is measured with R_2, X_1, or X_2 as numeraires

$$\begin{array}{rl} C_{R2} = & 4R_2 \\ C_{X1} = & 4X_1 \\ C_{X2} = & 4X_2. \end{array} \tag{17}$$

Similarly, for the modeified version, equations (13) and (15) lead to an overall capital stock of

$$\begin{aligned} C_{R1} &= R_1 + \tfrac{R_1+1}{R_2+1}R_2 + \tfrac{R_1+1}{X_1+1}X_1 + \tfrac{R_1+1}{X_2+1}X_2 \\ &= 4R_1 + \textstyle\sum_i(1-\tfrac{p_{Ri}}{p_{R1}}) + \sum_i(1-\tfrac{p_{Xi}}{p_{R1}}). \end{aligned} \tag{18}$$

which, measured with the other goods as numeraires, however, takes very different values.

$$\begin{aligned} C_{R2} = &\quad 4R_2 + \textstyle\sum_i(1-\tfrac{p_{Ri}}{p_{R2}}) + \sum_i(1-\tfrac{p_{Xi}}{p_{R2}}) \\ C_{X1} = &\quad 4X_1 + \textstyle\sum_i(1-\tfrac{p_{Ri}}{p_{X1}}) + \sum_i(1-\tfrac{p_{Xi}}{p_{X1}}) \\ C_{X2} = &\quad 4X_2 + \textstyle\sum_i(1-\tfrac{p_{Ri}}{p_{X2}}) + \sum_i(1-\tfrac{p_{Xi}}{p_{X2}}). \end{aligned} \tag{19}$$

Note that the two sums are small and have limited impact on the scale of the resulting value; the capital values are dominated by the first part ($4R_1$ in the case of C_{R1} etc.).

It is obvious that with very different developments of the availability (number or endowment) of these different resources and goods in the economy, the four possible bases of measurement will drift apart as well. As the resource depletion dynamics ultimately dominates the rest of the system (no matter how fast the technology τ grows), the capital stock as measured with one of the resources as numeraire are relatively good indicators of the true state of the system, while the capital value measured with one of the final goods as numeraires may be very misleading. (See the simulation section for an assessment of how misleading this may exactly be.)

Real world economic systems do not adhere the precepts of such a simplified neoclassical system but there is nevertheless some basis of measurement for evaluating the capital stock (the price level). It is not generally certain that this basis is an accurate indicator for the state of the system, especially in the light of the current argument and the complete lack of consensus regarding the incidence and severity of resource depletion that is to be expected in the near future. This is why Marxian and Chartalist[12] economists can treat the value of capital or money as a relatively illusive quantity; it may also contribute to the explanation why again and again in times of crisis large sums of money can easily and unanticipatedly simply vanish into thin air.

[12]For a detailed discussion of this issue from a Chartalist perspective see for instance Wray (1993).

3.4 Overstated Expectations and Centralized Walrasian Competitive Exchange

Expectations have always played a crucial role in modern economy, crisis after crisis hits the economy unexpectedly, generations of investors have lost their money and the memory of the financial system is fairly short. Overstated expectations are, as observed by Minsky (1980) and others, probably a major factor in keeping growth waves in the economy going until both the growth and the expectation collapses.[13] Much of the Keynesian and especially the post-Keynesian theory discusses processes of alternating overvaluation and undervaluation of financial assets with overvaluation obviously preceeding crashes, crises, and economic downturns.[14] As the main point of interest here is the crash following the depletion of a resource, this part of the model does not concern itself with modeling undervaluation and concentrates on introducing an effect of overvaluation.

While Minsky's and other models in this field are more complex, an overvaluation mechanism (isolated from other aspects of said theories) can easily be included in the above model. For this, the real part of the model, resource extraction, technology, and production are retained exactly as before; what changes is the expectations of the agents, thus the utility function they use for optimization and as a consequence also the prices and the capital measures. Let the overstated expectation be that the technology level affects production of the final good not linearily but exponentially, i.e. the utility function (1) becomes

$$\begin{aligned} U &= log(X_1) + log(X_2) + log(R_1^\tau) + log(R_2^\tau) \\ &= log(X_1) + log(X_2) + \tau log(R_1) + \tau log(R_2). \end{aligned} \tag{20}$$

The first order conditions change accordingly, equation (4) is altered to

$$\frac{1}{\tau} p_{R1} R_1 = \frac{1}{\tau} p_{R2} R_2 = p_{X1} X_1 = p_{X2} X_2 \tag{21}$$

[13]This is followed in Minsky's theory by a period of lack of confidence in the economy on the part of the financial sector which delays the recovery. Of course, Minsky's theory has other and more complex aspects which cannot be discussed in this context.

[14]For a more recent example, consider the recent special issue of the Journal of Economic Methodology on 'Reflexivity and Economics', for instance the article by Shaikh (2013) that also includes a brief historical overview.

leading to the offer curves (5) being now

$$\begin{array}{rl} R_1 = & \frac{\overline{W}}{2p_{R1}}\frac{\tau}{1+\tau} \\ R_2 = & \frac{\overline{W}}{2p_{R2}}\frac{\tau}{1+\tau} \\ X_1 = & \frac{\overline{W}}{2p_{X1}}\frac{1}{1+\tau} \\ X_2 = & \frac{\overline{W}}{2p_{X2}}\frac{1}{1+\tau} \end{array} \tag{22}$$

and yielding prices (again with R_1 as numeraire, $p_{R1} = 1.0$)

$$\begin{array}{rl} p_{R2} = & \frac{\sum W_{R1}}{\sum W_{R2}} \\ p_{X1} = & \frac{\sum W_{R1}}{\sum W_{X1}}\frac{1}{\tau} \\ p_{X2} = & \frac{\sum W_{R1}}{\sum W_{X2}}\frac{1}{\tau}. \end{array} \tag{23}$$

As a consequence, the capital stock measurement (equations (16) and (17)) changes into

$$C_{R1} = R_1 + \frac{R_1}{R_2}R_2 + \frac{R_1}{X_1}X_1\frac{1}{\tau} + \frac{R_1}{X_2}X_2\frac{1}{\tau} = 2R_1 + \frac{2}{\tau}R_1. \tag{24}$$

and

$$\begin{array}{rl} C_{R2} = & 2R_2 + \frac{2}{\tau}R_2 \\ C_{X1} = & 2(1+\tau)X_1 \\ C_{X2} = & 2(1+\tau)X_2. \end{array} \tag{25}$$

i.e. it is understated when measured with resources as a numeraire, but greatly overstated when final goods are used as numeraire. Moreover, the development of the capital stock gains here an exponential shape resembling real-world growth data (see also the according simulation in the following section).

Note that in this setting again the limit case of absolute depletion of one (or more) of the goods is undefined as the utility function results in a $log(0)$ expression. As in the above case with realistic expectations, a modified utility function can easily be obtained by adding 1 to the terms in the logarithmic functions. This results in a more complicated set of equations with the same properties as those discussed in this section. That model will be used in the simulations preseted in section 4.2.

3.5 Local Exchange

Another, possibly more realistic setting is that without a neoclassical almighty benevolent auctionator who provides the agents with true and fair market

prices for free. Rather, agents will in this setting only engage in trade (exchange) with their direct neighbors. As a simple regular network structure a ring network shall be assumed. Such settings are known from studies by Albin and Foley (1992) to slower adjustment to altered environmental conditions (such as the depletion of a resource) and less erratic behavior but possibly more complex dynamics and likely multiple equilibria. It is not conveniently possible to analyse this system deterministically; as a hypothesis, it may, however, be expected that the dynamics described above especially the greatly diverging observed capital stock dynamics depending on which good is used as a numeraire will be retained.
Two settings with local exchange will be simulated in the following section, one using the pairwise Walrasian competitive equilibrium (as described above, just computed separately for any two neighbors) and a setting with a heuristic (non-neoclassical) mode of exchange.

4 A Simple Computer Simulation

The model described in the previous section is for all four settings simulated with a Python program[15] with 128 agents for 5000 periods. Exploitation of resources starts with an expected value of 24 per iteration and agent (each of the two resources), they deplete stepwise by a factor of 0.2 every few periods,[16] resource R_2 about twice as fast as resource R_1, leading to the overall production of R_2 (and consequently also X_2) being reduced to 0 shortly before iteration 4000. A stochastic disturbance is applied to the number of resources extracted by every agent so that the endowments of the agents become heterogenous[17]; otherwise no exchange would occur. The disturbance is a uniform distribution between 0 and twice the expected value. While final goods last some time (each instance has a degeneration probability of 10% per iteration), resources are consumed immediately to produce final goods. The technology level τ which governs how many units of final goods may be produced by a unit of resource follows the function $\tau = 1.001^t$. This yields

[15]The source code is available from the author on request.

[16]The expected value for the extraction of R_2 is reduced by 2% whenever the time index is a multiple of 30, 40, or 70. This expected value for R_1 is reduced by the same factor (0.2) in only every second of those time steps, i.e. about half as fast.

[17]The quantity of extracted resources is distributed uniformly between 0 and twice the expected value. As this occurs on the agent level the law of large numbers applies and total number of resources follows the expected value very closely.

as the "real" part of the simulated economy the development of the stocks of goods (depicted in figure 2). This "real" part is common to all four simulation settings.
The simulation program records relative prices as well as the capital stock as measured in four different ways (with the four goods as numeraires). For the centralized exchange settings (the first two), this is straightforward; for the latter two (local exchange) settings, it requires averaging. To approximate system-wide price-levels, the arithmetic mean of the prices of all interactions shall be used in the first case, the geometric mean in the second case.[18] In case the price is unmeasurable (no exchange of the respective good occurs) it is (for technical purposes) recorded as $p_i = 1.0$ (which means that the measure breaks down when no exchange of that good occurs).

4.1 Centralized Walrasian Competitive Exchange with Accurate Expectations

The basic neoclassical setting[19], shown in figures 3a, 4a, and 4b, confirms the theoretical analysis given above. Capital development is falling exponentially if measured with resources or the second final good (the one depending on the more quickly depleting resource) as numeraires, growing exponentially for the other final good as numeraire. Prices develop smoothly until the availability of the second resource grows very low around iteration 3850 (about 10 to 20 units extracted in the entire economy) at which point the price moves sharply upward and the capital stock statistics respond accordingly (though this is not as evident as in the below setting with overstated expectations). When the resource depletes, followed by the corresponding final good as soon as all remaining units of this good have degenerated, the capital stock breaks down by 25% (each, in all 50%) which is also obvious from the theoretical analysis above. This is followed by a return to normal patterns in the capital measurements with the remaining two goods as numeraires, specifically by a

[18] This is necessary in this setting in order to keep the price matrix symmetric. In this setting, all 12 relative prices between the 4 goods are treated as independent and have to be aggregated independently. As an example: Suppose two interactions of the exchange of two goods X_1 and X_2 are observed; the "prices" are $X_1/X_2 = 1/4$ and $X_1/X_2 = 1/9$. Arithmetic averaging would yield $X_1/X_2 = (1/4 + 1/9)/2 = 13/72$ but $X_2/X_1 = (4+9)/2 = 13/2$, hence $X_1/X_2 \neq \frac{1}{X_2/X_1}$; using the geometric mean, however, $X_1/X_2 = \sqrt{1/4 \times 1/9} = 1/6$ but $X_2/X_1 = \sqrt{4 \times 9} = 6$.

[19] This simulation follows the model given in section 3.2.

newly resuming growth when measured in final goods (since the technology level continues to rise).

4.2 Overstated Expectations and Centralized Walrasian Competitive Exchange

Changing the agent's expectations as described in the above section[20] 3.3 leaves the dynamics largely intact with the effects however becoming more pronounced (figures 5a, 6a, and 6b). The capital statistics with both final goods as numeraires are now exponential; the dynamics start to slow considerably earlier, at about iteration 3400. However, this is at first limited to the statistics concering the second, more quickly depleting resource and the corresponding final good. The breakdown occurs at the same time as in the above setting, which is also when the slowdown is first apparent in the capital stock time series as measured with the other final good as numeraire. As in the above setting, exponential growth resumes (with the first final good as numeraire) after the depletion of the first resource (and the accordingly significant reduction in overall capital) and - for final good 1 as numeraire - easily surpasses the level that had been attained immediately before the downturn.

4.3 Local Walrasian Competitive Interaction

In a setting without the global almighty auctionator, but with local interaction on a regular ring topology (i.e. every agent has exactly two neighbors with whom she may engage in good exchange, figures 7a, 8a, and 8b), the dynamics are similar.[21] The scarcity in the depleting resource is more quickly translated into the capital statistics with other goods as numeraire. The growth even when measured with final good 1 (X_1) as numeraire is only linear, not exponential and is curbed from about period 2500 on. This is presumably because scarcity will also be more pronounced (and thus quicker apparent) if the radius of possible exchanges is limited to the immediate neighbors. Just preceeding the depletion of R_2, the capital development

[20] The model followed here is that from section 3.3 with a modified utility function to allow depletion of resources (i.e. 1 is added to the terms in the logarithm functions).

[21] This model does again use the equations from section 3.2, though locally, between just two agents at a time.

(measured with X_1 as numeraire) experiences a sharp spike upward. However, as the system comes closer to the depletion of resource R_2, the price mechanism starts to break down (as indicated by prices for R_2 and X_2 approaching 1.0, the default value if no exchange occurs).
It may be seen as a more realistic model of economic reality taking into account that a central auctioneer does not really exist, that optimization on a global scale does probably not happen, and that geography and transport and transaction costs matter (as does the institutional environment of the agents). Even if not as pronounced as in the above settings, the general dynamics analysed above can be reproduced for the current setting.

4.4 Local Heuristic Interaction

A setting with local (still on a ring topoligy) and heuristic (as opposed to competitively optimized) exchange (figures 9a, 10a, and 10b) remains close to what has been observed for the other local interaction setting. Price setting in this treatment is not based on optimization of any kind but rather chosen from a uniform distribution between the reservation prices of the two agents involved in the exchange. Goods are traded pairwise (i.e. exchange of e.g. R_1 against X_2) successively in a random order instead of all 4 at once. The reservation price, in turn corresponds to the agent's current endowment with two goods.[22] Here, the transition that comes with the downturn brought about by the final depletion of a resource is a little less intense as if it were anticipated by earlier local shortages. Also, the erratic spike of the capital measurement time series for the treatment with Walrasian local exchange does not happen. Otherwise, the dynamics are exactly reproduced with the variance being a somewhat larger.

[22]That is, if an agent has 10 units if R_1 and 2 of X_2, her reservation price for X_2 in R_1 is $\frac{R_1}{X_2} = \frac{10}{2} = 5$ (since $p_{R1}R_1 = p_{X2}X_2$). She will thus pay no more than 5 units of R_1 for one unit of X_2. And conversely, she will not trade one unit of X_2 for less than 5 units of R_1. If an agent has no units of one good, the reservation price (theoretically infinite, using this price setting method) is set to twice the price set by othe other agent to avoid erratic price dynamics.

5 Conclusion

The crisis of 2008 put a sudden end to the smooth growth pattern that could be observed since the second world war.[23] A number of economies have been in and out of technical recession several times since 2008 and it remains uncertain if, when, and to what degree the world's economy will completely recover from the current recession. For the time before 2008, the national incomes (usually in chained US dollars) largely followed an exponential growth pattern which in turn gave rise to the expectation of constant growth rates and constant returns to investments. While the limits-to-growth discussion since the 1970s called the future persistence of this pattern into question, the shift of much of the economic acticity of developed countries to the service sector the true contribution of which is - as noted by Kuznets (1934) - particularly elusive, also raises some questions. It is beyond doubt that income and wealth have increased; people in the 1950s did not have mobile phones, very few owned a car and they lacked access to a universal knowledge base such as the internet that facilitates education, scientific research and a wide range of economic activities today. The pattern, this growth process took is, however, less incontrovertible.

As has been shown in detail in the current paper, the very basis of measurement of capital, income, wealth, growth and even prices suffer from a structural problem: The choice of the numeraire (simple or complexly combined in the case of currency) for the determination of the value of income and capital also determines its dynamic development. For most real historical or theoretically possible situations this did and does not constitute a problem, but in some cases the resulting patterns may differ sharply. One such case - which might become real sometime during the coming decades - is the depletion of natural resources.

To study this case as an example, the current paper assessed the price dynamics resulting from depletion of exhaustable resources in a simple model. A number of settings ranging from those with well established and widely-used assumptions, mechanisms, and expectations (centralized Walrasian exchange) to very different agent-based models have been considered. Capital statistics as measured in an economy have to rely on some value base of

[23]For the time before, the measure was still under development and the methodology was subject to change. For the time before 1929 (the starting point of Kuznets' (1934) data series) no time series on national income and economic growth have been consistently recorded. All GDP and national income data for earlier times are retrospective estimates.

this computation, particularly the value of different available goods, capital goods, etc. It is by no means certain to which degree this value base remains stable if the system is unanticipatedly shaken by profound changes in for instance the resource inputs. The model approximates this by evaluating the economy's capital stock using different goods as numeraires. It was found that those statistics may not only diverge greatly for the same economy with different numeraires, some of those capital measures (i.e. those with numeraires other than the depleting resources) may show exponential growth sometimes up to the moment when the resource runs out, the production breaks down and the capital stock suddenly loses (has to be re-evaluated to be lower by) a significant share of its value.

This result could be confirmed from simulation studies for all settings: centralized and local interaction, Walrasian price optimization with realistic an overly positive expectations as well as with a purely heuristic price mechanism. It has been found that the effects are less pronounced (and the eventual crash less extreme) for settings with only local instead of global exchange interactions. This is not surprising since centralized systems are known to be more vulnerable as a whole to disturbances of any kind.

While a potential problem was identified and discussed in this article, no solution to it has been suggested and it may indeed be very difficult to come up with such a solution. Any possible solution would involve the inclusion of long-term sustainability into the measurement of capital, income etc. or at least into their interpretation; they may however, not generally be known in advance. If they are, a potential way to deal with this might be to use long-term replacement costs instead of production costs as a base for the value measurement of goods.[24] This, however, is obviously impractical under current circumstances as higher replacement costs are in principle intertemporal externalities which do not have an effect on the prices producers are currently able to offer.[25] Economic selection (in any competitive setting) operates on real prices and real costs, never on externalities or costs inflicted on the environment or an uncertain future. As such, this would be a classical

[24]Also, for the purpose, GDP and similar measures are currently predominantly used, i.e. to measure welfare, it is appropriate to include other indicators reflecting, for instance, the quality of life. Measures such as the human development index have taken this into account. Nevertheless the GDP component of this index is still subject to the issues discussed above.

[25]In the current setting firms do also not have an incentive to lie about prices which might change if value statistics were were decoupled from the economic selection mechanism.

social dilemma situation and while a better alternative for measuring capital, income, and wealth would be desirable, it is beyond doubt that either steps toward policy measures or a collectively organized effort will have to be made to resolve the dilemma problem. The same may likely be true for other options to deal with this kind of problem.

Acknowledgements

The author would like to thank two anonymous referees as well as the discussants and participants in several workshops and conferences where earlier versions of the current work were presented. The usual disclaimers apply.

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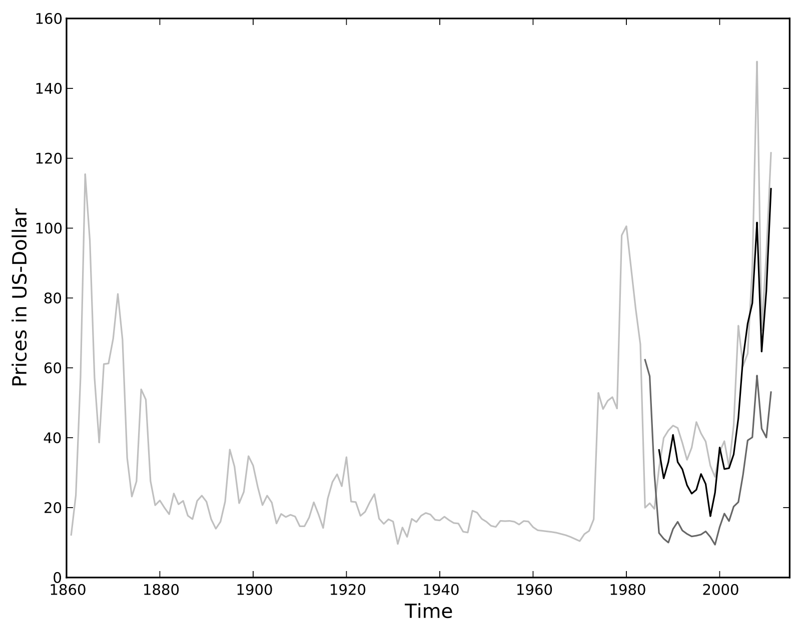


Figure 1: Price development of selected natural resources: coal (black, US-Dollar per tonne), gas (light gray, US-Dollar per 5 milion British thermal units), crude oil (dark gray, constant 2011 US-Dollar per barrel); data from the BP (2012).

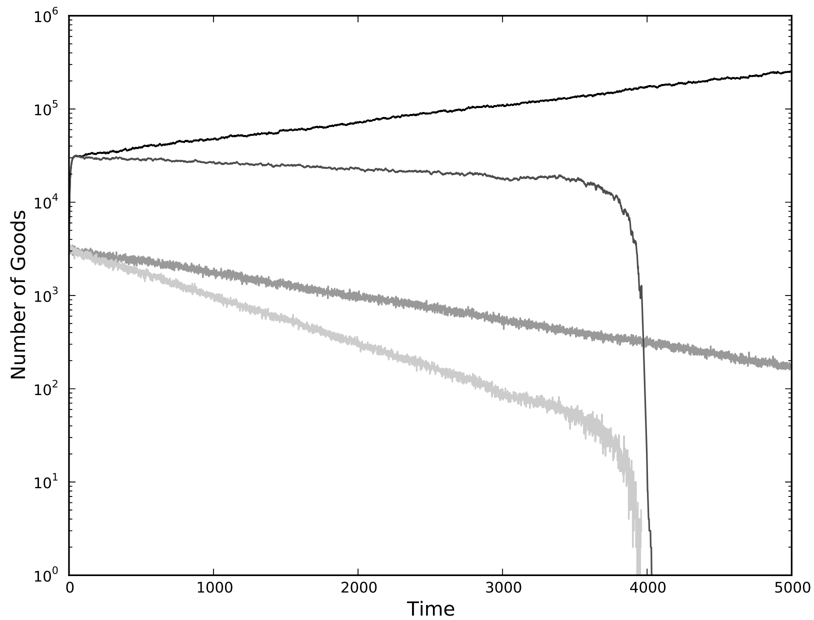
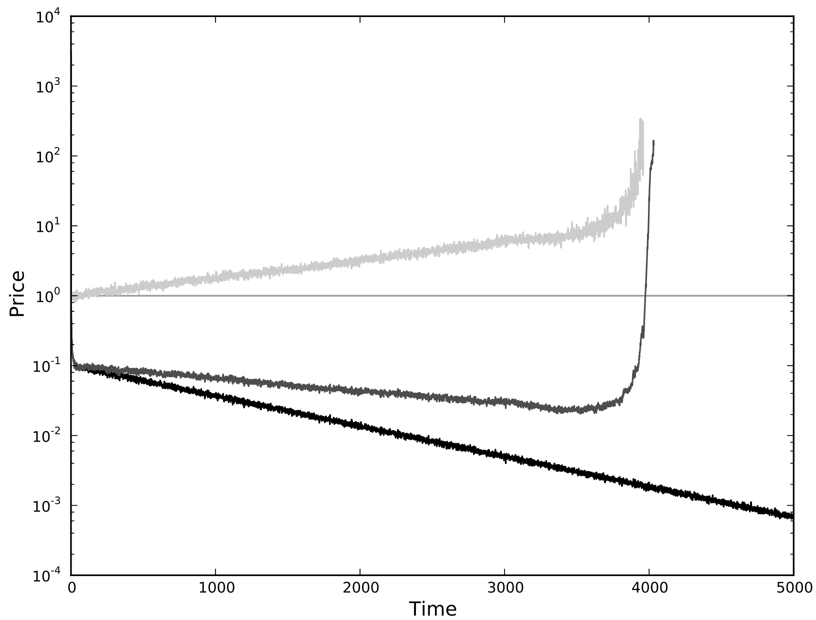
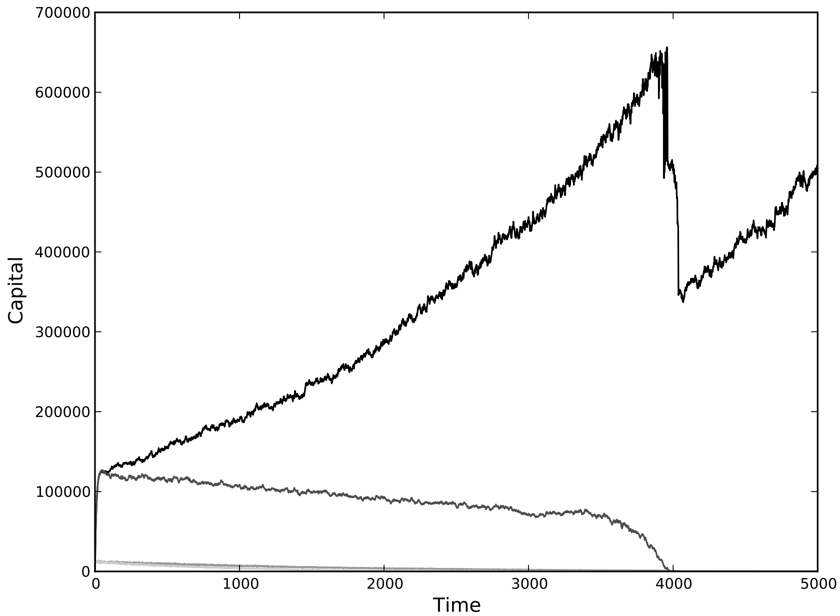


Figure 2: Development of resource and final good stock in the simulations. Black (X_1) and dark gray (X_2) the final goods, light gray (R_1) and very light gray (R_2) the resources.

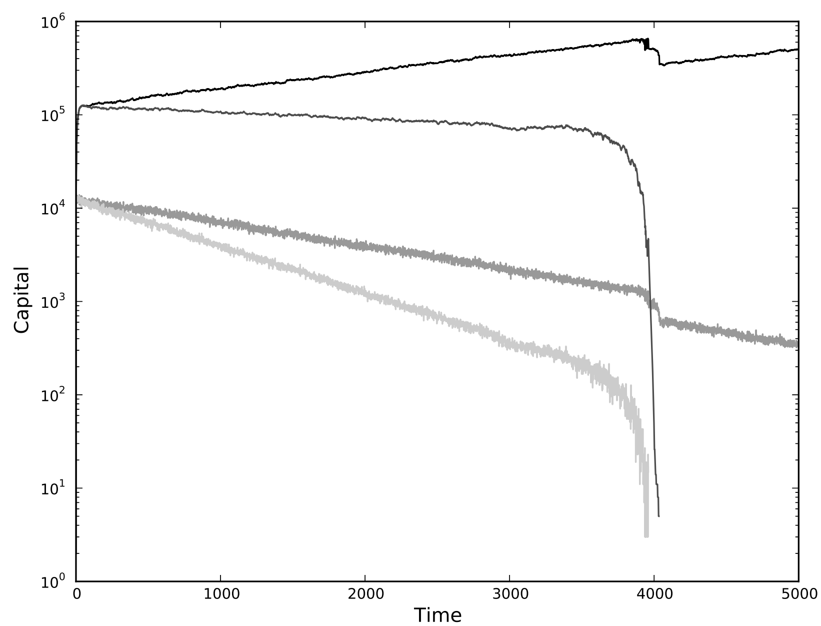


(a) Price (logarithmic scale), accurate expectations

Figure 3: Development of the price in the simulations with neoclassical market and accurate expectations with R_1 as numeraire. Black (X_1) and dark gray (X_2) the final goods, light gray (R_1) and very light gray (R_2) the resources.

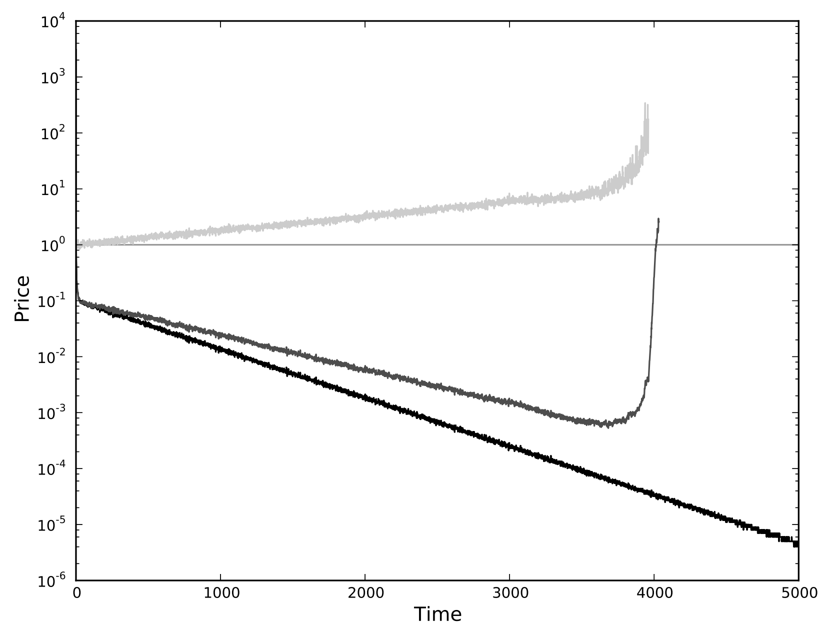


(a) Capital, accurate expectations



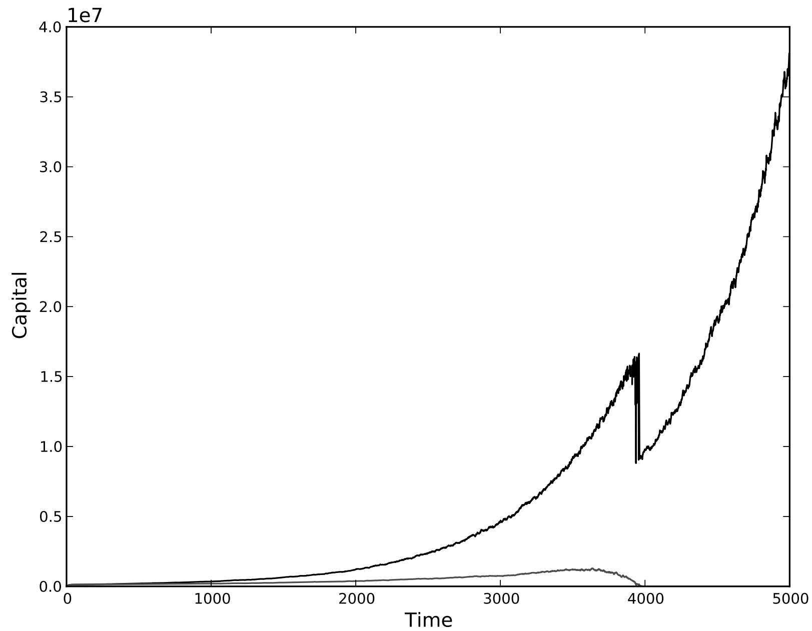
(b) Capital (logarithmic scale), accurate expectations

Figure 4: Development of capital in simulations with neoclassical market and accurate expectations measured with the different goods as numeraires. Black (X_1) and dark gray (X_2) the final goods, light gray (R_1) and very light gray (R_2) the resources.

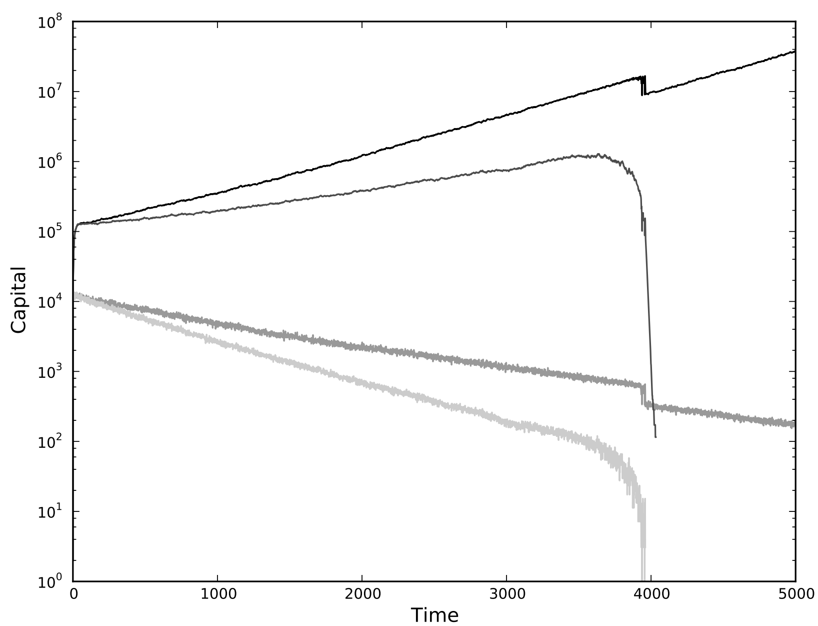


(a) Price (logarithmic scale), overstated expectations

Figure 5: Development of the price in simulations with neoclassical market and overstated expectations with R_1 as numeraire. Black (X_1) and dark gray (X_2) the final goods, light gray (R_1) and very light gray (R_2) the resources.

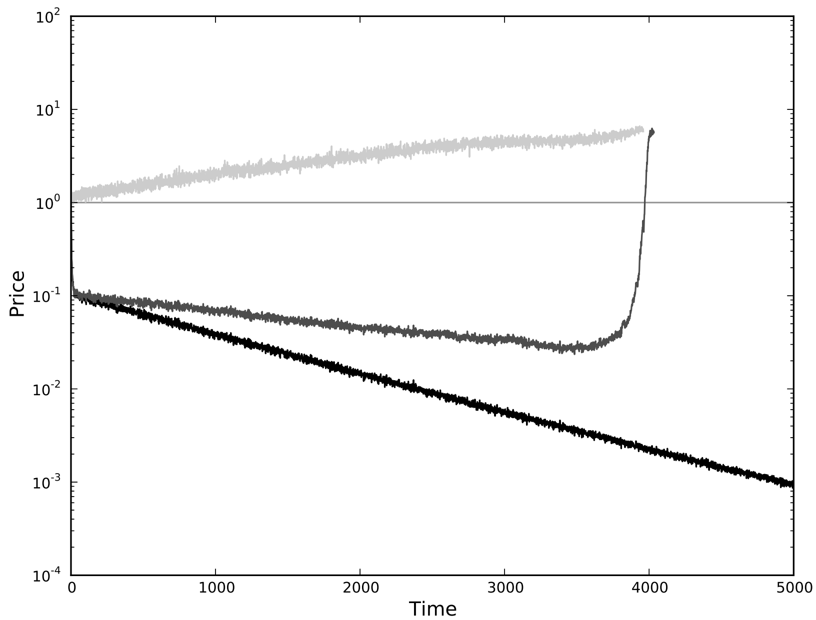


(a) Capital, overstated expectations



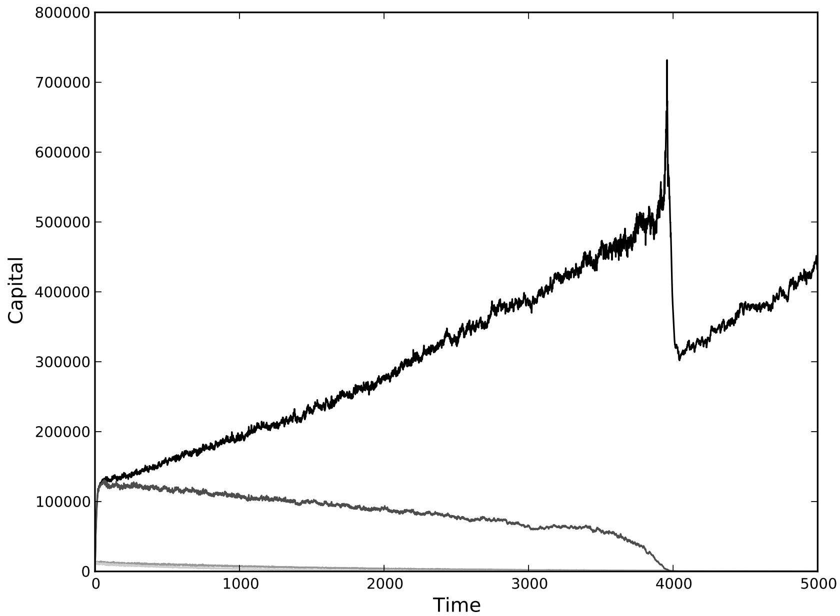
(b) Capital (logarithmic scale), overstated expectations

Figure 6: Development of capital in simulations with neoclassical market and overstated expectations measured with the different goods as numeraires. Black (X_1) and dark gray (X_2) the final goods, light gray (R_1) and very light gray (R_2) the resources.

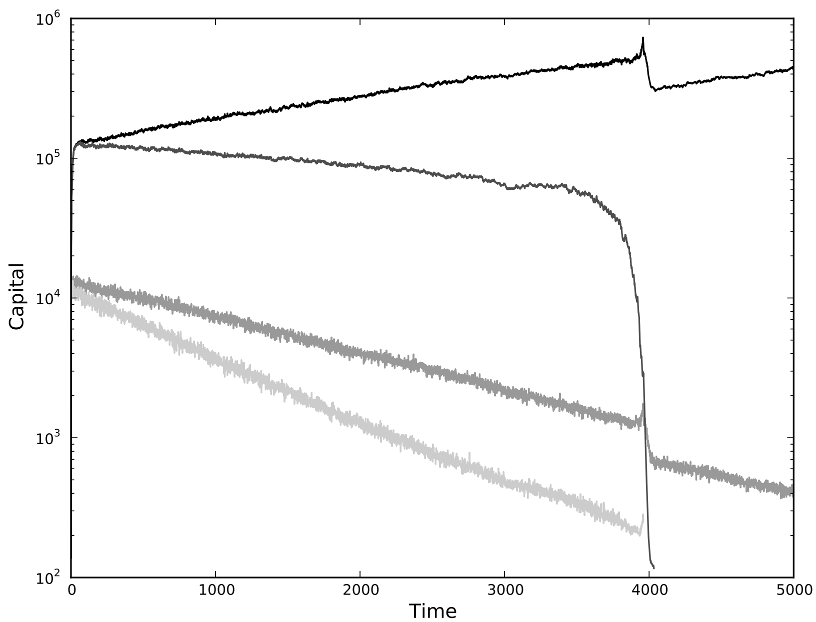


(a) Average price, local competitive exchange

Figure 7: Development of the price in simulations with local exchange on a ring topology with acurate expectations and local neoclassical Walrasian competitive exchange with R_1 as numeraire. Black (X_1) and dark gray (X_2) the final goods, light gray (R_1) and very light gray (R_2) the resources.

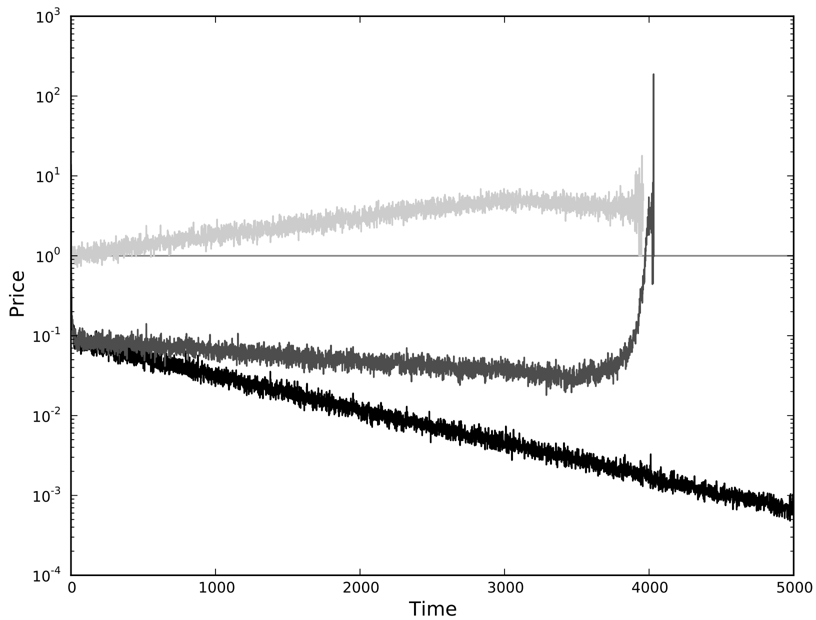


(a) Capital, local competitive exchange



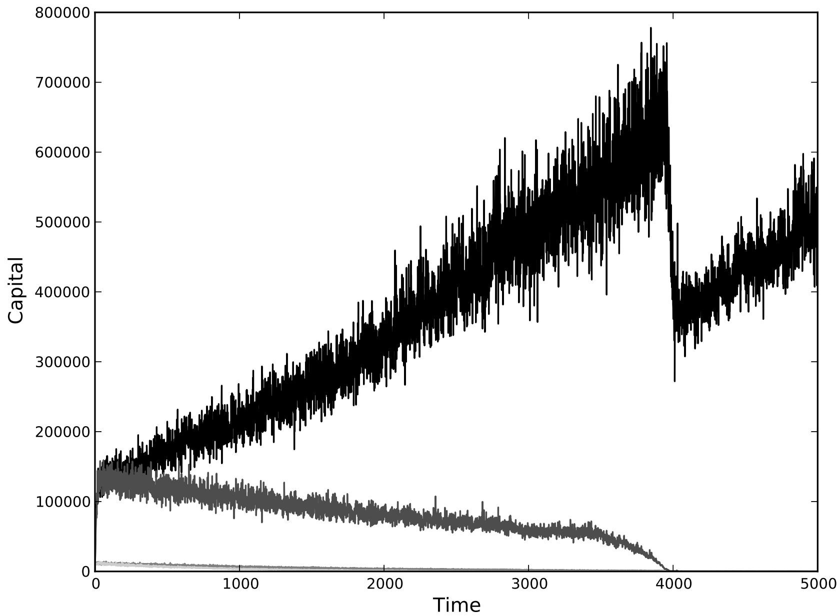
(b) Capital (logarithmic scale), local competitive exchange

Figure 8: Development of capital in simulations with local exchange on a ring topology with acurate expectations and local neoclassical Walrasian competitive exchange measured with the different goods as numeraires. Black (X_1) and dark gray (X_2) the final goods, light gray (R_1) and very light gray (R_2) the resources.

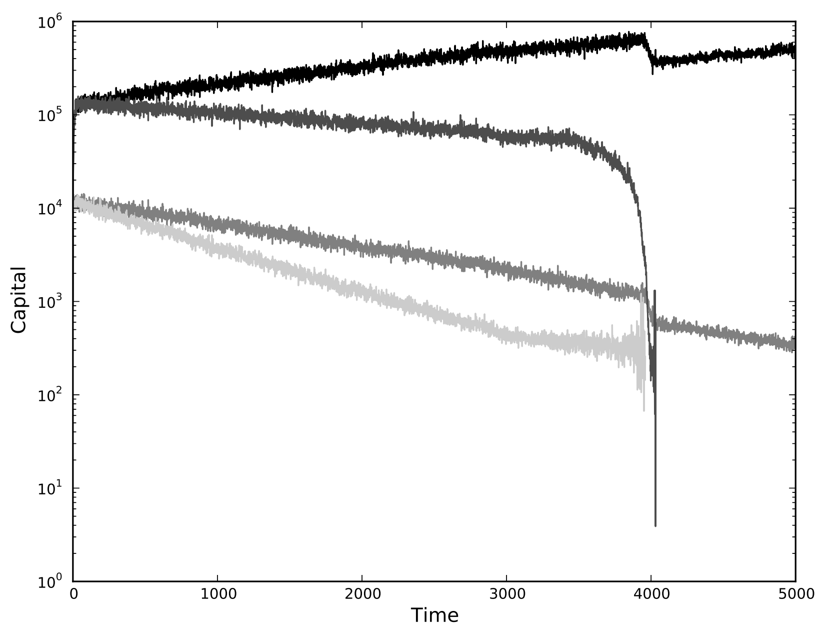


(a) Average price (logarithmic scale), local heuristic exchange

Figure 9: Development of the price in simulations with local exchange on a ring topology with acurate expectations and local heuristic exchange with R_1 as numeraire. Black (X_1) and dark gray (X_2) the final goods, light gray (R_1) and very light gray (R_2) the resources.



(a) Capital, local heuristic exchange



(b) Capital (logarithmic scale), local heuristic exchange

Figure 10: Development of capital in simulations with local exchange on a ring topology with acurate expectations and local heuristic exchange measured with the different goods as numeraires. Black (X_1) and dark gray (X_2) the final goods, light gray (R_1) and very light gray (R_2) the resources.