Amplitude-Duration-Persistence Trade-off Relationship for Long Term Bear Stock Markets

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July 2013

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Abstract:

We study the mechanism that controls the shape of the bear market through an information diffusion perspective, and establish a frontier of market decline, in terms of a trade-off between amplitude, duration and the rate of information diffusion. Empirical analysis using data from 15 stock markets confirms the existence of this trade-off relationship. An algorithm for generating the frontier using real data is proposed and applied in several market scenarios. The results suggest that the behaviour of international stock markets during the current US credit crunch is similar to that in previous bear markets in terms of the trivariate trade-off.

JEL classification: G14; G15; G17.

Keywords: Amplitude; Duration dependence; Volatility persistence; Bear markets; Information diffusion.

1. Introduction

In this paper, we study the mechanism that controls the shape of bear markets and attempt to establish a trade-off relationship among amplitude, duration and volatility persistence in it. Here amplitude refers to the total loss for stock indices in a bear market; duration relates closely to the concept of duration dependence in business cycle analysis, and is defined as the length of time the stock market is in the bearish phase. This bearish phase is identified through using the standard turning points detection approach in business cycle analysis, developed by Bry and Boschan (1977).

Amplitude, duration and volatility persistence are important risk characteristics for stock markets. Although the idea of a trivariate trade-off relationship, which we are about to establish in this article, has not been specified by scholars, amplitude, duration and volatility persistence have been regarded as important concepts in finance. Specifically, amplitude is related to the return-based risk measure, which has been conceptualised as the risk-return trade-off principle following landmark work by Markowitz (1952), Sharpe (1964) and Merton (1973). Duration dependence, although first identified in business cycle analysis, has been gaining prominence in stock market analysis, (see, for example, McQueen and Thorley 1994; Lunde and Timmermann 2004; Maheu and McCurdy 2000; Chen and Shen 2007; and Chong *et al.,* 2010). Moreover, empirical studies about aggregate stock markets (see, for example, Engle and Lee 1999; Cuñado *et al.,* 2008; Cuñado *et al.,* 2009) have suggested that linkages might exist between bear markets and volatility persistence.

There are some mixed studies relevant to the trade-off relationship, which we are about to establish in this paper. Specifically, Woodward and Marisetty (2005)'s study implies that the length of time spent in bull and bear markets is a key determinant of the risk-return relationship of risky assets. Lunde and Timmermann (2004) found that the longer the duration, the higher the volatility of a security's return, if in a bear market, and the lower the volatility, if in a bull market. This trade-off between amplitude and duration in bear stock markets can also be illustrated through reference to recent movements in stock markets. For example, the Dow Jones Index (DJI) during the 2000 dot-com crisis was characterized by an ultimate decline of 44% and duration of up to 38 months; while the DJI during the 2007–08 credit crunch exhibited a decline of 77%, but a much shorter duration of only 17 months. As Fisher (2007, pp. 76) observed, "Big bear markets are a trade-off between magnitude and duration." However, researchers have not been able to formally establish the trade-off relationship, with the exception of some descriptive analysis (see, for example, Edwards, Biscarri and de Gracia 2003).

Our paper attempts to supplement the traditional amplitude-duration trade-off phenomenon (Fisher, 2007) by incorporating information diffusion/volatility persistence, and build a theoretical framework for trade-off between amplitude, duration and volatility persistence.

The paper is structured as follows. In section two, the asset pricing model with information diffusion is proposed and the theoretical relationships between amplitude, duration and volatility persistence are discussed. In section three, we examine the trivariate trade-off relationship using data from 15 stock markets. Finally, in section four, we outline the main contributions of the paper and issues for future research.

2. The Model

In this section, an asset pricing model based on an information diffusion mechanism is presented. This information diffusion mechanism can also be regarded as an endogenous information generating process, as information is only public when it is available to the entire investors' population. To understand why bear market persist a considerable length of time, we need to recognise that the forces that can drive stock markets into bearish phases emerge from fundamental structural changes to the real economy or the financial sector, and that the information about these changes, even from the perspective of industry practitioners, could be vague and slowly digested (Black, 1986). This coincides with the framework developed by Hong and Stein (1999), who specified an information diffusion framework, in which information emerges gradually and circulates across investors' population (see. Appendix A.1). However, Hong and Stein (1999) assumed a constant rate of information diffusion. Li, Sun, and Wang (2012) relaxed the assumption of a constant rate of information diffusion and allowed for it to vary as a function of time *t*. Here we further relax the assumptions of Hong and Stein (1999) and Li *et al.* (2012) and assume that each piece of information is infinitely small, and define the information diffusion process over a continuous time domain (see Appendix A.2). This setting is consistent with our finding that volatility decays as price approaches the trough (see Appendix E for the case of the fall of Lehman Brothers). Moreover, it also implies that volatility persistence coincides with the rate of information diffusion and can, therefore, be used as its proxy.

2.1. Basic Assumptions

The key assumptions of the model are specified below:

2.1.1. Asset payoff.

A risky asset is issued at time 0 and pays a liquidating dividend at the end of the horizon $\overline{T}$, where $\overline{T}$ is sufficiently large. The ultimate value of the liquidating dividend at the end of period $\overline{T}$ can be written as $D_{\overline{T}} = D + d_{\overline{T}}$, where $d_{\overline{T}}$ ~N$(0, \sigma^2)$, and D is a constant term or the unconditional mean of $D_{\overline{T}}$. The supply of the asset in the entire market is Q and is assumed to be fixed.

2.1.2. Homogenous investor population.

There exists a continuum of investors *x*, $x \in X = [0,1]$. Each investor has identical constant absolute risk aversion (CARA) utility functions:

$$\max_{N_{xt}} \quad E_{xt}\{-e^{-\beta(N_{xt}(D_{\bar{T}}-P(t)e^{r(\bar{T}-t)}))}\}, \tag{1}$$

where β is the coefficient of absolute risk aversion, N_{xt} is the number of shares held by investor *x* at time *t*, *r* is the discount rate and *P*(*t*) is the stock price at time *t*. At every time *t*, each representative investor formulates his asset demands based on the static-optimization notion that he buys and holds until the liquidating dividend is paid out at the end of horizon $\bar{T}$, as in Hong and Stein (1999).

2.1.3. Continuous information diffusion.

The information innovation $d_{\bar{T}}$ is decomposed into a continuum of stochastic subinnovations:

$$d_{\bar{T}} = \int_{s=0}^{1} de^{(s)}, \; de^{(s)} = \sigma dW(s), \tag{2}$$

where dW is a Wiener process; and $W(s) - W(0)$ ~ $N(0,s)$; and s is a time sequence $s \in S = [0,1]$. Subinnovations spread symmetrically across the continuum of newswatchers *X* and over time sequence *S*. Hong and Stein's (1999) assumption of a constant rate of information flow is relaxed to a more flexible version: at time *t*, $f(t)$ percentage of $d_{\bar{T}}$ is revealed cumulatively to the investor. $f(t)$ is a continuous increasing function defined on [0, $\bar{T}$], with $f(0) = 0$ and $f(\bar{T}) = 1$. $f'(t)$ can be considered as a proxy for information efficiency. Moreover, following Veldkamp (2005)'s endogenous information production framework, the rate of information diffusion rises/falls as the asset value increases/decreases.

$f'(t)$ is, therefore, specified as monotonically decreasing in a bear market with $f''(t) < 0$, which is also consistent with the findings in Cowan and Jonard (2004) and Hong, Hong and Ungureanu (2010). At time *t*, when a proportion $f(t)$ of information has been revealed, the residual uncertainty, as measured by variance, is decreased to $[1 - f(t)]\sigma^2$.

2.2. The Pricing Equation

From assumptions *(i)*, *(ii)* and *(iii)*, we can derive total optimal demand for the risky asset, which determines the pricing equation. They are summarised in Proposition 1.

Proposition 1. *At any time t, investor x's optimal demand for the risky asset is given by* $N_{xt} = \dfrac{E_{xt}(D_{\bar{T}} - P(t)e^{r(\bar{T}-t)})}{\beta \operatorname{var}_{xt}(D_{\bar{T}})}$ *. The equilibrium price is set such that* N_t *and asset supply Q are equal. From this we obtain the pricing equation:*

$$P(t) = \left\{D + f(t)d_{\bar{T}} - \beta[1 - f(t)]\sigma^2 Q\right\}e^{-r(\bar{T}-t)}. \tag{3}$$

See Appendix B for proof.

Proposition 1 establishes the relationship between asset price and information diffusion. The price trajectory is determined by two components: The first component $D + f(t)d_{\bar{T}}$ is a drift, represented by an innovation multiplied by the rate of information diffusion. The second component $\beta(1 - f(t)\sigma^2 Q$ is the price discount due to the representative investor's expectation of the time varying uncertainty of future payoff.

2.3. Characteristics of Price Trajectory in a Bear Market

In this section, we explore the properties of the bottom of a downward market. For the stock price to attain its minimum value at time T, we must ensure that the first order condition $\left.\frac{dP(t)}{dt}\right|_{t=T} = 0$ and second order condition $\left.\frac{d^2P(t)}{dt^2}\right|_{t=T} \geq 0$ are satisfied. After some manipulation (see Appendix C for proof), we obtain Proposition 2, which formally characterizes the timing of the bottom of the price trajectory.

Proposition 2. *For a sufficiently large, negative shock* $d_{\bar{T}}$ *, and more specifically,* $d_{\bar{T}} < -\theta Q$ *, if* T *satisfies* $(D-\theta Q) = \frac{f'(T)}{r}(-d_{\bar{T}} - \theta Q) + f(T)(-d_{\bar{T}} - \theta Q)$ *and* $f''(T) + f'(T)r \leq 0$ *, then* $P(t)$ *has a minimum value at time* T[1].

2.4. The Trade-off among Amplitude, Duration and the Rate of Information Diffusion and Implications for ex ante Overvaluation

In this section, we formally establish the trivariate trade-off relationship, as summarised in Proposition 3. (See Appendix D for its derivation.)

Proposition 3. *Under the conditions specified in Proposition 2, there exists a trade-off relationship among amplitude, duration and the rate of information diffusion. Moreover, the relationship below holds:*

$$A = -rT - \ln\left(\frac{f'(T)}{r}\right) - \ln\left(\frac{-d_{\bar{T}} - \theta Q}{D - \theta Q}\right) \tag{4}$$

where $A = -\ln\frac{P(T)}{P(0)}$ *measures the amplitude of stock price decline,* $\frac{-d_{\bar{T}} - \theta Q}{D - \theta Q}$ *measures the degree of overvaluation or the gap between the correct price and the actual price at time 0, and* $f'(T)$ *is the rate of information diffusion.*

Note that $\frac{-d_{\bar{T}}}{D - \theta Q}$ is the relative shock and can be decomposed into two components,

[1] Similarly, we can also derive the conditions for P(t) to attain its maximum, which are available from the authors on request.

$\frac{\theta Q}{D-\theta Q}$ and $\frac{-d_{\bar{T}}-\theta Q}{D-\theta Q}$. The former component is the relative risk premium, whilst the latter measures the dividend shock that is not covered by the risk premium. In other words, the initial price of the stock ($D-\theta Q$) is overvalued because the risk premium (θQ) required by investors at time 0 is not large enough to cover the downside risk.

Equation (4) establishes the trade-off relationship: amplitude is negatively related to duration and the rate of information release $f'(T)$. This trade-off relation can be interpreted intuitively as follows. At the beginning, the stock price is set at an overvalued level since future negative news has not yet been made public and incorporated into price. As time passes, negative information diffuses slowly across the investors' population. This decreases the expected dividend payoff and lowers the stock price. Uncertainty is also reduced as "news" unavailable to investors declines. This reduces the risk premium and pushes up the stock price. If information diffuses quickly at the beginning, the main driving force of the stock price is, initially, the decreasing expected dividend payoff, which quickly switches to the decreasing risk premium. As a result, a sharp drop in stock prices and short duration is observed. If information diffuses slowly at the beginning, the bear market is characterized by weaker amplitude and longer duration.

While amplitude and duration can be obtained from market data, the rate of information diffusion cannot be observed. This restricts the empirical application of the trade-off relationship specified in Proposition 3. Hence, a proxy variable for the rate of information diffusion is needed. A review of the literature suggests that information arrival is the cause of conditional expected volatility and its clustering behaviour. See, for example, Andersen (1996), Fleming *et al.* (2006) (common stocks); Andersen *et al.* (2003), Berger *et al.* (2009) (exchange rates); Flannery and Protopapadakis (2002), Rangel (2011) (aggregate stock markets). More noticeably, Peng and Xiong (2003) relate volatility persistence to the speed of information digestion, while Li *et al.* (2012) associate volatility persistence with information diffusion. Our model is consistent with the existing literature, as the model setting implies that the process of information diffusion $f(t)$ coincides with the process of decaying conditional volatility $[1-f(t)]\sigma^2$, thus the rate of information diffusion can be measured by the rate of volatility decay. Furthermore, the empirical modelling of volatility dynamics has been extensively studied in the literature. Volatility persistence, primarily defined by a GARCH model or one of its many derivatives, is a widely accepted parametric measure of the rate of volatility decay. It is used in this article as a proxy for the rate of information release $f'(t)$ (see Section 3.2). From this basis, a modified trivariate trade-off relationship can be obtained in which amplitude is negatively related to duration and positively related to volatility persistence.

Moreover, the points of possible market bottoms form a frontier. We term this frontier - the "amplitude-duration-persistence frontier". An illustration of the pricing mechanism and the frontier is shown in Fig. 1.

[Insert Fig. 1About Here]

3. Empirical Evaluation

3.1. Data Selection and Preliminary Analysis

3.1.1. Data and their Turning Points

We select three of the most significant recent financial/economic crises (big bear markets) as our estimation sample, specifically the 1997 Asian Financial Crisis, the burst of the dot-com bubble in 2000 and the 2007–08 U.S. Sub-Prime Financial Crisis. Each of the crises led to widespread waves of stock market decline. Data for 15 stock markets, consisting of four European market indices, one US index, seven Asia market indices, one Middle East index and two Latin American indices, during these three crises, are used to test the trivariate trade-off relationship.

Before an empirical analysis can be undertaken, we must first identify the bearish and bullish phases. Traditionally, bear and bull markets are identified based on *ex post* assessment of peaks and troughs of the stock indices, which usually involves specifying dating algorithms. Noticeably, the methods adopted in Gonzalez *et al. (*2005) and Pagan and Sossounov (2003) are related to the standard business cycle detection technique developed by Bry and Boschan (BB) (1977). In this article, we follow Gonzalez *et al. (*2005) and Pagan and Sossounov (2003) and apply the BB algorithm. There are two main drawbacks of the BB algorithm. First, the lack of consensus regarding some of the bull/bear market turning points (Pagan and Sossounov, 2003). Second, that a turning point can only be identified through several observations after it occurs (Maheu *et al.,* 2012). Our article utilises data from three big bear markets; because there is common consensus regarding to the cause, the start and the end of them, the first criticism does not apply to our analysis. Moreover, three of these financial crises have ended, giving us enough observations to specify the trough, which means the second drawback would not have much impact on our analysis.

The existing BB algorithm for bear and bull market detection are for monthly or quarterly observations[2]. Using the BB algorithm to measure daily fluctuations would magnify the number of turning points. Before applying the original BB algorithm, we, therefore, smooth the daily observations using the Hodrick-Prescot (HP) filter (Hodrick and Prescot, 1997). We find that the identified dates of peaks and troughs of these smoothed series roughly coincide with common knowledge. Then the exact peaks and troughs of the indices are obtained through locating the recorded daily highs and lows of the unsmoothed stock market indices around the start and

[2] For any time series $\{y_t\}$.The heart of the BB algorithm is to define a local peak (trough) as occurring at time *t*. whenever $y_t > y_{t\pm\bar{k}}$, $\bar{k} = 1,2,3,...K$, where *K* is generally set to 5 for monthly data and 2 for quarterly data.

the end dates obtained through the BB algorithm. Moreover, to retain as much information as possible, we use the unsmoothed series in our empirical analysis.

However, we exercise some discretion in relation to the determination of these turning points. Specifically, the KS11 index exhibited a long term continuous downward trend before the 1997 Asia Financial Crisis, we identify the starting date according to the local recorded high during June–July, 1997, when most of the Asia indices peaked. Furthermore, the troughs of the developed markets were reached in approximately late 2002 and early 2003, while some developing markets exhibited a double dip with the first trough occurring in approximately 2001 with the second trough around the end of 2002, In addition, the BB algorithm cannot distinguish between the first and the second dip. We, therefore, manually select the second dip (see Fig. 1). The dating of each trough is presented in Table 1.

[Insert Table 1 About Here]

3.1.1. Calculating Amplitude, Duration and Volatility Persistence

Once the dating of bear market is completed, the calculation of amplitude and duration becomes straightforward. Namely, the amplitude of stock market decline is measured by percentage change from peak to trough, while duration is simply the number of trading days from peak to through. This section, therefore, focuses on volatility persistence, which is used as a proxy for the rate of information diffusion.

Typically, volatility persistence is calculated as the sum of the parameters in GARCH models[3]. Among the existing GARCH models, the Component-GARCH (CGARCH) (Engle and Lee, 1999) model possesses an outstanding advantage because it automatically filters out both a slow moving volatility component and a short-term component. The low

[3] Alternatives would be the stochastic volatility (SV) models; however the efficient estimation of these models is a non-trivial task, especially when rolling window estimation is concerned, under a two-component volatility specification.

frequency/slow moving volatility component can be linked to economic fundamentals and is more often known as business cycle risks (see Engle and Rangel (2007), Adrian and Rosenberg (2008)), while the short term variations are usually determined by liquidity shocks and strategic trading (Peng and Xiong, 2003). Moreover, there is evidence suggesting that the CGARCH model might be better suited to describe recent decades' world stock market data (McMillan and Ruiz, 2007)[4].

Thus in this paper, we use Engle and Lee's (1999) CGARCH model to measure the persistence of the slow moving volatility component. Serial correlations are also considered, in light of the empirical findings from Claessens *et al.* (1995) on the significance of serial correlations in equity returns. Thus we use an ARMA (1, 1)-CGARCH (1, 1) to model the effect of the unexpected shocks on return and volatility. It is defined as follows:

$$\begin{cases} r_t = c + \varepsilon_t \\ \varepsilon_t = AR \times \varepsilon_{t-1} + u_t + MA \times u_{t-1} \\ u_t = \sigma_t v_t \\ \sigma_t^{\,2} = \mu_t + (\alpha + \gamma\theta_{t-1})(\varepsilon_{t-1}^2 - \mu_{t-1}) + \beta(\sigma_{t-1}^2 - \mu_{t-1}) \\ \mu_t = \varpi + \rho(\mu_{t-1} - \varpi) + \phi(\varepsilon_{t-1}^2 - \sigma_{t-1}^2) \end{cases} . \quad (5)$$

Here the volatility dynamics can be decomposed into a transitory or short term component (σ_t^2) which converges to μ_t with power $\alpha + \beta$; and a long-run, trend or permanent component (μ_t) which converges to ϖ with power ρ .[5] ρ is termed volatility persistence. It measures the rate of long run volatility decay. c and ω are constants in the mean and variance

[4] To support the validity of using the persistence of the slow moving volatility component in the CGARCH model in our empirical investigation, we have also undertaken a conventional GARCH estimation of volatility persistence and examined its explanatory power in the trade-off relationship examined. It can be shown that the long run volatility persistence estimator outperforms that from a conventional GARCH model, results are available upon request.

[5] ρ is typically between 0.99 and 1 so that μ_t approaches ϖ very slowly. ϕ indicates the impact of unexpected shocks on the long-run component. ρ and ϕ together reflect the dynamic features of the long-run component.

equations, respectively. AR, MA, α, β and γ are coefficients. θ_{t-1} is a dummy variable. θ_{t-1}=1 if the return shock ε_{t-1}<0, otherwise θ_{t-1}=0. v_t is an independent, identically distributed, random variable. The daily return is calculated using the equation $r_t = \log(P_t) - \log(P_{t-1})$, where P_t is the index value at time t.[6]

The results are presented in Table 2.

[Insert Table 2 About Here]

3.3. The Empirical Model

Assuming that $\ln(f'(T))$ can be approximated by a function of volatility persistence ρ, we can derive our empirical model as follows. $-\ln(f'(T)) = C(2)\times\ln(\rho) + C(3)'$, where $C(2)$ and $C(3)'$ are the coefficient and constant term respectively. Then (4) can be transformed into:

$$A = \text{-}r \times T + C(2)\times\ln(\rho) + \ln(r) + C(3)' - \ln\left(\frac{-d_{\bar{T}} - \theta Q}{D - \theta Q}\right) \tag{6}$$

Here $\ln(r) + C(3)' - \ln\left(\frac{-d_{\bar{T}} - \theta Q}{D - \theta Q}\right)$ is a measure of *ex-ante* overvaluation, it corresponds to the intercept of the frontier on the amplitude axis in the theoretical model. Substituting $C(3)' - \ln\left(\frac{-d_{\bar{T}} - \theta Q}{D - \theta Q}\right)$ by $C(3)$ and r by $C(1)$ yields our empirical model:

$$A_{ik} = C(1)\times T_{ik} + C(2)\times\ln(\rho_{ik}) + \ln(-C(1)) + C(3) + \varepsilon_{ik} \tag{7}$$

Where subscript i denotes country i, and subscript k (=1,2 or 3) indicates the financial crisis

[6]In the theoretical model and the following frontier algorithm, t is an indicator of the process of a bear market rather than real time. Here, we maintain this setting, t is to denote the time process after the peak, equivalently t=0 indicates the beginning of the bear market (the previous peak). The estimation sample is composed of the observations between the peak and trough of a bear market.

period. Equation(7) treats the discount rate and the extent of overvaluation as estimators. Based on our theoretical model, $C(1)$, representing $-r$, should be negative and $C(2)$ should be positive. $\ln(-C(1))+C(3)$indicates the extent of overvaluation.

3.4. Empirical Results

To test our theory, four types of regression model were used to estimate (7). They comprise OLS panel regression, OLS panel regression with fixed period effect, OLS panel regression with fixed cross-sectional effect, and GLS (cross-sectional weights) panel regression with fixed cross-sectional effect. Table 3 reports the results. The estimation results confirm our theory. In terms of the signs of the estimates, the estimates for $C(1)$ are negative and those for $C(2)$ positive. They are also statistically significant. To be more precise, amplitude is negatively related to duration, positively related to volatility persistence and both relationships are statistically significant. According to R-squared, adjusted R-squared and Akaike information criterion, OLS panel regression with fixed cross-sectional effect and GLS regression outperform the other two specifications. This implies that the frontier is, to some extent, country specific and time independent.

The GLS estimates for (7) with fixed cross-sectional effect and cross-sectional weight can be expressed as:

$$A_{ik}=C(1)\times T_{ik}+C(2)\times\ln(\rho_{ik})+\ln(-C(1))+C(3)+cross_i \qquad (8)$$

Where $cross_i$ is the cross-sectional effect of country *i.* Here the degree of *ex ante* overvaluation is measured by the estimated country specific factor plus a constant term $cross_i+C(3)$ (see also(6)). The model's explanatory power (R-squared) is as high as 0.8658, indicating that duration, volatility persistence and country specific factors can explain a large part of the market decline. The estimate for $-C(1)$ or r indicates a daily discount rate of

0.0002 approximately.

An empirical implication of the trade-off relationship is that government intervention may not work in stock markets. Specifically, if government intervenes to limit the decline in stock prices then lower amplitude will potentially be compensated by a much longer duration. From a theoretical point of view, the trade-off relationship also suggests that the risk of *ex ante* overvaluation can be quantitatively measured by *ex post* observed amplitude, duration and volatility persistence of the bear market.

[Insert Table 3 About Here]

4. Algorithm for Generating the Frontier and its Application in Tracking the Market Bottom

Our findings suggest that the *ex ante* market overvaluation will be corrected by the *ex post* downward market cycle, characterized by the cumulated negative returns and the time spent in the downturn. The shape of the bear market is accordingly determined by the trade-off relationship between amplitude and duration as well as the rate of information diffusion. The trade-off relationship, shown by the amplitude-duration-persistence frontier, can be quantified by (4) and the empirical model (7), in which the constant term $C(3)$ indicates the degree of the *ex ante* overvaluation and determines the distance of the frontier from the origin. In this section, the algorithm that generates the amplitude-duration-persistence frontier is proposed and its potential for real-time indication of the bottom of the bear market demonstrated.

4.1. Frontier Algorithm

Following (8), we define

$$\bar{I}_i(t) = C(1)\times t + C(2)\times\ln(\rho_i(t)) + \ln(-C(1)) + C(3) + cross_i. \tag{9}$$

Here the subscript k seen in (8) is omitted since the expression holds across all three crisis periods. Moreover, we use t to measure how further the market has progressed into a specific bear market. That is to say, at the beginning of each bear market, or equivalently the previous peak, t is set to be 0. This is also consistent with the definition of t in the theoretical model. $\bar{I}_i(t)$ can be regarded as a real time indicator of the frontier amplitude with *ex ante* overvaluation level at $C(3)$ $+cross_i$. When $t=T_i$, $A_i=\bar{I}_i(T_i)$. In other words, if the stock price decline (A_i) reaches the frontier amplitude ($\bar{I}_i$), this would signal the end of downward market cycle. If the trade-off relationship is stable throughout crisis periods, then the end of a downward cycle can be gauged by observing the dynamics of $\bar{I}_i(t)$ and $P_i(t)$.

The price, time coordinates of the frontier curve for market i are generated by the following algorithm:

(i) Identification of the start of the bear market for country i.

The dating of the previous market peak, denoted by $\overleftarrow{t}_i$, is used as the start date of the bear market and the index value at that date $\bar{P}_i(\overleftarrow{t}_i)$ is used as its baseline price

(ii) Dynamic calculation of duration and volatility persistence.

As t increases, duration which is denoted by $D_i(\overleftarrow{t}_i + t) = t$; and volatility persistence which is denoted by $\rho_i(\overleftarrow{t}_i + t)$ can be estimated from the model (5) for the time interval ($\overleftarrow{t}_i$, $\overleftarrow{t}_i + t$).

(iii) Calculation of the frontier stock price.

Moreover, the frontier amplitude, which is denoted by $I_i(\overleftarrow{t}_i + t)$, can be formulated using a similar expression to (9):

$$I_i(\overleftarrow{t}_i + t) = C(1) \times t + C(2) \times \ln[\rho_i(\overleftarrow{t}_i + t)] + \ln(-C(1)) + C(3) + cross_i \tag{10}$$

The frontier stock price is denoted by $\overline{P}_i(\overleftarrow{t}_i + t)$, where $\overline{P}_i(\overleftarrow{t}_i + t) = e^{I_i(\overleftarrow{t}_i + t) + \ln(P(\overleftarrow{t}_i))}$, since amplitude is defined as the log return of price. The coordinates ($\overline{P}_i(\overleftarrow{t}_i + t)$, $\overleftarrow{t}_i + t$) define the frontier.

4.2. Frontier and Market Bottom: Global Evidence

The frontiers for the 15 stock market indices calculated using the above algorithm are shown in Fig. 2. Each frontier tilts upward, indicating a positive trade-off relationship between duration and amplitude. When stock index trajectories reach or cross the frontier, the downward trends quickly reverse. Our analysis of the frontier implies that the reaction of world stock markets during the US sub-prime crisis is similar to those during previous financial crisis in terms of the trivariate frontier.

[Insert Fig. 2 About Here]

4.3. Case Study of the Chinese Stock Market

The Chinese stock market is a relatively new emerging market, and, largely due to lacking of free float[7], the price movement constraints[8] and monetary policies[9], its market cycles do not share the same cyclical phases as international stock markets. Some of the bearish periods of the Shanghai Composite Index, therefore, lie outside the sample period used in our empirical test. For example, the bear markets existing in China between 1993 and 1996 do not appear to be related to the international stock market crashes of 1997 to 1998 and 2000 to 2003. During 1996 to 2001, China's stock market experienced a bullish trend and the East Asia financial crisis only had a limited impact on it. For the period 2003 to 2005, China's stock market experienced severe decline, while international stock markets were more bullish (See table 4).

[Insert Table 4 About Here]

Assuming that the Shanghai Composite Index shares the same trade-off relation (measured by coefficients $C(1)$ and $C(2)$) in terms of amplitude, duration and volatility persistence, as estimated from the 15 international stock indices, the only difference lies in the degree of *ex ante* overvaluation measured by ($C(3)+C_i$), these assumptions postulate that the trivariate trade-off relationship is a general phenomenon.

Given the historical bear market statistics, the value of $C(3)+C_i$ can be calibrated using (8). Using A_i, T_i and ρ_i from the 1993–96 bear market (see Table 4), $C(3)+C_i$ is calculated to be 9.572 and is then used to produce the amplitude-duration-persistence based frontier. From this we can analyze China's 2001–05 and 2007-08 bear markets.

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[7] Prior to share structure reform in 2005, the share structure of China's listed firms prohibited the trading of state shares of the listed firms, which resulted in only one third of the total shares being free floating.

[8] These mainly refer to the 5% constraint on daily price changes since inception of the Shanghai Stock Exchange, its subsequently removal in 1992, and the reestablishment of a 10% price movement constraint in 1996.

[9] During 1996–2001, China's macro economy entered into a period of deflation and slow growth. The central government adopted stimulatory monetary policies that saw capital move into the stock market rather than the real economy.

As shown in Fig. 3, the frontier is an important support line for Shanghai Composite Index.

[Insert Fig. 3 About Here]

4. Conclusions

In this article, we have introduced a general theoretical framework that is capable of capturing the dynamics of bear markets. Under this framework, the price of an asset is modelled in an environment where information about negative shocks relating to future asset payoffs leak in advance and diffuse slowly to the investing public. Our theory establishes a trivariate frontier, a trade-off relationship between amplitude, duration and rate of information diffusion as proxied by volatility persistence. The intercept of the frontier on the amplitude-axis measures the degree of investors' *ex ante* overvaluation, which is incorporated into the asset pricing error and consequently corrected by the subsequent bear market.

We also test the theory using data from three financial crises across 15 countries. The results confirm the existence of the trade-off relationship. Moreover, the frontier is found to be country specific and time independent. An algorithm for dynamically generating the frontier is proposed. We demonstrate that it can be applied successfully to trace the bottom of bear markets. An implication of our study is that the recent market crashes due to the US sub-prime crisis are similar to previous crashes in terms of the amplitude-duration-persistence frontier. Empirical analysis based on analysis of the Chinese stock market also indicates that the trivariate trade-off relationship is a general phenomenon.

Our analysis has also contributed to the literature on volatility persistence. Several studies have attempted to determine its cause, the core explanations being specified in terms of either market structure (Kavajecz and Odders-White, 2001) or investors' preferences (Barberis *et al.,* 1998; McQueen and Vorkink, 2004). Our model offers a new view on the underlying mechanism of persistent volatility from the perspective of information diffusion. In addition, our analysis provides an insight about the mechanism through which volatility persistence affects stock prices, which has been empirically, but inconclusively, investigated by Engle and Lee (1999), Cuñado *et al. (*2008), Cuñado *et al. (*2009).

For tractability, our study assumes a simple asset pricing mechanism that allows for a limited number of unobservable explanatory factors. Studies that include other important observable pricing factors (such as macroeconomic factors and sentiment factors) and consider more sophisticated transaction rules (for instance, the interaction between heterogeneous agents) are clearly required. Such factors are potentially correlated with information diffusion (Veldkamp, 2005) and may contribute to our understanding of the mechanisms of a bear market. With the increasing trend in global financialization, many other markets (for example, commodity market and artworks market) will come to share similar pricing characteristics as stock markets. Our amplitude-duration-persistence analysis can be extended to these areas for future research.

Appendix.

A. Information Diffusion Process

A.1 Hong and Stein's (1999) Framework

In Hong and Stein's (1999) framework, information innovation $d_{\bar{T}}$ can be decomposed into z i.i.d. subinnovations, each with identical variance $\frac{\sigma^2}{z}$:

$$d_{\bar{T}} = e^1 + e^2 + ... + e^z \tag{A.1}$$

When $d_{\bar{T}}$ begins to diffuse, say at time sequence s=1, newswatcher 1 observes e^1, newswatcher 2 observes e^2 , and so on, through to newswatcher z, who observes e^z. At the next time sequence s=2, the newswatchers "rotate," so that newswatcher 1 now observes e^2, newswatcher 2 observes e^3, and so on, through to newswatcher z, who now observes e^1. This rotation process continues until $d_{\bar{T}}$ becomes totally public. The information diffusion mechanism can also be shown by the arranging the stream of subinnovations as a matrix:

$$\begin{array}{l|ccccccc}
 & s=1 & s=2 & s=3 & \ldots & s=m & \ldots & s=z \\ \hline
\text{Newswatcher1}\rightarrow & e^1 & e^2 & e^3 & \cdots & e^m & \cdots & e^z \\
\text{Newswatcher2}\rightarrow & e^2 & e^3 & e^4 & \cdots & e^{m+1} & \cdots & e^1 \\
\text{Newswatcher3}\rightarrow & e^3 & e^4 & e^5 & \cdots & e^{m+2} & \cdots & e^2 \\
\text{Newswatcher4}\rightarrow & e^4 & e^5 & e^6 & \cdots & e^{m+3} & \cdots & e^3 \\
 & \vdots & & & & & & \vdots \\
 & \vdots & & & & & & \vdots \\
 & \vdots & & & & & & \vdots \\
\text{Newswatcher } z\rightarrow & e^z & e^1 & e^2 & \cdots & e^{m-1} & \cdots & e^{z-1}
\end{array} \tag{A. 2}$$

The subinnovations matrix $e(x,s)$ (x denotes newswatcher x, and s denotes sequence) diffuses sequentially to the investment population one column at a time, such that for any x, $\sum_{s=1}^{z} e(x,s) = d_{\bar{T}}$. Moreover, each column contains all the subinnovations of $d_{\bar{T}}$: $\sum_{x=1}^{z} e(x,s) = d_{\bar{T}}$.

Thus, on average, everyone is equally informed.

Suppose at sequence s=m, and subinnovations column vector $e(x, s=1)$, $e(x, s=2)$,…, $e(x, s=m)$, that a percentage $\frac{m}{z}$ of $d_{\bar{T}}$, has been released to newswatchers. Then $d_{\bar{T}}$ can be decomposed into two parts: $\sum_{s=1}^{m} e(x, s)$, which is what newswatcher x receives, and $\sum_{s=m+1}^{z} e(x, s)$, which is still unknown. His belief about $d_{\bar{T}}$ becomes : $d_{\bar{T}} = \sum_{s=1}^{m} e(x, s) + \sum_{s=m+1}^{z} e(x, s)$, which is normally distributed with mean $\sum_{s=1}^{m} e(x, s)$ and variance $\mathrm{var}[\sum_{s=m+1}^{z} e(x, s)] = \frac{(z-m)}{z}\sigma^2$. Note that $\sum_{s=1}^{m} e(x, s)$ is the realization of the sum of subinnovations.

A.2 A continuous Time Version

The information diffusion structure of Appendix1.1 can be extended into continuous time: Here the information innovation $d_{\bar{T}}$ is decomposed into a continuum of stochastic subinnovations $e(s): d_{\bar{T}} = \int_{s=0}^{z} de^{(s)}$, where s is defined on (0, z) representing the time sequence of diffusion. $de^{(s)} = \frac{\sigma}{\sqrt{z}} dW(s)$ where dW is a Wiener process. $W(s) - W(0)$ ~ $N(0, s)$. For simplicity, we normalize $z = 1$ to obtain:

$$d_{\bar{T}} = \int_{s=0}^{1} de^{(s)}, \; de^{(s)} = \sigma dW(s) \tag{A.3}$$

Subinnovations spread symmetrically across a continuum of newswatchers x, $x \in [0,1]$, and over time sequence s. The information diffusion structure can be described by a two dimensional stochastic process $de(x, s)$. Similar to the above discrete time information structure, we specify a symmetrically continuous diffusion process, with features as listed below:

(i) Each newswatcher x receives a continuous innovation sequence, such that

$$\int_{s=0}^{1} de(x,s) = \int_{s=0}^{1} e^{(s)} = d_{\overline{T}} \ . \tag{A.4}$$

(ii) For any sequence *s*, we have $\int_{x=0}^{1} de(x,s) = \int_{x=0}^{1} de^{(x)} = d_{\overline{T}}$. Everyone is equally informed.

(iii) $d_{\overline{T}}$ begins to spread at *t*=0. At time *t*, a percentage $f(t)$ of $d_{\overline{T}}$ is revealed cumulatively to the investment population. Each newswatcher's belief about $d_{\overline{T}}$ can then be decomposed into two parts: newswatcher *x* receives $\int_{s=0}^{f(t)} de(x,s)$ leaving $\int_{s=f(t)}^{1} de(x,s)$ unknown. The belief of newswatcher *x* about $d_{\overline{T}}$ becomes:

$$\begin{aligned} d_{\overline{T}} &= \int_{s=0}^{f(t)} de(x,s) + \int_{s=f(t)}^{1} de(x,s) \\ &= \int_{s=0}^{f(t)} de(x,s) + \int_{s=f(t)}^{1} \sigma dW(s) \\ &= \int_{s=0}^{f(t)} de(x,s) + \sigma W(1) - \sigma W(f(t)), \end{aligned} \tag{A.5}$$

which is normally distributed with mean $\int_{s=0}^{f(t)} de(x,s)$ and variance $\mathrm{var}[\sigma W(1) - \sigma W(f(t))] = [1 - f(t)]\sigma^2$

$f(t)$ is a continuously increasing function defined on (0, $\overline{T}$), with $f(0) = 0$ and $f(\overline{T}) = 1$. $f'(t)$ can be considered as a proxy for information efficiency. A lower value of $f'(t)$ implies a lower rate of information revelation and less informational efficiency.

B. Proof of Proposition 1

Newswatchers choose their optimal holdings by maximizing the utility function

$$\max_{N_{xt}} \quad E_{xt}\{-e^{-\beta(N_{xt}(D_{\overline{T}} - P(t)e^{r(\overline{T}-t)}))}\},$$

which implies

$$N_{xt} = \frac{E_{xt}(D_{\bar{T}} - P(t)e^{r(\bar{T}-t)})}{\beta \operatorname{var}_{xt}(D_{\bar{T}})}. \tag{B.1}$$

(B.1) can be rearranged as

$$P(t)e^{r(\bar{T}-t)} = E_{xt}(D_{\bar{T}}) - \beta \operatorname{var}_{xt}(D_{\bar{T}}) N_{xt}. \tag{B.2}$$

Due to the assumption of homogenous investors, (B.2) implies

$$\int_{x=0}^{1} P(t)e^{r(\bar{T}-t)}dx = \int_{x=0}^{1} E_{xt}(D_{\bar{T}})dx - \beta \int_{x=0}^{1} N_{xt} \operatorname{var}_{xt}(D_{\bar{T}})dx. \tag{B.3}$$

For a terminal payoff $d_{\bar{T}}$, the assumption that information about $d_{\bar{T}}$ diffuses symmetrically across newswatchers implies that at time t, each newswatcher x faces a fraction $1 - f(t)$ of residual subinnovations $\int_{s=f(t)}^{1} de(x,s)$, and his/her belief about the terminal payoff is:

$$D_{\bar{T}} = D + d_{\bar{T}} = D + \int_{s=0}^{f(t)} de(x,s) + \sigma W(1) - \sigma W(f(t)), \tag{B.4}$$

which is normally distributed with mean $D + \int_{s=0}^{f(t)} de(x,s)$ and variance $[1 - f(t)]\sigma^2$.

From (B.3) we can obtain:

$$\begin{aligned} P(t)e^{r(\bar{T}-t)} &= \int_{x=0}^{1} \{D + \int_{s=0}^{f(t)} de(x,s)\}dx - \beta(\int_{x=0}^{1} N_{xt}dx) \operatorname{var}_t(D_{\bar{T}}) \\ &= D + \int_{x=0}^{1} \int_{s=0}^{f(t)} de(x,s)dx - \beta Q \operatorname{var}_t(d_{\bar{T}}) \\ &= D + f(t)d_{\bar{T}} - [1 - f(t)]\beta Q \sigma^2. \end{aligned} \tag{B.5}$$

C. Proof of Proposition 2.

For the stock price to reach its extreme value, the first derivative $\frac{dP(t)}{dt}$ must be equal to 0.

Differentiating (3) we obtain:

$$\begin{aligned}&\frac{dP(t)}{dt}=(D-\theta Q)re^{-r(\bar{T}-t)}+\\&f'(t)(d_{\bar{T}}+\theta Q)e^{-r(\bar{T}-t)}+f(t)(d_{\bar{T}}+\theta Q)re^{-r(\bar{T}-t)}\end{aligned}\quad. \tag{C.1}$$

The condition $\left.\frac{dP(t)}{dt}\right|_{t=T}=0$ is equivalent to:

$$(D-\theta Q)=\frac{f'(T)}{r}(-d_{\bar{T}}-\theta Q)+f(T)(-d_{\bar{T}}-\theta Q),T\le\bar{T}\,. \tag{C.2}$$

Since $f(t)$ is an increasing function of t, $f'(t)>0$. If $D>\theta Q$, then $d_{\bar{T}}+\theta Q<0$ is required to ensure (C.2) holds. Furthermore, we know that for a bear market to exist, $d_{\bar{T}}$ should be sufficiently less than 0, i.e. $d_{\bar{T}}<-\theta Q$.

Moreover, for a stock price to attain its minimum then, $\left.\frac{d^2P(t)}{dt^2}\right|_{t=T}\ge 0$ must also hold.

$$\frac{d^2P(t)}{dt^2}=r\frac{dP(t)}{dt}+(d_{\bar{T}}+\theta Q)e^{-r(\bar{T}-t)}[f''(t)+f'(t)r]\,. \tag{C.3}$$

Combining this with (C.2), we get:

$$\left.\frac{d^2P(t)}{dt^2}\right|_{t=T}=(d_{\bar{T}}+\theta Q)e^{-r(\bar{T}-T)}[f''(T)+f'(T)r]\,. \tag{C.4}$$

If $f''(T)+f'(T)r\le 0$, then $\left.\frac{d^2P(t)}{dt^2}\right|_{t=T}\ge 0$, indicating that price reaches a minimum value at time T. This is the typical case observed in most large bear markets, in which the stock price hits its bottom before bad news is fully revealed to investors.

D. Proof of Proposition 3.

Based on (3) the price dynamic function of Proposition 1, we can obtain:

$$\frac{P(t)}{P(0)} = e^{rt}(1 - \frac{-d_{\bar{T}} - \theta Q}{D - \theta Q} f(t)) \ . \tag{D.1}$$

At time T, and by using the first order condition defined in (C.2), (D.1) can be rewritten as:

$$\frac{P(T)}{P(0)} = e^{rT} \frac{f'(T)}{r} (\frac{-d_{\bar{T}} - \theta Q}{D - \theta Q}) \ . \tag{D.2}$$

Taking logarithms of both sides of (D.2), we obtain the trade-off relationship:

$$\ln \frac{P(T)}{P(0)} = rT + \ln(\frac{f'(T)}{r}) + \ln(\frac{-d_{\bar{T}} - \theta Q}{D - \theta Q}) \ . \tag{D.3}$$

where $\ln \frac{P(T)}{P(0)}$ is the log return of the stock from time 0 to T. Its negative $-\ln \frac{P(T)}{P(0)}$, denoted by A, would be a good measure of the amplitude of stock price decline. Simplifying, we obtain:

$$A = -rT - \ln(\frac{f'(T)}{r}) - \ln(\frac{-d_{\bar{T}} - \theta Q}{D - \theta Q}) \ , \tag{D.4}$$

where θQ equals the risk premium at time 0; $D - \theta Q$ is the initial stock price at time 0 and $-d_{\bar{T}}$ represents the magnitude of the negative dividend shock. Note that $-d_{\bar{T}} < D$ is required to ensure that the ultimate stock value is positive.

E. The DJI Index around the Fall of Lehman Brothers.

[Insert Fig. E.1 About Here]

F. Comparison of Stock Market Behaviours during Financial Crisis.

From Fig. F.1, we can observe that the 1997 Asian Financial Crisis had little impact on the second group of markets. During the 1997 East Asia financial Crisis, the trend component of the DJI, AEX, CAC40, TEL and FTSE indices exhibited only slight falls, while their Asian and Latin American counterparties experienced severe declines.

[Insert Fig. F.1 About Here]

TABLE 1

Dating of the Trough across Stock Markets

Market	The 1997 Asian Financial Crisis	Dot-com Crisis	U.S. Sub-Prime Financial Crisis
AEX	——	09/04/2000–03/12/2003	07/16/2007–03/09/2009
BSE	08/05/1997–10/20/1998	02/11/2000–10/28/2002	01/08/2008–03/09/2009
BVSP	07/29/1997–10/01/1998	03/27/2000–10/16/2002	05/20/2008–10/27/2008
CAC	——	09/04/2000–03/12/2003	06/01/2007–03/09/2009
DJI	——	01/14/2000–03/11/2003	10/09/2007–03/09/2009
FTSE	——	09/04/2000–03/12/2003	06/15/2007–03/03/2009
HSI	06/16/1997–08/13/1998	03/28/2000–04/25/2003	12/05/2007–10/29/2008
JKSE	07/08/1997–09/21/1998	01/17/2000–10/14/2002	01/09/2008–10/29/2008
KLSE	02/20/1997–09/01/1998	02/18/2000–05/21/2001	01/08/2008–10/29/2008
KS11	06/17/1997–06/16/1998	01/04/2000–09/17/2001	10/31/2007–11/20/2008
MERV	08/20/1997–09/10/1998	03/03/2000–06/14/2002	10/31/2007–11/21/2008
MMX	10/21/1997–09/10/1998	03/09/2000–11/19/2002	10/18/2007–03/02/2009
SMI	——	08/23/2000–03/12/2003	06/01/2007–03/09/2009
STI	01/20/1997–09/04/1998	01/03/2000–03/10/2003	10/11/2007–03/09/2009
TEL	——	03/06/2000–02/13/2003	11/01/2007–12/29/2008

Note: This table presents the sample periods for the three crises. The stock market indices under consideration comprise the SMI (Swiss Market Index); CAC 40 (Cotation Assistée en Continu 40 index, France); AEX (Amsterdam Exchange index, Netherland); FTSE (Financial Times Stock Exchange Index, UK); DJI (Dow Jones Industrial Average index, US); TEL (Tel Aviv 100 Index, Israel); STI (Straits Times Index, Singapore); HIS (Hang Seng Index, Hongkong); SET (Thailand composite index); JKSE (the Jakarta Index, Indonesia); KLSE (Kuala Lumpur Stock Exchange Index, Malaysia); BSE SENSEX (Bombay Stock Exchange Sensitive Index, India); KS11 (KOSPI Composite Index, South Korea); BVSP (Bovespa index, Brazil); MMX (Mexico); and MERV (Merval Buenos Aires Index, Argentina). In analysing the 1997 Asian financial crisis, the DJI, AEX, CAC40, TEL and FTSE indices were omitted from the estimation sample as this crisis had little impact on Western stock markets (see Appendix E).

TABLE 2

Amplitude, Duration and Volatility Persistence for 15 Stock Market Indices during the Three Crisis Periods.

	Amplitude			**Duration**			**Volatility Persistence**		
	1997 Asian Financial Crisis	**Dot-com Crisis**	**U.S. Sub-Prime Financial Crisis**	**1997 Asian Financial Crisis**	**Dot-com Crisis**	**U.S. Sub-Prime Financial Crisis**	**1997 Asian Financial Crisis**	**Dot-com Crisis**	**U.S. Sub-Prime Financial Crisis**
AEX	——	1.1668	1.0368	——	640	420	——	0.9978	0.9949
CAC	——	1.0580	0.8954	——	648	452	——	0.9732	0.9966
DJI	——	0.4434	0.7717	——	798	355	——	0.9584	0.9724
FTSE	——	0.7267	0.6507	——	646	435	——	0.9626	0.9824
SMI	——	0.8238	0.7942	——	657	443	——	1.0010	0.9959
HSI	0.7707	0.7777	0.8374	286	756	219	0.9872	0.9537	0.9512
JKSE	1.0594	0.7345	0.9328	301	690	195	0.9777	0.9626	0.9629
KLSE	1.5728	0.6031	0.5856	380	313	199	1.0363	0.5313	0.5017
KS11	1.0401	0.8150	0.7777	293	418	262	0.9988	0.9405	0.9840
STI	1.0375	0.7552	0.9784	407	805	341	0.9868	0.9232	0.9856
BSE	0.4980	0.7388	0.9392	294	671	285	0.6061	0.9330	0.8949
BVSP	0.7833	0.8171	0.9153	296	633	113	0.9954	0.7729	0.9976
MERV	1.0549	0.8797	1.0426	262	545	262	0.9692	0.9992	0.9794
MMX	0.6313	0.3884	0.6625	223	668	341	0.9330	0.9944	0.9849
TEL	——	0.6442	0.7653	——	574	268	——	0.9025	0.9847
Average	——	1.1668	1.0368	——	640	420	——	0.9978	0.9949

Note: This table reports the values of amplitude, duration and volatility persistence for each sample outlined in Table 1. The amplitude of stock market decline is measured by the percentage change from peak to trough. Duration is calculated as the number of trading days between peak and trough. Bear stock market duration is much longer during the dot-com crisis than those of the 1997 East Asia Financial Crisis and the 2007 U.S. Sub-Prime Financial Crisis. Volatility persistence is calculated using the CGARCH model.

TABLE 3

Estimation Results

Model	(1)Panel regression	(2)Panel regression with fixed period effect	(3)Panel regression with fixed cross-sectional effect	(4)Panel regression with fixed cross-sectional effect (GLS cross sectional weight)
	$A_{tk}=C(1)\times T_{ik}+C(2)\times\ln(\rho_{ik})$ $+\ln(-C(1))+C(3)+\varepsilon_{ik}$	$A_{ik}=C(1)\times T_{ik}+C(2)\times\ln(\rho_{ik})$ $+\ln(-C(1))+C(3)+period_k+\varepsilon_{ik}$	$A_{ik}=C(1)\times T_{ik}+C(2)\times\ln(\rho_{ik})$ $+\ln(-C(1))+C(3)+cross_i+\varepsilon_{ik}$	$A_{ik}=C(1)\times T_{ik}+C(2)\times\ln(\rho_{ik})$ $+\ln(-C(1))+C(3)+cross_i+\varepsilon_{ik}$
C(1)	-0.0003*	-0.0003	-0.0002*	-0.0002**
	(0.0778)	(0.4310)	(0.0590)	(0.0100)
C(2)	0.6050***	0.5947**	1.1476***	1.1452 ***
	(0.0042)	(0.0102)	(0.0000)	(0.0000)
C(3)	9.1003***	9.2032***	9.3806***	9.4004 ***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
R^2	0.2349	0.2659	0.8159	0.8658
$\bar{R}^2$	0.1924	0.1795	0.6821	0.7682
AIC	-0.3472	-0.2859	-1.0539	——

Note: This table reports the estimation results from four panel regressions. $period_k$ is the period effect, where k(=1, 2, 3) indicates the crisis period, and $cross_i$ represents the cross-sectional effect. We report both estimated coefficients and corresponding p-values. Coefficients that are significant at the 1%, 5%, and 10% levels are marked indicated by ***, ** and * respectively. R^2, $\bar{R}^2$ and Akaike's information criterion (AIC) are also reported to aid assessment of the model performance.

TABLE 4

Statistical Description of each Downward Market Cycle

	First Downward Cycle	Second Downward Cycle	Third Downward Cycle
Dating	04/28/1993–01/22/1996	06/13/2001–06/03/2005	10/16/2007–11/04/2008
Amplitude	0.9774	0.7941	1.2623
Duration	695	957	259
Volatility Persistency(ρ)	0.9545	0.9785	0.6572

Note: This table reports the dating and the characteristics of each of the three bear markets.

TABLE G.1

Panel Regression Estimation Results, with $\bar{\rho}$ as an Explanatory Variable.

Model	(1)Panel regression	(2)Panel regression with fixed period effect	(3)Panel regression with fixed cross-sectional effect	(4)Panel regression with fixed cross-sectional effect (GLS cross-sectional weights)
	A_{it}=$C(1)\times T_{ik}$+$C(2)$ $\times\ln(\bar{\rho}_{ik})$+ln(-$C(1)$)+$C(3)$ +$C(4)\times\ln(\bar{\rho}_{ik})$+ ε_{ik}	A_{it}=$C(1)\times T_{ik}$ +$C(2)\times\ln(\bar{\rho}_{ik})$ +ln(-$C(1)$) +$C(3)$ +$C(4)$ $\times\ln(\bar{\rho}_{ik})$+$period_k$+$\varepsilon_{ik}$	A_{it}=$C(1)\times T_{ik}$ +$C(2)\times\ln(\bar{\rho}_{ik})$ + ln(-$C(1)$)+$C(3)$ +$C(4)$ $\times\ln(\bar{\rho}_{ik})$+$cross_i$+$\varepsilon_{ik}$	A_{it}=C(1)$\times T_{ik}$ +$C(2)\times\ln(\bar{\rho}_{ik})$ +ln(-$C(1)$) +$C(3)$+$C(4)$ $\times\ln(\bar{\rho}_{ik})$+$cross_i$+$\varepsilon_{ik}$
C(1)	-0.0003	-0.0003	-0.0002	-0.0002
	(0.0832)	(0.4417)	(0.0741)	(0.0741)
C(2)	0.5797	0.5758	1.0068	1.0068
	(0.0347)	(0.0383)	(0.0001)	(0.0001)
C(3)	9.1034	9.1507	9.4496	9.4496
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
C(4)	0.0327	0.0314	0.2043	0.2043
	(0.8836)	(0.8996)	(0.1776)	(0.1776)
R^2	0.2354	0.2662	0.8315	0.8705
$\bar{R}^2$	0.1698	0.1550	0.6952	0.7656
AIC	-0.2965	-0.2351	-1.0913	——

Note: This table reports the estimation results from four panel regressions with $\bar{\rho}$as an explanatory variable. $period_k$ is the period effect and $cross_i$ represents the cross-sectional effect. We report both the estimated coefficients and corresponding p-values. R^2 , $\bar{R}^2$ and Akaike's information criterion (AIC) are also reported to aid the assessment of the model performance.

Fig. 1. The Amplitude-Duration-Persistence Frontier.

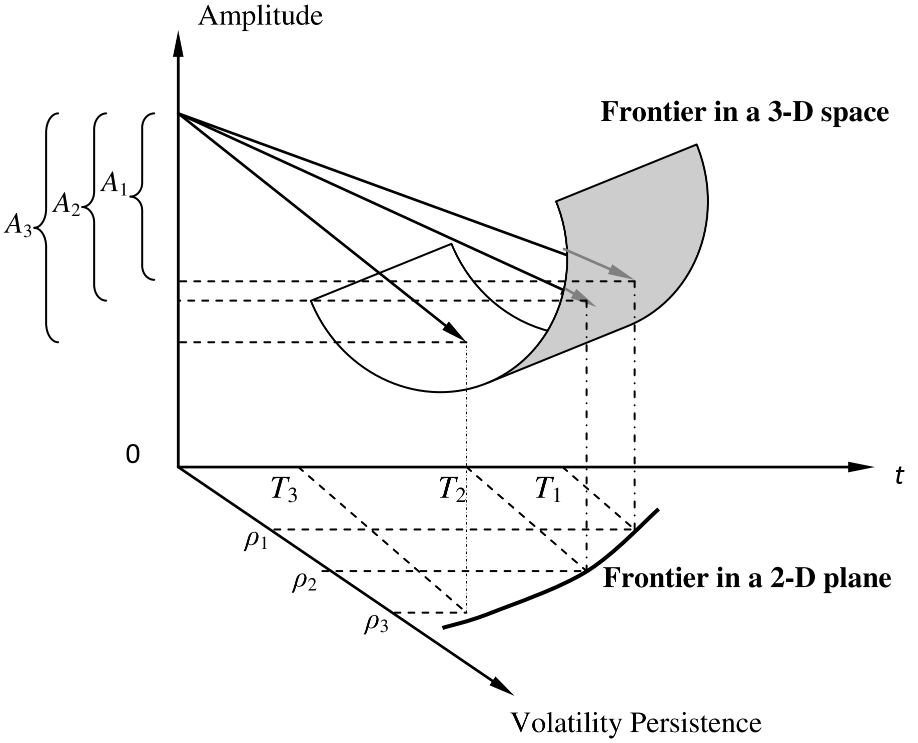

Fig. 1 presents the trivariate trade-off frontier between amplitude, duration and volatility persistence. The grey surface represents the amplitude-duration-persistence frontier for a bear market with an *ex ante* overvaluation. There is a projection of the surface onto each of the three planes: persistence-time, price-persistence and price-time.

Fig. 2. Index Frontiers for Each of the Financial Crises.

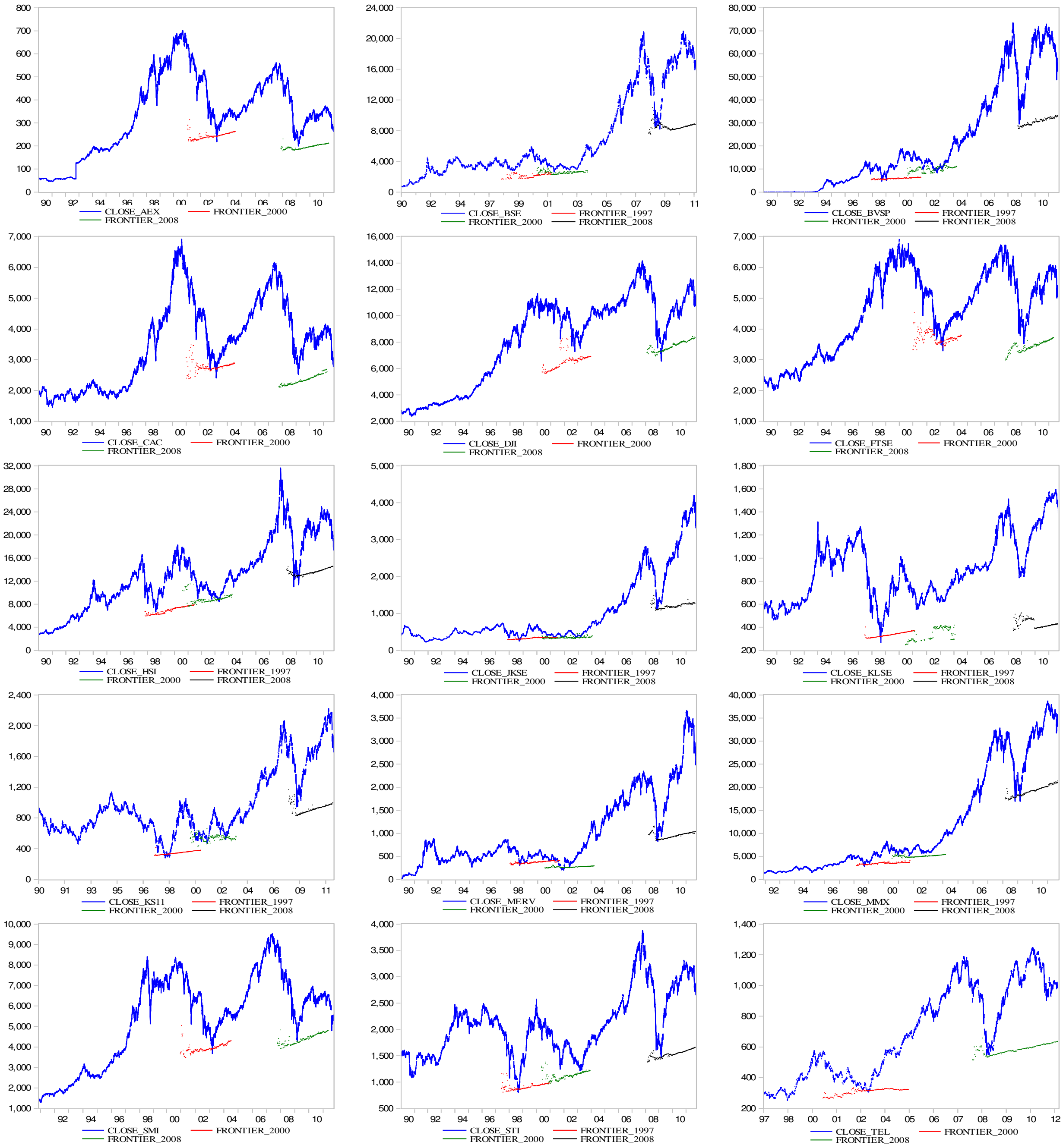

Fig. 2 shows for each of the 15 stock markets analysed the closing value of the stock market index (labelled as CLOSE_X, where X is the index name) together with the frontier for the 3 crisis periods (labelled by FRONTIER_1997, FRONTIER_2000 and FRONTIER_ 2008, respectively). The AEX, CAC40, DJT, FTSE and TEL were omitted from the analysis of the 1997 Asian Financial Crisis as they were relatively unaffected by it.

Fig. 3. The Index Frontier for China's 2001–05 Bear Market and 2007–08 Crash.

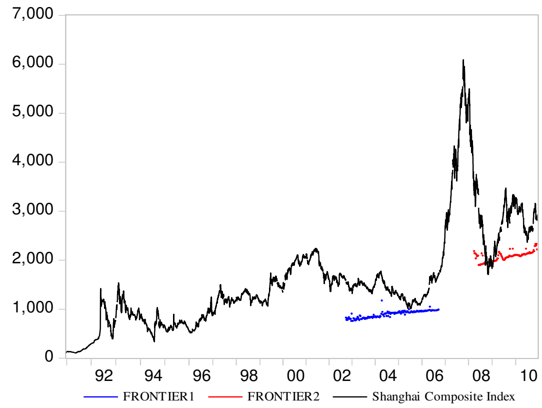

Fig. 3 shows the Shanghai Composite Index and its frontiers for the 2001–05 bear market (Frontier 1) and 2007–08 crash (Frontier 2). It shows that when index came close to the frontiers, the market rebounded in both mid-2005 and late 2009. Thus the frontiers build important support lines for the index.

Fig. E.1. Behaviour of the DJI Index during the Fall of Lehman Brothers.

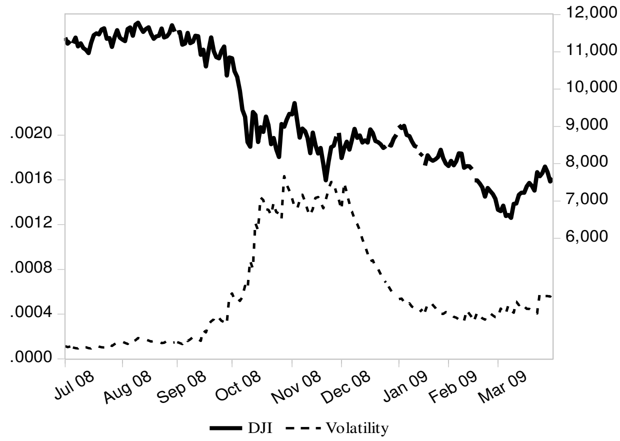

Fig. E.1 illustrates the volatility dynamics (dashed line, left hand scale) and DJI index (solid line, right hand scale) during the fall of Lehman Brothers in September 2008. It can be observed that the long run volatility level, derived from an ARMA(1,1)-CGARCH(1,1) model, increased dramatically on news of the bail out of Lehman Brothers, and the index value approached the trough with the decay of long run volatility.

Fig. F.1. Comparison of Stock Market Behaviour during Financial Crises.

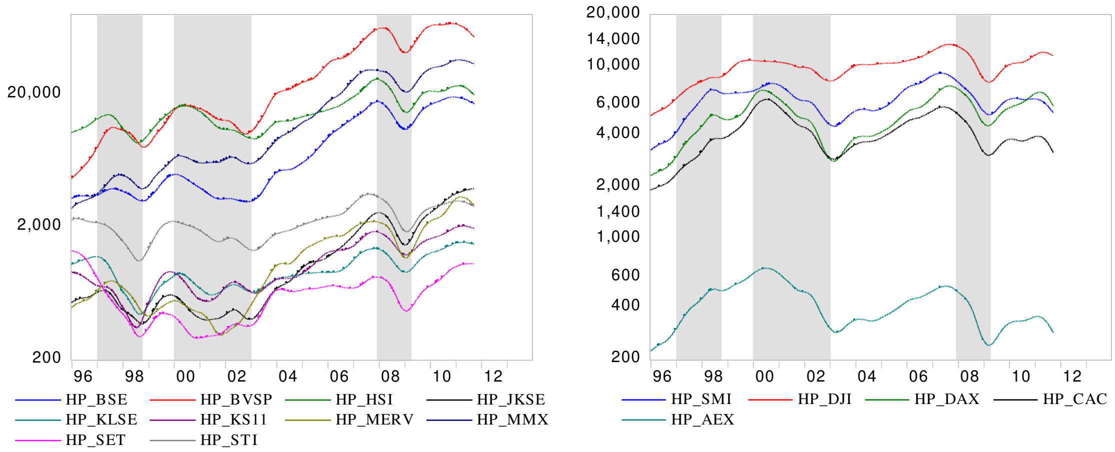

Fig. F.1 plots the trends of the 15 stock market indices using a HP filter (Hodrick and Prescott (1997)). The left hand panel shows the indices for the four developed markets and single Middle Eastern market (DJI, AEX, CAC40, TEL and FTSE). The right hand panel shows the indices for the Asian markets and emerging markets in South America. The trends are plotted on a logarithm scale. The shaded areas denote the timescale of three financial crises.

Acknowledgement

The authors would like to thank Dr Michael Cole and Professor Brendan for their constructive feedback and Mr Simon Blackman for proof reading the manuscript.

References

1. Adrian, T., Rosenberg, J., 2008. Stock returns and volatility: pricing the short-run and long-run components of market risk. Journal of Finance, 63, 2997–3030.
2. Andersen, T.G., 1996. Return Volatility and Trading Volume: An information flow interpretation of stochastic volatility. Journal of Finance, 51, 169–204.
3. Andersen, T.G., Bollerslev, T., Diebold, F.X., Vega C., 2003. Micro effects of macro announcements: real-time price discovery in foreign exchange. American Economic Review, 93, 38–62.
4. Barberis, N., Shleifer, A., Vishny, R., 1998. A model of investor sentiment.Journal of Financial Economics, 49, 307–343.
5. Berger, D., Chaboud, A., Hjalmarsson, E., 2009. What drives volatility persistence in the foreign exchange market? Journal of Financial Economics, 94, 192–213.
6. Black, F., 1986. Noise. Journal of Finance, 41, 529–543.
7. Bry, G., Boschan, C., 1977.Cyclical Analysis of Time Series: Selected Procedures and Computer programs. New York, NBER.
8. Chen, S., Shen, C., 2007. Evidence of the duration-dependence from the stock markets in the Pacific Rim economies. Applied Economics, 39, 1461–1474.
9. Chong, T.T., Li, Z., Chen, H., Hinich, M., 2010. An investigation of duration dependence in the American stock market cycle. Journal of Applied Statistics, 37, 1407–1416.
10. Claessens, S., Dasgupta, S., Glen, J., 1995. Return behavior in emerging Stock Market. The World Bank Economic Review, 9, 131–151.
11. Cowan, R., Jonard, N., 2004. Network Structure and the Diffusion of Knowledge. Journal of Economic Dynamics and Control, 28, 1557–1575.
12. Cuñado, J., Gil-Alana, L.A., de Gracia, F.P., 2008. Stock market volatility in US bull and bear markets. Journal of Money, Investment and Banking, 1, 24–32.

13. Cuñado, J., Gil-Alana, L.A., de Gracia, F.P., 2009. US stock market volatility persistence: evidence before and after the burst of the IT bubble. Review of Quantitative Finance and Accounting, 33, 233–252.

14. Engle, R.F., Rangel, J.G., 2008. The Spline-GARCH Model for Low-Frequency Volatility and Its Global Macroeconomic Causes. Review of Financial Studies, 21 (2007) 1187–1222.

15. Engle, R.F., Lee, G.G.J., 1999. A Permanent and Transitory Model of Stock Return Volatility. in R. Engle H. White, ed.: Cointegration, Causality, and Forecasting: A Festschrift in Honor of Clive W. J. Granger. Oxford University Press.

16. Diebold, F.X., Inoue, A., 2001. Long memory and regime switching. Journal of Econometrics, 105, 131–159.

17. Gonzalez, L., Powell, J. G., Shi, J., Wilson, A., 2005. "Two centuries of bull and bear market cycles." International Review of Economics & Finance 14, 469-486.

18. Fisher, K.L., 2007. The Wall Street waltz: 90 visual perspectives, illustrated lessons from financial cycles and trends. Hoboken, New Jersey: John Wiley and Sons, Inc.

19. Fleming, J., Kirby, C., Ostdiek, B., 2006. Stochastic volatility, trading volume, and the daily flow of information. Journal of Business, 79, 1551–1590.

20. Hodrick, R.J., Prescott, E.C., 1997. Postwar U.S. business cycles: An empirical investigation. Journal of Money, Credit and Banking, 29, 1–16.

21. Hong, D., Hong, H., Ungureanu, A., 2010. Word of Mouth and Gradual Information Diffusion in Asset Markets. Working Paper, Singapore Management University, available http://www.princeton.edu/~hhong/diffusion.pdf.

22. Hong, H., Stein, J. C., 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. Journal of Finance, 54, 2143–2184.

23. Li, Z., Sun J., Wang S., 2012. An Information Diffusion-Based Model of Oil Futures Price.Energy Economics, 36, 518–525.

24. Lunde, A., Timmermann, A., 2004. Duration dependence in stock prices: an analysis of bull and bear markets. Journal of Business and Statistics, 22, 253–321.

25. Maheu, J.M., McCurdy, T.H., Song, Y., 2012. "Components of bull and bear markets: bull corrections and bear rallies." Journal of Business & Economic Statistics 30, 391-403.

26. Maheu, J., McCurdy, T., 2000. Identifying bull and bear markets in stock returns. Journal of Business and Economic Statistics, 18, 100–112.

27. Markowitz, H.M., 1952. Portfolio Selection. The Journal of Finance, 7, 77–91.

28. McQueen, G., Thorley, S., 1994. Bubbles, stock returns, and duration dependence. Journal of Financial and Quantitative Analysis, 29, 379–401.

29. McQueen, G., Vorkink, K., 2004. Whence GARCH? A preference-based explanation for conditional volatility. Review of Financial Studies, 17), 915–949.

30. McMillan, D.G., Ruiz, I., 2009. Volatility persistence, long memory and time-varying unconditional mean: Evidence from 10 equity indices. The Quarterly Review of Economics and Finance, 49, 578–595

31. Merton, R.C., 1973. An Intertemporal Capital Asset Pricing Model. Econometrica, 41, 867–887.

32. Pagan, A.R., Kirill, A.S., 2003. A simple framework for analysing bull and bear markets. Journal of Applied Econometrics 18, 23-46.

33. Peng, L., Xiong, W., 2003.Time to Digest and Volatility Dynamics. Working Paper, Baruch College and Princeton University, available at http://www.princeton.edu/~wxiong/papers/volatility.pdf

34. Rangel, J.G., 2011. Macroeconomic news, announcements, stock market jump intensity dynamics. Journal of Banking & Finance, 35, 1263–1276.

35. Sharpe, W.F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. Journal of Finance, 19, 425–442.

36. Veldkamp, L., 2005. Slow boom, sudden crash. Journal of Economic Theory, 124, 230–257.

37. Woodward, G., Marisetty, V.B., 2005. Introducing non-linear dynamics to the two-regime market model: Evidence. Quarterly Review of Economics, 45, 559–581.