Borrowing Constraints, College Aid, and Intergenerational Mobility

Hanushek, Eric and Leung, Charles Ka Yui and Yilmaz, Kuzey

Stanford University, City University of Hong Kong, University of Rochester

March 2014

Borrowing Constraints, College Aid, and Intergenerational Mobility *

Eric Hanushek, Charles Ka Yui Leung and Kuzey Yilmaz†

March 6, 2014

1 Introduction

Education holds a special position in most societies around the world. Governments quite uniformly subsidize schooling heavily, often making it free to the student. The treatment of higher education is an especially contentious political subject, in part because fiscal pressures have led many governments to reconsider their subsidy policies. Coming out of the 2008 recession, for example, U.S. states have reduced their subsidies to college education (College Board 2011, 2012). Internationally, the movement from generally free higher education to having student fees – even with heavy subsidies – has led to a variety of political conflicts and student protests. Yet, any suggestion that a government contemplates raising student fees for higher education frequently brings a wave of protests. In the United States, political concerns about rising tuition costs have led the U.S. Congress to hold hearings and contemplate legislation and the U.S. Secretary of Education to establish a commission on higher education, even though tuition policies are the province of state governments.[1] Nonetheless, it is not obvious why governments intervene to this extent they do. Nor is it clear why a government might choose one form of college subsidy over another. This paper explores the implications of alternative college subsidy schemes both from efficiency and equity perspectives.

We consider three aspects of college subsidy policies. First, in a classic concern about market imperfections, we analyze whether different policies yield efficiency gains through reducing any financial constraints that stop high ability students from attending college. Second, we look at how the distribution of income in society is affected by various college subsidies. Finally, we look at whether subsidies affect the amount of intergenerational income mobility through modifying the intergenerational pattern of educational attainment.

To analyze these issues, we develop a general equilibrium model that is calibrated to existing U.S. subsidy policies. In this, individuals of varying ability make optimal schooling decisions in the face of uncertainty about completing college and, for a subset, in the face of financial constraints that limit otherwise rational college attendance. The college subsidies considered – general tuition subsidies, need-based and merit-based aid, and income contingent loans – alter the cost of education to the individual, and thus college decisions respond to the specific aid regimes. We focus on general equilibrium affects because different college subsidies have large impacts on college attendance and completion and thus on the wages observed in the economy. Any change of wages, however, will influence the incentives for individuals to make their human capital investment and may further magnify or dampen the effect of the educational policies. In fact, we show in a later section that the partial equilibrium approach for educational policies could generate misleading conclusions.

*The authors are grateful to the editor (Lance Lochner) and anonymous referees, and a variety of seminar participants for helpful discussion. For financial support, Leung is grateful to the City University EF Department Professional Grant. The usual disclaimer applies.

†Correspondence: Hanushek, Hoover Institution, Stanford University, Stanford, CA 94305-6010, USA, hanushek@stanford.edu; Leung, Department of Economics, City University of Hong Kong, Hong Kong, kycleung@cityu.edu.hk; Yilmaz, Department of Economics, University of Rochester, Rochester, NY 14627, kyilmaz@z.rochester.edu.

[1] The pattern of net tuition by income groups in the U.S. is analyzed in Congressional Budget Office (2004). The Secretary of Education's Commission on the Future of Higher Education (2006), while addressing a variety of issues in higher education, emphasized affordability and financial aid.

The impact of the varying types of college subsidies on output and social mobility is very different. While each tends to improve output compared to the credit-constrained case, need-based policies lead to significantly greater equality than merit-based policies. Further, targeted need-based policies have desirable properties compared to the most common support for higher education – uniformly reduced tuition at public colleges. Income contingent loans act quite differently by taxing high ability poor people, reflecting natural adverse selection. Nonetheless, they have considerable appeal in terms of both efficiency and distributional outcomes.

While other analyses of college subsidies have addressed similar aspects of the problem, this paper includes a number of advances of the existing literature. First, it provides an integrated treatment of all of the common subsidy schemes currently employed in higher education: uniform tuition subsidies, need-based aid, merit scholarships, and income contingent loans. Thus, it is possible to compare policy alternatives directly in terms of overall enrollment and completion effects, the impact on aggregate economic efficiency, and the distribution of rewards. Second, it places the analysis of college subsidies into the more traditional perspective of the incidence of taxes and benefits on households, albeit analyzed in a life-cycle manner. Third, it provides an analysis of key features of the analytical methodology commonly employed to consider college subsidies. Specifically, it evaluates the impact of considering a general equilibrium solution where wages adjust to aggregate schooling choices and of evaluating the intergenerational transmission of ability in conjunction with financial linkages of generations Both of these considerations make a significant difference in identifying the quantitative impact of subsidies and in providing relative evaluations of different programs.

2 Existing Literature

The economic impact of government intervention in education has received relatively little systematic research attention, particularly given the magnitude of programs. For K-12 education, government subsidy can be rationalized by arguments about externalities related to socialization, facilitating democratic government, and reducing crime.[2] But such externalities appear considerably less important when considering college education. Our earlier paper (Hanushek, Leung and Yilmaz (2003)) considered pure redistributional motives along with externalities of education in production, but provided limited general support for this form of government subsidization. Consideration of direct distributional objectives is also the main thrust of Benabou (2002), Caucutt and Kumar (2003), and Restuccia and Urrutia (2004). Those papers are directly related to our work here in that they explicitly consider the dynamics of the problem, and we return to them below.

A remaining argument for subsidization revolves around capital market imperfections and the inability to borrow against human capital (e.g., Becker 1993[1964] or Garratt and Marshall 1994). Because human capital is not good collateral for loans, individuals can find it difficult to fund college if the family cannot readily self-finance. Further, because any borrowing constraints are likely to be related to parental income, the resulting decisions on college tend to reinforce existing patterns of intergenerational mobility. To the extent that society wishes to disentangle opportunities of individuals from the socioeconomic status of their parents, subsidizing college may directly meet societal goals for distributional outcomes.

The existence and importance of credit constraints has been the subject of debate. In an influential set of papers (Cameron and Heckman 1999, 2001; Carneiro and Heckman 2002), Heckman and his coauthors argue that short run credit constraints are small even if longer run constraints deriving from transmission of achievement are more substantial.[3] Similarly, Restuccia and Urrutia (2004) consider both factors, although

[2] Of course as raised previously by Friedman (1962), these arguments do not establish the case either for the magnitude of current intervention or for the form that involves direct production as opposed to subsidy.

[3] Note that we do not attempt to ascertain empirically the importance of borrowing constraints but simply concentrate on the implications that such constraints would have for the economy and the distribution of welfare. For other discussions of the magnitude and nature of constraints on college attendance, see Keane and Wolpin (2001) and Kane (1999). Other discussions such as the effect on enrollment of eliminating Social Security tuition support for children with a deceased, disabled, or retired parent are relevant to our development of models where some children are constrained by insufficient parental support (Dynarski (2003)).

they focus most on early skills as opposed to any financial constraints. More recently, Lochner and Monge-Naranjo (2011) expand consideration of budget constraints to consider both private lending and government loan programs. They develop a model of educational attainment, family resources, and ability that is consistent with our structure below. While we do not try to estimate the magnitude of any credit or skill constraints directly, we base our analysis on a presumption that both exist.

This analysis delves explicitly into the intergenerational outcomes of various college subsidy schemes in the presence of financial constraints. Systematic study into aspects of both the efficiency and distributional impacts of educational policies has been growing over time (see, for example, Loury (1981), Glomm and Ravikumar (1992), Iyigun (1999), Maoz and Moav (1999), Galor and Moav (2000), and Fender and Wang (2003)), and the analysis here is a natural extension of these inquiries.[4]

3 An Intergenerational Model of College Choice

We develop a dynamic general equilibrium model that can provide insights into the implications of various commonly discussed and commonly employed college aid policies for both the efficiency of the economy and for the distribution of outcomes over time. We begin in a world where short run borrowing constraints can stop some families from making optimal schooling decisions, implying that society will not achieve its first best outcome without government intervention. Government has, however, a variety of instruments for subsidizing education, and these instruments have different implications for the economy in both the short and long run.

The focus of our work is college decision making. We abstract from pre-college and post-college investments in order to understand better how ability, families, and opportunities affect college attendance and completion.[5] We pay particular attention to the implications of college investments on the life-cycle patterns of earnings and on the income correlations across generations. Further, since government is heavily involved in higher education, we look beyond the impacts of subsidies on individual skills to consider aggregate outcomes including eliminating investment distortions and altering the distribution of income.

To capture the dynamic nature of the problem, we employ an overlapping generations model where the economy is populated by a continuum of agents who live three periods and are part of a continuum of three-agent families. In each family (or "dynasty"), there is an old agent ("grandparent"), a middle-aged adult ("parent"), and a young adult ("grandchild", or simply child). The population of the economy is constant over time. Heterogeneity of agents enters through ability differences that affect both the probability of completing college and subsequent labor market productivity.[6] Agents make optimizing decisions with respect to enrolling in school faced with uncertainty of successful completion. The relevance of uncertainty of college success is easy to see from the fact that the completion rate of U.S. high school graduates of the 1992 cohort enrolling in college was only 45% (Bound, Lovenheim, and Turner (2010)). Moreover, this completion rate was less than that for the 1972 cohort of graduates, a fact that is consistent with our analysis below. Each family must, however, fully fund the education of the child through resources or borrowing, so that in the absence of outside funding the child cannot attend college whenever tuition exceeds the parent's educational bequest and borrowing. Each child's ability is probabilistically related to parental ability, and the parent passes along pecuniary bequests that interact with children's ability to determine education in much the spirit of Loury (1981).[7]

The baseline is calibrated to an economy with subsidized tuitions and loans similar to the U.S. situation where borrowing may still be insufficient to remove all credit constraints on individuals. We then consider

[4]Consideration of distributional issues have been more common when discussing K-12 education. In the work closest to the spirit of ours, Fernández and Rogerson (1997, 1998, 2003) consider alternative funding mechanisms of schools and trace the implications for future earnings in a dynamic model. Similarly, Restuccia and Urrutia (2004), while emphasizing the comparison of early education and later education, consider the dynamic outcomes of policies including alternative college funding policy.

[5]For a discussion of how investments relate across the life-cycle, see Cunha, Heckman, Lochner, and Masterov (2006).

[6]Throughout the analysis, college quality is assumed constant. Thus, we ignore any possible feedback from altered college attendance to college quality (see Bound, Lovenheim, and Turner (2007)). It also ignores any potential impacts of peers on quality or prices (Epple, Romano, and Sieg (2003)).

[7]Restuccia and Urrutia (2004) also relate parent and child ability both through nature and nurture, where prior investments in schooling enter into the transmission of ability.

alternative governmental programs mirroring the most common governmental interventions: low tuition, need-based grants, merit awards, and income contingent loans. While the government can intervene in the college market in these alterantive ways, it must maintain a balanced budget each period (which is a generation in our three-period OLG model).[8] Government intervention distorts the economy through taxes and through changing college decisions with varying efficiency losses across types of subsidy. This basic economy permits us to trace out the dynamics of the income distribution along with the impacts of government intervention on overall output.[9]

3.1 Individual Decision Making

The individual decision process is to maximize expected lifetime utility subject to the cost of college, the available sources for funding college tuition, the probability of successfully completing college, and the wages of those completing college and those not completing. During the first period of life, children (and their parents) make schooling decisions; during the second and third period, they work at a wage determined by their educational attainment (college completion or not) as well as their ability. Agents supply one unit of labor inelastically in the second and third periods. Those who choose not to attend college can work for a fraction of the first period, implying that schooling has a time cost in addition to any direct tuition payment. Heterogeneity of agents enters through ability differences that affect both the probability of college completion and subsequent labor market productivity.

Each middle-aged adult has a child endowed with ability that is correlated with the parent's and will leave a bequest either to finance the child's college education or to permit further child consumption. Hence, agents differ in terms both of the bequests they receive and of the ability inherited from parents. The solution to the student optimization problem is easiest to see by starting with the old workers and working backwards. For notation, we subscript old agents by o, middle-aged agents by m, and youth by y.

3.1.1 The Old Agent's Problem

The old agent (in the third period of life) has ability x and provides $(k_0 + k_1 x)$ efficiency units of labor, where $x \in [0, 1]$ and $k_o > 0$ and $k_1 > 0$ are constants. The lowest ability agents with $x = 0$ provide k_0 efficiency units of labor, which is interpreted as a measure of basic skills and is sometimes thought of as the productivity of "raw labor". k_1 measures individual differences in productivity in the labor market associated with higher ability. Whether individuals provide work in a skilled or unskilled job depends on being a college graduate and is denoted by an indicator $I^e = 1$ for a college graduate and $I^e = 0$ for nongraduates. A skilled worker gets a wage of w_e while an unskilled worker gets a wage of w_u. Thus, the "type" of an agent is represented by the state vector, $\overrightarrow{x_o} = (x, I^e)$. The old agent is simply maximizing consumption subject to income (g) and accumulated savings. Utility is assumed to follow a standard constant relative risk aversion form as:

$$\begin{array}{ll} V_o(\overrightarrow{x_o}; s) & = \max_{c_o \geq 0} \frac{c_o^{1-\sigma} - 1}{1-\sigma} \\ s.t. & c_o = g_o + rs \\ & g_o = (1-\tau)(k_0 + k_1 x) w_o \\ & w_o = w_u(1 - I^e) + w_e I^e, \end{array} \tag{1}$$

where $\sigma > 0$ is a constant, r is the market interest rate, and τ is the tax rate on labor income. The agent will clearly consume all the income at the last period of her life (i.e. $c_o = g_o + rs$).

3.1.2 The Middle-Aged Adult's Problem

The middle-aged agent (in the second period of life) provides labor and pays back at the market interest rate, r, any loans made to fund the prior college decisions. The middle-aged agent has a child in this period

[8]While overall GDP can rise because of better use of resources, we do not allow for altered economic growth as might occur with, say, an endogenous growth model with skill-biased technological change.

[9]A related analysis is the seminal study by Caucutt and Kumar (2003) that also considers the dynamics of college attendance, although it differs in significant ways concerning both methodology and policy issues as described below.

and, must decide how much to consume, c_y, how much to save, s, and the amount of bequest, b', to be left for her child's basic needs and college education.[10]

There is a government loan program in the economy to help pay for college tuition, ϕ. We assume the government can perfectly observe the bequest left for any child in the first period. A child is then eligible for the minimum amount of loan in the loan program that provides him with enough resources to be financially unconstrained and to attend college (i.e., bequest - minimum consumption + loan > tuition).[11] Put differently, the loan amount available is conditional on a child's bequest level, b, and the loan can only be used to pay tuition. Clearly, if the bequest net of minimum consumption $(b-\bar{c}_{min})$ is larger than tuition, the child is not eligible for a loan (but is also financially unconstrained in deciding upon schooling).

For simplicity, we assume there are three different loan levels a child can choose: 25%, 50% and 75% of tuition. We introduce a new indicator function I^l, that defines the loan status of a child/adult:

$$I^l(b) = \begin{cases} 1 & \text{if } 0.75\phi \leq (b-\bar{c}_{min}) < \phi \\ 2 & \text{if } 0.5\phi \leq (b-\bar{c}_{min}) < 0.75\phi \\ 3 & \text{if } 0.25\phi \leq (b-\bar{c}_{min}) < 0.5\phi \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Given the definition of the indicator function for loans, the amount of loan made is given by $0.25*\phi*I^l(b)$. For instance, if a child with a bequest of $(b-\bar{c}_{min}) = 0.55*\phi$ takes the loan and attends college, then it must be the case that $I^l(b = 0.55*\phi) = 2$ and the loan amount will be $0.25*\phi*2 = 0.5*\phi$. Effectively, we are segregating the population of children into four different groups according to the amount of bequest they have (relative to the tuition), or equivalently the loans they need to attend college. If a young agent attends college without a loan or does not attend college at all, then $I^l(b) = 0$.

We can now characterize the problem facing a middle-aged agent in the second period (ex-post) by a state vector with three state variables, namely $\overrightarrow{x_m} = (x, I^e, I^l)$:

$$\begin{aligned} V_m(\overrightarrow{x_m}) = \max_{c_m\geq 0,\, b'\geq \bar{c}_{min},\, s} \quad & \frac{\left((c_m/\alpha)^{\alpha}\left(b'/(1-\alpha)\right)^{1-\alpha}\right)^{1-\sigma}-1}{1-\sigma} + \beta_3 V_o(\overrightarrow{x_o}; s) \\ & s.t. \\ & c_m + b' + s = g_m \\ & g_m = (1-\tau)(k_0 + k_1 x)w_m - r\cdot 0.25\cdot\phi\cdot I^l \\ & w_m = w_u(1-I^e) + w_e I^e \end{aligned} \tag{3}$$

where $0<\alpha<1$ is a constant, and $0<\beta_3<1$ is the discount factor. Clearly, the amount of bequest left to a child, b', is a function of the parent's state variables, and we write $b' = b'(x, I^e, I^l) = b'(\overrightarrow{x_m})$.

Moreover, generations are directly linked because parent's ability x affects her child's ability, x'. Unfortunately, we do not have direct observations of ability. In the calibration, we use test scores to construct our ability measure. More specifically, we assume that the transmission of the test score from a parent, z, to an child, z', follows a simple AR(1) process:[12]

$$z' = \lambda z + \epsilon \quad \epsilon \sim N(0, \sigma^2_{\epsilon}) \tag{4}$$

where λ is the correlation between the test scores of the parent and the child, and ϵ is the white noise. Ability x is related to test scores by $x = \Phi(z)$, where $\Phi(z)$ is the cumulative distribution function of the standard normal distribution. By construction, our ability measure x lies between zero and unity, $x\in[0,1]$. And in the spirit of Ben-Porath (1970), x' determines the college completion probability along with labor market skills. A child completes college successfully with probability x' and fails with probability $1-x'$.

[10]In terms of notation, a variable without a prime relates to the parent, while a prime indicates the relevant variable for a child. For example, consider ability: a parent has ability x and a child has ability x'. Similarly, the bequest from a parent to a child is b' while the bequest from a grandparent to a parent is b.

[11]We generally speak of being financially constrained as a binary condition, i.e., having sufficient funds above those needed for minimum consumption to pay for college. Obviously, being just at minimum consumption may not be a desirable state and individuals may still feel constrained if pushed to such low levels. They might optimally borrow to smooth consumption if such loans were available but in the absence of such loans may decide against attending college even though they have sufficient funds to do so.

[12]See Yilmaz (2011) for a more detailed model of ability formation

3.1.3 The Young Agent's Problem

The decision facing the "young agent" (in the first period of life) is whether to attend college or not. In this decision, ability plays a key role, because it directly indicates the probability of successfully finishing college given that the child enrolls.

Toward the end of the first period, the young agent makes the college attendance decision. A young agent is best described by the state vector, $\overrightarrow{x_y}' = (x', b')$, where b' depends on $\overrightarrow{x_m} = (x, I^e, I^l)$,which is the state vector of the young agent's parent. In addition to the young agent's ability x', $\overrightarrow{x_y}'$ also contains information about the bequest b', left by her parent and the young agent receives a bequest when her parent is in the second generation, $b' = b'(\overrightarrow{x_m})$. Moreover, it is easy to see that, given the amount of bequest, b' for the young agent, her loan status is already determined if she attends college. For instance, if a young agent with a bequest of $(b' - \bar{c}_{min}) = 0.85 * \phi$ attends college, then it must be the case that $I^{l'}(b') = 1$. The subsequent labor market outcome depends, however, on whether the young agent succeeds in education, rather than whether a loan is made. For future reference, let us define another indicator function, $I^{r'}$, that shows the college enrollment:

$$I^{r'} = \begin{cases} 1 & \text{if attends college} \\ 0 & \text{otherwise} \end{cases}$$

Given the fact that the college attendance is a risky decision and the child could fail, the (ex-ante) expected utility of the child attending college depends on the bequest and loan amounts. Formally, it means that

$$EU(I^{r'} = 1, \overrightarrow{x_y}') = \frac{\left(c_y\left(I^l\right)\right)^{1-\sigma} - 1}{(1-\sigma)} + \beta_2\left\{\overline{V_m}\left(x',\ I^l(b')\right)\right\} \tag{5}$$

where $c_y\left(I^l\right) = b' - 0.25 \cdot \phi \cdot I^l$, I^l being the short hand of $I^l(b')$, and V_m is the value function for a young agent when she is a middle-aged adult, which depends on her state variables of ability x', college graduation status $I^{e'}$, and loan category $I^l(b')$, as given by equation (**??**). Thus, $\overline{V_m}\left(x', I^l(b')\right)$ is the expected value of the value function given ability x' and loan type $I^l(b')$ and is given by $\overline{V_m}\left(x', I^l(b')\right) = x'V_m(x', 1, I^l(b')) + (1-x')V_m(x', 0, I^l(b'))$.[13]

A few observations are in order. Notice that for young agents with a bequest net of minimum consumption being less than tuition, attending college is only possible with a loan. For instance, consider again a young agent with $(b' - \bar{c}_{min}) = 0.55\phi$. The young agent can attend college with a loan of 0.5ϕ (i.e. $I^{l'} = 2$). Recall that $V_m(x', I^{e'} = 1, I^{l'} = 2)$ and $V_m(x', I^{e'} = 0, I^{l'} = 2)$ stand for the value functions for middle-aged adults with a loan of 0.5ϕ and ability of x', who complete college successfully and who fail, respectively. Since ability, x' determines the probability of success, the expected utility of college outcome at the beginning of second period is $x'V_m(x', 1, 2) + (1-x')V_m(x', 0, 2)$. In the first period, the young agent gets the loan, 0.5ϕ in addition to bequest, b' left by her parents. The young agent pays the tuition, ϕ, and consumes what is left. Following (**??**), a young agent faces one of the four possible expected utilities of attending college, depending on her bequest, b'.

We assume that a fraction φ of the first period can be spent in the labor market if the young agent does not attend college (but not if the young agent attends college). Any work by the young agent involves unskilled labor during that period. Therefore, the (ex ante) utility of not attending college is given by:

$$EU(I^{r'} = 0, \overrightarrow{x_y}') = \frac{(c_y^n)^{1-\sigma} - 1}{1-\sigma} + \beta_2 V_m(x', 0, 0), \tag{6}$$

where $c_y^n = b' + \varphi \cdot (1-\tau)(k_o + k_1 x')w_u$.

The young agent then starts the second period as an unskilled worker without any loan. The college attendance decision is determined by whether attending college provides a higher level of utility than that of not attending college at all, and the maximum utility level is given by:

[13] Notice that the state vector for the young agent is $\overrightarrow{x_y}' = (x', b')$, which appears on the left hand side of (**??**). Hence, it is not surprising that the right hand side of (**??**) would contain $I^l(b')$, which also depends on b'.

$$EU(\overrightarrow{x_y}') = \max_{I^{r'}} \left\{ EU(I^{r'} = 1, \overrightarrow{x_y}'),\ EU(I^{r'} = 0, \overrightarrow{x_y}') \right\} \tag{7}$$

3.2 Economic Environment

The key elements of the economic environment are the wages received with and without successful college completion and the governmental programs that are available to help pay for college. We first describe the aggregate dynamics that determine the macroeconomic variables in the economic environment.

3.2.1 Aggregate Dynamics

The aggregate dynamics of this model are simultaneously simple and complicated. They are "simple" because there is no aggregate uncertainty in this model. In fact, with a continuum of agents, the laws of motion for different types are deterministic, despite the fact that there is an idiosyncratic (education) risk for each young agent enrolled in college. On the other hand, the aggregate dynamics are "complicated" because the macroeconomic variables in this model, such as the equilibrium wages, depend on the distribution of the agents. Thus, it is necessary to keep track of the evolution of the distribution in order to characterize the dynamics. Furthermore, there are three endogenous participation constraints in the model: whether the young agent receives a sufficiently large bequest for college ($(b' - \bar{c}_{min}) \lesseqgtr \phi$), whether she gets a loan, $I^{l'} \in \{1, 2, 3\}$, to attend college, and whether the young agent has enough ability to make college attendance rational (i.e. $EU(I^{r'} = 1, \overrightarrow{x_y}') \lesseqgtr EU(I^{r'} = 0, \overrightarrow{x_y}')$). In this, college tuition, wages, and college attendance decisions are all endogenous.

Recall that we have assumed the transmission of ability between two consecutive generations is characterized by an AR(1) process (equation (**??**)). However, AR(1) has an infinite state-space and hence is difficult to compute. Therefore, we follow Tauchen (1986) to approximate it with a first-order Markov Chain with finite state-space. More importantly, we find that the aggregate dynamics of the model economy can be described by a first-order Markov Chain.

More formally, we use $f_t\left(\overrightarrow{x_m}\right)$ to represent the ex-post (i.e. college outcome realized) probability distribution function (pdf) for the parent cohort when they are middle-aged adults at time t, over the state space, Ω, and where $\overrightarrow{x_m} \in \Omega = \{(x, I^e, I^l) | x \in [0,1], I^e = \{0,1\}, I^l = \{0,1,2,3\}\}$ is the vector of the state variables for the parent. To fix the idea, let $F_t(\Omega)$ be a vector representation of the probability distribution over all different types of middle-aged agents (parent cohort) at time t (i.e. $F_t(\Omega) = f_t\left(\Omega\right)$ with abuse of notation). In other words, $F_t(\Omega)$ contains all the information for the probability distribution function of parents when they are middle-aged agents at time t, $f_t(\overrightarrow{x_m})$ over the state space Ω. Then, the evolution of the economy can be captured by a matrix equation:

$$F_{t+1}(\Omega) = \Pi_t F_t(\Omega) \tag{8}$$

where Π_t is the transition matrix of time t, incorporating the information of the transition probabilities of abilities $x'|x$, the wages, the distribution of wealth, and, perhaps more subtly, the previously mentioned endogenous participation constraints. (A description of the determination of the matrix Π_t, which is technically involved, is available from the authors). $F_{t+1}(\Omega)$ is clearly the vector representation of the probability distribution function for the grandchild's cohort when they are middle-aged agents. Put differently, the transition matrix Π_t shows the transition probabilities of time t between the state of parent when she is middle-aged, $\overrightarrow{x_m}$ and the state of her child when she is middle-aged, $\overrightarrow{x_m}'$ (i.e. $Prob(\overrightarrow{x_m}'|\overrightarrow{x_m}) = Prob(x', I^{e'}, I^{l'}|x, I^e, I^l)$ for any $\overrightarrow{x_m} \in \Omega$ and $\overrightarrow{x_m}' \in \Omega$. Note that we can drop time indices in both transition matrix and probability distribution vectors/functions because we focus on the stationary equilibrium. (i.e. $\Pi = \Pi_t$, $F(\Omega) = F_{t+1}(\Omega) = F_t(\Omega)$ and $f(\overrightarrow{x_m}) = f_{t+1}(\overrightarrow{x_m}) = f_t(\overrightarrow{x_m})\ \forall\ \overrightarrow{x_m} \in \Omega$). One important distribution that we can create, which will prove quite useful later, is a joint probability distribution function, $Prob(x', I^{e'}, I^{l'}, x, I^e, I^l)$ that shows the proportion of parents at state $\overrightarrow{x_m}$ with a child at state $\overrightarrow{x_m}'$ as $g_t(\overrightarrow{x_m}', \overrightarrow{x_m}) = Prob(\overrightarrow{x_m}', \overrightarrow{x_m}) = Prob(\overrightarrow{x_m}'|\overrightarrow{x_m}) f_t(\overrightarrow{x_m})$.

Also, it is easy to calculate the number of adults enrolled in the college who succeed or fail:[14].

[14]Notice that $\int_{I^e=1} f_t(\overrightarrow{x_m})\, d\overrightarrow{x_m}$ is simply a shorthand for $\int_{I^e=1} f_t(x, I^e, I^l)\, d\left(x, I^e, I^l\right)$.

$$
\begin{aligned}
N^e &= \int_{I^e=1} f_t(\overrightarrow{x_m})\, d\overrightarrow{x_m} \\
N^r &= \int_{I^e=1} \frac{f_t(\overrightarrow{x_m})}{x}\, d\overrightarrow{x_m}
\end{aligned}
\quad . \tag{9}
$$

To calculate the enrollment ratio, we integrate over skilled workers and make use of the fact that attendees in the parent cohort with ability x succeed in college with probability, x. Clearly, the difference between the college enrollment and the college success is the college failure.

3.2.2 Wage Determination.

Wage determination depends on the mix of skilled workers and unskilled workers in the labor market. At any time t, three successive generations in a dynasty coexist: a grandparent as an old agent, a parent as a middle-aged adult, and a grandchild as a young agent. For a grandparent, we use $\overrightarrow{x_m^g} = (x^g, I^{eg}, I^{lg}) \in \Omega$ to represent the state variables when they were middle-aged. The production side of this model economy is characterized by a CES production function that has both the efficiency units of skilled and unskilled labor as inputs. For skilled and unskilled labor, labor markets clear, implying that wage determination depends on the mix of college completers and noncompleters in the goods market. We consider the total skilled workers (skilled "old" grandparents and skilled "middle-aged" parents) employed in either goods production ($\mathbb{E}^{s_p}$) or colleges ($\mathbb{E}^{s_c}$) in efficiency units:

$$
\mathbb{E}^{s_p} + \mathbb{E}^{s_c} = \mathbb{E}^s = \int_{I^{eg}=1} (k_o + k_1 x^g) f(\overrightarrow{x_m^g})\, d\overrightarrow{x_m^g} + \int_{I^e=1} (k_o + k_1 x)\, f\,(\overrightarrow{x_m})\, d\overrightarrow{x_m}. \tag{10}
$$

where $f(\overrightarrow{x_m^g})$ shows the probability distribution function of grandparents when they are middle aged agents.[15] (The wages of college graduates are equalized across the goods sector and the education sector, so we do not consider work choices of graduates). Unskilled "old" grandparents, unskilled "middle-aged" parents, and "young" grandchildren in their college years not attending college provide unskilled labor. Total unskilled labor in good production, $\mathbb{E}^u$ is given by

$$
\begin{aligned}
\mathbb{E}^u = &\int_{I^{eg}=0} (k_o + k_1 x^g) f(\overrightarrow{x_m^g})\, d\overrightarrow{x_m^g} + \int_{I^e=0} (k_o + k_1 x)\, f\,(\overrightarrow{x_m})\, d\overrightarrow{x_m} \\
&+ \varphi \int_{I^{e'}=0} (k_o + k_1 x')\, f(\overrightarrow{x_m}')\, d\overrightarrow{x_m}'.
\end{aligned} \tag{11}
$$

Where once again, $f(\overrightarrow{x_m}')$ is the probability distribution function of grandchildren when they are middle aged agents. The production side of this model economy is kept simple with a CES production function, which depends on both the efficiency units of skilled and unskilled labor,

$$
Y = A[\xi(\mathbb{E}^{s_p})^{(\rho)} + (1-\xi)(\mathbb{E}^u)^{(\rho)}]^{1/\rho}, \tag{12}
$$

where $0 < \xi < 1$, and the elasticity of substitution is $\eta = 1/(1-\rho)$. When $\rho = 0$, this is the Cobb-Douglas case. When $\rho = 1$, $\mathbb{E}^{s_p}$ and $\mathbb{E}^u$ are perfect substitutes, and when $\rho \to -\infty$, the two factors are perfect complements and the production function is Leontief.

The labor market is assumed to be competitive, and the representative firm maximizes profits. Demands for the efficiency units of skilled and unskilled labor are given by:

$$
\frac{\partial Y}{\partial \mathbb{E}^{s_p}} = w^e, \qquad \frac{\partial Y}{\partial \mathbb{E}^u} = w^u. \tag{13}
$$

3.2.3 Colleges

Educating children at colleges has substantial costs that cannot be ignored. We assume all costs come from teacher salaries. The crucial assumption is that an agent can be a teacher if she has a college degree (skilled worker). It takes γ efficiency units of skilled labor to provide a child with a college education. Given that the college enrollment is $N^{r'}$, the total cost of running the college sector is $\mathbb{E}^{s_c} w_e = \gamma N^{r'} w_e$.

[15] Recall that since we focus on stationary equilibrium, we simply write $f(\overrightarrow{x_m^g}) = f_{t-1}(\overrightarrow{x_m^g})$

3.2.4 Governmental Support

The government enters in a variety of ways. We previously described a loan program. We do not consider any default, which implies that the loans are set by the government with its ability to follow students and collect on loans. These loans are unsubsidized but enable attendance for financially constrained individuals.

In practice today, however, the largest and most common subsidy to college students is the reduced tuition that students receive. State run public colleges and universities invariably maintain tuition below production costs, even for nonresidents of the state.[16] In this paper, these tuition subsidies are labeled the "uniform subsidy regime," since they do not vary by the characteristics of the prospective student. Formally, the government levies a uniform rate of tuition ϕ on those attending college and a uniform tax rate τ from all agents. The tax proceeds are used exclusively to cover the costs of education (which is the wage bill for the teachers), as in:

$$\gamma N^{r'} w^e = \phi N^{r'} + \tau(w^e \mathbb{E}^s + w^u \mathbb{E}^u) \tag{14}$$

Notice that, when the income tax rate is zero ($\tau = 0$), the regime is reduced to purely private education, and the tuition is equal to the social cost of college education, i.e., $\gamma w^e = \phi$.

3.3 Equilibrium

We focus on the stationary equilibrium in which all prices and aggregate variables are constant over time.

Definition: A stationary equilibrium in this economy is a set of policy and value functions $c_m(\overrightarrow{x_m})$, $s(\overrightarrow{x_m})$, $b'(\overrightarrow{x_m})$, $c_o(\overrightarrow{x_o}; s)$, $I_r(\overrightarrow{x_y})$, $V_o(\overrightarrow{x_o}; s)$, $V_m(\overrightarrow{x_m})$, $EU(I^r, \overrightarrow{x_y})$, $EU(\overrightarrow{x_y})$, a distribution of efficiency units of labor across economy $\mathbb{E}^{u_p}$, $\mathbb{E}^{s_p}$, and $\mathbb{E}^{s_c}$, wage rates (w^u and w^s), and a probability distribution vector ($F(\Omega)$) such that:

(i) Given wages, taxes, and college tuition, the young adults, middle-aged agents and old agents solve their optimization problems;

(ii) Given wages, the representative firm maximizes its profits;

(iii) Government always balances its budget;

(iv) The labor market clears;

(v) A Markov chain of first order accounting for the evolution of exogenous states links the probability distribution vector for children when middle-aged, $F_{t+1}(\Omega)$ to that of their parents' when middle-aged, $F_t(\Omega)$.

(vi) All other variables, functions, and probability distribution functions are time invariant (i.e., a fixed point).

4 Benchmark

While our main focus is alternative college aid schemes, it is important to understand the characteristics of this basic economy and the general role for government intervention. We calibrate this basic model to mimic key elements of the U.S. labor and college markets. Importantly, the benchmark begins with the current system of higher education finance in the U.S. where there is substantial involvement of the government. This benchmark economy has both basic college subsidies through low public tuition and a loan program for those who could not otherwise attend college.

[16]Nonresident tuition, applying both to U.S. citizens who are residents of other states and to nonU.S. citizens, is typically above that charged to state residents but below the total production costs. Private universities also tend to price tuition below total production costs, largely through subsidies from endowment (see McPherson, Schapiro, and Winston 1993), although the support from past private donations are difficult to include in this analysis.

4.1 Calibration

(Table 1 about here)

Each period is assumed to be 20 years. Moreover, it takes four years to get a college degree. Some parameters are easy to calibrate: We can either get them directly from the data or empirical literature:

- The interest rate is assumed to be 2% per year, implying $r = (1 + 0.02)^{20} = 1.4859$ each period. The discount factor controls the amount of saving, although the model does not explain the savings patterns per se. Given the normalizations in the second period term, we set the discount factor $\beta = \frac{1}{1.4859} = 0.673$ so that, when the net period 2 and period 3 incomes are the same, savings are zero. However, note that, when there are loans to be paid back in the second period, an income differential between the second period and third period is generated. We allow for negative saving in order to smooth out consumption. It is also consistent with the casual observation that many college students have credit card debt.

- It is hard to calibrate the parameter for intertemporal preference parameter σ because the length of a period is 20 years in our model. Based on a micro model that explicitly allows for borrowing constraints, Keane and Wolpin (2001) reports a value about 0.5. We choose $\sigma = 0.68$.

- The opportunity cost of attending college, φ is set to be $0.25 \times \frac{4}{20}$, based on the assumption that an individual provides part time unskilled labor during college years if she does not attend college.

- Based on the data extracted from the NLSY79 and NLSY79 Children Cohorts of the National Longitudinal Survey of Youth (NLSY), a Galtonian regression yields the transmission of ability parameters as $\lambda = 0.4$ and $\sigma_\epsilon^2 = 0.52$.

- Katz and Murphy (1992) report a value of 1.41 for the elasticity of substitution between skilled and unskilled efficiency units. Hence, setting $\rho = 0.31$ yields an elasticity of 1.45 in the model.

Unfortunately, there is no direct way to calibrate the remaining parameters. At this point, we take an indirect approach and calibrate the remaining parameters simultaneously to be consistent with calibration targets. The calibration targets are the values of endogenous variables in the model to match several important observations for the United States.

- The productivity parameters k_0 and k_1 are assumed to be 1 and 3, respectively. With these, the lowest ability agent makes a positive salary, the income taxes are consistent with the data (see the government budget for tuition), and the response of the wage ratio as the skilled and unskilled worker compositions changes due to altered tuition policy. We use the labor market clearing condition to set the production function parameter $\xi = 0.591$, which yields an stationary equilibrium wage ratio about $\frac{w^e}{w^u} \approx 1.61$ in the benchmark. The scale parameter, A is normalized to be $A = 1$.

- While patterns of transfers at death have been studied extensively, much less is known about the in vivo transfers that are relevant for our model. Moreover, the literature on bequest motives is controversial (for instance, see Behrman and Rosenzweig, 2004). Observing that it controls the amount of bequest a child receives, and hence her need for a loan and the loan amount, we choose the value of α to be 0.7. It implies that the agent will leave about 17.9% of second period income as bequest and generates a college attendance with a loan pattern that seems reasonable (see the education outcomes by the tuition loan amount at the benchmark).

- The income taxes (which go solely to support college aid) are calibrated to be $\tau = 0.8\%$ in the benchmark. With no government involvement ($\tau = 0$), expenditure per pupil, which comes entirely from teacher labor costs, equals tuition. The key parameter driving cost is the fraction of educated workers needed for work in the college sector, γ, and this is chosen such that ϕ/w^e – the ratio of tuition to wages of educated workers with an average ability – is set approximately to be 0.045 (0.072) when τ is 0.08% (0%).[17] These values yield an enrollment ratio about 58.2%, a college completion rate about 64.6%, and a reasonable pattern of loan enrollees in the stationary equilibrium.

- Minimum consumption is set at 32% of the wage of uneducated workers as an approximation of the poverty level.

4.2 Characteristics of the Benchmark

The following tables summarize some key statistics generated by the benchmark calibration, which will be referred as "tuition loan equilibrium" in later sections. Figure 1 indicates how the population splits into skilled and unskilled workers based on their schooling choices and their success in school.

(Figure 1, Table 2 about here)

Table 2 presents the enrollment and corresponding education outcomes of groups identified by their level of eligibility for loans. Only those agents who enroll in college are granted a tuition loan, and in the benchmark only a small portion of the population, 0.9%, receives a loan of 75% of tuition.[18] The aggregate enrollment rate across the entire population is 58.2% in the benchmark calibration, which is close to what we observe in U.S. data. The rate of successful completion given the college enrollment is 64.6%, which is also close to the data.

Several observations are in order. Notice that about 15% of the population is eligible for a tuition loan in this model economy, suggesting that the financial constraint is not binding for most of the population, a fact consistent with earlier works. 4.3% of the population is eligible for a 25% tuition loan and 4.8% of the population is eligible for a half-tuition loan. The combination of low tuition and tuition loans acts to eliminate any financial constraints in the benchmark economy. Second, the enrollment rate dramatically decreases when moving from the unconstrained population to those who require a tuition loan for attendance. For those not needing a tuition loan, 64.9% enroll in college, but this falls to less than half that when we get to households needing even a one quarter tuition loan. Third, the size of the tuition loan is inversely related to the amount of bequest, and those with low bequests will come from uneducated parents who on average have lower ability. In terms of loan usage in the total population, 1% enter college with a 25% loan (i.e., $4.3\% * 24.2\% = 1\%$) and 1.3% enter with a half-tuition loan.

Moreover, not everyone who attends college will graduate. Completion of college, conditional on enrolling, also varies across loan classes. For those not using a tuition loan, 64.6% complete. This falls to 60.4% for the group taking a three-fourth tuition loan. These differences are noticeably smaller than the enrollment rate differences, suggesting that the observed differential in representation of college graduates across different family income groups is mainly due to the dramatic difference in enrollment rates rather than the success rate (conditional on the enrollment).

The results for both enrollment and completion are driven by the intergenerational correlation of ability and the correlation between ability and bequests. More capable parents tend to have more capable offspring, to have higher incomes, and to leave higher levels of bequests. Thus, the younger generation from a parent with a college education will be more likely to enroll in college and, given their enrollment, more likely to be successful.

This model not only generates predictions of the cross-sectional education outcomes for the population but also of the dynamic correlations of education outcomes within the same family tree. Table 3 summarizes both the intergenerational mobility (measured by the probability of a child with an uneducated parent successfully completes college), as well as intergenerational persistence (measured by the probability of a

[17] When τ is 0, $\gamma N^r w^e = \gamma N^r \Rightarrow \frac{\phi}{w^e} = \gamma$. Note that the average ability is 2, so $\frac{\phi}{2w^e} = 0.072$

[18] For agents who would need to borrow 75% of tuition to enroll in college, only 27.7% of the population do so. And even if they do, their success rate is relatively low. Nobody is permitted to borrow 100% of tuition, so a few potential candidates are eliminated.

child with an educated parent successfully completes college). As it is shown in the table, educational mobility improves over generations and after about 5 generations (100 years) the two probabilities to be equalized.

(Table 3 about here)

5 Different College Aid Schemes

We can now use our calibrated model to investigate the potential impact of commonly proposed alternatives. To facilitate the comparison, we assume as in the benchmark that the government raises funds for college student aid with a proportional income tax and maintains a balanced budget every period in all policy regimes. In order to highlight the implications of each policy regime, we also shut down the tuition loan program and consider the alternatives one by one.

We begin by looking at different levels of uniform subsidies. The benchmark was calibrated to U.S. public colleges and universities, where heavily subsidized tuitions are prevalent. We look at the impacts of altering these public tuition levels from free to almost full cost. We then compare the outcomes in terms of enrollment and completion under different magnitudes of subsidies in need-based and merit-based programs.

These are, however, not the only alternative programs. An increasingly popular form of college subsidy around the world uses income contingent loans to support individuals who otherwise would not be able to attend because of constraints on funds available. In this, individuals borrow for the purpose of paying tuition, and their repayment rate depends on their future income. Individuals who have high incomes pay back the full loan plus, in most cases, an additional amount. Individuals with low incomes pay back less than the value of their loans. In the strongest form, where the loan pool is required to be balanced, this can be viewed as income insurance, where those with low incomes are subsidized by the high income borrowers. In pure form, there is no reason for the government to be involved, although because of collection purposes or the desire to provide additional overall subsidies, government may be involved.[19] These plans have been introduced in both developed and less developed countries around the world, particularly because they provide a politically feasible way to introduce tuition and fees in countries that previously offered college education at no cost to all students (Chapman 2006).

5.1 Uniform Subsidies of Varying Magnitude

The uniform subsidy is a significant component of current U.S. public tuition policies but can of course be an amount greater or smaller than that in the baseline. As with the other subsidy policies that we consider, the easiest way to describe the program size is simply by the governmental tax rate that is used to support the program. Varying the governmental subsidy directly alters the tuition level seen by students and thus affects attendance, completion, and intergenerational mobility. Such variations have recently become very relevant for policy as states consider varying levels of tuition and fees, particularly when faced with fiscal pressures on overall state budgets. The government budget constraint remains the same as in the baseline.

5.2 Need-Based Subsidy Schemes

Many college subsidies are targeted to poor students through means-tested schemes based on student needs. We consider two alternative versions of need-based subsidies defined by the amount of information about parental ability to pay and by whether the subsidy is constant across individuals or not. In each, only those who attend college and are identified as "poor" will be subsidized.

[19]In the early 1970s, Yale University developed a tuition-postponement plan that provided for privately sponsored income-contingent loans. It subsequently abandoned the plan, in part because of difficulties in collecting on the loans. See Nerlove (1975).

5.2.1 Imperfect information with flat subsidy

The parent's bequest might not be perfectly observable to the government. Here we take to the extreme and assume that the government cannot observe the income of the parents (middle-aged agents) but can observe their education levels. In the current setting, high ability people tend to enroll in college and to get higher wages that are proportional to ability, $(k_0 + k_1 x)w^i$, $i = e, u$. Thus, the group of more educated people and highly paid people overlap significantly in this setting.

Formally, if the parent succeeded in college ($I^e = 1$), then the child is not eligible for the subsidy, implying that the child pays the full cost of the college education, ϕ. The children of unskilled parents ($I^e = 0$) receive a lump sum amount m for enrolling in college, implying their tuition is $\phi - m$. (Note that without being able to observe true need it is not possible to vary the subsidy with need level).

There is a financially constrained group where tuition less any subsidy exceeds the bequest, implying that the constrained child cannot enroll regardless of ability. This subsidy produces a decision rule for attendance that is correlated with parental education. Since kids with educated parents are denied any education subsidy, their financial gains of attending college are less than those for children of uneducated parents, and thus only the more capable ones will try. For children with an uneducated parent, there is an ability cutoff determining whether attendance is optimal: $x^*(I^e = 0)$. For children with an educated parent, since they must pay the full tuition, there is a different cutoff that is higher such that $x^*(I^e = 1) \geq x^*(I^e = 0)$.

The government budget constraint is straightforward. Since the young agents with educated parents are not eligible for subsidy, the government expenditure is concentrated on those whose parents are unskilled. The students in this category are:

$$\int_{I^{e\prime}=1} \int_{I^e=0} \frac{f(\overrightarrow{x_m}', \overrightarrow{x_m})}{x'} d\overrightarrow{x_m}' d\overrightarrow{x_m} \equiv N^{r,m'}, \tag{15}$$

To find the number of subsidy recipients, once again we use the fact that the probability of success is the ability of the child, x', if the child attends college. Hence, the government budget constraint is such that the aggregate of lump sum transfers to the poor students is covered by the income tax revenue,

$$mN^{r,m'} = \tau(w^e \mathbb{E}^s + w^u \mathbb{E}^u),$$

and the tuition is equal to its social cost,

$$\gamma w^e = \phi.$$

5.2.2 Perfect information with variable subsidy

An alternative is to have students with larger bequests pay more in tuition so that tuition is (weakly) increasing in wealth. In practice, this scheme, which resembles much of the current U.S. aid, will lead to "false reporting" and other problems with information asymmetry. Moreover, while ignored here, there are obvious incentives for parents to adjust their bequests, since the government will partially compensate for any lesser funds for the child. In this paper, however, we only want to examine the case where wealth can be perfectly observed, and we assume away both the informational asymmetry issue and any behavioral bequest response.

In practice, need-based subsidies vary significantly in details, including typically being very nonlinear. We nonetheless focus on the linear case, so that the intuition is more transparent and the calculations are simplified. We characterize tuition as an increasing function of the bequest each child receives,

$$\phi(b') = \phi_1 \cdot w^e + \phi_2 b',$$

where $\phi_1 \cdot w^e \geq 0$ is the minimum level of tuition to be paid, and ϕ_2, $0 \leq \phi_2 \leq 1$, is the incremental increase in tuition for each additional unit of bequest.[20] Notice that the bequest a young agent receives will

[20]We write this in terms of w^e in order to emphasize that tuition is determined in general equilibrium by the wages paid to teachers.

depend on the state variables, or the "type" of his/her parent, $b' = b'\left(\overrightarrow{x_m}\right)$. Thus, unlike the prior imperfect information case, the amount of subsidy can be directly related to actual need.

Under this policy regime, a young agent is constrained if the bequest b' is less than the tuition $\phi(b')$,

$$\begin{array}{rlll} & b' & \leq & \phi(b') = \phi_1 \cdot w^e + \phi_2 b' \\ \Leftrightarrow & (1-\phi_2)\, b' & \leq & \phi_1 \cdot w^e \\ \Leftrightarrow & b' & \leq & \frac{\phi_1}{(1-\phi_2)} \cdot w^e \equiv \phi^* \end{array}.$$

For those who can afford college, the attendance decision depends on the ability they inherited, and further the ability cutoff for attendance is directly related to the child's bequest. Each agent has a critical value of ability/productivity, $x^*(b')$ that determines attendance, but attendance is increasing in the level of bequest the young agent receive. An implication is that this scheme will facilitate social mobility in ex ante terms because, other things being equal, kids from poor families are more likely to enroll in college with this subsidy (compared to no subsidy). They thus will have a higher chance of becoming a skilled worker in the later stage of life.

To close the model, it is necessary to introduce the government budget constraint,

$$\gamma N^{r'} w^e = \Phi^n + \tau(w^e \mathbb{E}^s + w^u \mathbb{E}^u) \tag{16}$$

where Φ^n is the total amount of tuition collected under this variable subsidy scheme:

$$\Phi^n = \int \int_{I^{e'}=1} \phi(b'\left(\overrightarrow{x_m}\right)) \frac{f(\overrightarrow{x_m}', \overrightarrow{x_m})}{x'} d\overrightarrow{x_m}' d\overrightarrow{x_m} \tag{17}$$

We investigate the effect of this subsidy under different tax rates. To maintain a balance budget (**??**), we are left with only one degree of freedom. We exogenously set the value of $\phi_2 = 0.06$ and use the budget constraint to identify the remaining parameter, ϕ_1. This value implies that the bottom 25 percent tuition payers (the "needy"), on average, pay 80.4% of college cost per child (i.e. γw^e) as tuition, while the top 25 percent tuition payers, on average, pay 111.9% of college cost per child as tuition. It implies that the average subsidy rate for the "needy" is about 20% and on average those "needy" are paying around 72% of the top 25 percent counterpart. As ϕ_2 approaches zero, the corresponding policy gets closer to the uniform subsidy case.

5.3 Merit-based subsidies

The prior aid regimes are directly related to the financial needs of the college student, but a variety of colleges and universities provide scholarships to those who perform better without regard to financial need.[21] This section considers merit-based aid. While merit-based subsidies can be very non-linear (with, for instance, only the top few students receiving a scholarship), we will again focus on the linear case for tractability:

$$\phi(x') = (\phi_3 - \phi_4 x') \cdot w^e,$$

where $\phi_3 > 0$ is the maximum level of tuition to be paid, and ϕ_4, $0 \leq \phi_4 \leq 1$, is the incremental decrease in tuition for each additional unit of ability. The tuition is restricted to be non-negative, $\phi(x') = (\phi_3 - \phi_4 x')\, w^e \geq 0$. Again, we emphasize that the level of the tuition, and hence the subsidy, is proportional to the wage rate of the teachers (which is also the wage rate of the skilled workers in the economy). This formulation enables us to express most variables as a ratio of the the teacher wage rate w^e, which means the economy essentially "unit-free" and makes solution easier.

The government budget constraint under the merit-based subsidy is very similar to the one under need based subsidies:

$$\gamma N^{r'} w^e = \Phi^m + \tau(w^e \mathbb{E}^s + w^u \mathbb{E}^u), \tag{18}$$

[21] Merit scholarships have also been introduced at the state level, such as the HOPE scholarship that was introduced in Georgia in 1993 for students who performed well in seconary school grades.

where Φ^m is the total amount of tuition collected under merit-based subsidies:

$$\Phi^m = \int \int_{I^{e'}=1} \phi(x') \frac{f(\overrightarrow{x_m}', \overrightarrow{x_m})}{x'} d\overrightarrow{x_m}' d\overrightarrow{x_m}. \tag{19}$$

As in the perfect information with variable subsidy case, we exogenously set $\phi_4 = 0.2$ and use the budget to identify ϕ_3. The bottom 25 percent tuition payers (the "outstanding students"), on average, pay 71.1% of college cost per capita as tuition while the top 25 percent tuition payers, on average, pay 130.7% of the same college cost. In other words, the average subsidy rate for the "outstanding students" is slightly below 30% and on average the "outstanding students" are paying slightly more than a half (which is 54.4%) of the average tuition of the top 25 percent tuition payers. Taking into account the "academic elites" and "college athletic stars" who virtually pay zero in practice, these numbers seem to us being conservative. As ϕ_4 approaches zero, the corresponding policy gets closer to the uniform subsidy case.

5.4 Outcomes of Alternative Subsidy Policies

The clear question is whether these alternative approaches to subsidizing college education lead to much difference in the outcomes for individuals or society. While there are obvious differences in the incentives facing individuals, the impact in both the short run and the long run depends crucially on how the dynamics of the choice problem change. The benchmark case provides the starting point from which we introduce alternative subsidies.

(Table 4 about here)

We simulate each of the subsidies under a range of tax rates (our index of the size of the governmental intervention), and the results are found in Table 4. Consider first how varying levels of the uniform tuition subsidy (i.e., tuition reduction) impact the economy. With no subsidy, tuition starts at 7.2% of the wage for an educated worker with average ability. As the government collects income taxes to subsidize college, two things happen. First, the tuition decreases, implying that the cost of college to the students decreases. But, as more people enroll in college and succeed, the proportion of skilled workers increases, and hence the "college premium" in wages decreases, implying that the benefit of college education also decreases. In our calibration, the tuition effect dominates so that higher taxes (larger subsidies) lead to greater college enrollment. Tuition drops by approximately 70% at a tax rate of 1.6%, and this fall in tuition induces a large increase in college attendance – from less than 48% to 66%.

A significant portion of this increase comes from a fall in "constrained" agents (ones who have the ability to attend optimally but lack sufficient funds to cover tuition in the absence of governmental intervention). Roughly 22% of the increased attendance comes from previously constrained people; the remainder comes from unconstrained people who find attendance to be optimal at the lower price. Without subsidy, 4.8% of agents are financially constrained, but this shrinks to 2.1% at a tax rate of 0.8%. With further reductions in tuition, it falls to 0.7%.

Nonetheless, a large portion of the increase in attendance translates into failure to complete college successfully. Parallel to the 18% increase in the college enrollment, college success only increases by half that, from about 31% to 40%. Importantly, the lowered cost of attending leads more people to conclude that attendance is optimal, but the newly induced attendees are ones with lower ability, making the chance of success consistently lower as the subsidy rises. While the marginal effect on attendance with greater subsidies is quite constant as the tax rates increase, the marginal impact on college success falls consistently.

It is instructive to compare the outcomes of the uniform subsidy scheme to the alternative need-based and merit-based subsidies. We begin with the two need-based programs. In the first, it is presumed that the government has perfect information about the needs of the student and provides a linear subsidy based on this need. In the second, the government cannot readily observe the bequest and resorts to using a proxy of the schooling completion of the parent, giving a constant subsidy because it is unsure of need levels. The final alternative, the merit-based scheme, pays no attention of student financial needs but subsidizes those with higher ability.

With perfect information about the financial situation of the family, the linear needs-based scheme does two things: it introduces differential tuition policies across students and offers a way of subsidizing all tuition. In the no-government case ($\tau = 0$), simply charging a lower relative tuition to more needy families helps to correct the distortions from the credit constraint by lessening the tuition requirements for the children from families with insufficient bequests. At low tax rates, the differential tuition dominates the overall subsidy effects, and the need-based scheme leads at comparable levels of subsidy to both greater attendance and greater completion rates when compared to the uniform subsidy. At higher tax and subsidy rates, the need-based program continues to have higher attendance and completion but it looks closer to the uniform subsidy. For the case with imperfect information with its flat subsidy, the college enrollment increases from 48% to 70% as the income tax rate increases from 0% to 1.6%. Thus, the need-based subsidy performs better than the uniform subsidy in encouraging college enrollment. The need-based subsidy with imperfect information yields the highest attendance rate of the different schemes, reflecting the fact that a number of unconstrained people are now receiving subsidies that induce them to attend.

However, the increase in college success is not comparably encouraging. For the imperfect information case, completion increases from 31% to slightly more than 41%, leaving the college success rate just about a percent greater than with the uniform subsidy even though 4% more enter college. The need-based scheme where actual needs can be observed falls in between the imperfect information needs-based scheme and uniform subsidy in terms of enrollment rates but gets the highest completion rate. The divergence of attendance and completion underscores the importance of being very careful in comparing the "performance" of alternative schemes.

The merit-based scheme, however, yields quite different results. Since this scheme implicitly reinforces existing differences in bequests and families (where ability and family incomes are correlated), noticeably lower percentages of students attend college at all levels of government intervention. These students, being more able, succeed in college at higher rates than with the alternative subsidy schemes. Thus, while the merit aid is "more efficient" at producing skilled employees for the economy, it does so with significantly reduced opportunities as only 28% of the population succeed in the no-government case, rising to 38% with government subsidies set at $\tau = 1.6\%$.

There are nonetheless other dimensions on which the subsidy schemes should also be compared. One important dimension – consistent with the overall notion of equalizing opportunities – is how they affect the intergenerational mobility of the population. We discuss this below after we have introduced the final aid scheme – an income contingent loan.

5.5 Income-Contingent Loans (ICL)

Income contingent loans (hereafter ICL) allow young agents to borrow for tuition (though it is not compulsory) with the condition that the repayment depends on their future income. As surveyed by Chapman (2006), some variant has been has been instituted in a variety of countries. To simplify the analysis, it is assumed that young agents either borrow the full amount of the tuition or do not borrow at all. Young agents who do borrow will consume the bequest inherited from their parents in the first period. In the second period, they repay a fixed fraction of their after-income tax income, $\tau_\gamma * \left[(1-\tau)(k_0 + k_1 x')w^i\right]$, $0 < \tau_\gamma < 1$, $i = e, u$. Hence, they are left with only $(1-\tau_\gamma) * \left[(1-\tau)(k_0 + k_1 x')w^i\right]$, $i = e, u$, for their second period consumption and any bequest for their kids. Notice that, while the amount of repayment depends on the second period income (and hence the education outcome), the repayment rate as a fraction of the income is independent of income. Thus, the income-contingent loans formulated here can also be interpreted as a type of "profit-sharing" of the return on the risky individual human capital investment.

With a balanced budget, the total income-contingent loans made in each period do not exceed the total repayment collected. A portion of the young people enroll in the college with an ICL, taking the income tax rate τ and college tuition ϕ as given. Their tuition must be paid by those who took an ICL in the previous period and are now in the working period of their lives, and the total loan pool TL (the total tuition for the currently young agents who take ICL) will equal the total revenue TR (the repayments from those who took

ICL in their first period of life paid back through a proportional tax with the rate τ_γ),

$$TL = TR, \tag{20}$$

where

$$TL \equiv \phi \int \int_{I^{c'}=1} g_t(\overrightarrow{x_m}', \overrightarrow{x_m}) d\overrightarrow{x_m}' d\overrightarrow{x_m},$$

$$TR \equiv \tau_\gamma \int \int_{I^{c'}=1} (1-\tau)(k_0 + k_1 x)(I^e w_e + (1-I^e) w_u) g_t(\overrightarrow{x_m}', \overrightarrow{x_m}) d\overrightarrow{x_m}' d\overrightarrow{x_m}.$$

I^c is an indicator function that takes the value one if an agent borrows from the loan pool. The marginal tax rate, or the repayment rate, τ_γ, will be set to ensure (**??**) holds. Clearly, parents who attended college with an ICL make repayment, while their children who attend college with ICL get the tuition loan.

A pure income contingent loan plan does not require the government except as a way of enforcing loan repayment, but here we extend the role of the government in order to understand how changing the tuition constrained group affects the overall results. We permit the governments to add income taxes to subsidize the college tuition uniformly. Formally, the government budget constraint is given by the following equation (which we previously saw in the benchmark model):

$$\gamma N^{r'} w^e = \phi N^{r'} + \tau(w^e \mathbb{E}^s + w^u \mathbb{E}^u) \tag{21}$$

In a pure form, the loan pool maintains a zero balance. If anybody can join the loan pool, very low ability people will have incentives to "take a chance" on college – because they are highly subsidized and the only cost to doing so is the small opportunity costs.[22] But, if somebody has a low income and repays less than tuition, there must be others in the loan pool who pay more than they borrowed for tuition, so that the loan pool remains solvent. This has important implications for the budget, because, if there is to be a viable loan pool, there must be high income participants who subsidize anybody who repays less than tuition.

The income insurance aspect of an ICL can introduce adverse selection that is potentially severe. For any level of tuition, there are some agents (with lower ability) who are financially unconstrained but decide that college is not a good investment. However, if the tuition can be paid through an ICL, even if the probability of success is slim, agents might attempt college with an ICL because there is no sacrifice in the first-period consumption and there is income smoothing. The loss in second-period consumption in the form of ICL repayment may not be much as the wage is low when college is not successful. A completely unrestricted income contingent loan system cannot really operate very well (at least within the structure of this model). Therefore, in our calculations, and in reality for an operational ICL scheme, access would almost certainly also include separate eligibility criteria for joining the pool. Here, we introduce a restriction into the scheme: an agent is eligible for an ICL if her "college fund" (i.e. $b - \bar{c}_{min}$) is less than $1.5 * \phi$ (where each young agent has minimum consumption $\bar{c}_{min}$).

To our knowledge, the ICL has not been explored in a dynamic general equilibrium setting. We therefore provide figure 1 as an illustration. Figure 2 shows a few "stylized facts" for an example of a specific form of an ICL with no governmental subsidy. Under an ICL, most of those who are originally financially unconstrained would not be eligible to take the ICL even if they might want to for consumption smoothing reasons (see the first column). They either self-finance their college tuition (solid blue area) or choose not to attend college at all (diagonal striped purple area). But, there is a group of unconstrained people that enters to the ICL program. For these, a portion ex post will succeed in completing college, giving them higher income and meaning that they will actually pay back more than tuition (checked red area). Another group of the unconstrained does not succeed and becomes part of the group subsidized in their tuition

[22] With no constraint on eligibility for a loan, approximately 69 percent of all students would take the loan and attend college. This response is slightly greater than the percentage attending at a high level of uniform subsidy (where tuition is zero); see Table 5.

payments (green vertical bars). But both of these groups gets a utility boost from the ability to smooth first period consumption. For those who are financially constrained (the second column), none can attend with self-financing. Among the attendees, there are people who receive more subsidy than the tax they pay at the end (”subsidized loan”) as well as people who receive less subsidy than the tax they pay (”taxed loan”). The large group of attendees with a taxed loan are the higher ability constrained people who succeed in college and end up subsidizing the others in the loan pool. A significant portion of the constrained, however, will still choose not to attend college because they have lower ability and less chance of success. In the aggregate, about 52% of all initially constrained students will attend once the ICL program eliminates the credit constraints.

(Figure 2 about here)

Throughout the simulation of the income contingent loan program, smart poor people end up as the main source of funds that supports the less able others in the pool. This is ameliorated to some extent by having the government subsidize tuitions, because less aggregate tuition is collected and thus the total loan pool is smaller. But on the margin the insurance element of the ICL implies that there will be adverse selection into the loan pool by less able but financially well-off individuals (as well as less able people who have lower financial capacity).

5.6 Efficiency, Equality, and Mobility

Questions about the distribution of income, both cross-sectionally and over time, have received considerable public and media attention. These discussions sometime but not always make the linkage to school attainment, but, when they do, they usually do not consider any underlying causes of differences in attainment and the impact of college aid policies. The analysis highlights the major problem with simply looking at the marginal conditions for attendance under the different scenarios. The subsidy programs can have large effects on the schooling behavior of the population, and this results in substantial changes in the cost of schooling and in the wages of people who enter the labor market with different skills. Further, when considering the characteristics of intergenerational mobility, it is clear that any changes in patterns of college enrollment and completion accumulate across generations. In the absence of any government intervention, substantial inefficiencies might exist – because smart poor kids cannot afford schooling and remain uneducated. Moreover, the financial constraints would tend to lock in family status across generations.

The benchmark case has both uniform subsidies and loans, but it is not clear that these are optimal in either design or level. Here we consider the implications of alternative subsidy schemes on the overall economy in terms of efficiency and distribution. We are particularly interested in the impact on intergenerational mobility.

5.7 Aggregate outcomes

The benchmark and the alternative subsidies here improve the efficiency of the economy by substantially reducing if not eliminating the group of financially constrained potential students. The first aggregate outcome that we consider is how each of the subsidies alters the overall efficiency of the economy relative to the baseline as measured by the sum of expected utilities in the economy – AEU or aggregate expected utility. Figure 3 plots the efficiency loss that occur at each tax rate for the alternative subsidies. Relative to the benchmark ”tuition loan equilibrium” (TLE), with the exception of the ICL each new regime does worse than the benchmark in terms of AEU at low tax rates (i.e., with smaller sized programs). Even with a zero tax rate (no government intervention), the ICL shows improvement with a negative efficiency loss. In other words, an ICL generates more aggregate utility than the TLE. As we increase the tax rate, all regimes improve. Above an 0.8% tax rate, which is the tax rate in the benchmark, all alternate regimes do as well or better than the TLE. In terms of the relative performance among different subsidy regimes, the merit-based subsidy is consistently worst, followed by the uniform subsidy regime. Although the relative positions among the two need-based subsidies and the ICL change as the tax rate is raised to 0.8% or above, the difference among them is relatively small in that range of tax rates.

The merit aid program does less well than this other subsidy schemes in large part because it still leaves a number of individuals credit constrained. Its focus is higher-ability students who come disproportionately from the unconstrained families but does not lead to large changes among the group of students who would be closer to the ability cut-off for attendance.

(Figure 3 about here)

A prime motivation of government subsidies for college education remains the potential impact on the distribution of welfare in the society.[23] The distributional aspect has two dimensions. First, one might ask whether government policy leads to more equality in addition to the efficiency gains already discussed. Figure 4 provides some insight into this. It plots steady state values of Aggregate Expected Utility against one minus the Gini coefficient for subsidy programs of differing sizes. Since the Gini measures distance from equality, higher values of (1-Gini) indicate more equality of utility. At any level of AEU, a regime is "better" if its equality lies above other regimes. In fact, if the locus of a given regime lies entirely above another locus, it implies that the upper regime can always generate a higher level of equality for any aggregate outcome. In Figure 4 we plot all five regimes along with the TLE, which is a point in the graph. Consistent with Figure 3, we find that virtually all regimes can generate a higher level of efficiency and equality above a certain level of tax and subsidy. It is also clear that the merit-based regime underperforms relative to the others in that it maintains the highest inequality for each level of aggregate utility.

Interestingly, the income contingent loan generate considerable equality along with high aggregate utility. While dominated by the means-tested programs at very large levels of governmental involvement, it does well against lower levels of means-tested programs and against uniform subsidies.

(Figure 4 about here)

5.8 Intergenerational Mobility

Our model allows us to compare different subsidy regimes not only in terms of their capacities in generating efficiency and cross-sectional equality but also in terms of altering intergenerational mobility. Our measure of intergenerational mobility is very simple: it is the probability that an n-th generation offspring ($n = 1, 2, ..$) attends college while the matriarch is a non-attender. Clearly, the intergenerational mobility may vary with the tax rate (which indexes the size of government involvement) as well as across regimes. Figure 5 shows that under the uniform subsidy regime, the intergenerational mobility increases with the income tax rate. The idea is simple. With a higher level of tax rate, the government is able to subsidize more people to attend college, and hence improve the chance that the offspring of nonattendees attend college. As the tax rate increases, fewer people are financially constrained, and thus the inertia of having an uneducated parent is partially broken. The transmission of ability, however, maintains a certain amount of inertia. The full impact is felt after roughly five generations.

Figure 6 compares the intergenerational mobility across different subsidy regimes at a moderate level of income tax rate ($\tau = 0.6\%$). While the qualitative patterns are actually very similar across regime, it is clear that ICL generates a higher level of mobility, followed by the imperfect-information need-based

[23] These of course are not the only objectives of policies toward higher education. The HOPE scholarship program in Georgia, for example, shows a broader objectives of providing incentives to students to do well in primary and secondary school by giving students a scholarship to higher education if they meet certain performance standards. Moreover, this scholarship has a secondary goal of keeping good students within the state in the hopes of spurring future economic development. This program has been introduced in various forms both within other states (e.g., Kentucky and South Carolina) and within local jurisdictions (e.g., Bufalo and Syracuse).

(constant subsidy), the perfect-information (linear subsidy) need-based, and uniform subsidy.[24] As expected, the merit-based subsidy regime is the worst in terms of fostering mobility, because it tends to reinforce parental outcomes and incomes. The prior figure shows clearly that mobility increases with more government intervention, but the pattern across subsidy regimes is maintained at different sizes of governmental programs.

(Figure 5, 6 about here)

5.9 Net Tax Incidence

It is also instructive to understand the tax and subsidy incidence by tracing the exact pattern of subsidies to people in different parts of the income distribution. There are different ways to think about the incidence of the government programs. We begin with the most straightforward incidence calculation based on lifetime taxes and benefits. Figure 7 displays the net subsidy under different schemes with varying tax rates according the distribution of outcomes for the young agents. Specifically, we calculate the present value of the lifetime income for each agent and place the young agents into different deciles of lifetime income. We then calculate the average tuition discount and taxes paid for each groups. We display the distributions of net taxes for the uniform subsidy, need-based subsidy, and merit-based subsidy.

Several observations are immediate from the graphs. First, in all three regimes, the poorest people typically do not benefit from college aid. For all three regimes and all the tax rates we considered, the 1st and 2nd deciles of the distribution always receive negative net subsidy – i.e., they are subsidizing the better off people. Second, the regime does matter for the "marginal group". If the uniform subsidy regime is imposed, then the 3rd decile will receive positive net subsidy for the tax rates we consider. If the need-based subsidy regime is implemented instead, the 3rd decile will face negative net subsidy and only the 4th and higher decile will get positive net subsidy. The merit-based regime is the most onerous for the poor. The 1st to 4th deciles of the distribution always receive negative net subsidy – that is, they subsidize the better off people. And even for the 5th decile, it is only when the income tax rate is increased to 1% that the group as a whole receive positive net benefit. Third, for both uniform and need-based subsidy regimes, the net benefit distribution displays an "inverted U-shape" in the sense that the net benefit would first increase with the income, and then there is a tendency of decreasing benefit near the top end of the income distribution. The most affluent decile receives less than the 9th decile, and the 9th decile tends to receive less than the 8th decile, and so on. All these seem to be consistent with the empirical research (such as Atkinson and Bourguignon, 2000; Hillman, 2003; Salverda, Nolan and Smeeding, 2009; Slemrod, 1996).

The merit based system seems tracks very differently. It is clear from Figure 6(c) that the positive net benefits are concentrated in the 5th to 10th deciles. Moreover, the distribution of benefit shows a tendency for increasing benefits near the top end of the income distribution, with the wealthiest receiving the largest net subsidy. Thus, the merit based subsidy regime is regressive since the most able receive the largest tuition subsidies and tend to get the largest benefits from completing college. These effects exceed the added income taxes that they will pay.

(Figure 7 about here.)

In our general equilibrium economy, a variety of forces are operating. The previous discussions described the patterns of net subsidies by the income distribution, but it is also possible to describe how people situated in different parts of the ability-bequest distribution fare under the subsidies. The largest winners under the subsidy schemes are high ability constrained people who could not attend school without intervention. But

[24]It may be surprising that the imperfect information need-based scheme leads to more intergenerational mobility than the perfect-information version, but the imperfect information case specifically focuses on parental education – thus leading to large impacts on the intergenerational transmission.

the government actions also improve the efficiency of the economy, while leading to some substantial changes in wages for both educated and uneducated. To understand the total impact, we compare the no-government economy with varying levels of uniform subsidies. In all cases, the consistent loser from introducing the subsidy is the group of unconstrained people (in the no-government case) who would attend college without any intervention. For them, wages fall and taxes increase sufficiently to overcome any benefits of paying lower tuition. Among the remaining unconstrained groups, both those who go to school with the subsidy and those who do not attend school in any case gain. The latter group is interesting, because their wage improvements are sufficient to cover any taxes to subsidize others. Finally, initially constrained people who do not attend college even with subsidies gain with smaller programs (where wage gains exceed taxes), but lose with larger programs (where the opposite holds). These results reinforce the importance of considering significant subsidy policies in a general equilibrium setting, where the changes in wages and overall performance of the economy have significant impacts.

6 Analytical Notes: General vs. Partial Equilibrium

This entire analysis has been placed in a general equilibrium context where wages and subsidies adjust when the tax rate is changed. The question still remains, "What analytical difference does all of this make?" We start with the importance of considering these questions in a general equilibrium setting. Each of these subsidies has large implications for the distribution of workers in the labor market, and this in turn directly affects wages. The increase in college completion drives down the wage ratio for college educated compared to uneducated workers, leading to a substantially more equal distribution of income. Thus, simply calculating aggregate effects with constant prices would yield very misleading results. As an example, Figure 8 compares our general equilibrium model to a partial equilibrium model where we ignore wage changes as a result of the change in skill distribution in the economy for uniform subsidies. The picture clearly provides a support for our analytical choice. The partial equilibrium calculations pick up the efficiency outcomes that follow from lessening the credit constraints on individuals, but they miss the impacts on the side of income distribution. Specifically, the partial equilibrium estimates would suggest a worsening of the income distribution, while the general equilibrium calculations indicate an overall movement toward more earnings equality.

(Figure 8 about here)

The previous analyses ignore any excess burden that might result from government taxation. This is natural because our modeling has not included any labor supply response to programs and taxation. At the same time, even though there are some disputes about the quantitative magnitude, considerable research shows that the marginal cost of public funds (MCPF) – the cost to society of raising a dollar of tax revenue – is generally greater than one.[25] While we do not investigate this analytically here, we can provide some estimates of how the outcomes are affected by distortions that raise the MCPF above one.

The review of empirical evidence by Bovenberg and Goulder (2002) suggests that raising \$1 of governmental financing may have a cost to U.S. society of \$1.10-\$1.56. In an investigation of OECD countries, Kleven and Kreiner (2006) suggest that the MCPF may be substantially higher – around 2 for proportional tax increases in Denmark, Germany, or France.[26]

Our previous analyses implicitly assumes an MCPF of 1. To see the impact of this assumption, we set the MCPF=1.5 and recalculated the resulting impacts of the subsidy programs. The primary result is that for any tax rate there is less reduction in constrained individuals and less success in moving the economy toward its optimum outcome. For example, at each tax rate, proportionally fewer students successfully complete schooling under the uniform, needs based, and merit programs (than with MCPF=1). Importantly, however, these calculations do not alter the ranking of the programs seen before.

[25]See the reviews and discussion in Bovenberg and Goulder (2002) and Auerbach and Hines (2002).

[26]The focus of Kleven and Kreiner (2006) is consideration of how taxes affect labor force participation in addition to the more standard consideration of hours worked

7 Concluding Remarks

Many policy decisions about social programs are made without a clear understanding of the role of government or of the impact that any government program might have. This situation characterizes much of the discussion of government support for higher education. Even though governments dominate much of the financing and operation of colleges and universities, discussions of the form and of the impact of varying policies are quite thin and narrow.

We address the issue that financial constraints on some households may lead to underinvestment in schooling and may distort aggregate economic outcomes. Moreover, these constraints undoubtedly affect the distribution of income and the amount of intergenerational mobility. Our focus is how various college aid schemes alter the college decisions of households and ultimately the operation of the economy.

In the presence for credit constraints, there is strong justification for some kind of governmental intervention into college finance.[27] At the same time, not all policies have the same benefits. We trace the implications of common policies – general tuition subsidies, need-based aid, merit-based aid, and income contingent loans – for both the level and distribution of outcomes in the economy. Our framework also enables us to compare different regimes in terms of their capacities to improve equality and intergenerational mobility and allows us to compare the net benefit under different regimes and different tax rates.

A key analytical aspect of this work is the use of a dynamic general equilibrium framework. College policies have significant impacts on the schooling and skills of the workers in the economy, and the impact of these policies on wages cannot be ignored. Moreover, a motivation for many policy proposals toward college education is the potential impact on economic mobility across generations. These issues cannot be addressed within a static, partial equilibrium framework.

We begin with an individual household optimizing model in an overlapping generations framework and then place college attendance decisions within an overall labor market. Individuals, who live for three periods, decide on whether to make a risky investment in schooling based on the expected payoffs from different choices. Individuals who attend, however, are subject to the risk of not completing, a risk that depends on the individual' ability level. Wages adjust to the mix of college graduates and others in the economy.

We calibrate this model to match the U.S. economy in the basic stylized facts of college attendance and completion, college and noncollege wages, and intergenerational correlation of incomes. For the benchmark system we build on the current U.S. college situation where there are heavily subsidized tuitions and loan programs available to needy families. From this benchmark we can judge the alternative financial aid schemes.

It appears that each of the alternative college aid policies can, if properly implemented, generate noticeable gains over the current U.S. financing structure. Specifically, it is possible to improve on the efficiency of the economy by getting to a better set of college decisions by households. Moreover, these improved outcomes can come with more equal distribution of incomes in the economy and with increased intergenerational mobility.

Across the alternative schemes, a need-based aid program can generally lead to the best combination of aggregate economic performance and more equal income distribution. On the other hand, a merit-based system does systematically worse, particularly on distributional grounds.

[27]The importance of such constraints has been controversial and has been discussed in a variety of analyses; see Cameron and Heckman (2001), Carneiro and Heckman (2002), and Keane and Wolpin (2001).

References

Atkinson, A. B., and F. Bourguignon, eds. 2000. *Handbook of Income Distribution, Volume 1*. Vol. Volume 1. Amsterdam, The Netherlands: Elsevier Science B.V.

Becker, Gary S. 1993. *Human capital: A theoretical and empirical analysis, with special reference to education*. 3rd ed. Chicago: University of Chicago Press. Original edition, 1964.

Behrman, Jere R., and Mark R. Rosenzweig. 2004. "Parental allocations to children: New evidence on bequest differences among siblings." *Review of Economics and Statistics* 86, no. 2 (May): 637-640.

Ben-Porath, Yoram. 1970. "The Production of Human Capital Over Time." In *Education, Income and Human Capital*, edited by W. Lee Hansen. New York: National Bureau of Economic Research: 129-147.

Benabou, Roland. 2002. "Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency?" *Econometrica* 70, no. 2 (March): 481-517.

Bound, John, Michael F. Lovenheim, and Sarah Turner. 2010. "Why have college completion rates declined? An analysis of changing student preparation and collegiate resources." *American Economic Journal: Applied Economics* 2, no. 3 (July): 129-157.

Bound, John, and Sarah Turner. 2007. "Cohort crowding: How resources affect collegiate attainment." *Journal of Public Economics* 91, no. 5-6 (June): 877-899.

Bovenberg, A. Lans, and Lawrence H. Goulder. 2002. "Environmental taxation and regulation." In *Handbook of Public Economics*, edited by J. Auerbach Alan and Feldstein Martin. Amsterdam: Elsevier: 1471-1545.

Cameron, Stephen V., and James J. Heckman. 1999. "Can tuition policy combat rising wage inequality?" In *Financing college tuition: Government policies and educational priorities*, edited by Marvin H. Kosters. Washington, DC: The AEI Press: 76-124.

Cameron, Stephen V., and James J. Heckman. 2001. "The dynamics of educational attainment for Black, Hispanic and White males." *Journal of Political Economy* 109, no. 3 (June): 455-499.

Carneiro, Pedro, and James J. Heckman. 2002. "The evidence on credit constraints in post-secondary schooling." *Economic Journal* 112(October): 705-734.

Caucutt, Elizabeth M., and Krishna B. Kumar. 2003. "Higher education subsidies and heterogeneity: A dynamic analysis." *Journal of Economic Dynamics and Control* 27, no. 8 (June): 1459-1502.

Chapman, Bruce. 2006. "Income contingent loans for higher education: International reforms." In *Handbook of the Economics of Education*, edited by Eric A. Hanushek and Finis Welch. Amsterdam: North Holland: 1435-1503.

College Board. 2011a. *Trends in College Pricing, 2011*. New York: The College Board.

College Board. 2011b. *Trends in Student Aid, 2011*. New York: The College Board.

Congressional Budget Office. 2004. *Private and public contributions to financing college education*. Washington, DC: Congressional Budget Office (January).

Crumpler, Heidi, and Philip J. Grossman. 2008. "An experimental test of warm glow giving." *Journal of Public Economics* 92, no. 5-6 (June): 1011-1021.

Cunha, Flavio, James J. Heckman, Lance Lochner, and Dimitriy V. Masterov. 2006. "Interpreting the evidence on life cycle skill formation." In *Handbook of the Economics of Education*, edited by Eric A. Hanushek and Finis Welch. Amsterdam: Elsevier: 697-812.

Epple, Dennis, Richard E. Romano, and Holger Sieg. 2003. "Peer Effects, Financial Aid, and Selection of Students into Colleges." *Journal of Applied Econometrics* 18, no. 5 (September/October): 501-525.

Fender, John, and Ping Wang. 2003. "Educational policy in a credit contstrained economy with skill heterogeneity." *International Economic Review* 44, no. 3 (August): 939-964.

Fernández, Raquel, and Richard Rogerson. 1997. "Keeping people out: Income distribution, zoning, and the quality of public education." *International Economic Review* 38, no. 1 (February): 23-42.

Fernández, Raquel, and Richard Rogerson. 1998. "Public education and income distribution: A dynamic quantitative evaluation of education-finance reform." *American Economic Review* 88, no. 4 (September): 813-833.

Fernández, Raquel, and Richard Rogerson. 2003. "Equity and Resources: An Analysis of Education Finance Systems." *The Journal of Political Economy* 111, no. 4 (August): 858-897.

Friedman, Milton. 1962. *Capitalism and freedom*. Chicago: University of Chicago Press.

Galor, Oded, and Omer Moav. 2000. "Ability-biased technologoical transition, wage inequality, and economic growth." *Quarterly Journal of Economics* 115, no. 2 (May): 469-497.

Garratt, Rod, and John Marshall. 1994. "Public finance of private goods: The case of college education." *Journal of Political Economy* 103, no. 3 (June): 566-582.

Glomm, Gerhard, and B. Ravikumar. 1992. "Public vs. private investment in human capital: Endogenous growth and income inequality." *Journal of Political Economy* 100, no. 4 (August): 818-834.

Hanushek, Eric A., Charles Ka Yui Leung, and Kuzey Yilmaz. 2003. "Redistribution through education and other transfer mechanisms." *Journal of Monetary Economics* 50, no. 8 (November): 1719-1750.

Hillman, Ayre. 2003. *Public Finance and Public Policy: Responsibilities and Limitations of Government*. Cambridge, UK: Cambridge University Press.

Iyigun, Murat F. 1999. "Public education and intergenerational economic mobility." *International Economic Review* 40, no. 3 (August): 697-710.

Kane, Thomas J. 1999. *The Price of Admission: Rethinking How Americans Pay for College*. Washington, D.C. and New York, NY: Brookings Institution Press and Russell Sage Foundation.

Katz, Lawrence F., and Kevin M. Murphy. 1992. "Changes in relative wages, 1963-1987: Supply and demand factors." *Quarterly Journal of Economics* 107, no. 1 (February): 35-78.

Keane, Michael, and Kenneth I. Wolpin. 2001. "The effects of parental transfers and borrowing constraints on educational attainment." *International Economic Review* 42: 1051-1103.

Kleven, Henrik Jacobsen, and Claus Thustrup Kreiner. 2006. "The marginal cost of public funds: Hours of work versus labor force participation." *Journal of Public Economics* 90, no. 10-11 (November): 1955-1973.

Lochner, Lance J., and Alexander Monge-Naranjo. 2011. "The Nature of Credit Constraints and Human Capital." *American Economic Review* 101, no. 6 (October): 2487-2529.

Loury, Glenn C. 1981. "Intergenerational transfers and the distribution of income." *Econometrica* 49, no. 4: 843-867.

Maoz, Yishay D., and Omer Moav. 1999. "Intergenerational Mobility and the Process of Development." *The Economic Journal* 109, no. 458 (October): 677-697.

McPherson, Michael S., Morton Owen Schapiro, and Gordon C. Winston. 1993. *Paying the piper: Productivity, incentives, and financing in U.S. higher education*. Ann Arbor, MI: University of Michigan Press.

Mulligan, Casey B. 1999. "Galton versus the human capital approach to inheritance." *Journal of Political Economy* 107, no. 6, pt. 2 (December): S184-S224.

Restuccia, Diego, and Carlos Urrutia. 2004. "Intergenerational Persistence of Earnings: The Role of Early and College Education." *American Economic Review* 94, no. 5 (December): 1354-1378.

Salverda, Wiemer, Brian Nolan, and Timothy M. Smeeding, eds. 2009. *The Oxford Handbook of Economic Inequality*. Oxford: Oxford University Press.

Secretary of Education's Commission on the Future of Higher Education. 2006. *A test of leadership: Charting the future of U.S. higher education*. Washington: U.S. Department of Education (September).

Slemrod, Joel. 1996. *Tax Progressivity and Income Inequality*. Cambridge, UK: Cambridge University Press.

Tauchen, George. 1986. "Finite State Markov Chain Approximation to Univariate and Vector Autoregressions." *Economics Letters* 20, no. 2: 177-81.

Yilmaz, Kuzey. 2013. "On the Importance of Fertility Behavior in School Finance Policy Design." Working Paper.

Table 1. Calibration Parameters

Parameter	Value	Parameter	Value
r	1.49	β	0.67
σ	0.68	φ	0.05
λ	0.4	σ_{ε}^{2}	0.52
ρ	0.31	α	0.7
K_0	1	K_1	3
ϒ	0.144	$\overline{c}_{\min} / w_{u}$	0.32
A	1	ξ	0.591
τ	0.008		

Table 2 Distribution of Population by Loan Eligibility and Education Outcomes

	Loan eligibility (as percent of tuition)				
	0%	25%	50%	75%	100%
Percent of population	84.8%	4.3%	4.8%	3.2%	2.9%
Enrollment rate	64.9	24.2	26.3	27.7	0.0
Completion rate of loan group	41.9	16.6	17.0	16.7	n.a.
Completion rate of enrollees	64.6	68.7	64.8	60.4	n.a.

Table 3. Intergenerational Mobility and Persistence by Generation

Prob (the n-th generation offspring is skilled | parent education status)

Generation (n)	Upward mobility Prob(n=skilled|1st gen=unskilled)	Persistence of completion Prob(n=skilled|1st gen=skilled)
1	0.28	0.54
2	0.34	0.44
5	0.37	0.38
10	0.38	0.38

Note: Calculations of probability that the child in the nth generation completes college given status of beginning generation.

Table 4. Distribution of Schooling Outcomes under Different Aid Schemes

Tax Rate τ	Subsidy Schemes: Uniform		Need-based (Perfect Information)		Need-based (Imperfect information)		Merit-based	
	Enroll	Succeed	Enroll	Succeed	Enroll	Succeed	Enroll	Succeed
0%	47.5%	30.8%	50.4%	33.1%	47.5%	30.8%	41.4%	28.4%
0.4%	52.9%	34.0%	54.9%	35.7%	55.4%	36.1%	46.7%	31.7%
0.8%	57.5%	36.5%	59.2%	37.9%	60.5%	38.7%	50.9%	34.1%
1.2%	61.8%	38.6%	63.3%	39.9%	65.4%	40.4%	54.7%	36.1%
1.6%	66.0%	40.5%	67.7%	41.6%	69.9%	41.4%	58.7%	38.1%

Fig. 1: Benchmark College Patterns and Labor Market Outcomes

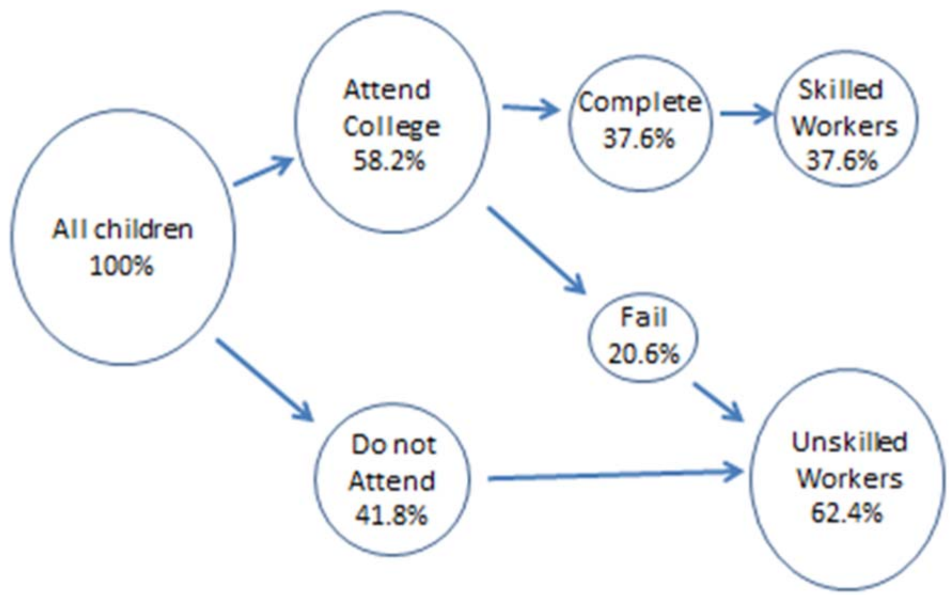
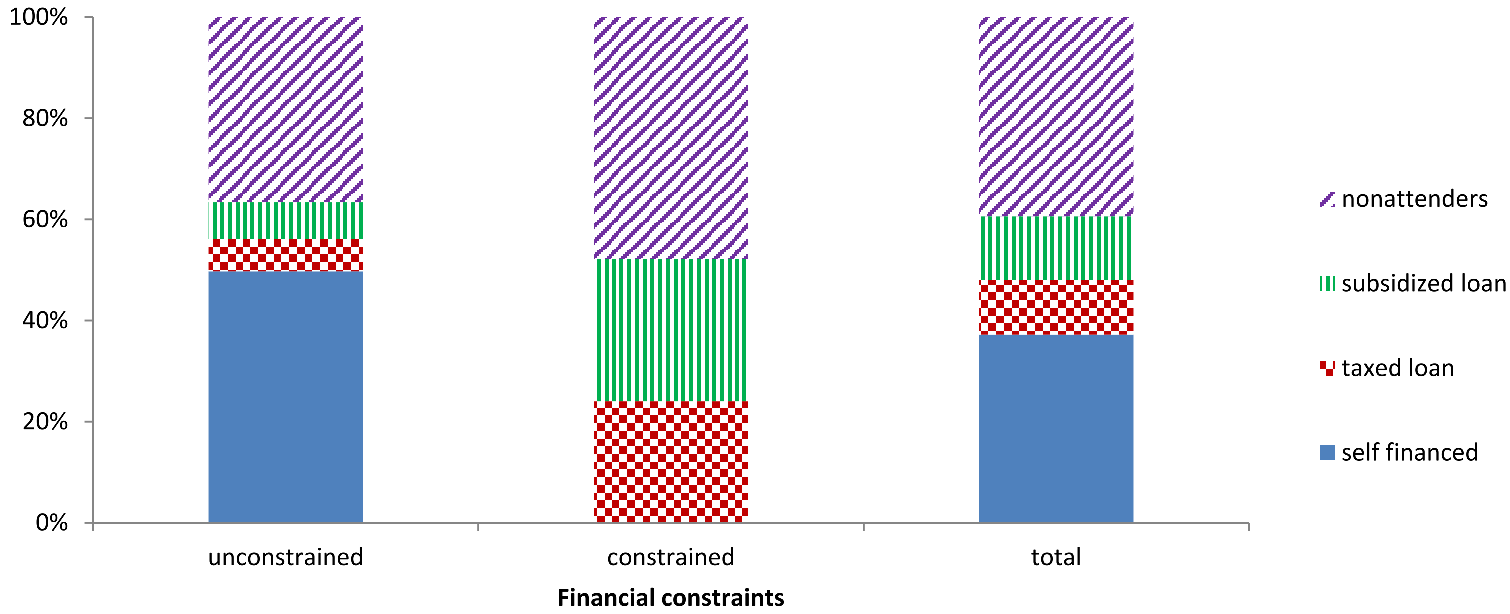


Fig. 2: Distribution of Choices with Unsubsidized Income Contingent Loans (ICL)



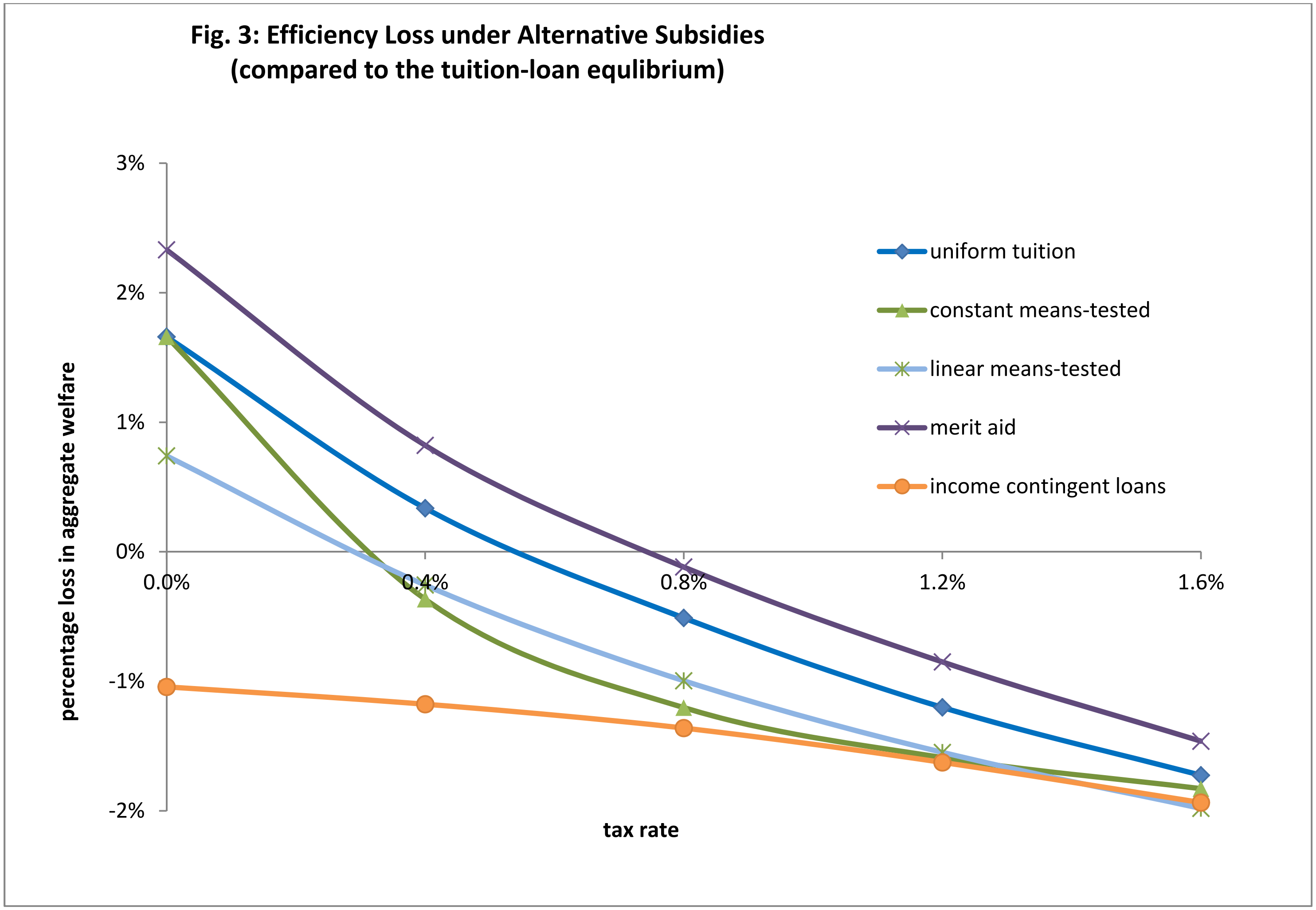
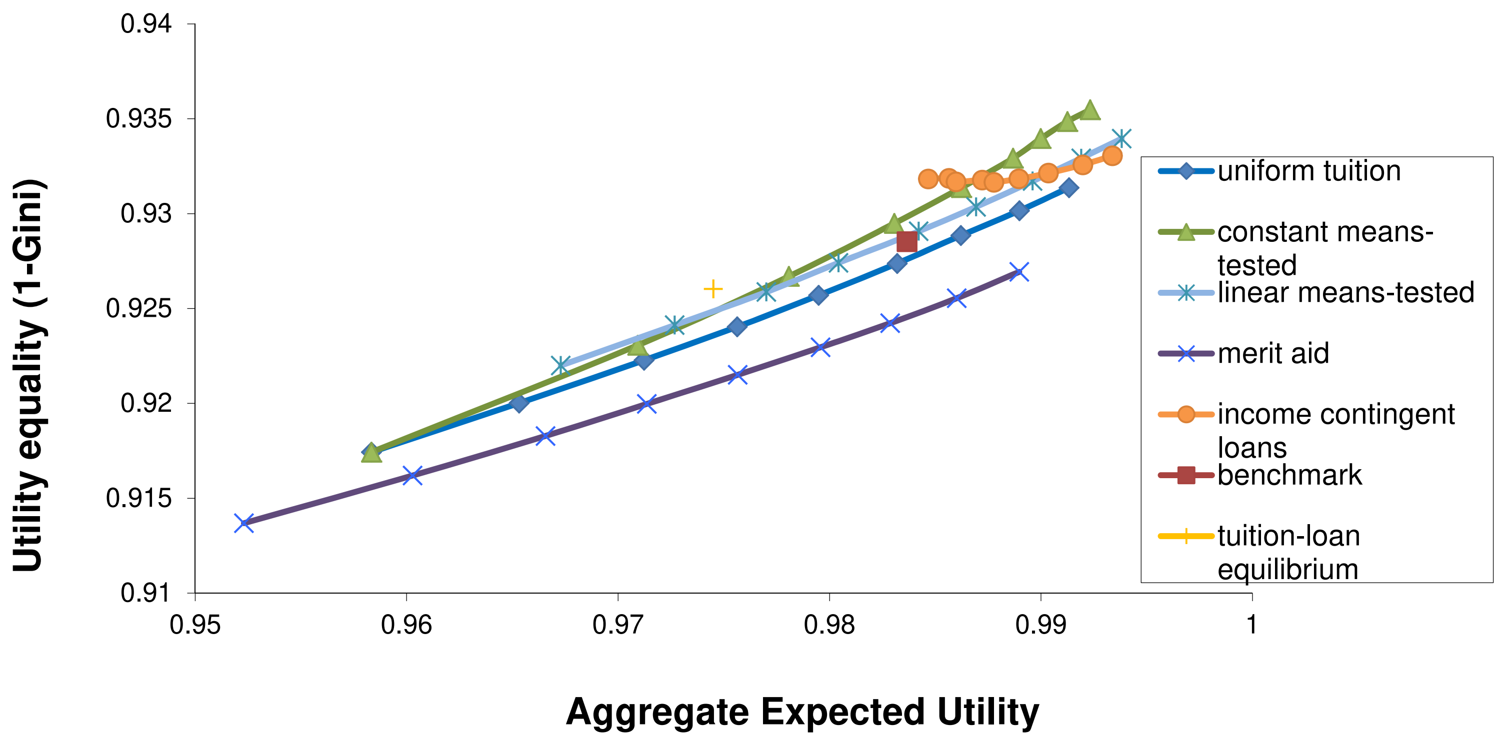


Fig. 3: Efficiency Loss under Alternative Subsidies
(compared to the tuition-loan equlibrium)
3%
2%
1%
0%
-1%
-2%
0.0%
0.4%
0.8%
1.2%
1.6%
percentage loss in aggregate welfare
tax rate
uniform tuition
constant means-tested
linear means-tested
merit aid
income contingent loans

Fig. 4: Aggregate Utility and Inequality



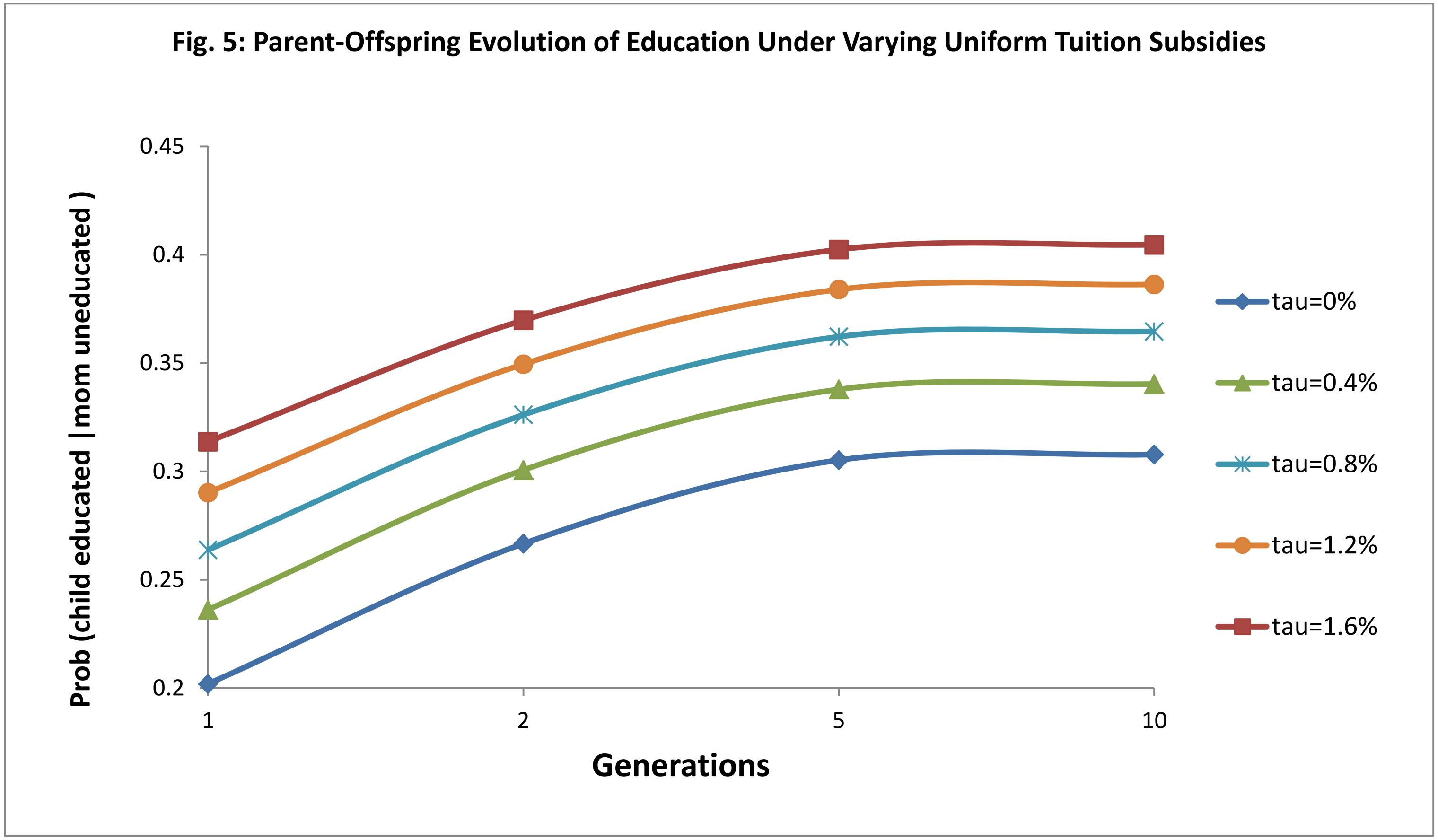


Fig. 5: Parent-Offspring Evolution of Education Under Varying Uniform Tuition Subsidies
Prob (child educated | mom uneducated)
0.45
0.4
0.35
0.3
0.25
0.2
1
2
5
10
Generations
tau=0%
tau=0.4%
tau=0.8%
tau=1.2%
tau=1.6%

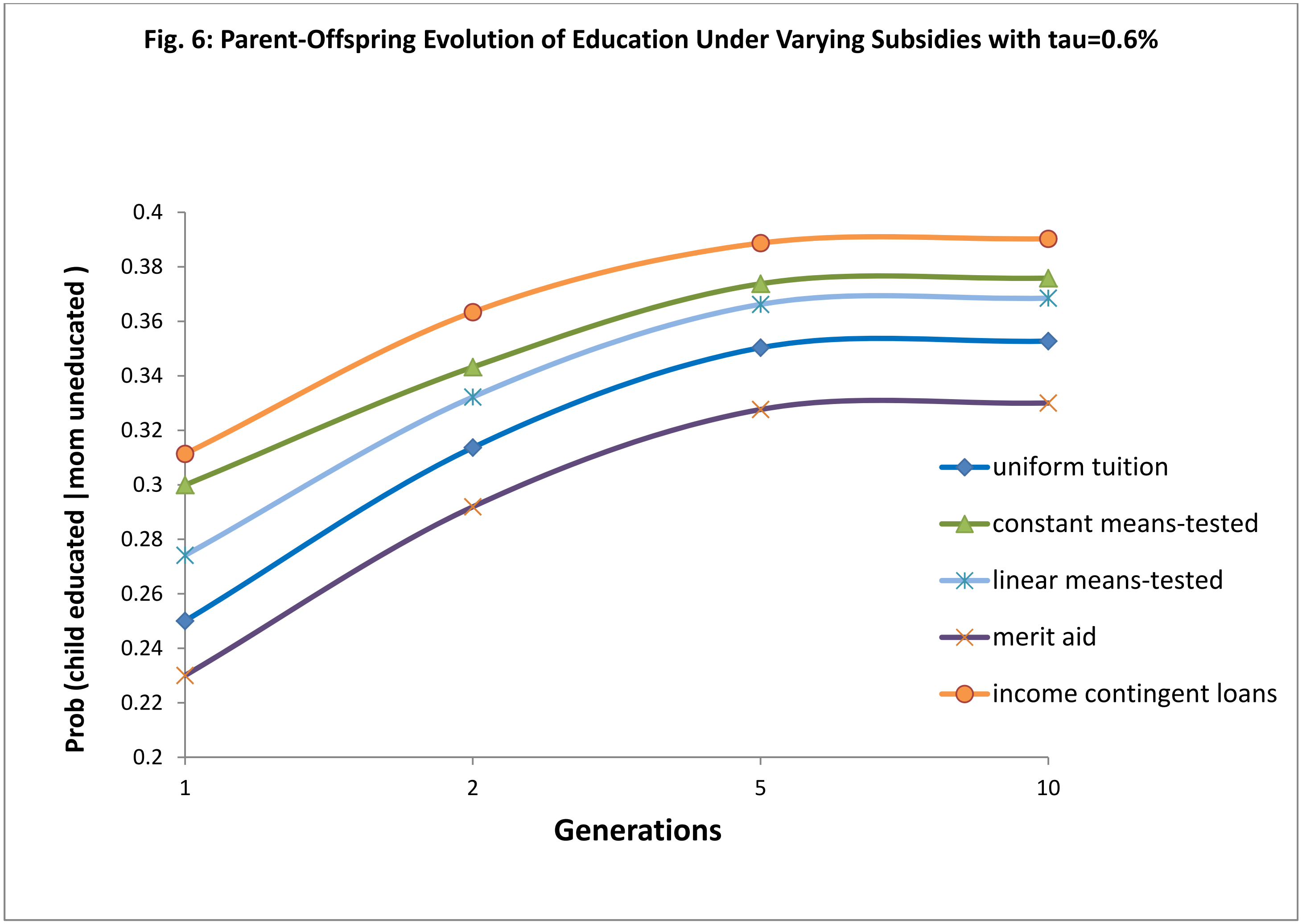
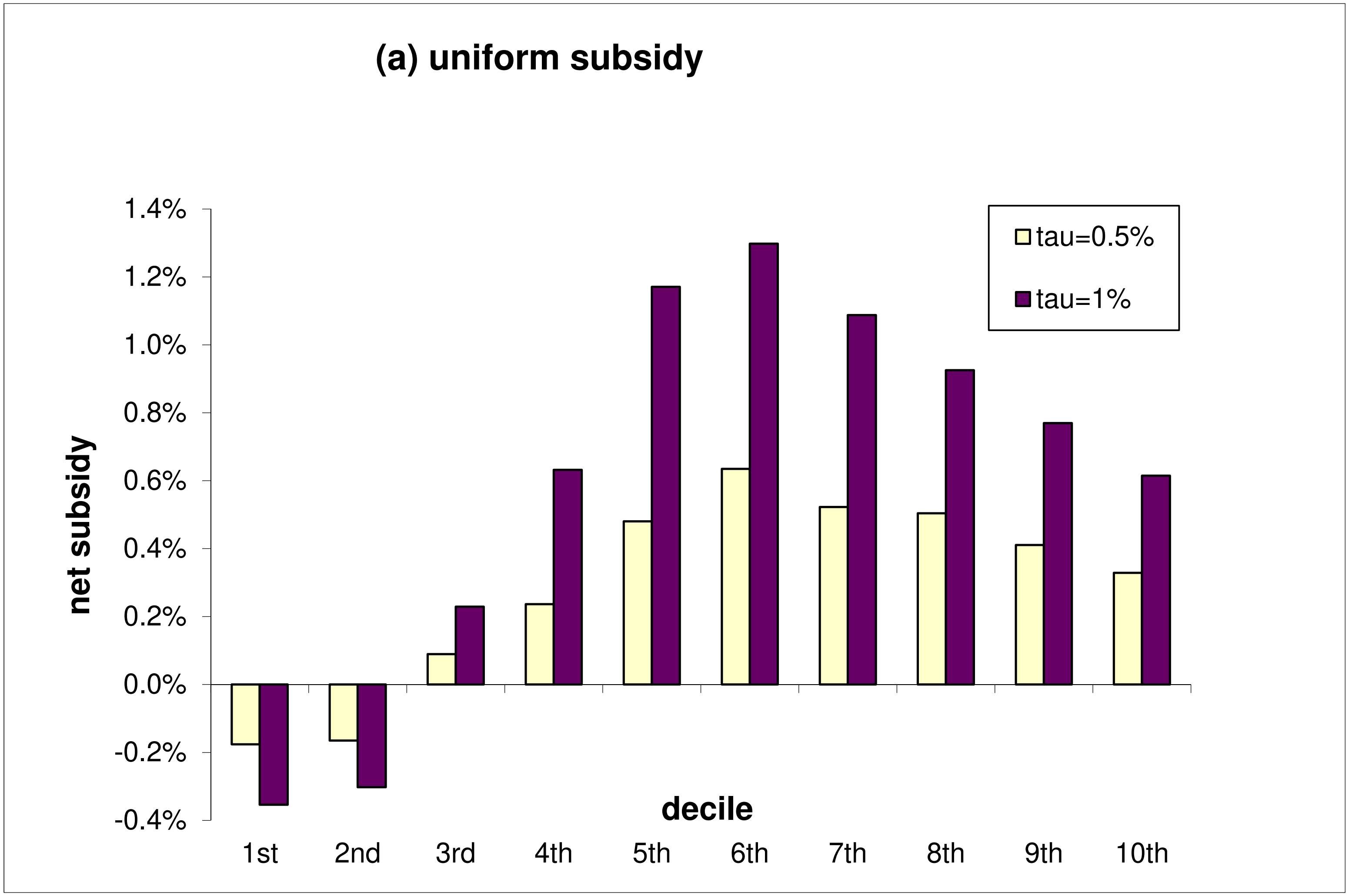
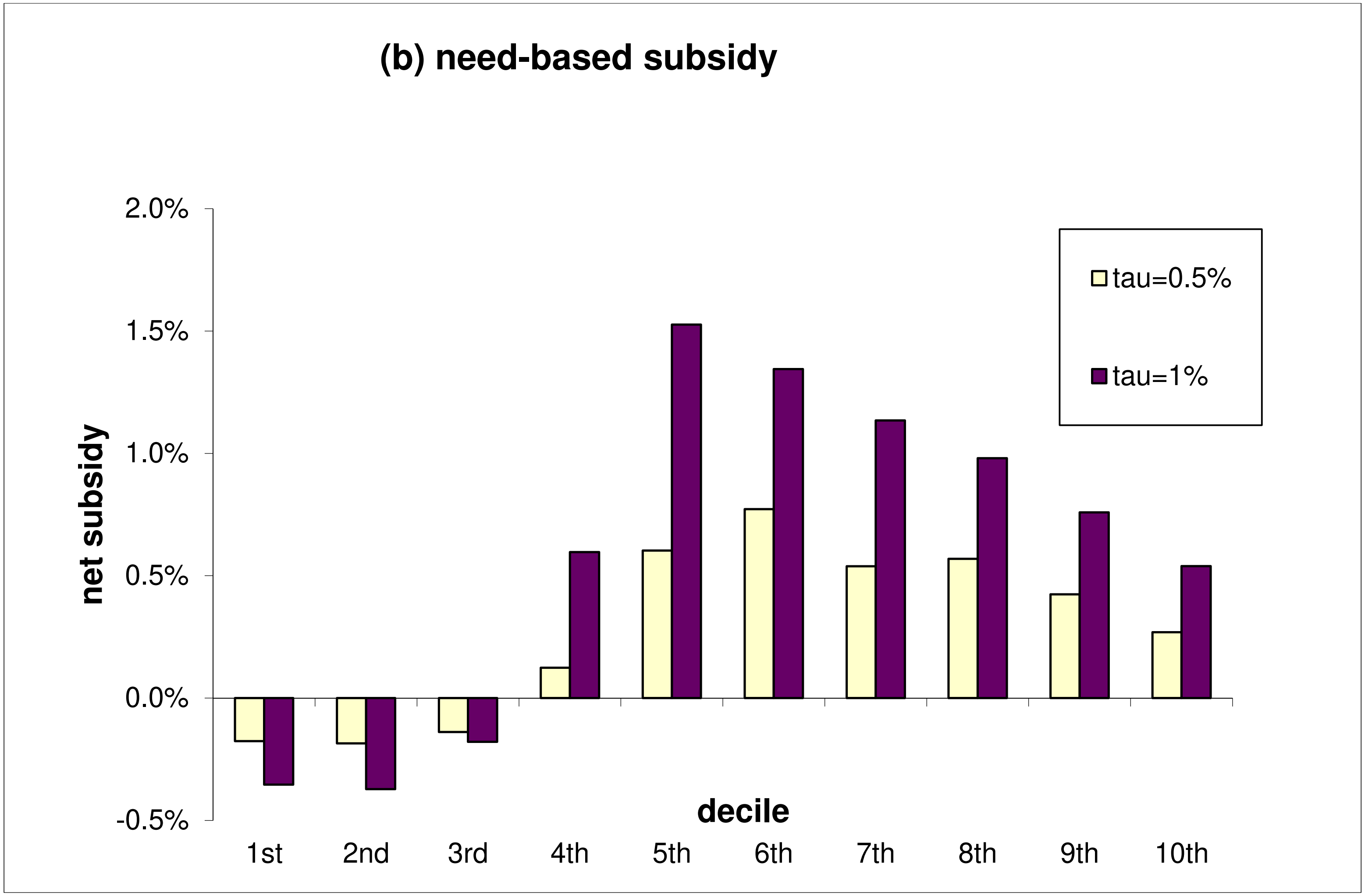


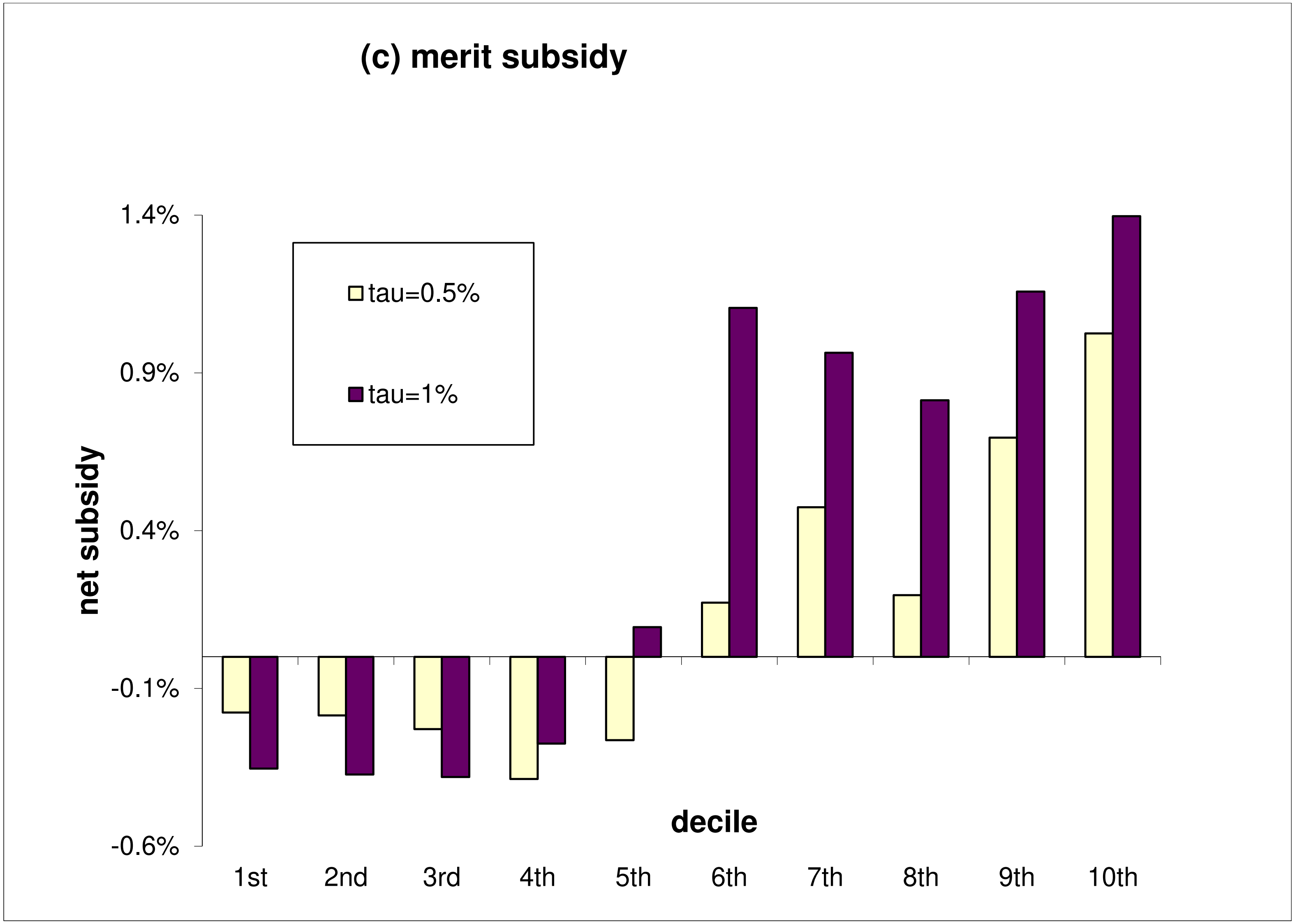
Fig. 6: Parent-Offspring Evolution of Education Under Varying Subsidies with tau=0.6%
Prob (child educated |mom uneducated)
0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
1
2
5
10
Generations
uniform tuition
constant means-tested
linear means-tested
merit aid
income contingent loans

Figure 7: Distribution of net subsidies by income decile of the child



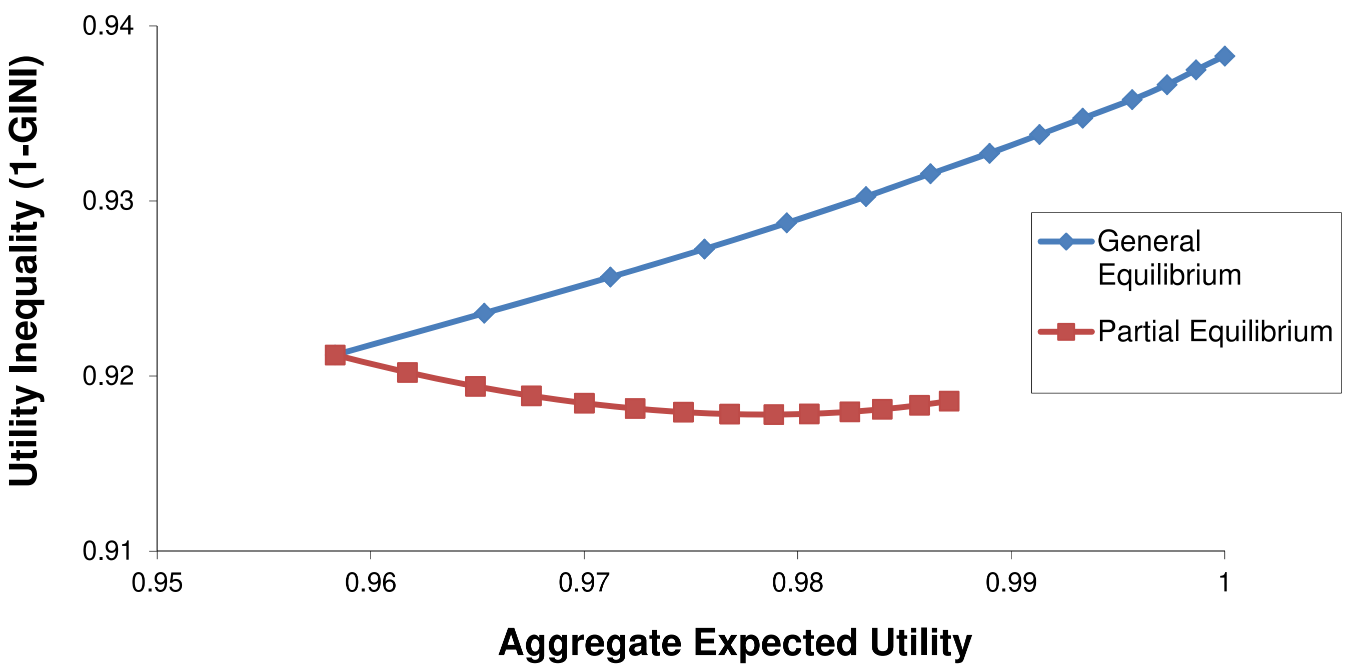


(b) need-based subsidy
net subsidy
2.0%
1.5%
1.0%
0.5%
0.0%
-0.5%
tau=0.5%
tau=1%
decile
1st
2nd
3rd
4th
5th
6th
7th
8th
9th
10th



(c) merit subsidy
tau=0.5%
tau=1%
net subsidy
1.4%
0.9%
0.4%
-0.1%
-0.6%
decile
1st
2nd
3rd
4th
5th
6th
7th
8th
9th
10th

Figure 8. Comparison of Uniform Subsidies in General Equilibrium and Partial Equilibrium with Varying Tax Rates



APPENDIX

This appendix has several parts.

- The first part explains the relationship between test score of an individual z and the ability x, and how they transfer across generations.
- The second part provides more details about the transition matrix which governs the aggregate dynamics of the model.
- The third part provides the empirical results for the test score.

The Transmission of Ability

In the model, the transmission of ability from a parent to her child is a nonlinear mapping.

Here are the details of the process: We assume the test score for a child, z' and her parent, z are measured by standardized test scores and are linked through a first order Markov process:

$$z' = \beta_0 + \beta z + \epsilon, \quad \epsilon \sim N(0, \sigma^2) \tag{1}$$

where β_0 is a constant, β is the correlation between the preschool abilities of parent and child, and ϵ is the white noise. To improve the credibility of our work, we employ the NLSY79 data to recover the corresponding parameter values. More details will be provided in later section of this appendix. Given this AR(1) process, it is easy to see that the stationary distribution of ability, z is $z \sim N\left(\frac{\beta_0}{(1-\beta)}, \frac{\sigma^2}{1-\beta^2}\right)$ under that assumption that $|\beta| < 1$.

We further assume that the probability of success in college, conditioning on the attendance, is governed by the variable x. Since x is not directly observable, we need a mapping from the test score z to "ability" x. Intuitively, ability x should be an increasing function of test score z. And as a probability, x needs to be in between zero and unity, $x \in [0, 1]$. Apparently, for our purposes, any cumulative distribution function suffices. Therefore, we assume

$$x = \Phi(z), \tag{2}$$

where Φ is the cumulative distribution function for standard normal distribution. By definition, Φ is a strictly increasing function. Thus, if $x_1 = \Phi(z_1)$, $x_2 = \Phi(z_2)$, and the statement "$z_1 = z_2$" would imply "$x_1 = x_2$". Hence the ability of a child, x', is a nonlinear function of her parent's ability, since

$$\begin{aligned} x' &= \Phi(z') && \text{by (2)} \\ &= \Phi(\beta_0 + \beta z + \epsilon) && \text{by (1)} \\ &= \Phi(\beta_0 + \beta\Phi^{-1}(x) + \epsilon) && \text{by (2)}. \end{aligned}$$

To sum up, children and parent's are endowed with ability measured in standardized test scores and linked through a first order Markov process. The monotone transformation we adopted imposes certain empirical discipline on our calibration. In addition, by using the method introduced by Tauchen (1986), we construct a mapping from continuous AR(1) process for the transmission of abilities in standardized test scores between parents and their children to a first order Markov chain with a discrete state space for $z'|z$.

More on Aggregate Dynamics

Recall that each individual is characterized by the ex-post state variable (x, I^e, I^l) where, as we have explained in the text, $x \in [0, 1]$, I^e and I^l are indicator functions, $I^e \in \{0, 1\}$, $I^l \in \{0, 1, 2, 3\}$. Then we construct the following markov chain.

$$F_{t+1}(\Omega) = \Pi \quad F_t(\Omega)$$

where Ω shows the state space of individuals and $F_t(\Omega)$ is the probability distribution function over the state space at time t (details are described below). This formulation implicitly assumes the variable, ability is discretized.

As an illustration, assume that the variable x can only take values on a finite set, $x \in \{0.1, 0.5, 0.9\}$.[1]. Therefore, the state space is simply the following collection:

[1] In the computational model, a much finer discretization has been done

$\Omega = \{(0.1, 0, 0), (0.5, 0, 0), (0.9, 0, 0), (0.1, 1, 0), (0.5, 1, 0), (0.9, 1, 0), (0.1, 0, 1), (0.5, 0, 1), (0.9, 0, 1),$
$(0.1, 1, 1), (0.5, 1, 1), (0.9, 1, 1), (0.1, 0, 2), (0.5, 0, 2), (0.9, 0, 2), (0.1, 1, 2), (0.5, 1, 2), (0.9, 1, 2),$
$(0.1, 0, 3), (0.5, 0, 3), (0.9, 0, 3), (0.1, 1, 3), (0.5, 1, 3), (0.9, 1, 3)\}$.

As expected, the size of state space is $3 \times 2 \times 4 = 24$. Let's label the states as $\omega_1 = (0.1, 0, 0)$, $\omega_2 = (0.5, 0, 0)$, $\omega_3 = (0.9, 0, 0)$, $\omega_4 = (0.1, 1, 0)$ .... With a little bit abuse of notation, let $f_t(\omega)$ be the proportion of mothers at state $\omega = (x, I^e, I^l)$, and hence it is a scalar. Correspondingly, $F_t(\Omega)$ is a vector representation (i.e a 24×1 column vector) of the probability distribution over the discretized state space, Ω. Clearly, $F_t(\Omega) = (f_t(\omega_1)\ f_t(\omega_2)\ f_t(\omega_3)\ ...\ f_t(\omega_{23})\ f_t(\omega_{24}))'$.

The size of transition matrix Π is a 24×24 matrix in which the entry π_{ij} shows the probability of that a mother at state ω_j has a child at state ω_i. For instance, the entry π_{72} show the probability of that a mother at state ω_2 (i.e. $(0.5, 0, 0)$) has a child at state ω_7 (i.e. $(0.1, 0, 1)$) In other words, π_{72} =Prob$(x' = 0.1, I^{e'} = 0, I^{l'} = 1)|(x = 0.5, I^e = 0, I^l = 0))$...... Note that the transition matrix, Π has 24×24 entries which captures all possible state transitions between the mother and child.

At time t, the number of people moving from state ω_2 (mother) to state ω_7 (child) at time t+1 is $= \pi_{72} \times \omega_2 = \pi_{72} \times f_t(x = 0.5, I^e = 0, I^l = 0)$. Or similarly, the number of people moving from state ω_3 to state ω_7 is $\pi_{73} \times \omega_3 = \pi_{73} \times f_t(x = 0.9, I^e = 0, I^l = 0)$. To find the proportion of children at state ω_7, we need to sum up all children coming to state ω_7 from all other states:

$$f_{t+1}(\omega_7) = \sum_j \pi_{7j} \times \omega_j$$

The entries of transition matrix incorporates the information of the transition probabilities of abilities $x'|x$, the wages, the distribution of wealth, and, perhaps more subtly, endogenous participation constraints. It is not as complicated as it seems once we combine the computation of this matrix with our understanding of the model. As an example, consider the probability of transition of mother at state 2 to kid at state (i.e. π_{72}). Recall that we know how much bequest the mother leaves $b' = b'(x = 0.5, I^e = 0, I^l = 0)$. If it is NOT the case that $0.75 * \phi \leq (b' - \bar{c}_{min}) < \phi$, $\pi_{72} = 0$. It is because at state 7, $I^{l'} = 1$. Next, check if the college attendance is profitable: I.e.

$EU(I^{r'} = 1, x' = 0.1, l' = 1) > EU(I^{r'} = 0, x' = 0.1, l' = 1)$. If it is not, $\pi_{72} = 0$. It is so because the loan in the model is a tuition loan and is available only when an agent attends college. Finally, the ratio of unskilled workers (note that $I^{e'} = 0$) is given by: $(1 - 0.1) * Prob(x' = 0.1|x = 0.5)$ where $Prob(x' = 0.1|x = 0.5)$ comes from the transmission of human capital, which is an AR(1) process.

A Galtonian Regression

To determine the transmission of ability, we look at the test scores of parents (mothers) and children. Our data are extracted from NLSY79 and NLSY79 Children Cohorts of National Longitudinal Survey of Youth (NLSY). A full account of the NLSY, its history, and main data files can be found in NLS Handbook (2000).

The NLSY79 is a nationally representative sample of 12,686 young men and women who were 14 to 22 years of age when first surveyed in 1979. The dataset includes the 1980 administration of the Armed Forces Vocational Aptitude Battery (ASVAB), which is used to create Armed Forces Qualifications Test (AFQT) Scores, as well as the highest grade they attended and their age at the time the survey conducted. We converted the AFQT scores in percentiles into the standardized test scores. To eliminate the bias due to age and schooling difference, we calculated the effect of age and schooling and adjusted the AFQT scores to their predicted values when young men and women were 23 years of age.

The NLSY79 Child Sample consists of all children born to female NLSY79 respondents who completed an interview during the even-year interviews beginning in 1986. The dataset includes Peabody Individual Achievement Test (PIAT), which measures ability in Math, Reading Recognition, and Reading Comprehension. We used PIAT scores when pupils were 14 years of age and standardized the Math, Reading Recognition, and Reading Comprehension scores in percentiles. As a measure of the ability of a child, the average of those three scores are taken.

We chose family income as a proxy for the family background. Family income is calculated by taking the average of CPI-adjusted income before the year the pupil took the test[2].

We estimated the following Galtonian Regression (1886),

$$z' = \beta_0 + \beta_1 z + \epsilon$$

Table 1 reports the findings. It is evident that a pupil and her mother's ability are correlated. The smarter a mother is, the more likely the pupil would be of high ability. Regression based on income quartiles provides support for a well-known fact -the importance of family background in the ability formation-. Apparently, the mobility of ability is higher for the second and third quartiles. We also conducted a Jack Berra test and looked at the residual plots to confirm that errors are normally distributed. Hence, $z'|z$ has a normal distribution.

	Full	*By Quartile*			
	Sample	*Bottom*	*2nd*	*3rd*	*Top*
Constant	-0.05 (1.62)	-0.31* (4.28)	-0.05 (0.79)	0.05 (1.03)	0.13 (1.84)
Child's Ability	0.40* (14.1)	0.27* (4.52)	0.35* (5.53)	0.31* (4.82)	0.30* (4.20)
Sample Size	814	301	215	159	139
R^2	0.20	0.06	0.13	0.13	0.11

[2]The change in family income due to a later change in the family background such as divorce would have nothing to do a pupil's achievement.